Logic is going to be the main topic of the course in the next 2 weeks. In this tutorial our goal is to review the basics of propositional logic. First review the following concepts about propositional logic:

- (Syntax of propositional logic) The syntax of propositional logic defines the rules for constructing valid propositions. More specifically, a proposition is defined inductively as follows:
  - An atomic proposition (also known as propositional variable), denoted as A, B, C, etc, is a proposition;
  - Propositional logic uses several logical connectives to combine propositions and create compound propositions: Given two propositions p, q, then the following are also propositions:
    * $(\neg p)$
    * $(p \vee q)$
    * $(p \wedge q)$
    * $(p \rightarrow q)$
    * $(q \leftarrow p)$
    * $(p \leftrightarrow q)$

  When the context is clear we sometimes omit the parenthesis.

- (Semantics of propositional logic)
  - Truth values (i.e., true and false) represent the possible states of a proposition. Atomic propositions are assigned one of these truth values to indicate whether they are true or false in a given interpretation.
  - An interpretation in propositional logic is a mapping that assigns truth values (T or F) to each atomic proposition. Given a set of atomic propositions $\{p, q, r, \ldots\}$, an interpretation I is a function $I\colon \{p, q, r, \ldots\} \rightarrow \{T, F\}$ that maps each atomic proposition to either T or F.
  - Truth tables are used to specify the truth values of compound propositions (based on their constituent atomic propositions' truth values.

A	B	$\neg A$	$A \wedge B$	$A \vee B$	$A \leftarrow B$	$A \rightarrow B$	$A \leftrightarrow B$
true	true	false	true	true	true	true	true
true	false	false	false	true	true	false	false
false	true	true	false	true	false	true	false
false	false	true	false	false	true	true	true

Now answer the following questions. If a question asks you to translate a sentence into propositional logic. Make sure you define the atomic propositions first.

1. (Basic Propositions) Translate the following sentences into propositional logic:

(a) The sun is shining, and it's a warm day.

(b) Either Alice will go to the concert, or Bob will go, but not both.

2. (Conditional Statements) Convert the following conditional statements into propositional logic:

(a) If you study hard, you will pass the exam.

(b) Whenever it snows, the roads become slippery.

3. (Negations and Conjunctions) Translate the following sentences, which involve negations and conjunctions, into propositional logic:

(a) It is not raining, but it is windy.

(b) Neither Sarah nor John will attend the party.

4. (Biconditionals) Convert the following sentences, which involve biconditional relationships, into propositional logic:

(a) You can enter the room if and only if you have the access card.

(b) A triangle is equilateral if and only if all its sides are of equal length.

5. (Complex Sentences) Translate the following more complex sentences into propositional logic, combining various logical operators:

(a) If it's not Monday or Tuesday and the weather is good, then we will have a picnic.

(b) You can borrow the book if and only if it's available, and you return it on time.

6. Write down the truth table of the following: $(((r \wedge (p \leftrightarrow q)) \rightarrow (p \vee q)) \rightarrow (p \wedge r))$.

7. Examine the following sentences and determine if they are logically equivalent. Provide a truth table to support your answer if necessary.

- Sentence A: "If it's raining, then I'll stay home."
  Sentence B: "I'll stay home only if it's raining."
- Sentence X: "All humans are mortal."
  Sentence Y: "No mortal beings are non-human."
- Sentence P: "I will attend the conference or the workshop."
  Sentence Q: "I won't miss both the conference and the workshop."
- Sentence M: "If the store is open, then I'll buy groceries."
  Sentence N: "I'll buy groceries unless the store is closed."