# CS 367: Tutorial Week 12
# Reasoning with Uncertainty & Planning with Uncertainty

The following examples are adapted from Kevin B. Korb and Ann E. Nicholson *Bayesian Artificial Intelligence* (Chapman & Hall, 2004).

## Q1: Bayesian Reasoning

Suppose the women attending a particular clinic show a long-term chance of 1 in 100 of having breast cancer. Suppose also that the initial screening test used at the clinic has a false positive rate of 0.2 (that is, 20% of women without cancer will test positive for cancer) and that it has a false negative rate of 0.1 (that is, 10% of women with cancer will test negative). The laws of probability dictate from this last fact that the probability of a positive test given cancer is 90%. Now suppose that you are such a woman who has just tested positive. Use Bayes' Theorem to calculate the probability that you have cancer.

Note that the prior probability of a positive test can be calculated by taking: (the probability of the true positive rate multiplied by the prior probability of having cancer) PLUS (the false positive rate multiplied by the prior probability of not having cancer).

*Answer:*

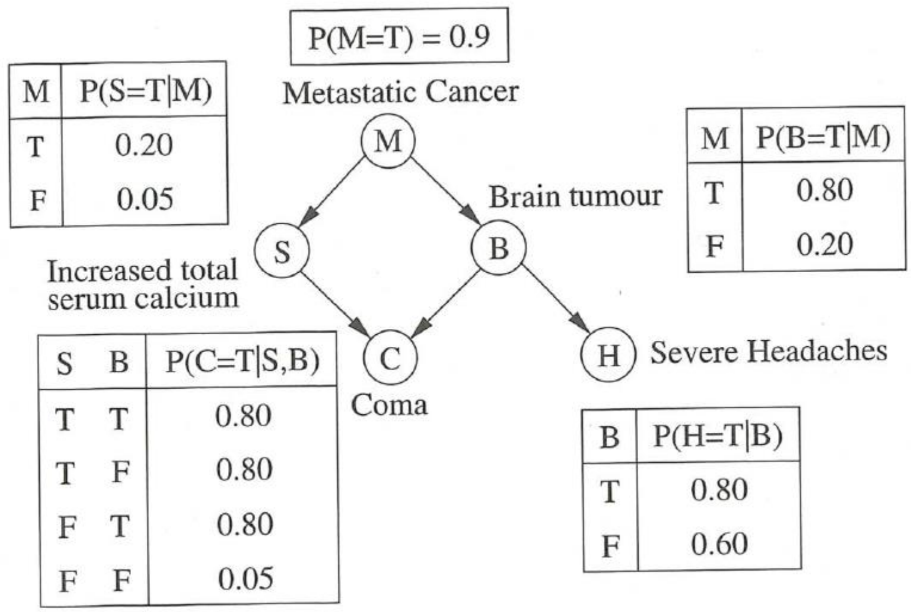
$$\begin{aligned}
P(Cancer|Pos) &= \frac{P(Pos|Cancer)P(Cancer)}{P(Pos)} \\
&= \frac{P(Pos|Cancer)P(Cancer)}{P(Pos|Cancer)P(Cancer)+P(Pos|\neg Cancer)P(\neg Cancer)} \\
&= \frac{0.9\times 0.01}{0.9\times 0.01+0.2\times 0.99} \\
&= \frac{0.009}{0.009+0.198} \\
&\approx 0.043
\end{aligned}$$

## Q2: Reasoning with Bayesian Networks

Metastatic cancer (M) is a possible cause of brain tumours (B) and is also an explanation for increased total serum calcium (S). In turn, either of these could explain a patient falling into a coma. Severe headache (H) is also associated with brain tumours.

(a) Draw the Bayesian Network Diagram for these facts.

*Answer: Note: the following network diagram includes probabilities that are not introduced until 2(b).*




(b) Consider the following facts:

P(M) = 0.9

P(B | M) = 0.8
P(B | ¬M) = 0.2

P(S | M) = 0.2
P(S | ¬M) = 0.05

P(C | S, B) = 0.8
P(C | S, ¬B) = 0.8
P(C | ¬S, B) = 0.8
P(C | ¬S, ¬B) = 0.05

P(H | B) = 0.8
P(H | ¬B) = 0.6

Given this information, draw the full conditional probability tables for the following probability distributions: **P**(*M*), **P**(*B*), **P**(*S*) , **P**(*C*), and **P**(*H*).
*Answer:*

| M | P(M) |
|---|---|
| 1 | 0.9 |
| 0 | 0.1 |

| M | B | P(B\|M) |
|---|---|---|
| 1 | 1 | 0.8 |
| 1 | 0 | 0.2 |
| 0 | 1 | 0.2 |
| 0 | 0 | 0.8 |

| M | S | P(S\|M) |
|---|---|---|
| 1 | 1 | 0.2 |
| 1 | 0 | 0.8 |
| 0 | 1 | 0.05 |
| 0 | 0 | 0.95 |

| B | H | P(H\|B) |
|---|---|---|
| 1 | 1 | 0.8 |
| 1 | 0 | 0.2 |
| 0 | 1 | 0.6 |
| 0 | 0 | 0.4 |

| S | B | C | P(C\|S,B) |
|---|---|---|---|
| 1 | 1 | 1 | 0.8 |
| 1 | 1 | 0 | 0.2 |
| 1 | 0 | 1 | 0.8 |
| 1 | 0 | 0 | 0.2 |
| 0 | 1 | 1 | 0.8 |
| 0 | 1 | 0 | 0.2 |
| 0 | 0 | 1 | 0.05 |
| 0 | 0 | 0 | 0.95 |

(c) Use the <u>variable elimination algorithm</u> to calculate the probability that a patient referred to your clinic has metastatic cancer (M) given that they have a severe headache (H)

*Answer:*

The relevant CPT tables are: P(M), P(B) and P(H). The other nodes have no influence on whether someone has a headache.

***Here is the Variable Elimination method:***

Step 1: Restriction of P(H | B) for H = TRUE to give $f$(B):

| B | $f$(B) |
|---|---|
| 1 | 0.8 |
| 0 | 0.6 |

Step 2: Eliminate B
First, multiply $f$(B) by P(B) to get a new table $g$(M,B).

| M | B | $g$(M,B) |
|---|---|---|
| 1 | 1 | $0.8 \times 0.8 = 0.64$ |
| 1 | 0 | $0.2 \times 0.6 = 0.12$ |
| 0 | 1 | $0.2 \times 0.8 = 0.16$ |
| 0 | 0 | $0.8 \times 0.6 = 0.48$ |

Second, sum out B to get table $h$(M).

| M | $h$(M) |
|---|---|
| 1 | 0.76 |
| 0 | 0.64 |

Step 3: Multiplication. $h$(M) × ***P***(M) to give $r$(M)

| M | $r$(M) |
|---|---|
| 1 | $0.9 \times 0.76 = 0.684$ |
| 0 | $0.1 \times 0.64 = 0.064$ |

Step 4: Normalisation to retrieve **P**(M | H)

P(M | H) = 0.684 / 0.748 = **0.9144**

***Here is the Naïve method:***

Step 1: Restriction of P(H | B) for H = TRUE to give $f$(B):

| B | $f$(B) |
|---|---|
| 1 | 0.8 |
| 0 | 0.6 |

Step 2: Multiplication of $f$(B) × P(B) × P(M) = $g$(B, M)

| B | $f$(B) |
|---|---|
| 1 | 0.8 |
| 0 | 0.6 |

| M | B | P(B\|M) |
|---|---|---|
| 1 | 1 | 0.8 |
| 1 | 0 | 0.2 |
| 0 | 1 | 0.2 |
| 0 | 0 | 0.8 |

| M | P(M) |
|---|---|
| 1 | 0.9 |
| 0 | 0.1 |

| M | B | $g$(B, M) |
|---|---|---|
| 1 | 1 | 0.8 x 0.8 x 0.9 = 0.576 |
| 1 | 0 | 0.2 x 0.6 x 0.9 = 0.108 |
| 0 | 1 | 0.2 x 0.8 x 0.1 = 0.016 |
| 0 | 0 | 0.8 x 0.6 x 0.1 = 0.048 |

Step 3: Sum to eliminate variable B on $g$(B, M) to get $h$(M):

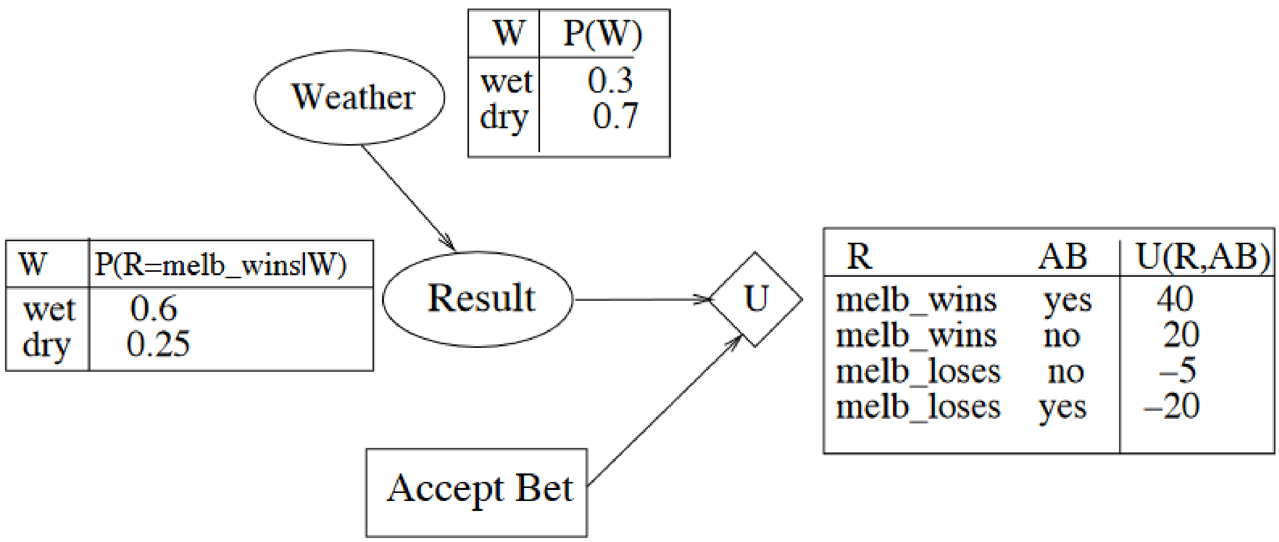
| M | $h$(M) |
|---|---|
| 1 | 0.684 |
| 0 | 0.064 |

Step 4: Normalisation. Extract conditional probabilities.

P(M | H) = 0.684 / 0.748 = **0.9144**

## Q3: Decision Networks

Clare's football team, Melbourne, is going to play her friend John's team, Carlton. John offers Clare a friendly bet: whoever's team loses will buy the wine the next time they go out for dinner. They never spend more than $15 on wine when they eat out. When deciding whether to accept this bet, Clare will have to assess her team's chances of winning (which will vary according to the weather on the day). She also knows that she will be happy if her team wins and miserable if her team loses, regardless of the bet.




A decision network for this problem is shown above. This network has one chance node *Result* representing whether Clare's team wins or loses (values {*melb wins, melb loses*}), and a second chance node *Weather* which represents whether or not it rains during the match (values {*rain, dry*}). It has a binary decision node *AcceptBet* representing whether or not she bets and a utility node *U* that measures the decision maker's level of satisfaction.

The priors for the *Weather* reflect the typical match conditions at this time of year. The CPT for *Result* shows that Clare expects her team to have a greater chance of winning if it doesn't rain (as she thinks they are the more skillful team).

There are arcs from *Result* and *AcceptBet* to *U*, capturing the idea that Clare's satisfaction will depend on a combination of the eventual match winner and the betting decision. Her preferences are made explicit in the utility table. The numbers given indicate that the best outcome is when her team wins and she accepted the bet (utility = 40) while the next best outcome is her team wins but she didn't bet on the result (utility = 20). When her team loses but she didn't bet, Clare isn't happy (utility = -5) but the worst outcome is when she has to buy the dinner wine also (utility = -20). Clearly, Clare's preferences reflect more than the money at risk. Note also that in this problem the decision node doesn't affect any of the variables being modeled, i.e., there is no explicit outcome variable.

*Should Clare accept the bet?*

*Answer:*

$P(R = \text{melb wins}) \quad = P(W = w) \times P(R = \text{melb wins}|W = w) + P(W = d)$

$= \times P(R = \text{melb wins}|W = d)$

and the losing probability is $1 - P(R = \text{melb wins})$.

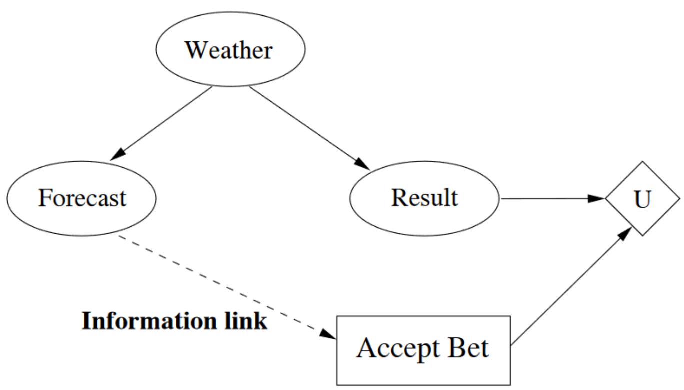
$$\begin{aligned}
EU(AB = yes) &= P(R = \text{melb wins}) \times U(R = \text{melb wins}|AB = yes) \\
&\quad + P(R = \text{melb loses}) \times U(R = \text{melb loses}|AB = yes) \\
&= (0.3 \times 0.6 + 0.7 \times 0.25) \times 40 + (0.3 \times 0.4 + 0.7 \times 0.75) \times -20 \\
&= 0.355 \times 40 + 0.645 \times -20 = 14.2 - 12.9 = 1.3 \\
EU(AB = no) &= P(R = \text{melb wins}) \times U(R = \text{melb wins}|AB = no) \\
&\quad + P(R = \text{melb loses}) \times U(R = \text{melb loses}|AB = no) \\
&= (0.3 \times 0.6 + 0.7 \times 0.25) \times 20 + (0.3 \times 0.4 + 0.7 \times 0.75) \times -5 \\
&= 0.355 \times 20 + 0.645 \times -5 = 7.1 - 3.225 = 3.875
\end{aligned}$$

Decision: With no other information available, Clare's decision is to not accept the bet.

## Extra for Experts:

Suppose that in the football team example, Clare was only going to decide whether to accept the bet or not after she heard the weather forecast. The network can be extended with a *Forecast* node, representing the current weather forecast for the match day, which has possible values {*sunny, cloudy or rainy*}. Forecast is a child of *Weather*. There should be an information link from *Forecast* to *AcceptBet*, shown using dashes in the figure below, indicating that Clare will know the forecast when she makes her decision. Assuming the same CPTs and utility table, compute a decision table for the decision node given the forecast.

| W | F | P(F\|W) |
|---|---|---|
| wet | rainy | 0.60 |
| | cloudy | 0.25 |
| | sunny | 0.15 |
| dry | rainy | 0.10 |
| | cloudy | 0.40 |
| | sunny | 0.50 |




**Decision Table**

| F | Accept Bet |
|---|---|
| rainy | |
| cloudy | |
| sunny | |

*Answer:*

Highest utility is in **BOLD**

Decisions calculated for football team, given the new evidence node *Forecast*.

| $F$ | $Bel(W = wet)$ | $Bel(R = melb\_wins)$ | EU(AB=yes) | EU(AB=no) |
|---|---|---|---|---|
| *rainy* | 0.720 | 0.502 | **10.12** | 7.55 |
| *cloudy* | 0.211 | 0.324 | -0.56 | **3.10** |
| *sunny* | 0.114 | 0.290 | -2.61 | **2.25** |