

Formula Sheet Additions

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1 Butterworth Formulae

Transfer Function of a Butterworth Filter:

$$|H(s)| = \frac{G_0}{\sqrt{1 + (\frac{\omega}{\omega_c})^{2n}}}$$

Order Calculation from Pass and Stop Band Values:

$$n = \log \frac{10^{-\frac{G_s}{10}} - 1}{10^{-\frac{G_p}{10}} - 1} \div 2 \log \frac{\omega_s}{\omega_p}$$

2 Chebyshev Formulae

Gain of the Transfer Function:

$$G(\omega) = |H(\omega)|^2 = \frac{G_0^2 k_0^2}{1 + \epsilon^2 T_N^2(\frac{\omega}{\omega_c})}$$

Order of Chebyshev Calculation:

$$n = \frac{1}{\cosh^{-1}(\omega_s/\omega_p)} \cosh^{-1} \left[\frac{10^{-G_s/10} - 1}{10^{-\hat{r}/10} - 1} \right]^{1/2}$$
$$\epsilon_{dB} = r = \sqrt{1 + \epsilon^2}$$

Gain Correction Factor:

$$k_N = \begin{cases} a_0 & N \text{ odd} \\ \frac{a_0}{\sqrt{1+\epsilon^2}} = \frac{a_0}{10^{\frac{\hat{r}}{20}}} & N \text{ even} \end{cases}$$

3 Low Pass Filters to Other Forms

Low Pass to Band Pass Filter:

$$s \rightarrow \omega_c \times \frac{s^2 + \omega_H \omega_L}{s \times (\omega_H - \omega_L)}$$

Low Pass to Low Pass Filter:

$$s \rightarrow \omega_c \times \frac{s^2 + \omega_H \omega_L}{s \times (\omega_H - \omega_L)}$$