

Formula Sheet Additions

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1 General Formulae

Band-Stop Stop Frequencies:

$$\omega_s = \frac{(\omega_{p2} - \omega_{p1}) \times \omega_{s1}}{\omega_{p1}\omega_{p2} - \omega_{s1}^2}$$

or

$$\omega_s = \frac{(\omega_{p2} - \omega_{p1}) \times \omega_{s2}}{\omega_{s2}^2 - \omega_{p1}\omega_{p2}}$$

2 Butterworth Formulae

Transfer Function of a Butterworth Filter:

$$|H(s)| = \frac{G_0}{\sqrt{1 + (\frac{\omega}{\omega_c})^{2N}}}$$

Order Calculation from Pass and Stop Band Values:

$$N = \log \frac{10^{-\frac{\bar{G}_s}{10}} - 1}{10^{-\frac{\bar{G}_p}{10}} - 1} \div 2 \log \frac{\omega_s}{\omega_p}$$

3 Chebyshev Formulae

Gain of the Transfer Function:

$$|H(\omega)| = \frac{G_0 k_0}{(1 + \epsilon^2 T_N^2(\frac{\omega}{\omega_c}))^{\frac{1}{2}}}$$

Chebyshev Polynomial:

$$T_N(\frac{\omega}{\omega_c}) = \begin{cases} \cos(N \times \arccos(\frac{\omega}{\omega_c})); & |\frac{\omega}{\omega_c}| \leq 1 \\ \cosh(N \times \operatorname{arccosh}(\frac{\omega}{\omega_c})); & |\frac{\omega}{\omega_c}| > 1 \end{cases}$$

or

$$T_N(\frac{\omega}{\omega_c}) = 2(\frac{\omega}{\omega_c})T_{N-1}(\frac{\omega}{\omega_c}) - T_{N-2}(\frac{\omega}{\omega_c}); N \geq 2$$

Order of Chebyshev Calculation:

$$N = \frac{1}{\cosh^{-1}(\omega_s/\omega_p)} \cosh^{-1} \left[\frac{10^{-\bar{G}_s/10} - 1}{10^{-\bar{r}/10} - 1} \right]^{1/2}$$

$$\epsilon_{dB} = r = \sqrt{1 + \epsilon^2}$$

Gain Correction Factor:

$$k_N = \begin{cases} a_0 & N_{odd} \\ \frac{a_0}{\sqrt{1+\epsilon^2}} = \frac{a_0}{10^{\frac{\epsilon}{20}}} & N_{even} \end{cases}$$

4 Low Pass Filters to Other Forms

Low Pass to Band Pass Filter:

$$s \rightarrow \omega_c \times \frac{s^2 + \omega_H \omega_L}{s \times (\omega_H - \omega_L)}$$

Low Pass to Low Pass Filter:

$$s \rightarrow \omega_c \times \frac{s}{\omega_B}$$