Formula Sheet Additions

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May 1, 2024

1 Butterworth Formulae

Transfer Function of a Butterworth Filter:

$$|H(s)| = \frac{G_0}{\sqrt{1 + (\frac{\omega}{\omega_c})^{2n}}}$$

Order Calculation from Pass and Stop Band Values:

$$n = \log \frac{10^{-\frac{G_s}{10} - 1}}{10^{-\frac{G_p}{10} - 1}} \div 2\log \frac{\omega_s}{\omega_p}$$

1.1 Low Pass Filters to Other Forms

Low Pass to Band Pass Filter:

$$s \to \omega_c \times \frac{s^2 + \omega_H \omega_L}{s \times (\omega_H - \omega_L)}$$

If you swap the numerator and denominator, you get a band stop filter

2 Chebyshev Formulae

Gain of the Transfer Function:

$$G(\omega) = |H(\omega)|^2 = \frac{G_0^2 k_0^2}{1 + \epsilon^2 T_N^2(\frac{\omega}{\omega_c})}$$

Order of Chebyshev Calculation:

$$n = \frac{1}{\cosh^{-1}(\omega_s/\omega_p)} \cosh^{-1} \left[\frac{10^{-\hat{G_s}/10} - 1}{10^{-\hat{r}/10} - 1} \right]^{1/2}$$

$$\epsilon_{dB} = r = \sqrt{1 + \epsilon^2}$$

$$\epsilon = G_{oudB}$$