Formula Sheet Additions

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May 1, 2024

1 Butterworth Formulae

Transfer Function of a Butterworth Filter:

$$|H(s)| = \frac{G_0}{\sqrt{1 + (\frac{\omega}{\omega_c})^{2N}}}$$

Order Calculation from Pass and Stop Band Values:

$$N = \log \frac{10^{-\frac{\hat{G_s}}{10}} - 1}{10^{-\frac{\hat{G_p}}{10}} - 1} \div 2 \log \frac{\omega_s}{\omega_p}$$

2 Chebyshev Formulae

Gain of the Transfer Function:

$$|H(\omega)| = \frac{G_0 k_0}{(1 + \epsilon^2 {T_N}^2 (\frac{\omega}{\omega_c}))^{\frac{1}{2}}}$$

Chebyshev Polynomial:

$$T_N(\frac{\omega}{\omega_c}) = \begin{cases} cos(N \times arccos(\frac{\omega}{\omega_c})); & |\frac{\omega}{\omega_c}| \leqslant 1 \\ cosh(N \times arccosh(\frac{\omega}{\omega_c})); & |\frac{\omega}{\omega_c}| > 1 \end{cases}$$

Order of Chebyshev Calculation:

$$N = \frac{1}{cosh^{-1}(\omega_s/\omega_p)} cosh^{-1} \left[\frac{10^{-\hat{G}_s/10} - 1}{10^{-\hat{r}/10} - 1} \right]^{1/2}$$

$$\epsilon_{dB} = r = \sqrt{1 + \epsilon^2}$$

Gain Correction Factor:

$$k_N = \begin{cases} a_0 & Nodd \\ \frac{a_0}{\sqrt{1+\epsilon^2}} = \frac{a_0}{10^{\frac{\hat{r}}{20}}} & Neven \end{cases}$$

3 Low Pass Filters to Other Forms

Low Pass to Band Pass Filter:

$$s \to \omega_c \times \frac{s^2 + \omega_H \omega_L}{s \times (\omega_H - \omega_L)}$$

Low Pass to Low Pass Filter:

$$s \to \omega_c \times \frac{s}{\omega_B}$$