Problem Chosen	2020 MCM/ICM Summary Sheet	Team Control Number
A		# 2010755

Escape From Scotland

Summary

Firstly, based on the data in the ICES working document[3][4],we use **Generalized Additivity Model (GAM)** to analyze the relationship between fish density and environmental factors. By utilizing the results of Generalized Additive Model ,we get the environmental fitness formula of fish. Then we use this formula to calculate the situation in 30 different sea areas,and finally we use **Continuous-time Markov Model** to calculate the situation in each area in the next 50 years. From the rectangle state, we can infer the living area of mackerel and herring.

Secondly, in order to figure out the affection on the fish's occurrence position caused by temperature, we collect the temperature data of North Sea from 2009 to 2019. Then we divide the ocean into grids by longitude and altitude. After analyzing we find that the change of the temperature is highly relevant with season, which tends to increase. We establish a **Seasonal Autoregressive Integrated Moving Average Model (SARIMA)** to predict the temperature change in the next 50 years. Next, we represent the position of the fish population by finding the clustering center. We use **Mean-Shift Algorithm** to get cluster centers of each years and look up the surrounding temperature. Then we build a liner regression to find out the relationship between year and fish's favorite temperature. After that we construct a **Score Function** to determine whether a region will become a cluster center in the future or not. The main idea of the function is to predict the temperature of one region and its surroundings. The higher score a region gets,the higher possibility it will be chosen by fish.

Thirdly, to provide constructive suggestions for fishing companies in Scotland, we build a **Similarity to an Ideal Solution (TOPSIS)** to evaluate the several strategies. We consider different factors such as the maintenance of facilities, the cost of relocation and fuel, the total catch of each strategy. We use **Analytic Hierarchy Process (AHP)** to determine the weight of each factor, then sort strategies in ascending order by calculating the distance from each strategy to the optimal one. In this way, we conclude that relocation is a reasonable and sustainable choice in the long term.

Next, considering the fact that territory seas of countries restricts the effective fishing, we collect data from the United Nations Convention on the Law of the Sea (LOSC). By calculating the frequency of fish appearance in available seas we calculate the quantity of total catch. Then we define the **loss rate** to describe the impact of territorial limits and get the conclusion that as mackerel and herring migrating to the north, the loss of fishing companies grow larger.

Besides the models we established, we complete the Sensitivity Analysis and prove the stability of our models. Finally, we write an article on the basis of results we get to help fishermen.

Keywords: **Generalized Additive Model (GMA)**, **Continuous-time Markov Model**, **Seasonal Autoregressive Integrated Moving Average Model(SARIMA Model)**, **Mean-Shift Algorithm**, **Similarity to an Ideal Solution (TOPSIS)**

Contents

1 Introduction

1.1 Restatement of the Problem

We need to build mathematical models to solve following problems:

1. From a number of perspectives, our mathematical model needs to determine where mackerel and herring are most likely to appear in the next 50 years, with temporal and spatial environmental factors considered.

2. Our second task is to establish two model, one of them is the ocean temperature prediction model, the other one is fish occur position prediction model.

3. Strategies require to be determined to help small companies.

1.2 Overview of Our Work

We searched a large number of papers that discuss the condition of herring and mackerel to help us deepen the understanding of the problem.

Firstly,based on the data in the ICES working document[3][4], we establish a **Generalized Additive Model** (GAM) to analyze the relationship between fish stocks and environmental impact factors.Then we find out the key factors that influenced the model and select the optimal spatial distribution prediction model to provide scientific basis for predicting the spatial distribution of herring and mackerel.We get the environmental fitness formula of fish. Then we used the formula to calculate the situation in 30 different sea areas, and finally we use **Continuous-time Markov model** to calculate the situation in each area in the next 50 years. From the rectangle state, we predict the mackerel's living area and herring's living area.

For the second question, firstly we choose North Sea as our target to implement temperature prediction. Considering the pattern of temperature change is highly relevant with season, we establish a Seasonal Autoregressive Integrated Moving Average Model to predict the temperature of North Sea in the next 50 years base on the temperature date from Hadley Centre observations office. Then we decide to use cluster center point to represent the distribution of fish and calculate the position of cluster centers by Shift Means algorithm base on the data from ICES. Finally we build a model to predict the occur position(cluster center) of fish in the future by analyzing the relationship between year and the temperature around the cluster center and combining the result of temperature prediction. We conclude that in the worst case, small fishery companies will get into trouble for the extra voyage distance after 2025 and the best case is after 2045.

2 Assumption

To simplify our model, we make following well-justified assumptions.

(1) Carbon emissions will increase at a certain rate as well as sea surface temperature in the next 50 years.

(2) Effect of the weather is neglected.

(3) Sea temperature can be represented by the sea surface temperature in this region.

(4) Fishing operation is always carried out on July or August.

(5) We divided the selected rectangular sea area ($44°N20°W \sim 68°N30°E$) equally into 30 equal parts.In order to simplify the calculation, it is assumed that the fish are evenly distributed in each rectangular area (the longitude span is 10° and the latitude span is 4°) with the same environmental and spatial factor.

(6) The population density of species remain relatively constant during a year.

(7) The quantity of mackerel and herring is in proportion to their frequency of occurrence.

2.1 Symbols and Definition

There are some symbols appear in the model. We show them below:

Table 1: Symbols in Chapter 3

Symbols	Description
i	Station variable
DS_i	Density of fish(mackerel or herring) at station i (kg/km^2)
H_i	Horizontal opening of trawl at station i (km)
TD_i	Distance of the trawl haul (km^2)
C_i	Catch at station i(kg)
λ_i	Longitude at station i $(°W)$
ϕ_i	Latitude at station i $(°N)$
SST_i	Sea Surface Temperature at station i $(°C)$
z_i	Zooplankton's dry weight at station i (kg)
SSB_i	Spawning-stock biomass at station i
b_i	Number of biological species at station i
y	Year
j	Rectangular number
t	Time
$M_j(t)$	State of the j_{th} rectangle at time t
P_{M_w,M_k}	Transition probability for the State $M_w$changing into State M_k

Table 2: Symbols in Chapter 4

Symbols	Description
B	Backshift operator
x_t	Temperature of a region of seawater in t_{th} month.
P, Q	Orders
T_b	Average temperature
R_s	Coastal region
R_t	Inland region
$r(lo, la)$	Temperature of a region with geographical coordinates (lo, la)

Table 3: Symbols in Chapter 5 & 6

Symbols	Description
$CPUE_y$	Catch Per Unit Effort in year y
F	Funds
M	Maintenance cost
T	Transportation fee
s_j	Area of j_{th} square
A	Comparison matrix
W	Weight
y	Year
L_y	Quantity of fish in the year y
C	Appearance of mackerel

3 Migration Model of Mackerel and Herring

We first establish a Generalized Additive Model(GAM) to predict the relationship between fish density changes and spatiotemporal factors.Then,we use the Continuous-time Markov model to calculate the state of each rectangular sea area in the next 50 years. From the rectangle state, we can infer the mackerels living area and herrings living area.In the figure below,Z stands for zooplanktons dry weight, B stands for the number of biological species.

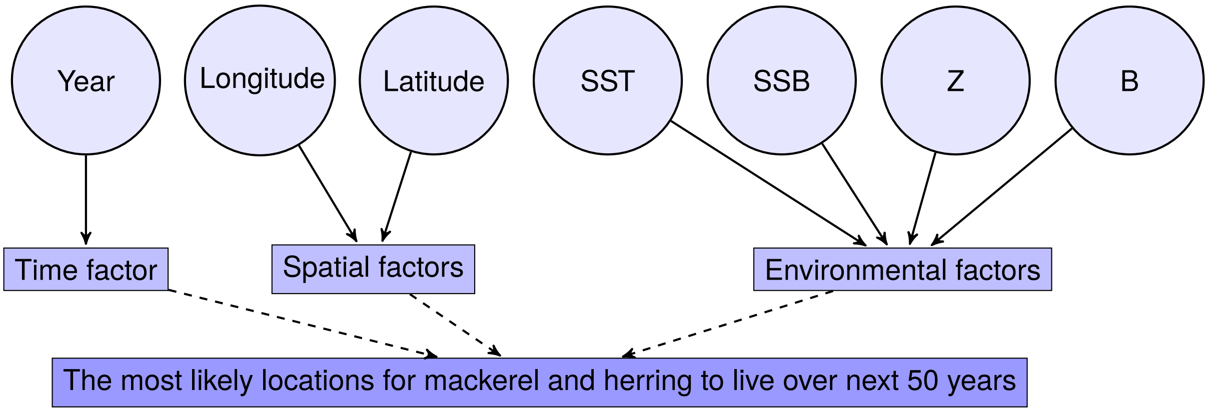


Figure 1: Parametric analysis of the model.

3.1 Prediction of Fish Migration based on Generalized Additive Model

The Generalized Additive Model (GAM), which is flexible and focuses more on the complex relationship between detection data, has been widely used in the fishery field.

3.1.1 Construction of GAM

At each station, Mackerel density per station ($DS; kg/km^2$) is calculated as:

$$DS_i = \frac{C_i}{TD_i * H_i} \tag{1}$$

Thanks to ICES Working Group for the data[1][4]. We get the horizontal opening of the trawl H_i, the distance towed by the trawl TD_i and the mackerel catch C_i used in calculations. The model expression of **GAM** is as follows:

$$InDS_{i,(\lambda,\phi)} = a + s1(\lambda_i, \phi_i) + s2(SST_i) + s3(b_i) + s4(z_i) + s5(y) + \beta * SSB_i + \varepsilon \tag{2}$$

where $InDS_{i,(\lambda,\phi)}$ is a link function, a is the model intercept, λ_i is longitude at station i, ϕ_i is latitude, SST_i is Sea Surface Temperature ,b_i is the number of biological species , z_i is zooplankton's dry weight , SSB_i is the estimated mackerel spawning stock biomass with linear coefficient β, s1 is a two-dimensional smoothing function, y is year, $s2 \sim s5$ are one dimensional smoothing functions for temperature and mesozooplankton, and ε is the error term.

3.1.2 Optimization of GAM

We select variables with significant influence on DS_i.The GAM model is used to test the significance of environment factors and annual resources of mackerel population, so as to obtain statistically significant influence factors.

Table 4: Test of GAM between the Environmental or Climatic Factors.

Factor	p-value	R^2
SST	0.0275*	0.677
latitude	0.0134*	0.606
longitude	0.0795·	0.329
y	0.00338**	0.76
B	0.0222*	0.541
Z	0.0163*	0.705

Note: ** indicates a significant correlation at a significance level of only α=0.05 (bilateral).

According to the comprehensive analysis of expert knowledge[2] , SST, SSB,latitude, longitude were retained in the model. The expression of optimized GAM is as follows:

$$InDS_{i,(\lambda,\phi)} = a + s_1(SST_i) + s_2(y) + s_3(z_i) + \beta * SSB_i + \varepsilon \tag{3}$$

3.1.3 Results of GAM model

Result is obtained by using the "mgcv" package in R 3.6.2. The following figure shows the relationship diagram of the mackerel density of each factor calculated by GAM. W stands for longitude, N stands for latitude.

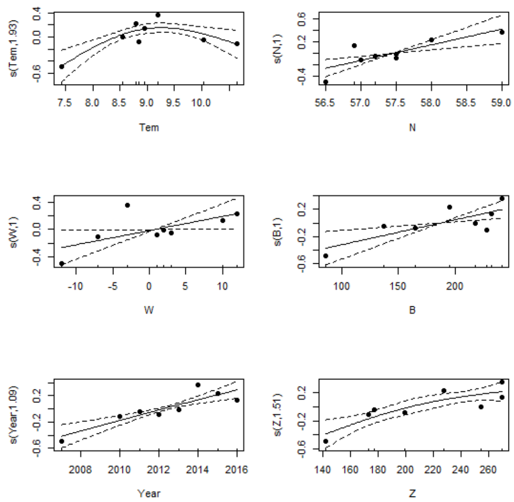


Figure 2: The relationship between factors and the density of mackerel

- **Steric factor :** It can be seen from the spatial factors that mackerel is concentrated in latitude $56.5°N \sim 58.5°N$, longitude $-7°W \sim 10°W$. Figure 2 shows the tendency of mackerel to migrate northward.In terms of longitude, Mackerel shoals tend to migrate eastward (toward the Norwegian sea) and westward (toward Greenland).

- **Environmental factor :** The results of GAM model shows that SST is the most important habitat indicator affecting mackerel migration. When the temperature is between $8°C$ and $10°C$, the effect of temperature on fish density increases with the increase of temperature.The greater the plankton biomass in the middle layer, the larger the fish population.It also shows that the greater the number of species, the greater the number of fish.

3.2 Future Location Prediction based on Continuous-time Markov Chain

3.2.1 Construction Continuous-time Markov Model

We divide the selected rectangular sea area ($44°N20°W \sim 68°N30°E$) equally into 30 equal parts.Each part has a longitude span of 10 degrees and a latitude span of 4 degrees. The results of the GAM model show that B and Z are positive indicators of fish's environmental fitness, and SST is an interval indicator. Hence, the environmental fitness formula of fish was obtained.

$$L_j(t) = \frac{b_j}{max\{b\}} + \frac{z_j}{max\{z\}} + \begin{cases} 1 - \frac{d-SST_j}{d-d^*} & SST_j < d \\ 1 & d \leq SST_j \leq e \\ 1 - \frac{SST_j-e}{e^*-e} & SST_j > e \end{cases} \tag{4}$$

where $L_j(t) \in [0,3]$ stands for the degree to which each square is suitable for fish to live, $[d,e]$ stands for the optimal stability interval of SST($8°C \sim 10°C$),$[d^*,e^*]$ stands for maximum tolerance interval($0°C \sim 25°C$).[1] Since $L_j(t)$ can be considered as a function of time t, we use Continuous-time Markov chain to simulate the distribution change of mackerel.The condition of each rectangle is closely related to the population density of mackerel.To simplify the problem, we set the following 5 states by calculating the value of $L_j(t)$.

Table 5: States of each rectangle

States	Meaning
M_1	most unsuitable for mackerel to survive ($0 \leqslant L_j < 0.6$) $M_1 = 1$
M_2	not suitable for mackerel to survive ($0.6 \leqslant L_j < 1.2$) $M_2 = 2$
M_3	suitable for mackerel to survive ($1.2 \leqslant L_j < 1.8$) $M_3 = 3$
M_4	quite suitable for mackerel to survive ($1.8 \leqslant L_j < 2.4$) $M_4 = 4$
M_5	most suitable for mackerel to survive($2.4 \leqslant L_j \leqslant 3.0$) $M_5 = 5$

Define $M_j(t)$ as the state of the j_{th} rectangle at time t, P_{wk} as the transition probability for the state M_w changing into state M_k. $M_w, M_k \in \{1,2,3,4,5\}$.
In random process$\{X(t),t\}$, for n=0,1,2..., in continuous time$t_0 < t_1 < ... < t_b < ... < t_{n+1}$, the following mathematical formula holds:

$$p(M_{wk}) = p(M_w|M_k) = p_{M_w M_k} \tag{5}$$

For any time $t \geq t_b$, the transfer probability can be written as:

$$p_{M_w M_k}(t_b, t) = P(M(t) = k|M(t_b) = w) \tag{6}$$

Then we can get the Transition probability matrix P:

$$P = \begin{bmatrix} p_{11} & \cdots & p_{15} \\ \vdots & \ddots & \vdots \\ p_{51} & \cdots & p_{55} \end{bmatrix} \tag{7}$$

Markov state transitin equation can be represented as:

$$Q_2 = Q_1 P \tag{8}$$

$$Q_2 = \begin{bmatrix} M_1(0) & \cdots & M_5(0) \\ \vdots & \ddots & \vdots \\ M_1(t-1) & \cdots & M_5(t-1) \end{bmatrix}_{t\times 5} \tag{9}$$

$$Q_1 = \begin{bmatrix} M_1(1) & \cdots & M_5(1) \\ \vdots & \ddots & \vdots \\ M_1(t) & \cdots & M_5(t) \end{bmatrix}_{t\times 5} \tag{10}$$

According to least square method, we can get

$$P = (Q_1^T Q_1)^{-1} Q_1^T Q_2 \tag{11}$$

Consequently, we can predict the state M of each rectangle in the next 50 years.

$$Q_{50} = Q_1 P^{50} \tag{12}$$

3.2.2 Results of Continuous-time Markov Model

Since M_5 represents the state most suitable for mackerel survival, the smart fish will live in this area intensively.From the rectangle state, we can predict the mackerel's living area and herring's living area. Figure 3 shows the results.

Figure 3: Population growth

The orange rectangle shows that the most likely place for herring to migrate is in **Gulf of Bothnia**. The pink rectangle shows where the mackerel is most likely to migrate is in **Norwegian Sea**. The results of GAM shows that SST is the most important habitat indicator affecting fish migration.

4 Time to Give a Warning

4.1 Temperature Prediction of Ocean

4.1.1 Introduction

The temperature of Ocean is hard to predict because it is affected by many factors, mainly are sun radiation and marine atmospheric heat exchange and the ocean current make an obvious affection on local seawater. However, we can still find that the change of temperature is highly linked to the season change which means we can predict the temperature by using the former data. As a sequence of that, we decide to build SARIMA model as our solution.

4.1.2 Prediction Model

Due to hypotheses, the temperature of seawater is influenced by autocorrelation in time series and highly related to the season. As a consequence, we choose Seasonal ARIMA (SARIMA) Model [1] to fit the curve. The general form of seasonal model SARIMA(p, d, q)(P, D, Q) is given by:

$$\Phi_p(B^s)\varphi(B)\nabla_s^D\nabla^d x_t = \Theta_Q(B^s)\theta(B)w_t \tag{13}$$

where $\{w_t\}$ is the nonstationary time series, usually is the usual Gaussian White noise process. s is the period of the time series, here represent the month after 2019-12. The ordinary autoregressive and moving average components are represented by polynomials $\varphi(B)$ and $\theta(B)$ of orders p and q. The seasonal autoregressive and moving average components are $\Phi_p(B^s)$ and $\Theta_Q B^s$, where P and Q are their orders.∇^d and ∇_s^D are ordinary and seasonal difference components. B is the backshift operator. The expressions are shown as follows:

$$\varphi(B) = 1 - \varphi_1 B - \varphi_2 B^2 - \cdots - \varphi_p B^p \tag{14}$$

$$\Phi_p(B^s) = 1 - \varphi_1 B^s - \varphi_2 B^{2s} - \cdots - \varphi_p B^{ps} \tag{15}$$

$$\theta(B) = 1 + \Theta_1 B^s + \Theta_2 B^{2s} + \cdots + \Theta_Q B^{Qs} \tag{16}$$

$$\nabla^d = (1 - B)^d \tag{17}$$

$$\nabla_s^D = (1 - B^s)^D \tag{18}$$

$$B^k x_t = x_{t-k} \tag{19}$$

x_t represent the temperature of a region of seawater in t_{th} month.
Firstly, we collect the time-series data of all regions. Here we choose 10 position and their analyze of temperature will write in the essay (appendix). In this character we will only show three of them (circle by red): Time series analysis requires that the series should be stationarity. The basic idea of stationarity is that the probability laws that govern the behavior of the process do not change over time. If it's not stationarity, then we should smooth it by using by using the difference equation. The result of the chosen place is shown below:

From the Autocorrelation Function (ADF) result and the different equation result above shows that d = 1 is well enough. As for the Autocorrelation Coefficient Function (ACF) and Partial Autocorrelation Fnuction (PACF) , there is no obvious tailing or truncation was found in the

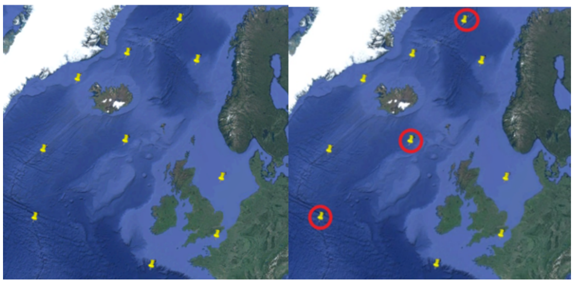
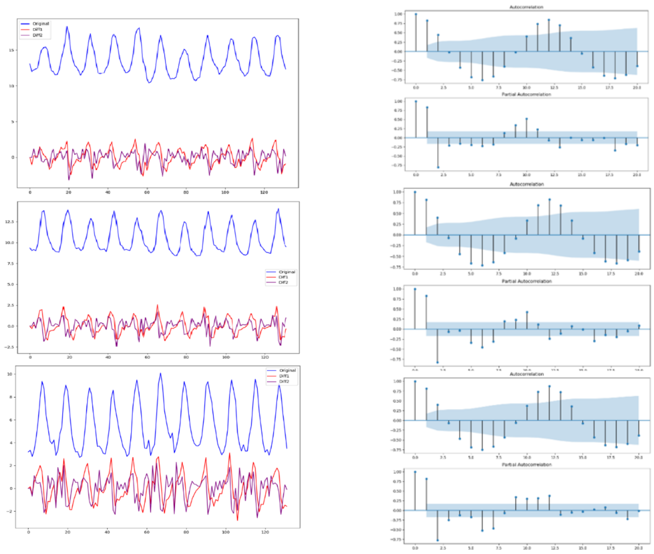


Figure 4: Position to Analyze



(a) different equation result (b) ACF and PACF Result

Figure 5: Results

diagrams of the sequences ACF and PACF, which indicates that for such sequences, it is not suitable to fit the SARIMA model. Here we will use the time series decomposition (STL) method, and then use the SARIMA model to fit the trend series and the residual series. Here we only take the second one to go on. From the picture above, there is a certain periodic waving of seasonal. As a result,

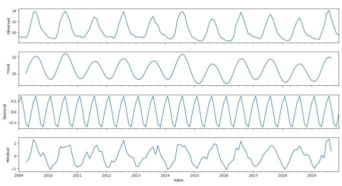
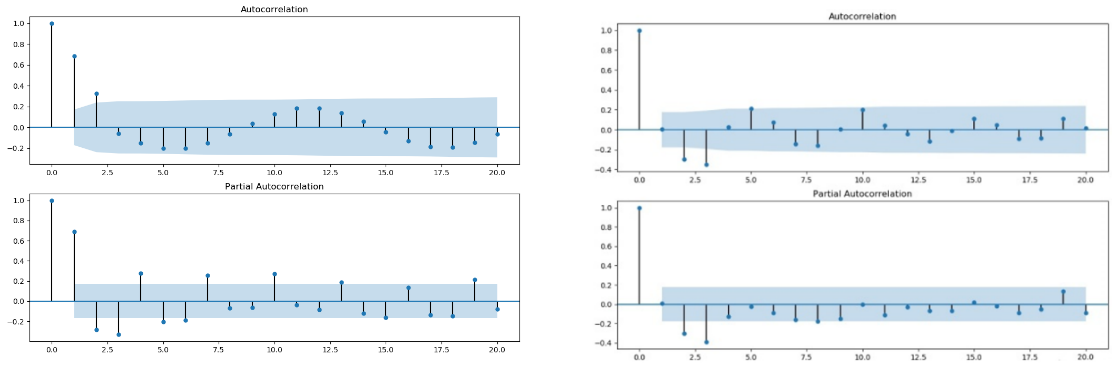


Figure 6: STL Result

we decide to use SARIMA to fit the trend and residual. According to the ADF and PADF result of the trend and residual, we can roughly estimate that :

(1) Trend sequence ACF has 3 order tailings, PACF has 2 order tailings, so we can choose $p = 2, q = 3$.

(2) Residual sequence ACF has 4 order tailing, PACF has 4 order tailing, so we can choose $p = 4, q = 4$.



(a) (b)

Figure 7: ADF and PADF of trend and residual

When choosing parameters, we need to balance prediction error with model complexity. We can determine the order of the model based on the information criterion function method. Here we introduce a Judging criteria called AIC raised by Akaike in 1971 [9].

$$AIC = -2ln(L) + 2K \tag{20}$$

If the model's error follows an independent normal distribution, then

$$AIC = 2K + nln(\frac{SSR}{n}) \tag{21}$$

Where K is the number of parameter, n is the number of sample and SSR is SUM SQAURE OF RESIDUE. After the test, for trend, $p = 0$, $q = 1$ and for residual $p = 2$, $q = 1$.
By calculation, we pick the location that is circled by red and use their data to train our model and predict the temperature from 2017.4 to 2019.12. The results are as following , blue is the true value and orange is the predict value. Now we can conclude that the prediction model is reasonable.

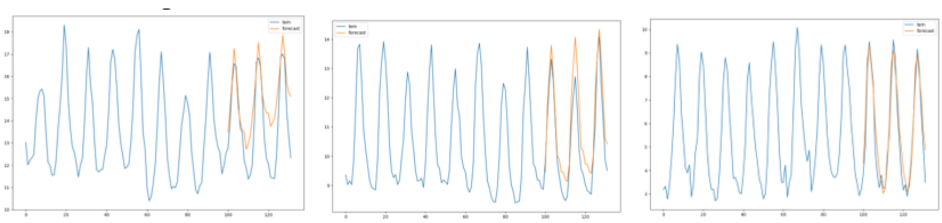


Figure 8: Predict result on test set

4.1.3 Results

Base on the prediction model show in 4.2.4, the heatmap of the seawater in different months and different years is shown below. We set the land to zero Celsius in order to distinguish seawater and land easily. When the color of one region looks dark, it means the temperature of seawater in this region is high.

According to the result, there is a noticeable change among seasons, which confirms the validity of our model from the side. In this model, the temperature of the area which is below the Britain in spring (April) and Winter (January) is becoming higher and higher in the next 50 years. At the same time, the temperature of the region close to Greenland and close to the Arctic Ocean is growing higher too. We think the increasement of the temperature is because of global warning which cause the large-scale melt of glacier, indirectly lead to the raise in the north area.

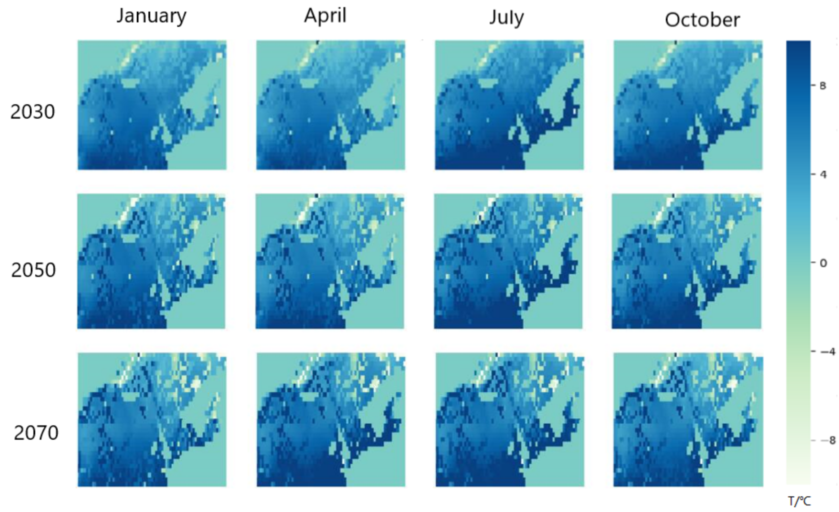


Figure 9: Predict result

4.2 Time for Warning

4.2.1 Introduction

The growth and development of fish require a suitable temperature and they are sensitive to the change of temperature. Extra expense will be paid by Scotland-based fishing companies because of the longer distance (the cost of fuel) which will make a side effect on fishery (small fishing companies especially). In this character we will use means shift clustering algorithm to find out the best position to fish and find out its' character on temperature. Therefore we can predict the gather point or distribution (roughly) of fish according to the future temperature of ocean we already got in last chapter.

4.2.2 Clustering Process

Clustering is a machine learning technique that involves grouping data points. Given a set of position points, we can use a clustering algorithm to classify each point into a particular group.
(1) Kernel Density Estimation
Given n data point, $x_i \in R^d$ where d is dimension (latitude, longitude, SPABUN), parameter h is the bandwidth of kernel function K. Then the KDE of the dataset is:

$$f(x) = \frac{1}{nh^d} \sum_{i=1}^{n} K(x) \tag{22}$$

where $K(x) = c_{k,d}k(x^2)$,$c_{k,d}$ is normalization constant which make the integral of $K(x)$ is 1. Here we choose Radially Symmetric Kernel as our kernel function.

$$K(x) = K(\frac{x - x_i}{h}) \tag{23}$$

(2) Mean Shift The basic goal of mean shift is to move the sample points in the direction that local density is increases. The mean moving vector refers to the direction of local density that increase fastest, so this vector point to the gradient direction of the data set density. The gradient of $f(x)$ is:

$$\nabla f(x) = \frac{2c_{k,d}}{nh^5}[\sum_{i=1}^{n} g(||\frac{x - x_i}{h}||^2)][\frac{\sum_{i=1}^{n} x_i g(||\frac{x-x_i}{h}||^2)}{\sum_{i=1}^{n} g(||\frac{x-x_i}{h}||^2)} - x] \tag{24}$$

Where $g(s) = -k'(s)$, so the gradient direction $m_h(x)$ is:

$$m_h(x) = \frac{\sum_{i=1}^{n} x_i g(||\frac{x-x_i}{h}||^2)}{\sum_{i=1}^{n} g(||\frac{x-x_i}{h}||^2)} - x \tag{25}$$

So at every steps of Mean-shift clustering algorithm is:
(1) Calculate $m_h(x)$ of every sample (2) $x_i = x_i + m_h(x_i)$ (3) Repeat (1) (2), until the sample point is convergent (4) The points which converge into the same point are considered they belong to a same cluster

4.2.3 Temperature of Cluster Center

In order to get the overall geography trend of the clustering centers, we decide to choose the period from 2009 to 2017 and calculate their clustering point relatively base on the data from ICES (International Association for the Physical Science of the Ocean)
Then we plot them in the picture below.

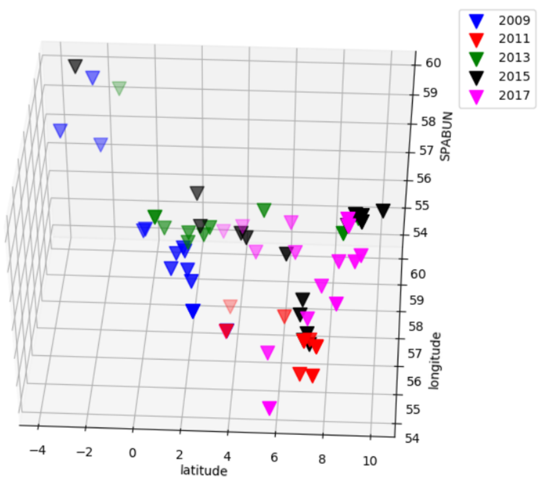


Figure 10: Clustering center distribution

From the result we can see that the position tendency of “best points” are leaving England, moving to the south-east. The average temperature of these cluster of each years is:

Year	Average Temperature(℃)
2009	14.16
2011	14.94
2013	14.28
2015	13.63
2017	14.02

We define the Average Temperature(℃) as T_b. Naturally, we can use liner regression to simulate the change of the average temperature of "best points".

$$T_b = \theta x + b + \varepsilon \tag{26}$$

The result of liner regression is $\theta = 0.0815$, $b = 14.61$ and ε is white noise subject to normal distribution.

4.2.4 Predict Model and Result

To determine a region is "best point" or not, we introduce a criterion S. We divided the sea into two kinds: region that all of its neighbor region is sea, called R_s and region that some of its neighbor region is land, called R_l. Then the criterion for each kind of sea is:

$$S = \frac{\sum_{j=-1}^{1}\sum_{i=-1}^{1} T_{c,ij}(r(lo+i, la+j))}{9} \tag{27}$$

Where $T_{c,ij}(r(lo, la))$ is score function:

$$T_{c,ij} = \begin{cases} 0 & \text{if r is } R_l \\ \frac{\sqrt{2}}{|r(lo,la)-T_b|} & \text{i=j} \\ \frac{1}{|r(lo,la)-T_b|} & \text{else} \end{cases} \tag{28}$$

And $r(lo, la)$ is the temperature of a region where its longitude is lo and its latitude is la. So for a particular year, in order to find the position of "best point", First is calculate S for every grid of the map. Then choose those who S is larger than our threshold. Here we test a group of years from 2020 to 2045, in steps of 2 and choose 5 as our threshold. The red grid represent that this

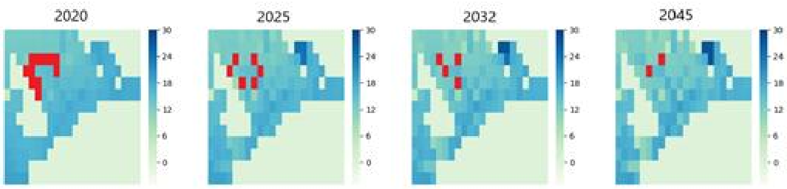


Figure 11: best point distribution in the future

place is probably a “best point” and the pink grid represent that this place has fewer possibilities to be a “best point”. Considering small fishery companies is lack of resource (money, facilities, government permission) to sail far, the places are “best point” but too far away from England is not shown on the picture. According to the prediction and compare with the location that they fishing now where is shown in Appendix 1, we can preliminary conclude that:

(1) In worst case, the fish in the location that they current operate will keep decline after 2025 and it may cost too much to go to the new "best point" fishing.
(2) In best case, the fish in the location that they current operate will slowly decline after 2020 but after 2045, the new "best point" is too far away.

5 Strategy of Fishing Companies

We collect several fisheries in Scotland according to the longitude and latitude on Google Map, combined with the prediction of mackerel's migration before, we can get following figure.

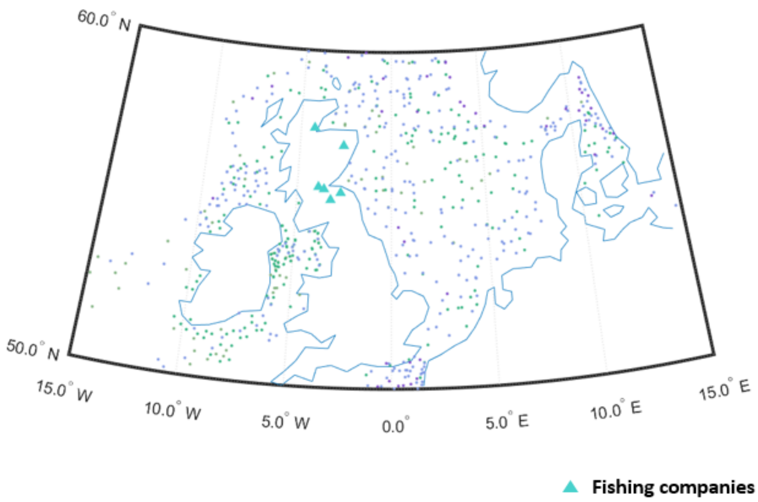


Figure 12: Fisheries in Scotland & distribution of mackerel

Small fishing companies should be aware of migration of herring and mackerel. However, to make reasonable decisions, a reasonable assessment system is needed. Firstly, we list 4 strategies according to given information as follows:

- Relocate the whole company
- Establish a branch company in Northern cities
- Employ lightweight fishing vessels which are able to work without land supplies
- Remain the same

5.1 Comparison between 2 Relocating Stretagies

Next we analyze the impact of each action. Relocation means moving the whole company to the north. We find main fisheries in Scotland distribute along the coastline in 2020, consequently we choose the center of these companies as **the initial location**, the nearest coastline of fish clustering as **the target location**. Assume that the company move in accordance with herring and mackerel, the effective fishing radius is 10 km. Consider the convenience of transportation, all the locations are set in big cities. The initial location is around Glasgow and the new address is around Inverness.

In comparison, if we establish a branch company instead, both two companies will have economic income. However, due to the split of the funds and fishing facilities, the fishing radius decreases to 5 km for each. The figures are as follows.

(a) Relocation (b) Branch company

Figure 13: Comparison of strategies 1&2

5.2 Applying lightweight fishing vessels

The advantage of small vessels lies in its flexibility. We assume a company introduce 3 small vessels which can reach 12 nautical miles(21.6 km) away from coastline . Following figure illustrates the situation.

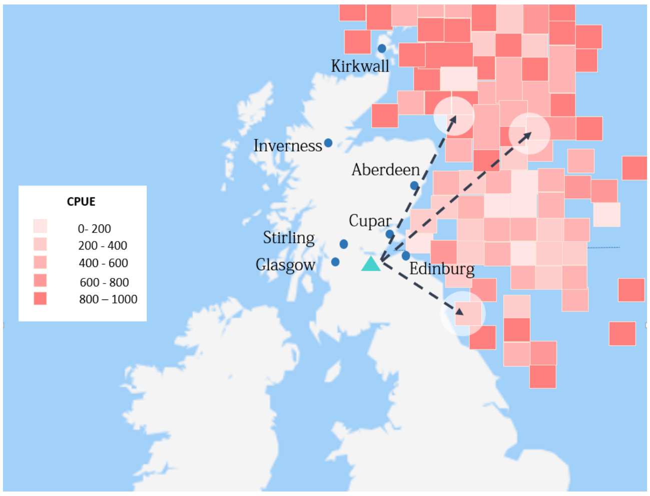


Figure 14: Small vessels applied

5.3 Assessment System with TOPSIS

(1) Factors involved

We use Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) to build an assessment system. We introduce 4 separate factors: Funds(F), Catch Per Unit Effort (CPUE), Maintenance Cost (M), Transportation Fee (T).

- The index Catch Per Unit Effort (CPUE) is used for measuring the richness of species in a fixed area. The CPUE in the year y can be represented as $CPUE_y$

$$CPUE_y = \frac{C_y}{E_y} \tag{29}$$

  C_y is the total catch yield of the year y, E_y is the quantity of effort in the year y. Assuming that the fishing area contains n parts of different $CPUE$. The average CPUE of an operation Can be calculated by summing up effective areas.

$$\overline{CPUE} = \sum_{j=1}^{n} \frac{s_j}{S} * CPUE_j \tag{30}$$

  s_j is the area of j_{th} part, S is the gross area of fishing area, which is related to the number of fishing vessels.
  This approach to calculating $CPUE$ is generalized and applied in our following model.

- F represents the expense of establishing a new company, relocation, spent on new facilities. CPUE is used to describe total quantity of fish.

$$F = F_{base} + F_{add} \tag{31}$$

  , here we set F_{add} for 3 strategies is 7,5,3 respectively. Additionally, we assume the cost of a new small vessel and a common fishing boat is 5 unit and 10 unit respectively.

- M is maintenance cost of facilities.

$$M = \sum_{k=1}^{n} *v_{small} + \sum_{k=1}^{m} *v_{common} \tag{32}$$

  n,m is the quantity of new small vessels and common boats respectively, v_{small} and v_{common} represents the value of two vessels.

- T is the expense on the fuel for traveling, which is closely related to the distance between fishing point to the coast and the distance from the company to selling point. T can be measured by calculating the gross length of one operation, here we get data from Google Map.

Strategy 1, 2, 3, 4 stands for relocating to the north, establishing a branch company in the north, applying lightweight fishing wessels and remaining the same respectively.

Strategy	Distance/55.4km	Maintenance	Funds	CPUE/10^3
1	23.12	17	9.41	7.25
2	18.48	15	9.55	10
3	12.61	13	15.23	8
4	12.61	10	10	6.3

Table 6: Factors Involved

(2)Weight of Each Factor

Using Analytic Hierarchy Process (AHP), we set comparison matrix

$$A = \begin{bmatrix} 1 & 1 & \frac{1}{2} & \frac{1}{2} \\ 1 & 1 & 1 & \frac{1}{2} \\ \frac{1}{2} & 1 & 1 & \frac{1}{4} \\ 2 & 2 & 4 & 1 \end{bmatrix}$$

Check value $CR = \frac{CI}{RI} = 0.02 < 0.1$, thus the matrix is reasonable.
Weight : $W = [0.224, 0.192, 0.137, 0.447]$.

(3) Assessment results

Initial data matrix X

$$X = \begin{bmatrix} 23.12 & 17 & 9.41 & 7.25 \\ 18.48 & 15 & 9.55 & 10 \\ 12.61 & 13 & 15.23 & 8 \\ 12.61 & 10 & 10 & 6.3 \end{bmatrix}$$

According to $z_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}$, we get standardized matrix Z

$$Z = \begin{bmatrix} 0.328 & 0.381 & 0.556 & 0.452 \\ 0.411 & 0.432 & 0.548 & 0.624 \\ 0.602 & 0.498 & 0.343 & 0.500 \\ 0.602 & 0.648 & 0.523 & 0.394 \end{bmatrix}$$

Define:

the Maximun Value

$Z^+ = (Z_1^+, Z_2^+, ..., Z_m^+) = (max\{z_{11}, z_{21}, ...z_{n1}\}, max\{x_{12}, z_{22}, ..., z_{42}\}, .., max\{z_{14}, z_{24}, ..., z_{44}\})$

the Minimum Value

$Z^- = (Z_1^-, Z_2^-, ..., Z_4^-) = (min\{z_{11}, z_{21}, ...z_{41}\}, min\{x_{12}, z_{22}, ..., z_{42}\}, .., min\{z_{14}, z_{24}, ..., z_{44}\})$

The distance between i_{th} (i=1,2,3,4) strategy and Z^+ is:

$$L_i^+ = \sqrt{\sum_{j=1}^{4} (Z_j^+ - z_{ij})^2} \quad L_i^- = \sqrt{\sum_{j=1}^{4} (Z_j^- - z_{ij})^2} \tag{33}$$

Score of the i_{th} (i=1,2,3,4) is represented as:

$$S_i = \frac{L_i^-}{L_i^+ + L_i^-} \tag{34}$$

We sort 4 strategies by S_i,$i \in \{1, 2, 3, 4\}$, the result is shown as follows:

Index	Distance	Maintenance	Funds	CPUE	D^+	D^-	S_i	**Rank**
1	0.328	0.381	0.556	0.452	0.209	0.088	0.296	4
2	0.411	0.432	0.548	0.624	0.131	0.178	0.576	1
3	0.602	0.498	0.343	0.500	0.132	0.156	0.542	3
4	0.602	0.648	0.523	0.394	0.155	0.187	0.546	2

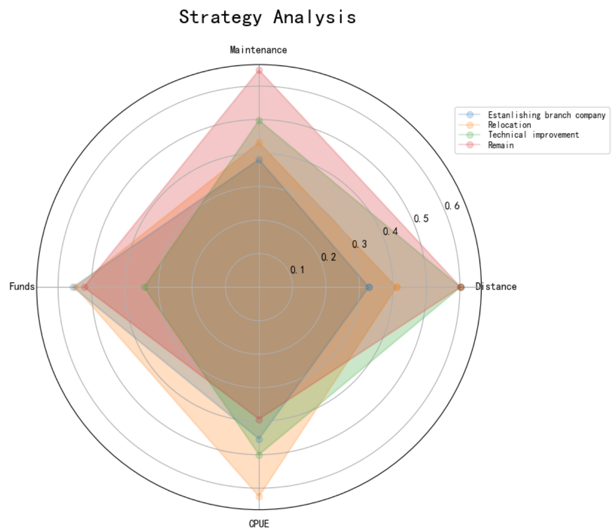


Figure 15: Strategy Analysis

6 Effect of Territorial waters

6.1 Range of territorial waters

The analysis of 4 responses above proves that Moving to the northern part of the country brings richer marine resources. However, the operation is restricted due to the limited domain of territorial seas. What we want to do is to build a model to analyze the economic loss of practical reason.

According to the United Nations Convention on the Law of the Sea (LOSC), maximum of 12 nautical miles from the coastlines (including the coastline of offshore islands) is the territorial seas of a country, where this country is able to utilize surrounding resources.
Countries around North Sea and Ireland Sea are Norway, Demark and Ireland. As a result, fishing within their territorial seas is prohibited.

6.2 Impacts analysis

Based on the prediction we made, we count the appearance frequency of mackerel during last years, define L_y as the quantity of mackerel affected by territorial limits in y_{th} year.

$$L_y = \frac{c_y}{C_y} * P_y \tag{35}$$

s_y is the frequency of appearance in available waters in y_{th} year, S_y is total appearance of mackerel in Northern Pacific in y_{th} year.

Statistics we collected is shown in following chart. Statistics from Fisheries & Aquaculture Organization(FAO) and International Council for the Exploration of the Sea (ICES).

Production loss due to territorial seas

Year	C(/times)	c(/times)	P(/t)	Hit Rate(s/S)	Loss	Percentage
1999	1507	856	711634	0.568	307414.6	0.432
2003	14499	5129	503669	0.354	325496.8	0.646
2005	3720	1328	554113	0.357	356300.6	0.643
2008	4141	1328	442669	0.321	300707.1	0.679
2010	13836	3437	503669	0.248	378552.6	0.752
2013	10181	2481	473980	0.244	358476.2	0.756
2017	8833	1824	581881	0.206	461723.5	0.794

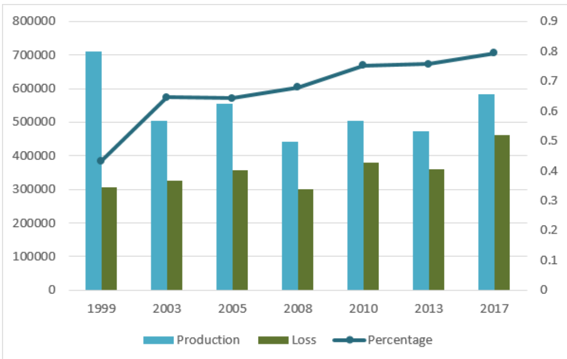


Figure 16: Change curve of the loss
* Pertentage is used for measuring the diminution casused by territorial problems.

As the chart illustrates, the loss quantity due to territorial seas keeps increasing year by year, which matches the migration of mackerel to the north. Due to the rising sea temperature, habitants of mackerel moves north from 55°N to 60°N. As a result, it is necesssary for small fishing companies around the North Sea to change operatng strategies and push forward innovation.

7 Model Test

7.1 Error Analysis of GAM

For GAM:

$$InDS_{i,(\lambda,\phi)} = a + s1(\lambda_i, \phi_i) + s2(SST_i) + \beta * SSB_i + \varepsilon \tag{36}$$

The estimated results of parameters are in the Table 7.

Table 7: The Estimated Results of Parameters

	Estimate	Std. Error	t value	Pr(>|t|)
a	13.29251	0.16794	79.151	0.000124
SSB	0.11987	0.03898	3.075	0.087775
	edf	Ref.df	F	p-value
λ	1	1	29.217	0.0295
ϕ	1.933	1.995	26.228	0.0320
SST	1.000	1.000	0.264	0.6568

$R - sq.(adj) = 0.981$ and deviance explained =99.4%.

$R - sq.(adj)$ is close to 1, and indicates that the model fitting effect is excellent.

7.2 Sensitivity Analysis of Continuous-time Markov Model

The Regression Square Sum:$S_reg = \Sigma_{i=1}^{n}(\hat{y}_i - \bar{y})^2$

The Residual Square Sum:$S_res = \Sigma_{i=1}^{n}(y_i - \hat{y}_i)^2$

Where n denote the sample size, m denote the number of countries. The total square sum can be divided into parts of the regression square sum (SSR) and the residual square sum (SSE)

$$MSR = SSR/(m-1) \tag{37}$$

$$MSE = SSE/(n-m-1) \tag{38}$$

$$F = \frac{MSR}{MSE} = \frac{SSR/(m-1)}{SSE/(n-m-1)} \tag{39}$$

We calculated that $F > F_{0.05}(m-1, n-m-1)$.The regression equation is significant under the significance level $\alpha = 0.05$.

7.3 Temperature prediction

The threshold is determine subjective, which will influence the number and position of “best point”. So we test the sensitivity by valuation. We change the value of threshold in steps of 0.02 from 5.06 to 4.92.

From the graph we can say that the have little affection on the number of best point. As for the position, the biggest change is very small, which can be seen in Appendix 2 for detail. The model is available, the worst case is around 2025, the best case is around 2045.

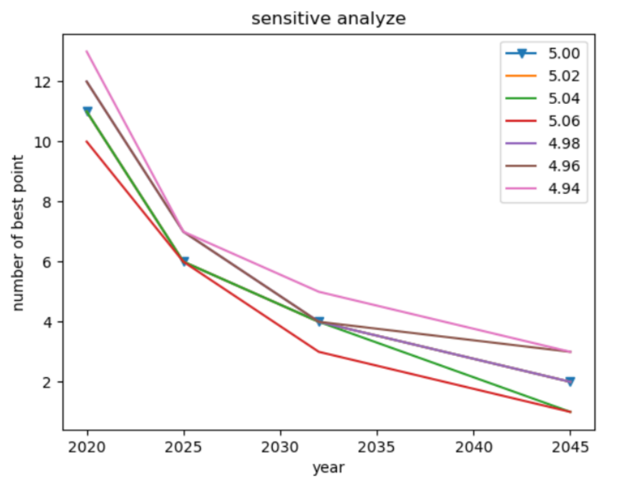


Figure 17: Sensitive analyze on threshold

8 Strengths and Weaknesses

8.1 Strengths

- **Applies widely**
  The model can be used to predict the future migration of other animals, such as African zebras and birds.

- **The model is stable and reliable**
  Our models are robust while the parameters changes. That is to say, a slight change of parameter will not cause a significant change of the results.

- **Great accuracy**
  The time series model represents the influence of the historical information of each variable on the situation of the system and entirely describes the dynamic law of the change of temperature.

- **Reliable data**
  Our data comes from the official website such as Fisheries & Aquaculture Organization(FAO) and International Council for the Exploration of the Sea (ICES).

- **Easy appliance**
  Do not need to choose the cluster number. Unlike traditional k-means algorithm, mean-shift don't need to choose how many cluster. TOPSIS provides a direct assessment of strategies. With weight of factors given, the rank is easy to understand.

8.2 Weaknesses

- **Factors neglected**
  Temperature predict model didn't consider the affect from salt. In the strategy assessment system, we only consider several primary factors impacting strategy choice.

- **Rough measurement**
  The measurement of distance on the sea is rough, the statistics about the distribution of mackerel is not complete.

- **Complex data**
  The data of some parameters is forecast data because we do not have real practical experience.

9 A Letter

MEMORANDUM
TO: Hook Line and Sinker
FROM:Team #2010755

With the increase of ocean temperature, marckerel and herring will migrate to higher latitudes, searching for a suitable place to settle down since the metabolism of fish accelerates once the surrounding temperature rises. That is the reason why they require more oxygen and food yet higher temperature reduces oxygen solubility of seawater, which leads to the death of fish. At the same time, some fish can't spawning when the temperature is higher than a threshold. Both Scottish herring and mackerel play an important role in the Scottish fishing industry. But unfortunately, according to our research, they are trying to escape from England.

Where do they want to go
Based on former data and consider the influence of temperature, our team build a model to predict the distribution of herring and mackerel in the next 50 years. The result shows that during the next 50 years, herring will migrate to Gulf of Bothnia and mackerel will migrate to Norwegian Sea — both of them will be more than 900km away from Scotland at that time.

Suggestions to small fishery company
In order to achieve higher profitability or just to maintain the profit, companies should take actions such as relocating the whole company, establishing a branch company in the Northern cities or employing lightweight fishing vessels. All of these methods work to some extent. With various factors taken into account, building a new branch company in the Northern is the best choice by which companies can get closer to fish resources. Additionally, it costs less compared with other solutions such as relocating the whole company.

Suggestions to a normal fisherman
With the depletion of fishery resources, there are two ways to for fishermen to increase or maintain their income. One of them is to change the traditional ways to catch fish. Fishermen are able to "farming" in the sea, cultivating economic fish in shallow waters. The other one is to develop recreational fishing for tourists. Picking tourists to the beautiful seaside and teaching them how to fish during the fishing off-season must be a great pleasure. At the same time, it brings enormous economic benefits as well.

Thank you for your consultation!

Best Wishes, Team #2010755

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Appendices

Appendix A First appendix

Here are simulation programmes we used in our model as follow.

Part 1

Limited by space, only the code for calculating the distribution of mackerel is listed.

Generalized additive model(GAM):

```
library(mgcv)
Data <- read.delim("fish.txt")
Data <- as.matrix(Data)
B<-Data[,1]
Q<-Data[,2]
Year<-Data[,3]
Tem<-Data[,4]
SSB<-Data[,5]
N<-Data[,6]
W<-Data[,7]
Z<-Data[,8]
result<-gam(log(Q) ~ s(B,k=3))
result1 <- gam(log(Q) ~ s(Tem,k=3))
result2<-gam(log(Q) ~ s(N,k=3))
result3 <- gam(log(Q) ~ s(W,k=3))
result4 <- gam(log(Q) ~ s(Year,k=3))
result5 <- gam(log(Q) ~ s(Z,k=3))
dev.off()
par(mfrow=c(3,2))
plot(result1,se=T,resid=T,pch=16)
plot(result2,se=T,resid=T,pch=16)
plot(result3,se=T,resid=T,pch=16)
plot(result,se=T,resid=T,pch=16)
plot(result4,se=T,resid=T,pch=16)
plot(result5,se=T,resid=T,pch=16)
summary(result1)
summary(result2)
summary(result3)
summary(result4)
summary(result)
summary(result5)
sink("AIC.txt")
AIC(result1,result2,result3,result,result4,result5)
sink()
sink("anova.txt")
anova.gam(result1,result2,result3,result4,result5,result,test = "Chisq")
sink()
```

Optimization of GAM

```
library(mgcv)
Data <- read.delim("fish.txt")
Data <- as.matrix(Data)
B<-Data[,1]
Q<-Data[,2]
Tem<-Data[,4]
SSB<-Data[,5]
N<-Data[,6]
W<-Data[,7]
result1 <- gam(log(Q) ~ SSB+s(W,k=3)+s(N,k=3)+s(Tem,k=3))
summary(result1)
plot(result1,se=T,resid=T,pch=16)
```

fish.txt

```
B   Q      year    Tem   SSB  N    W    Z
86  606793  2007    7.4275  2   56.5    -12  142
```

```
228 890861  2010    10.6467 5.25    57  -7  173
137 950226  2011    10.0367 4.25    57.2    3   177
165 914993  2012    8.845   4.21    57.5    1   199
218 986950  2013    8.5492  4.21    57.5    2   254
241 1424042 2014    9.1908  5   59  -3  269
195 1247666 2015    8.7992  4.75    58  12  227
232 1138053 2016    8.9425  4.7 56.9    10  269
```

Markov Model The following data are the 30 state transition matrices calculated.

```
[[0.75 0.25 0.   0.   0.  ]
 [1.   0.   0.   0.   0.  ]
 [0.   0.   0.   0.   0.  ]
 [0.   0.   0.   0.   0.  ]
 [0.   0.   0.   0.   0.  ]]
[[0.66666667 0.33333333 0.         0.         0.        ]
 [1.         0.         0.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]]
[[0.66666667 0.33333333 0.         0.         0.        ]
 [1.         0.         0.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]]
[[1. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]]
[[1. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]]
[[1. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]]
[[1. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]]
[[1. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]]
[[0.66666667 0.33333333 0.         0.         0.        ]
 [1.         0.         0.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]]
[[0.66666667 0.33333333 0.         0.         0.        ]
 [1.         0.         0.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]
```

```
 [0.         0.         0.         0.         0.         ]
 [0.         0.         0.         0.         0.         ]]
[[0. 0. 0. 0. 0.]
 [0. 1. 0. 0. 0.]
 [0. 1. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]]
[[0.5        0.5        0.         0.         0.         ]
 [0.66666667 0.33333333 0.         0.         0.         ]
 [0.         0.         0.         0.         0.         ]
 [0.         0.         0.         0.         0.         ]
 [0.         0.         0.         0.         0.         ]]
[[1. 0. 0. 0. 0.]
 [1. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]]
[[1. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]]
[[1. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]]
[[1. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]]
[[0.66666667 0.33333333 0.         0.         0.         ]
 [1.         0.         0.         0.         0.         ]
 [0.         0.         0.         0.         0.         ]
 [0.         0.         0.         0.         0.         ]
 [0.         0.         0.         0.         0.         ]]
[[0.5 0.5 0.  0.  0. ]
 [1.  0.  0.  0.  0. ]
 [0.  1.  0.  0.  0. ]
 [0.  0.  0.  0.  0. ]
 [0.  0.  0.  0.  0. ]]
[[0.         0.         0.         0.         0.         ]
 [0.         0.66666667 0.33333333 0.         0.         ]
 [0.         1.         0.         0.         0.         ]
 [0.         1.         0.         0.         0.         ]
 [0.         0.         0.         0.         0.         ]]
[[0.  0.  0.  0.  0. ]
 [0.  0.5 0.5 0.  0. ]
 [0.  1.  0.  0.  0. ]
 [0.  0.  0.  0.  0. ]
 [0.  0.  1.  0.  0. ]]
[[0.  1.  0.  0.  0. ]
 [0.5 0.5 0.  0.  0. ]
 [1.  0.  0.  0.  0. ]
 [0.  0.  0.  0.  0. ]
 [0.  0.  0.  0.  0. ]]
[[1. 0. 0. 0. 0.]
```

```
 [1. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]]
[[0.75 0.25 0.   0.   0.  ]
 [1.   0.   0.   0.   0.  ]
 [0.   0.   0.   0.   0.  ]
 [0.   0.   0.   0.   0.  ]
 [0.   0.   0.   0.   0.  ]]
[[1. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]]
[[0.66666667 0.33333333 0.         0.         0.        ]
 [1.         0.         0.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]]
[[0.         0.         0.         0.         0.        ]
 [0.         0.66666667 0.33333333 0.         0.        ]
 [0.         1.         0.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]]
[[0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 1.]
 [0. 0. 0. 0. 1.]]
[[0.         0.         0.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]
 [0.         0.         0.33333333 0.66666667 0.        ]
 [0.         0.         1.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]]
[[0.         0.         0.         0.         0.        ]
 [0.         0.66666667 0.33333333 0.         0.        ]
 [0.         1.         0.         0.         0.        ]
 [0.         1.         0.         0.         0.        ]
 [0.         0.         0.         0.         0.        ]]
[[0.   0.   0.   0.   0.  ]
 [0.   0.   0.   0.   0.  ]
 [0.   0.   0.75 0.   0.25]
 [0.   0.   0.   0.   0.  ]
 [0.   0.   1.   0.   0.  ]]
```

Appendix Appendix 1

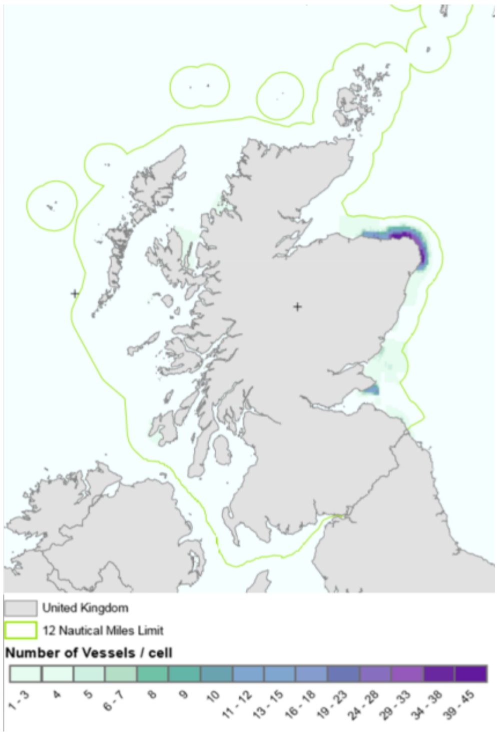

Figure 18: Number of vessels in Scotland

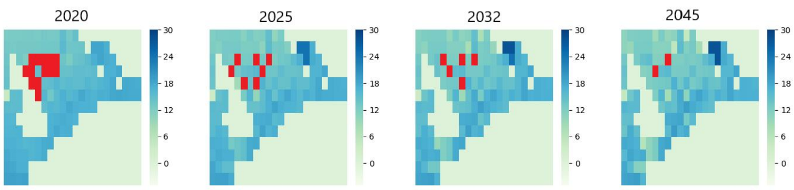

Figure 19: Fisheries in Scotland & distribution of mackerel