Excercise sheet 2

Machine Intelligence 2, SoSe 2016, The Nebenhörers:

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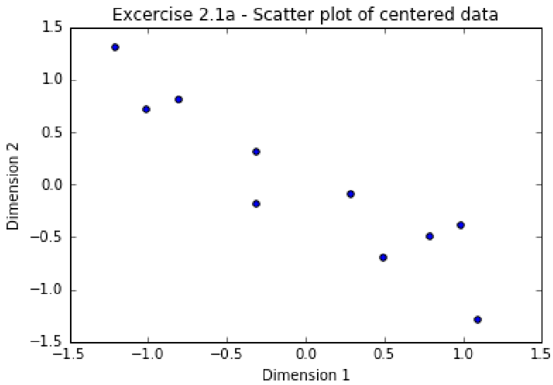
Recommended readable form: https://github.com/danijar/course-machine-intelligence-2/blob/master/EX2/EX2-MI2-Nebenhoerers.ipynb (https://github.com/danijar/course-machine-intelligence-2/blob/master/EX2/EX2-MI2-Nebenhoerers.ipynb)

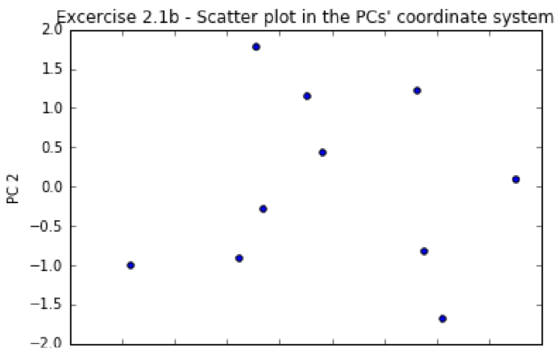
In [288]:
```
import itertools
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
%matplotlib inline
```

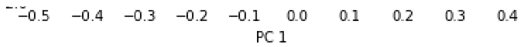
Excercise 2.1: PCA: 2-dimensional Toy Data

In [275]:
```
f = open('pca-data-2d.dat.txt')
data = pd.read_table(f, header=None, sep='   ',lineterminator='\n', engine='python')

#center
data = data - data.mean()

#plot
ax = data.plot(kind='scatter', x=0, y=1, title='Excercise 2.1a - Scatter plot of centered data')
ax.set_xlabel('Dimension 1')
ax.set_ylabel('Dimension 2');
```

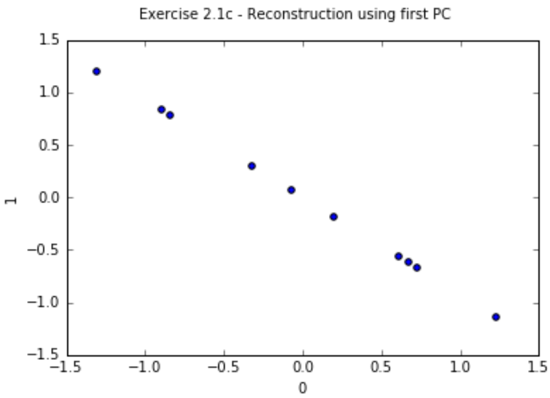


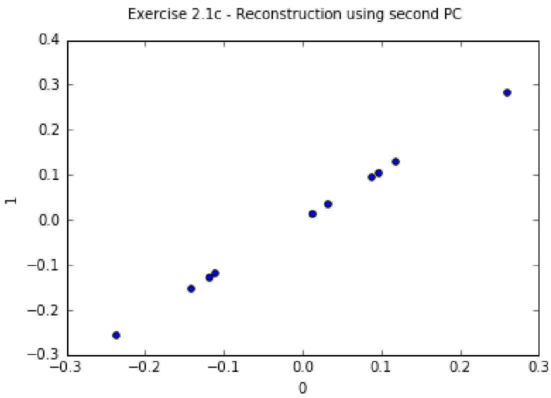
In [276]:
```
# calculate eigenvectors and eigenvalues
covmat = data.cov()
eigvals, eigvecs = np.linalg.eigh(covmat)

#transform into the coordinate system of the PCAs
datan = np.dot(eigvecs, data.T).T
datan = pd.DataFrame(datan)

#plot
ax = datan.plot(kind='scatter', x=0, y=1, title='Excercise 2.1b - Scatter plot in the PCs\' coord
inate system');
ax.set_xlabel('PC 1')
ax.set_ylabel('PC 2');
```





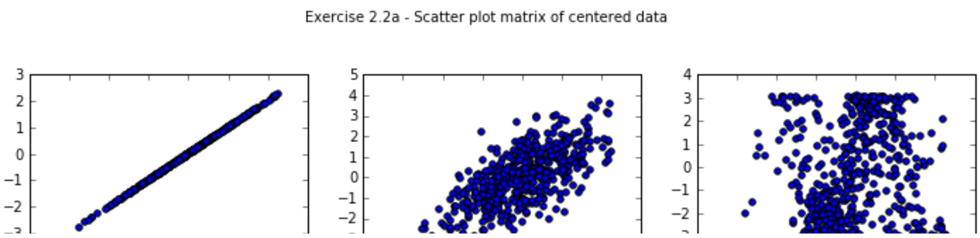
In [277]:
```
#reconstruction using the first principal component
datan = np.dot(eigvecs[1], data.T)
d = datan.reshape(10,1)
eig = eigvecs[1].reshape(2,1)
datan = np.dot(d,eig.T)

datan = pd.DataFrame(datan)
datan.plot(subplots=True, kind='scatter', x=0, y=1, title='Exercise 2.1c - Reconstruction using f
irst PC')

#reconstruction using the second principal component
datan = np.dot(eigvecs[0], data.T)
d = datan.reshape(10,1)
eig = eigvecs[0].reshape(2,1)
datan = np.dot(d,eig.T)

datan = pd.DataFrame(datan)
datan.plot(subplots=True, kind='scatter', x=0, y=1, title='Exercise 2.1c - Reconstruction using s
econd PC');
```

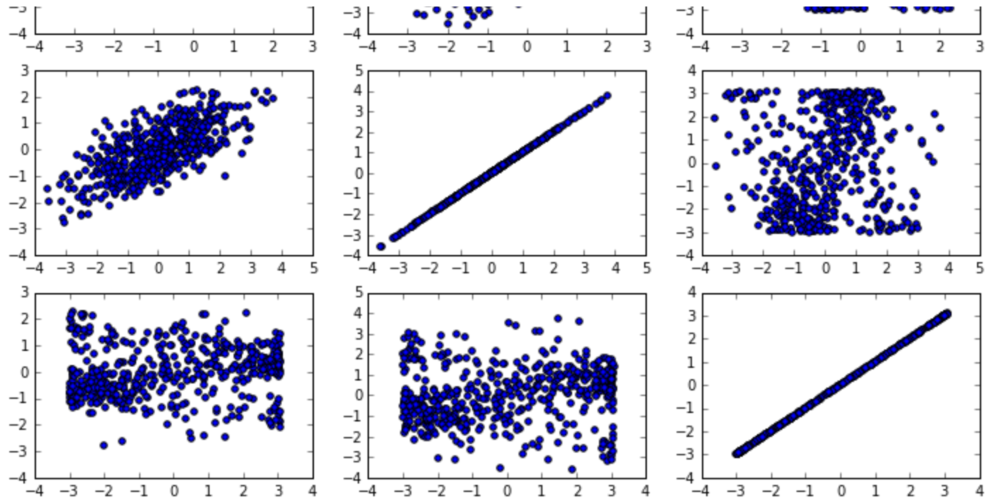




Excercise 2.2: PCA: 3-dimensional Toy Data

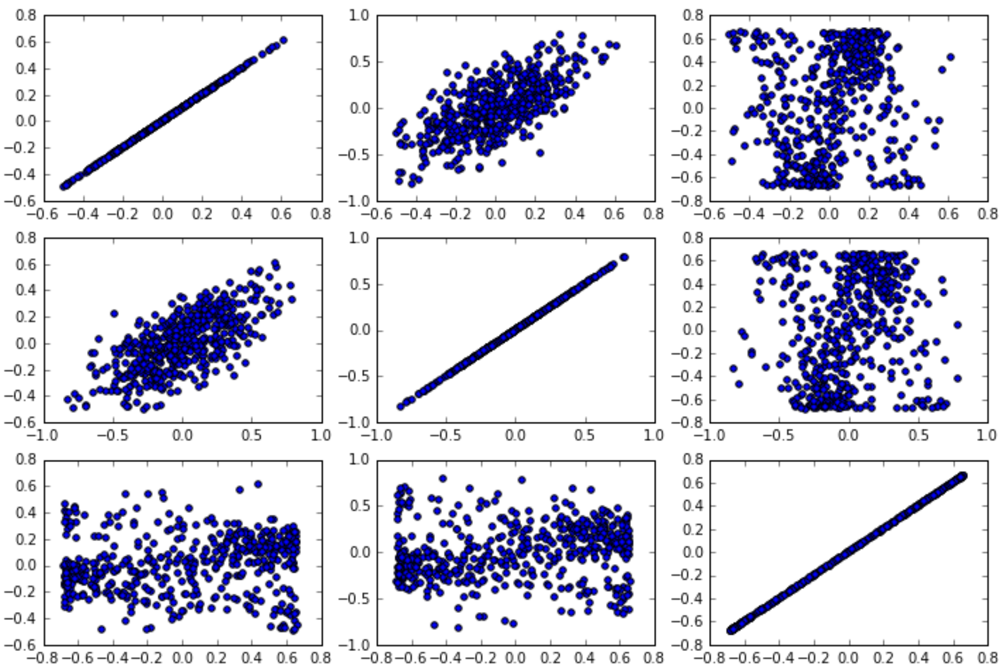
In [278]:
```
f = open('pca-data-3d.txt')
data = pd.read_table(f, sep=',',lineterminator='\n')

#center
data = data - data.mean()

def scatter_plot_matrix(data, title='Scatter Matrix'):
    data = data.as_matrix() if hasattr(data, 'as_matrix') else data
    rank = data.shape[1]
    fig, ax = plt.subplots(nrows=rank, ncols=rank, figsize=(12, 8))
    fig.suptitle(title)
    for x, y in itertools.product(range(rank), repeat=2):
        ax[x, y].scatter(data[:, x], data[:, y])

scatter_plot_matrix(data.as_matrix(), 'Exercise 2.2a - Scatter plot matrix of centered data')
```



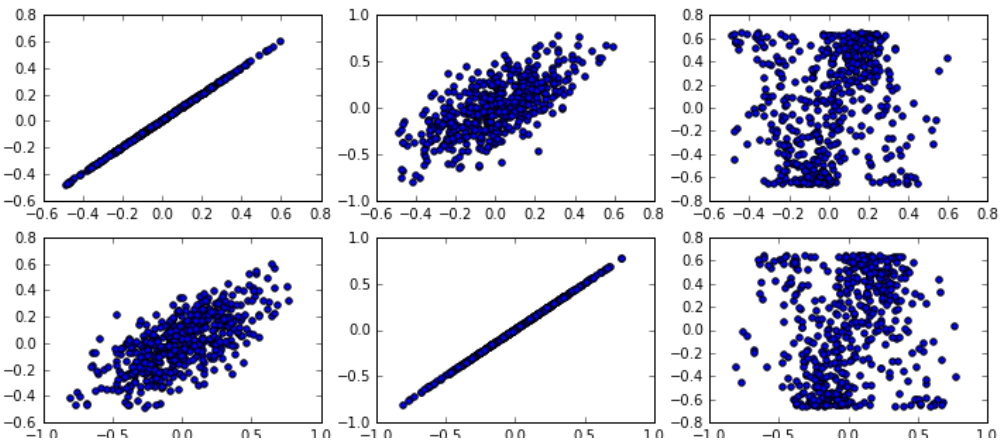


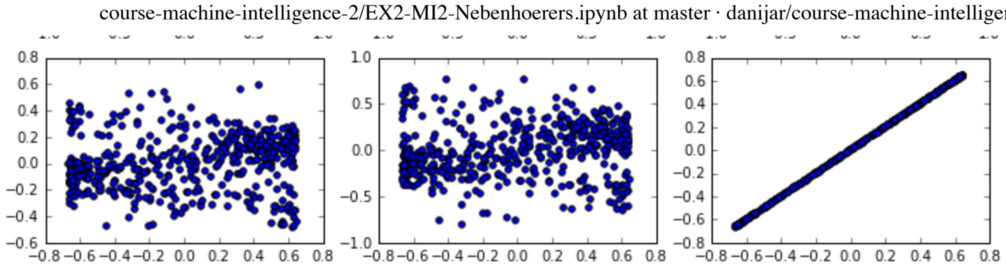
In [279]:
```
#calculate eigenvectors and eigenvalues
covmat = data.cov()
eigvals, eigvecs = np.linalg.eigh(covmat)

eigvecs = eigvecs[::-1]


#for every combination of the three PCs, make a scatter plot
for comb in itertools.combinations(range(3), 2):
    transformed = np.dot(eigvecs[comb],data.T).T
    data = pd.DataFrame(transformed)
    scatter_plot_matrix(data, title="Exercise 2.2b - Scatter plot matrix for PC {} and PC {}".for
mat(comb[0]+1,comb[1]+1))
```

Exercise 2.2b - Scatter plot matrix for PC 1 and PC 2

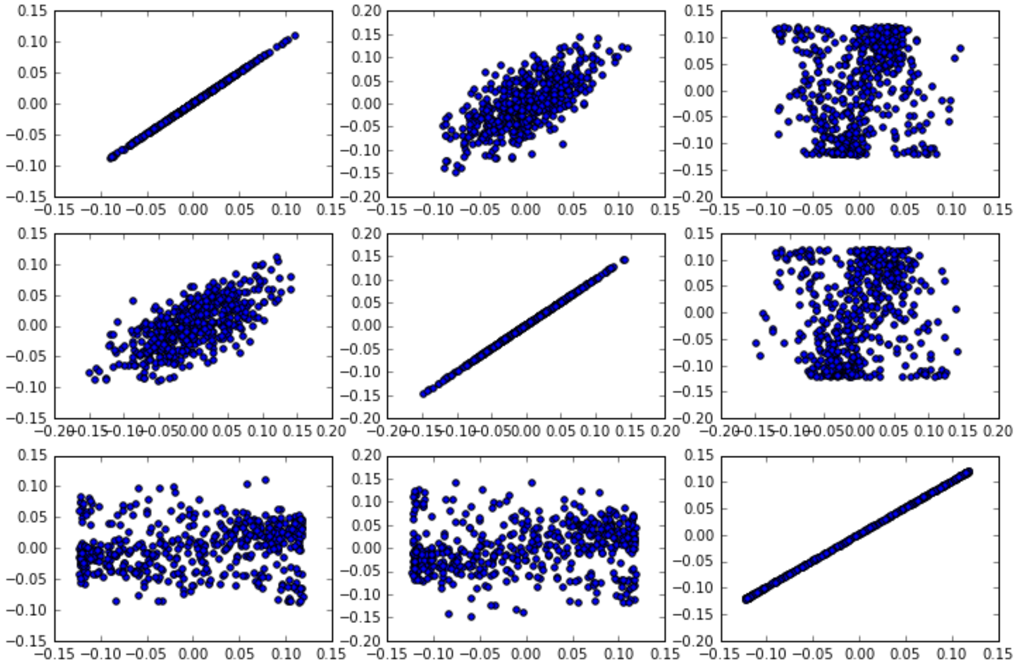


Exercise 2.2b - Scatter plot matrix for PC 1 and PC 3



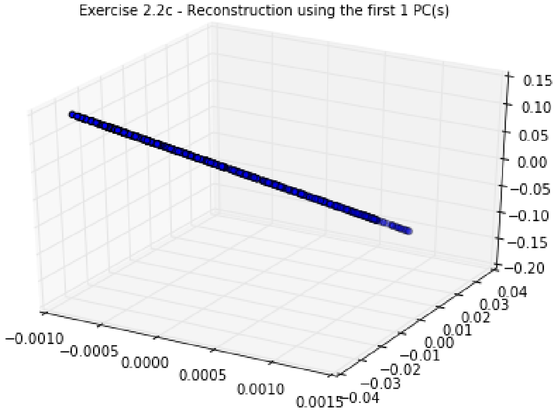


Exercise 2.2b - Scatter plot matrix for PC 2 and PC 3

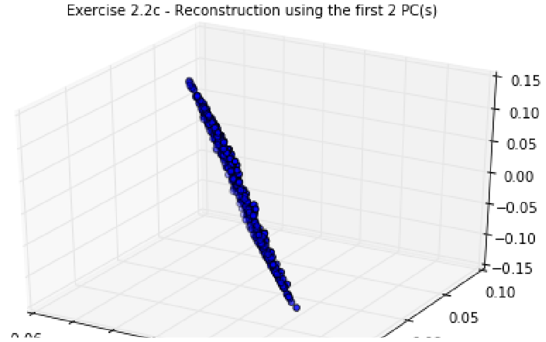


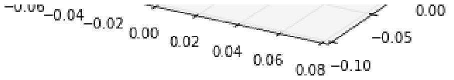
In [280]:
```
#2.2C
for num_pcs in range(3):
    pcs = eigvecs[0:num_pcs+1]

    transformed = np.dot(pcs,data.T).T
    reconstructed = np.dot(pcs.T, transformed.T).T

    fig = plt.figure()
    ax = Axes3D(fig)
    fig.suptitle("Exercise 2.2c - Reconstruction using the first {} PC(s)".format(num_pcs+1))

    ax.scatter(reconstructed[:, 0], reconstructed[:, 1], reconstructed[:, 2])
```

Exercise 2.2c - Reconstruction using the first 1 PC(s)

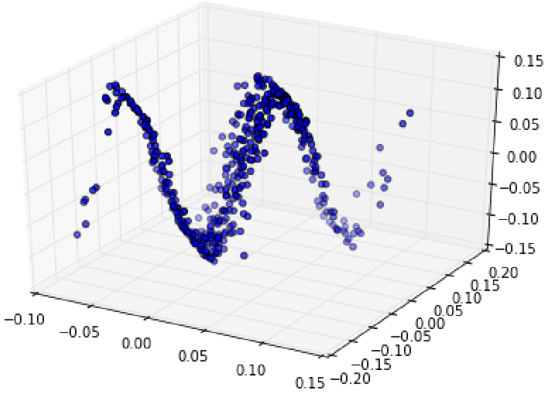


Exercise 2.2c - Reconstruction using the first 2 PC(s)





Exercise 2.2c - Reconstruction using the first 3 PC(s)



Reconstruction based on all three principal components creates a representation that exactly matches the original data (no information is lost).

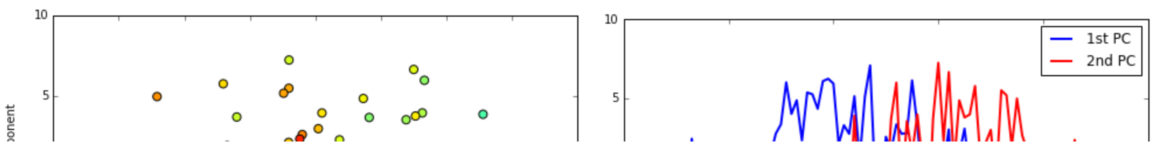
Reconstructions based on just one or just two of them do not preserve enough information to aptly represent the original distribution of the data.

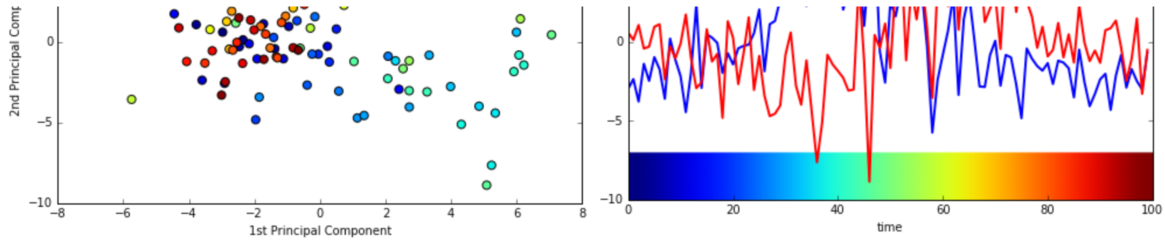
This is because the data is highly nonlinear, which PCA doesn't do well at representing.

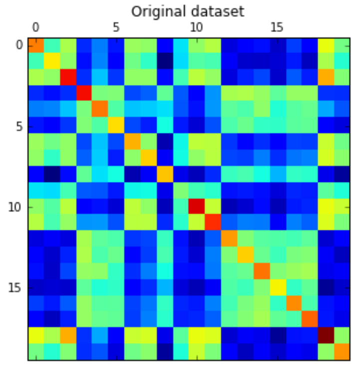
Excercise 2.3: Projections of a dynamical system

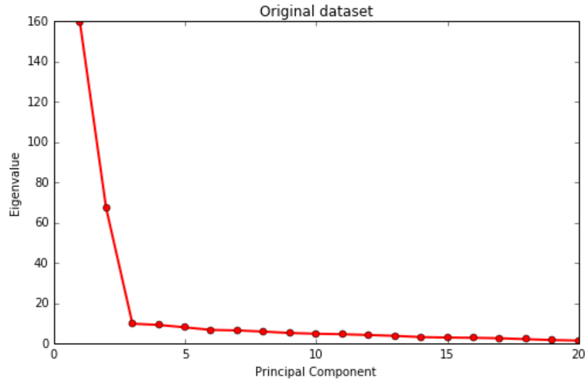
In [281]:

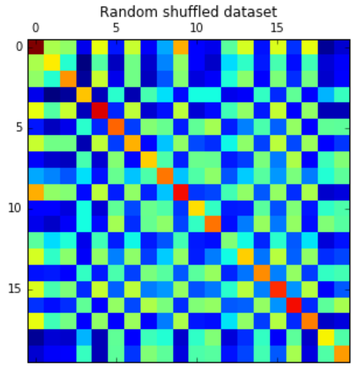
```
# 2.3(a)
data = pd.read_csv("expDat.txt", index_col=0).as_matrix()
data = data.astype(float)
data -= data.mean(axis=0)

cov = np.cov(data.T)
_, eigvecs = np.linalg.eigh(cov)
eigvecs = eigvecs[::-1]

fig, ax = plt.subplots(ncols=2, nrows=3, figsize=(14, 14))
timesteps = np.linspace(0, 1, 100)

# 2.3(b)i)
datan = np.dot(eigvecs[:2], data.T).T
ax[0, 0].scatter(datan[:, 0], datan[:, 1], c=timesteps, s=50)
ax[0, 0].set_xlabel('1st Principal Component')
ax[0, 0].set_ylabel('2nd Principal Component')

# 2.3(b)ii)
datan = np.dot(eigvecs[0], data.T).T
ax[0, 1].plot(np.arange(100), datan, c='blue', label='1st PC', linewidth=2)
datan = np.dot(eigvecs[1], data.T).T
ax[0, 1].plot(np.arange(100), datan, c='red', label='2nd PC', linewidth=2)
ax[0, 1].imshow([np.linspace(0, 1, 20), np.linspace(0, 1, 20)], cmap=plt.cm.jet, interpolation='b
icubic', extent=[0, 100, -10, -7], aspect=3)
ax[0, 1].axis((0, 100, -10, 10))
ax[0, 1].legend()
ax[0, 1].set_xlabel('time')

# 2.3(c)
new_data = np.random.RandomState(0).permutation(data.T).T

# 2.3(d)
def plot_cov_and_scree(data, index, title):
    cov = np.cov(data.T)
    ax[index, 0].matshow(cov)
    ax[index, 0].set_title(title)
    eigvals, eigvecs = np.linalg.eigh(cov)
    sing_vals = np.arange(len(eigvals)) + 1
    ax[index, 1].plot(sing_vals, eigvals[::-1], 'ro-', linewidth=2)
    ax[index, 1].set_title(title)
    ax[index, 1].set_xlabel('Principal Component')
    ax[index, 1].set_ylabel('Eigenvalue')

plot_cov_and_scree(data, 1, 'Original dataset')
plot_cov_and_scree(new_data, 2, 'Random shuffled dataset')

fig.tight_layout()
```

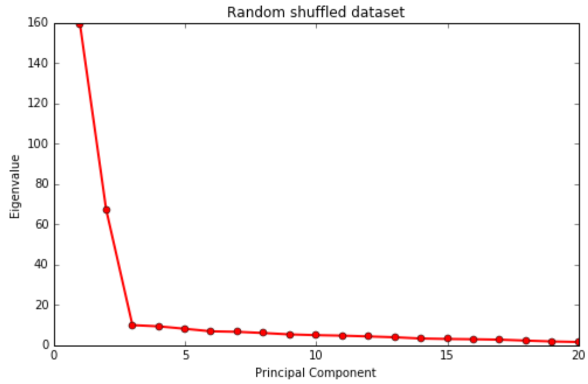












Ad. 2.3(d)

While the covariance matrices for scrambled data differ significantly, the scree plots remain the same. This means that shuffling does not affect the eigenvalues of the matrix.

Ad. 2.3(e)

Mean, covariance matrix, eigenvectors, eigenvalues and finally the scree plot would be the same because we only change the order of the collected data not the values themselves. A data point still has the same values for each dimension.

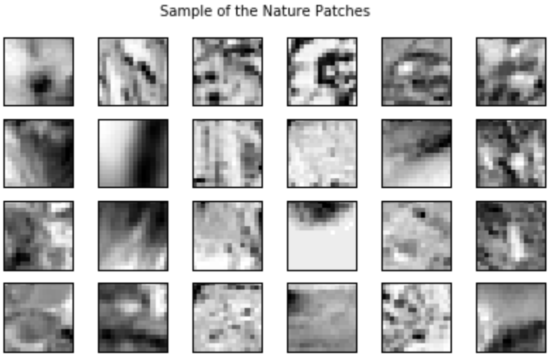
Excercise 2.4: PCA: Image Data

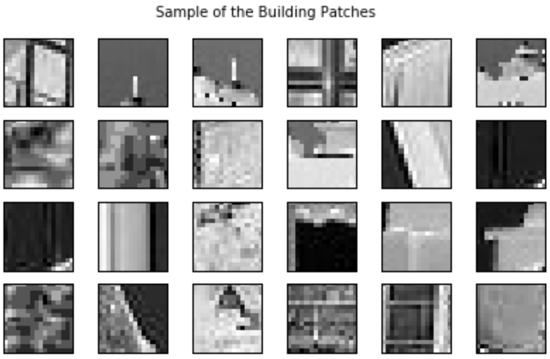
In [282]:
```
import random

def sample_patches(images, count=5000, size=16):
    patches = np.empty((count, size, size))
    for i in range(count):
        image = random.choice(images)
        x = int(random.random() * (image.shape[0] - size))
        y = int(random.random() * (image.shape[1] - size))
        patches[i] = image[x: x + size, y: y + size]
    return patches
```

In [283]:
```
def show_patches(patches, rows, cols, title='Patches', size=10):
    assert len(patches) == rows * cols
    fig, ax = plt.subplots(
        nrows=rows, ncols=cols, figsize=(size, size / cols * rows),
        subplot_kw={'xticks': [], 'yticks': []})
    fig.suptitle(title)
    for x, y in itertools.product(range(rows), range(cols)):
        patch = patches[x * cols + y]
        ax[x, y].get_xaxis().set_visible(False)
        ax[x, y].imshow(patch, cmap=plt.cm.gray_r, interpolation='nearest')
    plt.show()
```

In [284]:
```
import glob
from scipy import ndimage

nature = [ndimage.imread(x) for x in glob.glob('imgpca/n*.jpg')]
```

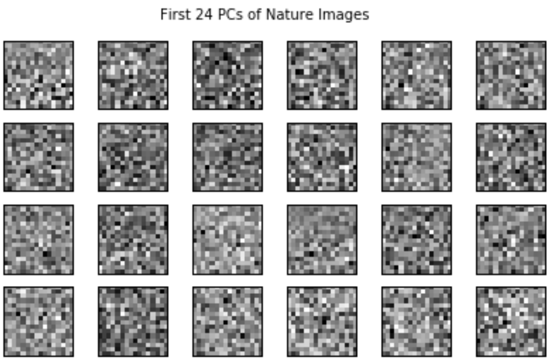
```
buildings = [ndimage.imread(x) for x in glob.glob('imgpca/b*.jpg')]

nature = sample_patches(nature, 5000)
buildings = sample_patches(buildings, 5000)

show_patches(nature[:24] / 256, 4, 6, 'Sample of the Nature Patches', size=7)
show_patches(buildings[:24] / 256, 4, 6, 'Sample of the Building Patches', size=7)

nature = nature.reshape((len(nature), -1))
buildings = buildings.reshape((len(buildings), -1))
```

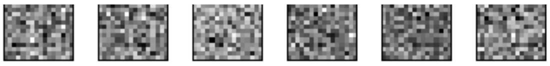




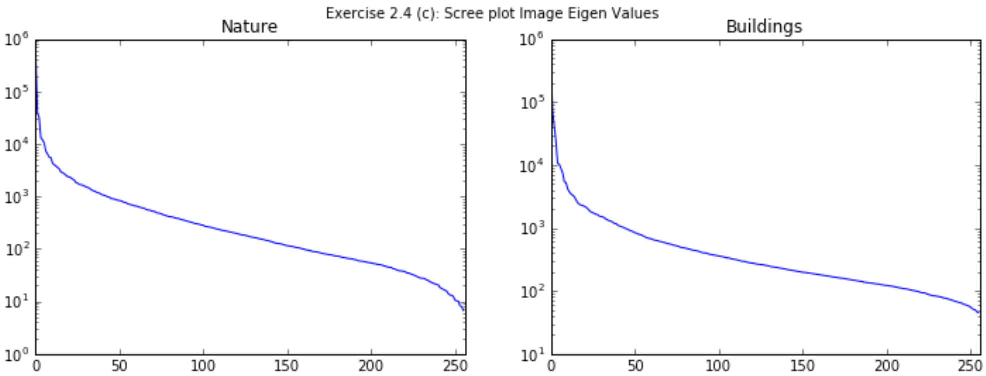
In [285]:

```
def compute_pcs(data):
    data -= data.mean(axis=0)
    cov = np.cov(data.T)
    eig, pcs = np.linalg.eigh(cov)
    pcs, eig = pcs[::-1], eig[::-1]
    return pcs, eig


nature_pcs, nature_eig = compute_pcs(nature)
buildings_pcs, buildings_eig = compute_pcs(buildings)

nature_pcs_images = nature_pcs.reshape((-1, 16, 16))
show_patches(nature_pcs_images[:24], 4, 6, 'First 24 PCs of Nature Images', size=7)

buildings_pcs_images = buildings_pcs.reshape((-1, 16, 16))
show_patches(buildings_pcs_images[:24], 4, 6, 'First 24 PCs of Building Images', size=7)
```





In [286]:
```
fig, ax = plt.subplots(ncols=2, figsize=(12, 4))
fig.suptitle('Exercise 2.4 (c): Scree plot Image Eigen Values')

ax[0].set_title('Nature')
ax[0].plot(nature_eig)
ax[0].set_yscale('log')
ax[0].set_xlim(0, 256)

ax[1].set_title('Buildings')
ax[1].plot(buildings_eig)
ax[1].set_yscale('log')
ax[1].set_xlim(0, 256)

plt.show()
```



In [287]:
```
nature_subspace = nature_pcs[:200]
nature_rec = np.dot(nature_subspace.T, np.dot(nature_subspace, nature.T)).T
nature_rec = nature_rec.reshape((-1, 16, 16))
show_patches(nature_rec[:24] / 256, 4, 6, 'Sample of Reconstructed Nature Patches', size=7)

buildings_subspace = buildings_pcs[:200]
buildings_rec = np.dot(buildings_subspace.T, np.dot(buildings_subspace, buildings.T)).T
buildings_rec = buildings_rec.reshape((-1, 16, 16))
show_patches(buildings_rec[:24] / 256, 4, 6, 'Sample of Reconstructed Building Patches', size=7)
```