Direct Geolocation Of Wideband Radio Signal based on Delay & Doppler

Alon Amar* and Anthony J. Weiss§

School of Electrical Engineering - Systems Department

Tel Aviv University, Tel Aviv 69978, Israel

Abstract

Contrary to the suboptimal (two-step) geolocation procedures, we propose a maximum likelihood estimation for the position of a stationary emitter which its delayed and Doppler shifted signal is observed by moving receivers. The position is estimated based on the same data used in common methods. However, the estimation is performed in a single step by maximizing a cost function that depends on the unknown position only.

1. INTRODUCTION

Passive geolocation of a stationary transmitter based on the delayed and Doppler shifted signal observed by at least a single moving platform found applications in radar [1]-[3], airborne systems [4], and satellites [5]. Since the receiver location and velocity are known, the emitter location can be estimated.

Common methods use two steps for localization. The system first estimate the delay and frequency differences from the observations of all receivers taken at several locations along their trajectories, for example, by maximizing the observations' complex ambiguity function [6]. This step ignores the constraint that all estimates correspond to the same position. In the second step the system estimates the position based on the results of the first step. Thus, these two step methods are not guaranteed to yield optimal location results.

* Corresponding author: Alon Amar, Tel: +97236408605, Fax: +97236407095, Email: amar@eng.tau.ac.il

§ Anthony J. Weiss, Tel: +97236407460, Fax: +97236407095, Email: ajw@eng.tau.ac.il

This research was supported by the Israel Science Foundation (grant No. 1232/04).

Herein we propose a maximum likelihood position estimation by considering the same data, however, using a single step. We show that the proposed method selects the position that maximize the sum of all distinct cost functions, each used in the common approach [6].

2. Problem Formulation

Consider a stationary radio emitter located at position $\mathbf{p}$ and L moving receivers. The receivers are synchronized in frequency and time. Each receiver intercepts the transmitted signal at K short intervals along its trajectory. Let $\mathbf{p}_{\ell,k}$ and $\mathbf{v}_{\ell,k}$ where $k = 1, \dots, K$ and $\ell = 1, \dots, L$ denote the position and velocity vectors of the ℓ-th receiver at the k-th interception interval, respectively. The complex signal observed by the ℓ-th receiver at the k-th interception interval at time t is

$$r_{\ell,k}(t) = b_{\ell,k} s_k(t - \tau_{\ell,k}) e^{j2\pi f_{\ell,k}(t-\tau_{\ell,k})} + w_{\ell,k}(t), \;\; 0 \le t \le T \tag{1}$$

where T is the observation time interval, $s_k(t)$ is the observed signal envelope during the k-th interception interval, $b_{\ell,k}$ is an unknown complex path attenuation such that $\tau_{\ell,q} = \frac{1}{c}\|\mathbf{p}_{\ell,k} - \mathbf{p}\|$ is the signal's delay where c is the signal's propagation speed, $w_{\ell,k}(t)$ is a wide sense stationary additive white zero mean complex Gaussian noise with flat spectrum and $f_{\ell,k}$ is given by,

$$f_{\ell,k} \triangleq [f_c + \nu_k]\,[1 + \mu_{\ell,k}(\mathbf{p})] \tag{2}$$

$$\mu_{\ell,k}(\mathbf{p}) \triangleq \frac{1}{c}\mathbf{v}_{\ell,k}^T[\mathbf{p} - \mathbf{p}_{\ell,k}]/\|\mathbf{p} - \mathbf{p}_{\ell,k}\| \tag{3}$$

where f_c is the known nominal frequency of the transmitted signal, and ν_k is the unknown transmitted frequency shift due to the source instability, during the k-th interception interval.

Since $\mu_{\ell,k} \ll 1$ and $\nu_k \ll f_c$, (2) can be approximated as $f_{\ell,k} \cong \nu_k + f_c[1 + \mu_{\ell,k}(\mathbf{p})]$. Also, since f_c is known to the receivers, each receiver performs a down conversion of the intercepted signal by f_c and (2) can be replaced by $\bar{f}_{\ell,k} \cong \nu_k + f_c\mu_{\ell,k}(\mathbf{p})$. Similarly to [6] we assume that $\tau_{\ell,k} \ll T$ and that $\bar{f}_{\ell,k} \ll B$, where B is the receiver bandwidth.

Let $\bar{r}_{\ell,k}(f)$, $\bar{s}_k(f)$ and $\bar{w}_{\ell,k}(f)$ be the Fourier transform of $r_{\ell,k}(t)$, $s_k(t)$ and $w_{\ell,k}(t)$, respectively. We get that

$$\bar{r}_{\ell,k}(f) = b_{\ell,k}\bar{s}_k(f - \nu_k - f_c\mu_{\ell,k})e^{-j2\pi f\tau_{\ell,k}} + \bar{w}_{\ell,k}(f) \tag{4}$$

We now Sample the Fourier transform in (4) every Δ frequency units. Assume that $\nu_k \cong u_k\Delta$ and $f_c\mu_{\ell,k} \cong g_{\ell,k}\Delta$ where u_k, $g_{\ell,k}$ are integers. Now equation (4) can be written as

$$\bar{r}_{\ell,k}[m] = b_{\ell,k}\bar{s}_k[m - u_k - g_{\ell,k}]e^{-j2\pi\tau_{\ell,k}m\Delta} + \bar{w}_{\ell,k}[m] \tag{5}$$

where $m = -M, \dots, M$ is the frequency bin index. Define $\bar{x}_k \triangleq \bar{s}_k[m - u_k]$. Also define,

$$\begin{aligned}
\bar{\mathbf{r}}_{\ell,k} &\triangleq [\bar{r}_{\ell,k}[-M], \dots, \bar{r}_{\ell,k}[M]]^T \\
\bar{\mathbf{w}}_{\ell,k} &\triangleq [\bar{w}_{\ell,k}[-M], \dots, \bar{w}_{\ell,k}[M]]^T \\
\bar{\mathbf{x}}_k &\triangleq [\bar{x}_k[-M] \dots, \bar{x}_k[M]]^T \\
\mathbf{A}_{\ell,k} &\triangleq \mathrm{diag}\{e^{j2\pi\tau_{\ell,k}M\Delta}, \dots, e^{-j2\pi\tau_{\ell,k}M\Delta}\}
\end{aligned} \tag{6}$$

We can now write (5) in a vector form as

$$\bar{\mathbf{r}}_{\ell,k} = b_{\ell,k}\mathbf{A}_{\ell,k}\mathbf{F}_{\ell,k}\bar{\mathbf{x}}_k + \bar{\mathbf{w}}_{\ell,k} \tag{7}$$

where $\mathbf{F}_{\ell,k}$ is a cyclic shift matrix; the product $\mathbf{F}_{\ell,k}\mathbf{y}$ shifts the vector $\mathbf{y}$ by $g_{\ell,k}$ indices.

The log likelihood of $\{\bar{\mathbf{r}}_{\ell,k}[m]\}$ conditioned on the unknown parameters $\{\bar{\mathbf{x}}_k\}$, $\{b_{\ell,k}\}$, $\{\nu_k\}$ and $\mathbf{p}$ is equivalent, up to an additive constant and scaling to

$$C(\mathbf{p}) = \sum_{k=1}^{K}\sum_{\ell=1}^{L} \|\bar{\mathbf{r}}_{\ell,k} - b_{\ell,k}\mathbf{A}_{\ell,k}\mathbf{F}_{\ell,k}\bar{\mathbf{x}}_k\|^2 \tag{8}$$

The path attenuation scalars that minimize (8) are given by

$$\begin{aligned}
\hat{b}_{\ell,k} &= [(\mathbf{A}_{\ell,k}\mathbf{F}_{\ell,k}\bar{\mathbf{x}}_k)^H\mathbf{A}_{\ell,k}\mathbf{F}_{\ell,k}\bar{\mathbf{x}}_k]^{-1}(\mathbf{A}_{\ell,k}\mathbf{F}_{\ell,k}\bar{\mathbf{x}}_k)^H\bar{\mathbf{r}}_{\ell,k} \\
&= (\mathbf{A}_{\ell,k}\mathbf{F}_{\ell,k}\bar{\mathbf{x}}_k)^H\bar{\mathbf{r}}_{\ell,k}
\end{aligned} \tag{9}$$

where we assume, without loss of generality, that $\|\bar{\mathbf{x}}_k\|^2 = 1$. Substituting (9) in (8) yields,

$$C_1(\mathbf{p}) = \sum_{k=1}^{K}\sum_{\ell=1}^{L} \|\bar{\mathbf{r}}_{\ell,k}\|^2 - |(\mathbf{A}_{\ell,k}\mathbf{F}_{\ell,k}\bar{\mathbf{x}}_k)^H\bar{\mathbf{r}}_{\ell,k}|^2 \tag{10}$$

Since $\|\bar{\mathbf{r}}_{\ell,k}\|^2$ is independent of the parameters, then instead of minimizing (10) we can now maximize the cost function $C_2(\mathbf{p})$ given by

$$C_2(\mathbf{p}) = \sum_{k=1}^{K} \bar{\mathbf{x}}_k^H\mathbf{Q}_k\bar{\mathbf{x}}_k \tag{11}$$

where we defined the $N \times N$ hermitian matrix we did not define N

$$\mathbf{Q}_k \triangleq \mathbf{V}_k \mathbf{V}_k^H \tag{12}$$

$$\mathbf{V}_k \triangleq [\mathbf{F}_{1,k}^H \mathbf{A}_{1,k}^H \bar{\mathbf{r}}_{1,k}, \cdots, \mathbf{F}_{L,k}^H \mathbf{A}_{L,k}^H \bar{\mathbf{r}}_{L,k}] \tag{13}$$

The cost function in (11) is maximized by maximizing each of the K quadratic forms w.r.t. $\bar{\mathbf{x}}_k$. Thus, the vector $\bar{\mathbf{x}}_k$ should be selected as the eigenvector corresponding to the largest eigenvalue of $\mathbf{Q}_k$ denoted by $\lambda_{max}\{\mathbf{Q}_k\}$ [7, p. 62]. The dimension of the matrix $\mathbf{Q}_k$ increases with the number of data samples. Determining the eigenvalues of $\mathbf{Q}_k$ can in turn result in high computation effort. However, using (12) and according to [7, pp. 42-43] the non-zero eigenvalues of $\mathbf{Q}_k$ and the $L \times L$ matrix $\bar{\mathbf{Q}}_k \triangleq \mathbf{V}_k^H \mathbf{V}_k$ are identical. This leads to a substantial reduction of the computation load whenever $L \ll N$. Therefore, (11) reduces to

$$C_3(\mathbf{p}) = \sum_{k=1}^{K} \lambda_{max}\{\bar{\mathbf{Q}}_k\} \tag{14}$$

The estimated emitter's position is then given by

$$\hat{\mathbf{p}} = \underset{\mathbf{p}}{\text{argmax}}\{C_3(\mathbf{p})\} \tag{15}$$

A possible algorithm is displayed in Algorithm 1.

3. How is the Direct Approach Related to the Indirect Estimation Approach

To simplify the exhibition consider the case of only two moving platforms ($L = 2$), as discussed in [8],[9]. Then $\bar{\mathbf{Q}}_k$ is a 2×2 hermitian matrix, and its largest eigenvalue is given by

$$\lambda_{\max}\{\bar{\mathbf{Q}}_k\} = \frac{1}{2}\left(\bar{\mathbf{Q}}_k(1,1) + \bar{\mathbf{Q}}_k(2,2) + \sqrt{(\bar{\mathbf{Q}}_k(1,1) - \bar{\mathbf{Q}}_k(2,2))^2 + 4|\bar{\mathbf{Q}}_k(1,2)|^2}\right) \tag{16}$$

Define the area of interest and determine a suitable grid of positions $\mathbf{p}_1, \mathbf{p}_2 \cdots \mathbf{p}_g$.

for $j = 1$ **to** g **do**

    Set $C_3(\mathbf{p}_j) = 0$

    for $k = 1$ **to** K **do**

        for $\ell = 1$ **to** L **do**

            Evaluate $\tau_{\ell,k}, \mu_{\ell,k}$ and $g_{\ell,k}$

            Evaluate $\mathbf{A}_{\ell,k}$

            Evaluate $\mathbf{F}_{\ell,k}$ based on $g_{\ell,k}$

        end

        Evaluate $\mathbf{V}_k$ according to (13)

        Evaluate $\bar{\mathbf{Q}}_k = \mathbf{V}_k^H \mathbf{V}_k$

        Let $C_3(\mathbf{p}_j) = C_3(\mathbf{p}_j) + \lambda_{\text{max}}\{\bar{\mathbf{Q}}_k\}$

    end

end

Find the grid point for which C_3 is the biggest. This grid point is the estimated position.

Algorithm 1: The DPD algorithm based on the Doppler Effect for wideband signals

where $\bar{\mathbf{Q}}_k(i,j)$, denotes the (i,j)-th element of $\bar{\mathbf{Q}}_k$ given by

$$\bar{\mathbf{Q}}_k(1,1) = \sum_{m=-M}^{M} |\bar{r}_{1,k}[m]|^2 \tag{17}$$

$$\bar{\mathbf{Q}}_k(2,2) = \sum_{m=-M}^{M} |\bar{r}_{2,k}[m]|^2 \tag{18}$$

$$\bar{\mathbf{Q}}_k(1,2) = e^{-j2\pi(g_{1,k}-g_{2,k})\Delta} \sum_{m=-M}^{M} \bar{r}^*_{1,k}[m]\bar{r}_{2,k}[m-(g_{1,k}-g_{2,k})]e^{j2\pi(\tau_{1,k}-\tau_{2,k})m\Delta} \tag{19}$$

Note that if $\bar{\mathbf{Q}}_k(1,1) \cong \bar{\mathbf{Q}}_k(2,2)$ then the cost function $C_3(\mathbf{p})$ can be replaced with

$$\begin{aligned}\tilde{C}_3(\mathbf{p}) &= \sum_{k=1}^{K} |\bar{\mathbf{Q}}_k(1,2)| \\ &= \sum_{k=1}^{K} \left| \sum_{m=-M}^{M} \bar{r}_{1,k}^*[m]\bar{r}_{2,k}[m-(g_{1,k}-g_{2,k})]e^{j2\pi(\tau_{1,k}-\tau_{2,k})m\Delta} \right| \end{aligned} \tag{20}$$

where the inner sum is the expression proposed by Stein in [6] for estimating the delay and Doppler frequency.

4. Conclusions

We presented a maximum likelihood estimation for locating a stationary radio emitter based on the delayed and Doppler shifted signals observed by moving receivers. Contrary to the common methods the position is determined by a single step. The proposed method selects the position that maximizes the sum of all distinct cost functions used in the common methods.

References

[1] D. J. Torrieri, “Statistical theory of passive location systems,” *IEEE Trans. on Aerospace and Electronic Systems*, Vol. AES-20, No. 2, pp. 183–198, Mar. 1984.

[2] P. C. Chestnut, “Emitter location accuracy using TDOA and differential Doppler,” *IEEE Trans. on Aerospace and Electronic Systems*, Vol. AES-18, No. 2, pp. 214–218, Mar. 1982.

[3] K. Becker, “An efficient method of passive emitter location,” *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 28, No. 4, pp. 1091–1104, Oct. 1992.

[4] N. Levanon, “Interferometry against differential Doppler: performance comparison of two emitter location airborn systems,” *IEE Proceedings*, Vol. 136, Pt. F, No. 2, pp. 70-74, Apr. 1989.

[5] K. C. Ho, and Y. T. Chan, “Geolocation of a known altitude object from TDOA and FDOA measurements,” *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 33, No. 3, pp. 770–783, Jul. 1997.

[6] S. Stein, “Differential delay/Doppler ML estimation with unknown signals,” *IEEE Trans. on Signal Processing*, Vol. 41, No. 8, pp. 2717-2719, Aug. 1993.

[7] C. R. Rao, *Linear Statistical Inference and Its Applications*, 2nd Ed., New York, NY: John Wiley & Sons, 2002.

[8] D. P. Haworth, N. G. Smith, R. Bardelli, and T. Clement, ”Interference localization for EUTELSAT satellites; The first european transmitter location system,” *International Journal of Satellite Communications*, Vol. 15, No. 4, pp. 155–183, Sep. 1997

[9] J. Mason, “Algebraic two satellite TOA/FOA position soluion on an ellipsoidal earth,” *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 40, No. 3, pp. 1087–1092, Jul. 2004.