TIME DELAY ESTIMATION IN THE PRESENCE OF RELATIVE MOTION

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ABSTRACT

Time delays between signals observed at one or more pair of receivers and originating from a remote acoustic point source, are useful for estimating source or receiver location. Consequently, numerous procedures have been proposed for passive time delay estimation in such cases. Past investigations apply, however, only when source and receiver motion is negligible. This paper extends previous results for maximum likelihood (ML) estimation of time delay to the situation where the source is in motion relative to one or both receivers. It is shown that ideally one of the received signals must be appropriately time scaled prior to computing a generalized cross correlation of the received signals.

INTRODUCTION

Pairwise delays between signals received at three or more locations from a common point-source are useful for estimating the location of the source. Similarly such delays can be used to estimate location of one receiver relative to two or more sources and a second receiver. Consequently, numerous procedures have been proposed for passive time delay estimation in such situations [1-6]. Previous investigations assume, however, that signal source and receivers are undergoing negligible relative motion. This note extends previous results for maximum likelihood (ML) estimation of time delay to situations where relative motion is not negligible.

Assume that $s(t)$ is the signal observed at receiver 1 in the absence of noise or relative motion. Then the actual waveforms observed at the two locations will be modelled by

$$\begin{aligned} x_1(t) &= s(\beta_1 t) + n_1(t), \\ x_2(t) &= \alpha s(\beta_2(t + D)) + n_2(t). \end{aligned} \tag{1}$$

$n_1(t)$ and $n_2(t)$ represent additive noise observed in the absence of signal; D is the time delay of interest; α is relative attenuation; and β_1, β_2 are time scale factors resulting from relative motion. For example if the source and receiver 1 are closing at velocity v_1, then $\beta_1 = 1 + v_1/c$, C being the propagation velocity. It is assumed throughout that $\beta_1, \beta_2 \cong 1$ as is the case with sound in water or electromagnetic waves in air. Furthermore, it is assumed that β_1, β_2 are constant over the observation interval T. This model is obviously rather primitive, but nevertheless contains the essential features of the problem. Most other investigations use (1) with $\alpha=\beta_1=\beta_2=1$. In the absence of motion, the maximum likelihood (ML) estimate is determined by calculating a generalized (i.e. weighted) crosscorrelation between $x_1(t)$ and $x_2(t)$, [1]. The time argument at which this function is a maximum yields the estimate of delay. In the presence of relative motion this crosscorrelation may exhibit no well defined maximum. What is required, as shall be shown, is to appropriately time scale one of the received signals to remove the effect of β_2/β_1 prior to crosscorrelation.

As in previous work, $s_1(t)$, $n_1(t)$, and $n_2(t)$ are assumed to be sample functions from uncorrelated, zero mean, Gaussian random processes with known correlation functions. $x_1(t)$ and $x_2(t)$ are observed over an interval T which is long compared to the delay D plus the correlation times of signal and noise. The objective is to develop an ML estimate of delay D, given the observations. The major complication over the case without relative motion is that $x_1(t)$, $x_2(t)$ are drawn from (wide sense) stationary processes, yet the processes are jointly nonstationary. That is

$$\begin{aligned} R_{x_1x_2}(t_1, t_2) &= E\left[x_1(t_1)x_2(t_2)\right] \\ &= \alpha R_s(\beta_1 t_1 - \beta_2 t_2 - \beta_2 D) \end{aligned} \tag{2a}$$

whereas

$$R_{x_1}(t_1, t_2) = R_s[\beta_1(t_1-t_2)] + R_{n_1}(t_1-t_2) \tag{2b}$$

$$R_{x_2}(t_1, t_2) = \alpha^2 R_s[\beta_2(t_1-t_2)] + R_{n_2}(t_1-t_2). \tag{2c}$$

The processes are jointly stationary only when $\beta_1=\beta_2$. Note, however, that if $x_2(t)$ undergoes a time scale change to give $y_2(t) = x_2(t\beta_1/\beta_2)$, then

$$R_{x_1y_2}(t_1, t_2) = \alpha R_s(\beta_1(t_1-t_2)-\beta_2 D),$$

indicating that x_1, y_2 are jointly stationary and that $R_{x_1y_2}$ has a maximum when $t_1-t_2 = \beta_2 D/\beta_1$.

Intuitively then, we expect that the ML processor must determine both a time scale change, β_1/β_2, and delay, t_1-t_2, that maximizes $R_{x_1y_2}(t_1-t_2)$. To show that this in part is borne out by the theory, the basic procedure followed in [1] will be used to determine a likelihood function to be maximized by D, β_1, β_2.

DEVELOPMENT OF THE LIKELIHOOD FUNCTION

In order to compute a likelihood function for observation $x_1(t)$, $x_2(t)$ given D, β_1, β_2, and all signal and noise spectra, $x_1(t)$ over $[0, T]$ can be represented by the Fourier coefficients

$$C_1(k) = \frac{1}{T}\int_0^T x_1(t)\, e^{-jk\Delta\omega t}dt, \qquad (3)$$

where

$$\Delta\omega = 2\pi/T. \qquad (4)$$

$x_2(t)$ can be represented similarly. It simplifies matters, however, in the developments to follow if the Fourier coefficients of $x_2(\beta_1 t/\beta_2)$ on $[0, \beta_2 T/\beta_1]$ are used. Since $x_2(t)$ may be obtained from $x_2(\beta_1 t/\beta_2)$, these coefficients also define $x_2(t)$ and are denoted by

$$\begin{aligned} C_2(k) &= \frac{\beta_1}{\beta_2 T}\int_0^{\beta_1 T/\beta_2} x_2(\beta_1 t/\beta_2)e^{-jk\Delta\omega t}dt \\ &= \frac{1}{T}\int_0^T x_2(\sigma)e^{-jk\Delta\omega\beta_2\sigma/\beta_1}d\sigma. \end{aligned} \qquad (5)$$

$C_1(k)$, $C_2(k)$ are Gaussian random variables since each is a linear transformation of a sample function of a Gaussian random process. Furthermore, if $T\to\infty$, $k\to\infty$ in such a way that $k\Delta\omega = 2\pi k/T = \omega_k$ is constant, then as $T\to\infty$

$$TC_1(k) \to X_1(\omega_k),\quad TC_2(k)\to X_2(\beta_2\omega_k/\beta_1),$$

where $X_i(\omega)$ is the Fourier transform of $x_i(t)$, [7]. If, as assumed, T is large compared to the correlation time of $x_1(t)$, it is well known [7] that

$$\begin{aligned} E\left[C_1(k)\ C_1^*(q)\right] &= \frac{1}{T}G_{x_1}(k\Delta\omega)\,\delta_{kq} \\ &= \begin{cases} \frac{1}{T\beta_1}G_s(\ ) + \frac{1}{T}G_{n_1}(\ ) & k = q \\ 0, & k \neq q \end{cases} \end{aligned} \qquad (6)$$

where * denotes complex conjugation and G_{x_1}, G_s, G_{n_1} are the power spectral densities of $x_1(t)$, $s(t)$, $n_1(t)$, respectively. Similarly,

$$\begin{aligned} E\left[C_2(k)\ C_2^*(q)\right] &= \frac{1}{T}G_{x_2}(k\Delta\omega\beta_2/\beta_1)\delta_{kq} \\ &= \begin{cases} \frac{\alpha^2}{T\beta_2}G_s(\ ) + \frac{1}{T}G_{n_2}(\ ), & k = q \\ 0, & k \neq q \end{cases} \end{aligned} \qquad (7)$$

The more difficult problem is to calculate $E[C_1(k)C_2^*(q)]$ where $x_1(t)$, $x_2(t)$ are jointly non-stationary. Proceeding in a direct manner, (3), (5) and (2a) combine to give

$$\begin{aligned} E\left[C_1(k)C_2^*(q)\right] &= \frac{\alpha}{T^2}\int_0^T dt_1\int_0^T dt_2 \\ &\cdot R_s(\beta_1 t_1-\beta_2 t_2-\beta_2 D)e^{-j\Delta\omega(kt_1-qt_2\beta_2/\beta_1)} \end{aligned} \qquad (8)$$

$R_s(\ )$ may be replaced by the inverse Fourier transform of spectral density $G_s(\ )$ and the integrations performed with respect to t_1, t_2. The result after some manipulation is

$$\begin{aligned} E\left[C_1(k)C_2^*(q)\right] &= \frac{\alpha}{2\pi}\int_{-\infty}^{+\infty} G_s(\Omega)e^{-j\Omega D\beta_2} \\ &\cdot e^{j\beta_1(\Omega-k\Delta\omega/\beta_1)T/2} \cdot e^{j\beta_2(\Omega-q\Delta\omega/\beta_1)T/2} \\ &\cdot \frac{\sin\beta_1(\Omega-k\Delta\omega/\beta_1)T/2}{\beta_1(\Omega-k\Delta\omega/\beta_1)T/2} \cdot \frac{\sin\beta_2(\Omega-q\Delta\omega/\beta_1)T/2}{\beta_2(\Omega-q\Delta\omega/\beta_1)T/2}\,d\Omega. \end{aligned} \qquad (9)$$

Now as T increases while holding $k\Delta\omega=\omega_k$ and $q\Delta\omega=\omega_q$ constant, the product of the sinc functions becomes negligible unless k=q. In that case as $T\to\infty$, the product of the sinc functions approaches an impulse area $\frac{2\pi/T}{\max(\beta_1,\beta_2)}$ at $\Omega = k\Delta\omega/\beta_1$. Thus for T large enough that $G_s(\Omega)e^{-j\Omega D}$ changes little over $[\omega_k - \frac{2\pi}{T}, \omega_k + \frac{2\pi}{T}]$,

$$E\left[C_1(k)C_2^*(q)\right] = \frac{\alpha}{T\max(\beta_1,\beta_2)}G_s(k\Delta\omega/\beta_1) \cdot e^{-jk\Delta\omega D\beta_2/\beta_1}\delta_{qk} \qquad (10)$$

$x_1(t)$ on $[0,T]$ will be represented by coefficients $C_1(k)$, $x_2(t)$ by the coefficients $C_2(k)$. In particular, define

$$\underline{C}^T(k) = \left[C_1(k),\ C_2(k)\right] \qquad (11)$$

$$\underline{Y}^T = \left[\underline{C}^T(-N),\ \underline{C}^T(-N+1),\dots,\ \underline{C}^T(N)\right] \qquad (12)$$

where superscript T denotes the transpose. By virtue of (6), (7), and (10), vectors $\underline{C}(k)$, $\underline{C}(n)$ are uncorrelated Gaussian vectors for $k \neq n$ and hence independent. Furthermore the covariance matrix

$$\frac{\underline{Q}(k\Delta\omega)}{T} \triangleq E\left[\underline{C}(k)\ \underline{C}^{*T}(k)\right] \qquad (13)$$

is defined by (6), (7), and (10). Thus the probability density function for $\underline{Y}$ given α, β_1, β_2, D, $G_s(\omega)$, $G_{n_1}(\omega)$, $G_{n_2}(\omega)$ and denoted as $p(\underline{Y}/\underline{Q})$ is

$$p(\underline{Y}/\underline{Q}) = \prod_{k=-N}^{N} h_k\, e^{-J_k/2} \qquad (14)$$

where

$$h_k = \left[\frac{2\pi}{T}\left|\underline{Q}(k\Delta\omega)\right|^{1/2}\right]^{-1}, \qquad (15a)$$

and
$$J_k = \underline{C}^{*T}(k) \left[\frac{Q(k\Delta\omega)}{T}\right]^{-1} \underline{C}(k). \qquad (15b)$$

The ML estimates of D, β_1, β_2, are those values which maximize $p(\underline{Y}/Q)$. Equivalently we may maximize the $\ln p(\underline{Y}/Q)$ which upon allowing T to get very large becomes [1]

$$J = -T/2\int_{-\infty}^{+\infty} \ln|Q(\omega)|d\omega$$
$$-\tfrac{1}{2}\int_{-\infty}^{+\infty} \frac{|X_1(\omega)|^2 Q_{22}(\omega) + |X_2(\frac{\beta_2}{\beta_1}\omega)|^2 Q_{11}(\omega)}{|Q(\omega)|}d\omega$$
$$+\int_{-\infty}^{+\infty} X_1(\omega)\, X_2^*\left(\frac{\beta_2}{\beta_1}\omega\right) \frac{Q_{12}^*(\omega)}{|Q(\omega)|}\, d\omega, \qquad (16)$$

where $X_1(\omega)$, $X_2(\omega)$ are Fourier transforms of $x_1(t)$, $x_2(t)$, respectively. Q_{11}, Q_{12}, Q_{22} are elements of the spectral matrix Q which by (6), (7), (10) and (13) are

$$Q_{11} = G_{n_1}(\omega) + \frac{1}{\beta_1} G_s(\omega/\beta_1),$$
$$Q_{22} = \frac{\alpha^2}{\beta_2} G_s(\omega/\beta_1) + G_{n_2}(\omega\beta_2/\beta_1), \qquad (17)$$
$$Q_{12} = \frac{\alpha}{\max(\beta_1,\beta_2)} G_s(\omega/\beta_1)e^{-j\omega D\beta_2/\beta_1} = Q_{21}^*$$

$X_2(\beta_2\omega/\beta_1)$ in (16) may be obtained by performing a time scale change on $x_2(t)$ yielding $x_2(\frac{\beta_1}{\beta_2} t)$. The Fourier transform of the latter is then $X_2(\beta_2\omega/\beta_1)$. In particular, if

$$y_2(t) = \sqrt{\beta_1/\beta_2}\, x_2(\beta_1 t/\beta_2), \qquad (18)$$

then
$$G_{x_1}(\omega) = Q_{11}(\omega),\ G_{y_2}(\omega) = Q_{22}(\omega),$$
$$G_{x_1y_2}(\omega) = \frac{\max(\beta_1, \beta_2)}{\sqrt{\beta_1\beta_2}} Q_{12}(\omega). \qquad (19)$$

Thus for β_1, $\beta_2 \cong 1$, Q is essentially identical to the spectral matrix of $[x_1(t)\ y_2(t)]^T$.

DISCUSSION

ML estimates of β_1, β_2, D are those values which maximize (16). Implementation of a practical estimator, however, requires some simplification of (16). Although each situation should be examined with care, in the usual case of a signal dominated by broadband noise, the first two terms of (16) are relatively insensitive to changes in β_1, β_2, D. For example, the difference between $G_{n_2}(\omega)$ and $G_{n_2}(\omega\beta_2/\beta_1)$ over the processed band is small if $\beta_2/\beta_1 \cong 1$ and $G_{n_2}(\omega)$ is broadband. This is also true of the signal if it is broadband and in this case the requirement for low signal to noise ratio may be dropped. Note that the first two terms of (16) are independent of delay D. Thus in the case of broadband noise and either low signal to noise ratio or broadband signal, maximization of (16) is approximately equivalent to maximization of

$$J_a = \int_{-\infty}^{+\infty} X_1(\omega)\, X_2^*(\omega\beta_2/\beta_1) W(\omega,\beta_1,\beta_2) e^{j\omega D\beta_2/\beta_1} d\omega, \qquad (20)$$

where
$$W(\omega, \beta_1, \beta_2) = \frac{|Q_{12}(\omega)|}{|Q(\omega)|} = \frac{\alpha G_s(\omega/\beta_1)}{\max(\beta_1, \beta_2)\, Q(\omega)}. \qquad (21)$$

When $\beta_1 = \beta_2 = 1$, i.e. there is no relative motion, the above expression reduces to previous results for the ML estimate of D, [1].

Implementation requires further approximation to determine the weighting $W(\omega,\beta_1, \beta_2)$ since $G_s(\omega)$, $G_{n_1}(\omega)$, $G_{n_2}(\omega)$ are known approximately at best. Two possible approaches are; (a) make an educated guess at the spectra, or (b) estimate $W(\omega, \beta_1, \beta_2)$ using observations $x_1(t)$, $x_2(t)$. In either case the resulting estimate must be considered an approximate ML estimate. If approach (a) is taken and low signal to noise ratio is assumed, then

$$W(\omega, \beta_1, \beta_2) \cong \frac{G_s(\omega/\beta_1)}{G_{n_1}(\omega)G_{n_2}(\omega)} \qquad (22)$$

for broadband noise. For broadband signal the numerator may be further approximated by $G_s(\omega)$. If $W(\omega, \beta_1, \beta_2)$ is estimated, $x_1(t)$ and $y_2(t)$ can be used as indicated by equation (19) so that

$$\hat{W}(\omega,\beta_1,\beta_2) \cong \frac{|\hat{G}_{x_1y_2}(\omega)|}{\hat{G}_{x_1}(\omega)\hat{G}_{y_2}(\omega) - |\hat{G}_{x_1y_2}(\omega)|^2} \qquad (23)$$

where ^ denotes an estimated quantity. The problem with (23) is that $y_2(t)$ as given by (18) depends on β_1/β_2. Thus $G_{x_1y_2}(\omega)$ (and perhaps $G_{y_2}(\omega)$) must be estimated each time β_1/β_2 changes. Use of (22) avoids this but assumes that an explicit expression is available for $G_s(\omega)$. In the case of a narrowband signal centered at ω_0, (20) and (22) indicate that x_1, y_2 should be filtered by a narrowband filter centered at $\omega_0\beta_1$. If the bandwidth of the signal spectrum is less than $2\omega_0\beta_1^{-1}$ then replacing $G_s(\omega/\beta_1)$ by $G_s(\omega)$ in (22) will produce substantially different results.

Maximization of (20) using (22), (23), or alternate approximations to $W(\omega, \beta_1, \beta_2)$ may be accomplished digitally or in analog fashion. A schematic realization is shown in Figure 1 where the frequency integral has been converted to a time integral using Parseval's theorem.

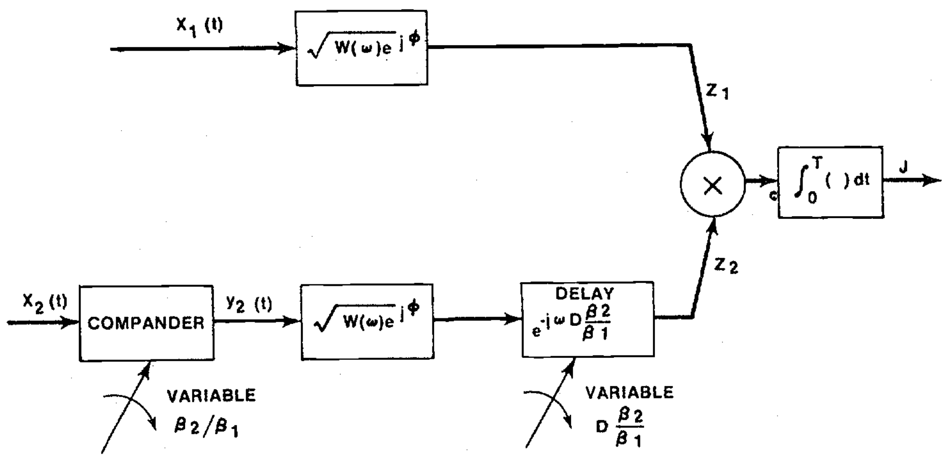

FIGURE (1). SCHEMATIC REALIZATION FOR ML ESTIMATION OF TIME DELAY AND TIME COMPRESSION

The time scaling of $x_2(t)$ can be achieved using a variable speed analog recorder. Alternately, if the processing is done digitally (except that $x_2(t)$ is recorded in analog form prior to sampling), time scaling can be realized by using a variable sampling rate $x_2(t)$. To illustrate, first sample $x_1(t)$ with sample period T_s yielding sequence $\{x_1(t)\}$ with period $\beta_1 T_s/\beta_2$ giving sequence $\{x_2(kT_s\beta_1/\beta_2)\}$. Next, calculate the Fast Fourier Transform (FFT) of $\{x_1(kT_s)\}$ and $\{x_2(kT_s\,\beta_1/\beta_2)\}$, denoted by $X_1(k\omega_s)$ respectively. Finally, compute the (inverse) FFT of $X_1(k\omega_s)\ X_2^*\ (k\beta_2/\beta_1\omega_s)W(k\omega_s, \beta_1,\beta_2)$ as required in (20) for $D\beta_2/\beta_1 = \pm T_s, \pm 2T_s, \ldots$, etc. The value $D\beta_2/\beta_1$ which maximizes this function is the best estimate of $D\beta_2/\beta_1$ for the particular value of β_1/β_2 used in sampling $x_2(t)$. This process must be repeated for various β_1, β_2 until an overall maximum is found. Suitable approximations to simplify evaluation of $W(k\omega_s, \beta_1, \beta_2)$ can be used as previously noted.

SUMMARY

The ML estimates of the relative delay D, and time compression factors β_1, β_2 for signals received at two different locations are the values which maximize (16). Implementation of an estimator based on (16) or simplified version (20) is complicated by the need to time scale one of the received signals, say $x_2(t)$. Under certain conditions, e.g., $(\beta_1 - \beta_2)T$ small and low frequency signal, time scaling may provide marginal improvement in estimates of delay. Under most conditions, however, time scaling appears to pay significant dividends in ability to estimate delay. In addition, estimates of β_1/β_2 may be of some benefit in estimating movement of source.

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