DIRECT POSITION DETERMINATION OF NARROWBAND RADIO TRANSMITTERS

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ABSTRACT

The most common methods for location of communications or radar transmitters are based on measuring a specified parameter such as signal Angle-of-Arrival or Time-of-Arrival. The measured parameters are then used to estimate the transmitter location. These methods are sub-optimal since they are indirect and involve two separate estimation steps. We propose a technique that uses exactly the same data as the common methods but the estimation of location is based on Maximum-Likelihood and the location determination is direct. Although there are many stray parameters, including the attenuation coefficients and the signal waveform, the method requires only a two-dimensional search. Monte-Carlo simulations indicate that the accuracy is superior to Angle-of-Arrival, Time-of Arrival and their combination

1. INTRODUCTION

The problem of emitter location attracts much interest in the Signal Processing, Vehicular Technology and Underwater Acoustics literature. Defense oriented location systems have been reported since word war I. Perhaps the first paper on the mathematics of emitter location, using Angle-of-Arrival (AOA), is due to Stansfield [1]. Many other publications followed including a fine review paper by Torrieri [2]. The papers by Krim and Viberg [3] and Wax [4] are comprehensive review papers on antenna array processing for location by AOA. Recently, Van-Trees [5] published a book on Array Processing. Positioning by Time-of-Arrival (TOA) is well known in radar systems [6], and in underwater acoustics [7]. In underwater acoustics Matched-Field Processing (MFP) is viewed as a promising procedure for source localization [8]. MFP can be interpreted as the Maximum *a Posteriori* (MAP) estimate of location given the observed signal at the output of an array of sensors [8,9]. Other interpretation of MFP is the well-known beam-forming applied to wide or narrow bandwidth signals, non-planar wave field and unknown environmental parameters.

In this correspondence we discuss a method that have some similarities with matched field processing. While the concept is similar the details are different. The models of underwater acoustic propagation are usually more complex than the models used in most AOA/TOA electromagnetic emitter location papers. Hence, the required processing for traditional matched-field processing is rather heavy. Moreover, the underwater sources distance from the sensors is usually the same order of magnitude as the sensor array size. Hence, the far-field assumption that is usually used in electromagnetic AOA does not hold for MFP.

The Direct Position Determination (DPD) method that we propose takes advantage of the rather simple propagation assumptions that are usually used for Radio Frequency (RF) signals. This enables us to obtain a simple, closed form, cost function. The cost function can be maximized using a two-dimensional search for an emitter known to be located on a plane or a three-dimensional search in general. The DPD belong to the least squares family if the noise statistics are unknown. If the noise is Gaussian, DPD is the Maximum Likelihood estimate of location. We demonstrate that DPD outperforms AOA, TOA and the combination of AOA and TOA. The DPD technique requires the transmission of the received signals (possibly sampled) to a central processing location. However, AOA and TOA require only the transmission of the measured parameters to the central processing location. This is the cost of employing DPD. The paper focuses on the single signal case. Extensions to multiple signals are published in a companion paper [10].

2. PROBLEM FORMULATION AND ALGORITHM

Consider a transmitter and L base stations intercepting the transmitted signal. Each base station is equipped with an antenna array consisting of M elements. Denote the transmitter position by the vector of coordinates, $\mathbf{p}$, and the l-th base station position by the vector of coordinates

$\mathbf{q}_l$. The signal observed by the l-th base station array is given by

$$\mathbf{r}_l(t) = b_l \mathbf{a}_l(\mathbf{p}) s(t - \tau_l(\mathbf{p}) - t_0) + \mathbf{n}_l(t); \tag{1}$$

where $\mathbf{r}_l(t)$ is a time-dependent $M \times 1$ vector, b_l is an unknown complex scalar representing the channel effect (attenuation), $\mathbf{a}_l(\mathbf{p})$ is the l-th array response to signal transmitted from position $\mathbf{p}$, and $s(t - \tau_l(\mathbf{p}) - t_0)$ is the signal waveform, transmitted at time t_0 and delayed by $\tau_l(\mathbf{p})$. The vector $\mathbf{n}_l(t)$ represents noise and interference, including multipath observed by the array.

The sampled version of the signal in (1) is given by

$$\begin{aligned} &\mathbf{r}_l(j) = b_l \mathbf{a}_l(\mathbf{p}) s_l(j) + \mathbf{n}_l(j); \quad 0 \le j \le N_s - 1 \\ &s_l(j) \triangleq s(t - \tau_l(\mathbf{p}) - t_0)\big|_{t=jT} \\ &\mathbf{r}_l(j) \triangleq \mathbf{r}_l(t)\big|_{t=jT} \\ &\mathbf{n}_l(j) \triangleq \mathbf{n}_l(t)\big|_{t=jT} \end{aligned} \tag{2}$$

We observe that information about the transmitter position is embedded in the observed signal in 2 different ways. The first is the array response. If the signal is in the far field (more than 10 times the array aperture) the array response becomes a function of the angle of arrival only. The position is also reflected by the time of arrival of the signal at the array $\tau_l(\mathbf{p})$, which depends on the distance between the transmitter and the station. Taking the Discrete Fourier Transform (DFT) of (2) we get

$$\begin{aligned} &\bar{\mathbf{r}}_l(k) = b_l \mathbf{a}_l(\mathbf{p}) \bar{s}(k) e^{-j\omega_k[\tau_l(\mathbf{p}) + t_0]} + \bar{\mathbf{n}}_l(k); \\ &0 \le k \le N_s - 1; \quad \omega_k \triangleq \frac{2\pi k}{N_s T} \end{aligned} \tag{3}$$

where the over-bar indicate the DFT coefficient of the corresponding time samples.
The least squares estimate of the position is given by minimizing the cost function

$$Q(\mathbf{p}) = \sum_{l=1}^{L} \sum_{k=0}^{N_s-1} \left\| \bar{\mathbf{r}}_l(k) - b_l \mathbf{a}_l(\mathbf{p}) \bar{s}(k) e^{-j\omega_k[\tau_l(\mathbf{p}) + t_0]} \right\|^2 \tag{4}$$

where $\|\bullet\|$ stands for the Frobenius norm. Note that the cost function can be represented by a sum over L terms as follows,

$$\begin{aligned} &Q(\mathbf{p}) = \sum_{l=1}^{L} Q_l(\mathbf{p}) \\ &Q_l(\mathbf{p}) \triangleq \sum_{k=0}^{N_s-1} \left\| \bar{\mathbf{r}}_l(k) - b_l \mathbf{a}_l(\mathbf{p}) \bar{s}(k) e^{-j\omega_k[\tau_l(\mathbf{p}) + t_0]} \right\|^2 \end{aligned} \tag{5}$$

Define the following vectors,

$$\begin{aligned} &\bar{\mathbf{r}}_l \triangleq [\bar{\mathbf{r}}_l^T(0), \bar{\mathbf{r}}_l^T(1), \cdots, \bar{\mathbf{r}}_l^T(N_s - 1)]^T \\ &\bar{\mathbf{s}}_l \triangleq [\bar{s}(0) e^{-j\omega_0[\tau_l(\mathbf{p}) + t_0]} \cdots \bar{s}(N_s - 1) e^{-j\omega_{N_s-1}[\tau_l(\mathbf{p}) + t_0]}]^T \\ &\mathbf{c}_l \triangleq \bar{\mathbf{s}}_l \otimes \mathbf{a}_l(\mathbf{p}) \end{aligned} \tag{6}$$

where $\otimes$ stands for the Kronecker product. Now equation (5) can be represented by

$$Q(\mathbf{p}) = \sum_{l=1}^{L} \left\| \bar{\mathbf{r}}_l - \mathbf{c}_l b_l \right\|^2 \tag{7}$$

The estimate of b_l that minimizes the cost function is given by

$$\begin{aligned} \hat{b}_l &= (\mathbf{c}_l^H \mathbf{c}_l)^{-1} \mathbf{c}_l^H \mathbf{r}_l \\ &= \left(\left[\bar{\mathbf{s}}_l^H \bar{\mathbf{s}}_l \otimes \mathbf{a}_l^H(\mathbf{p}) \mathbf{a}_l(\mathbf{p}) \right] \right)^{-1} \left[\bar{\mathbf{s}}_l \otimes \mathbf{a}_l(\mathbf{p}) \right]^H \bar{\mathbf{r}}_l \\ &= \frac{1}{\|\bar{\mathbf{s}}_l\|^2 \|\mathbf{a}_l(\mathbf{p})\|^2} \left[\bar{\mathbf{s}}_l \otimes \mathbf{a}_l(\mathbf{p}) \right]^H \bar{\mathbf{r}}_l \end{aligned} \tag{8}$$

Without loss of generality we assume that

$$\|\bar{\mathbf{s}}_l\|^2 = 1; \quad \|\mathbf{a}_l(\mathbf{p})\|^2 = 1; \quad \forall l \tag{9}$$

Substituting equations (8) and (9) in (7) we get

$$\begin{aligned} Q(\mathbf{p}) &= \sum_{l=1}^{L} \left\| \bar{\mathbf{r}}_l - \left[\bar{\mathbf{s}}_l \otimes \mathbf{a}_l(\mathbf{p}) \right] \left[\bar{\mathbf{s}}_l \otimes \mathbf{a}_l(\mathbf{p}) \right]^H \bar{\mathbf{r}}_l \right\|^2 \\ &= \sum_{l=1}^{L} \bar{\mathbf{r}}_l^H \bar{\mathbf{r}}_l - \bar{\mathbf{r}}_l^H \left[\bar{\mathbf{s}}_l \otimes \mathbf{a}_l(\mathbf{p}) \right] \left[\bar{\mathbf{s}}_l \otimes \mathbf{a}_l(\mathbf{p}) \right]^H \bar{\mathbf{r}}_l \\ &= \sum_{l=1}^{L} \|\bar{\mathbf{r}}_l\|^2 - \sum_{l=1}^{L} \left| \mathbf{a}_l^H(\mathbf{p}) \sum_{k=0}^{N_s-1} e^{j\omega_k[\tau_l(\mathbf{p}) + t_0]} \bar{s}^*(k) \bar{\mathbf{r}}_l(k) \right|^2 \end{aligned} \tag{10}$$

Instead of finding the minimum of $Q(\mathbf{p})$ we can find the maximum of $\tilde{Q}(\mathbf{p})$ defined by

$$\tilde{Q}(\mathbf{p}) = \sum_{l=1}^{L} \left| \mathbf{a}_l^H(\mathbf{p}) \sum_{k=0}^{N_s-1} e^{j\omega_k[\tau_l(\mathbf{p}) + t_0]} \bar{s}^*(k) \bar{\mathbf{r}}_l(k) \right|^2 \tag{11}$$

Define the vectors

$$\begin{aligned} &\mathbf{d}_l = [d_l(0), \cdots, d_l(N_s - 1)]^T \\ &d_l(k) \triangleq e^{j\omega_k \tau_l(\mathbf{p})} \mathbf{a}_l^H(\mathbf{p}) \bar{\mathbf{r}}_l(k); \\ &\bar{\mathbf{s}} = [\bar{s}(0) e^{-j\omega_0 t_0}, \cdots, \bar{s}(N_s - 1) e^{-j\omega_{N_s-1} t_0}]^T \end{aligned} \tag{12}$$

Using these definitions we can rewrite (11) as

$$\begin{aligned} \tilde{Q}(\mathbf{p}) &= \sum_{l=1}^{L} \left| \bar{\mathbf{s}}^H \mathbf{d}_l \right|^2 = \sum_{l=1}^{L} \bar{\mathbf{s}}^H \mathbf{d}_l \mathbf{d}_l^H \bar{\mathbf{s}} \\ &= \bar{\mathbf{s}}^H \left(\sum_{l=1}^{L} \mathbf{d}_l \mathbf{d}_l^H \right) \bar{\mathbf{s}} = \bar{\mathbf{s}}^H \mathbf{D} \bar{\mathbf{s}} \end{aligned} \tag{13}$$

Under the common assumption that the signal waveform is not known to the receivers the cost function in (13) is maximized by selecting the vector $\bar{\mathbf{s}}$ as the eigenvector corresponding to the largest eigenvalue of the matrix **D**. Hence, equation (13) reduces to

$$\tilde{Q}(\mathbf{p}) = \lambda_{\max}(\mathbf{D}) \tag{14}$$

where the right side of (14) denotes the largest eigenvalue of **D**, and the matrix **D** is a function of the data, the array response at each base station, the location of the base stations and the unknown emitter location **p**. It is clear that the maximization of (14) requires only a two-dimensional (or three dimensional) search although the estimator knows neither the channel response nor the signal. It is interesting to note that the dimensions of the matrix **D** are $N_s \times N_s$ which might be rather large for some cases. However, we can replace **D** with the $L \times L$ matrix $\tilde{\mathbf{D}}$ where,

$$\begin{aligned} &\mathbf{U} \triangleq [\mathbf{d}_1, \cdots \mathbf{d}_L] \\ &\mathbf{D} = \mathbf{U}\mathbf{U}^H \\ &\tilde{\mathbf{D}} \triangleq \mathbf{U}^H \mathbf{U} \end{aligned} \tag{15}$$

Thus, equation (14) becomes

$$\tilde{Q}(\mathbf{p}) = \lambda_{\max}(\tilde{\mathbf{D}}) \tag{16}$$

This result holds for a single observation of the signal for a period equivalent to N_s samples. Extension to multiple observations of the signal is straightforward.

In the case that the receivers know the signal waveform (*e.g.*, training signal or a synchronization signals are known to the receivers) we return to equation (13) and rewrite it as follows,

$$\begin{aligned} &\tilde{Q}(\mathbf{p}) = \overline{\mathbf{s}}^H \mathbf{D} \overline{\mathbf{s}} = \mathbf{z}^H \mathbf{S}^H \mathbf{U}\mathbf{U}^H \mathbf{S}\mathbf{z} = \mathbf{z}^H \tilde{\mathbf{U}}\tilde{\mathbf{U}}^H \mathbf{z} \\ &\mathbf{S} \triangleq \operatorname{diag}\{[\overline{s}(0), \overline{s}(1), \cdots, \overline{s}(N_s - 1)]\} \\ &\tilde{\mathbf{U}} \triangleq \mathbf{S}^H \mathbf{U} \\ &\mathbf{z} \triangleq [1, e^{-j\omega_1 t_0}, e^{-j2\omega_1 t_0}, \cdots, e^{-j(N_s-1)\omega_1 t_0}]^T \\ &\omega_1 \triangleq \frac{2\pi}{N_s T} \end{aligned} \tag{17}$$

The unknowns are the transmit time, t_0, and the emitter position, **p**. For any given p we can estimate t_0 by a one-dimensional search or by FFT of the columns of $\tilde{\mathbf{U}}$. If we choose the later method we get the following cost function,

$$\begin{aligned} &\mathbf{w} \triangleq \sum_{l=1}^{L} \left| \mathrm{FFT}\{\tilde{\mathbf{U}}_l\} \right|^2 \\ &\tilde{\mathbf{U}}_l \triangleq [\tilde{\mathbf{U}}(1,l), \tilde{\mathbf{U}}(2,l), \cdots \tilde{\mathbf{U}}(N_s, l)]^T \\ &\tilde{Q}(\mathbf{p}) = \max_k \{\mathbf{w}_k\} \end{aligned} \tag{18}$$

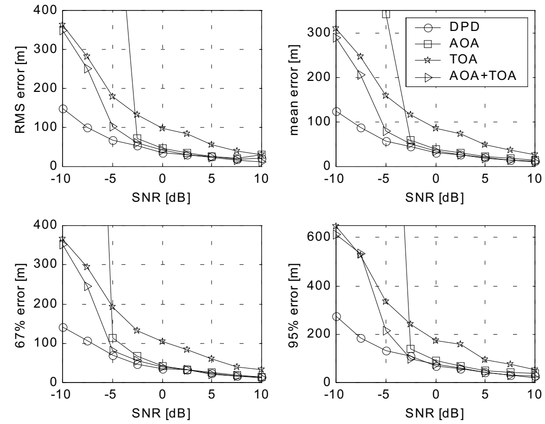

Figure 1: RMS, Mean, 67%, 95% of miss distance for 4 different methods, unknown signal

Stated in words, perform FFT on each of the columns of $\tilde{\mathbf{U}}$, sum the squared absolute value of the Fourier coefficient over the *L* results to obtain the vector **w**. The length of **w** corresponds to the FFT length, which may be a multiplicity of N_s depending on the desired resolution. The maximum element of **w** is the desired cost function.

3. NUMERICAL RESULTS

In order to examine the performance of the advocated method and compare it with the traditional approaches we performed extensive Monte-Carlo simulations. Some examples are shown here. Consider 4 base-stations placed at the corners of a 4 Km $\times$ 4 Km square. Each base-station is equipped with a circular array of 5 antenna elements. The radius of the array is one wavelength. The transmitter location is selected at random, uniformly, within the square formed by the base-stations. Each location determination is based on 32 samples of the signal. The SNR is varied between –10 dB and +10 dB. At each SNR value we performed 100 experiments in order to obtain the statistical properties of the performance. The path-loss attenuation magnitude is selected at random using normal distribution (mean=1, std=0.1) and the attenuation phase is uniformly distributed in $[-\pi, \pi]$. We applied 4 different techniques in order to locate the transmitter:

1. Angle of Arrival estimation using Maximum Likelihood (also known as beam-forming) and Maximum Likelihood emitter location estimation using the AOA estimates as the data.

2. Time of Arrival estimation using Maximum Likelihood (under the assumption that the signal waveform is known at the base stations and using all antenna elements) and Maximum Likelihood emitter location estimation using the TOA estimates as the data.

3. Maximum Likelihood emitter location estimation using both AOA and TOA as the data.

4. Direct Position Determination (DPD) according to equation (16).

The performance evaluation is based on the statistics of the miss distance i.e., the distance between the true emitter position and the estimated emitter position.
We used 4 different criteria:

1. Root Mean Square (RMS) of miss distance

2. Mean of miss distance

3. Miss distance that upper bounds 67% of the errors

4. Miss distance that upper bounds 95% of the errors

All the plots in Figure 1 indicate that DPD is superior to AOA, TOA and even combined AOA and TOA. The advantage of DPD is at low SNR. At high SNR all methods give excellent results. Figure 2 show similar results for known signal.

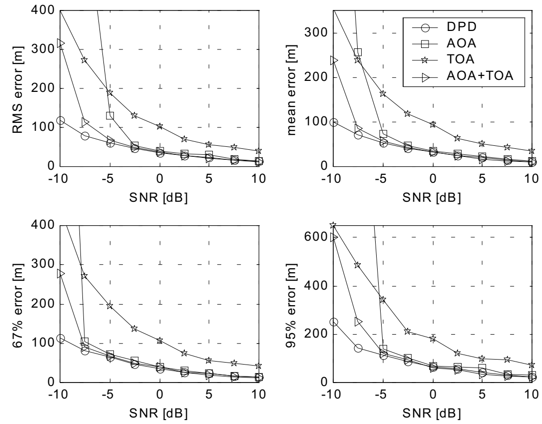

Figure 2: RMS, Mean, 67%, 95% of miss distance for 4 different methods, known signal.

4. CONCLUSIONS

We have proposed a direct position determination technique that outperforms AOA, TOA and their combination. The DPD is closely related to matched-field processing but it is suitable only for Radio Frequency signals and not for underwater emitter location. Further research is currently underway that explores the advantages and disadvantages of the proposed method for multiple signals and more complex propagation models. Small error analysis, threshold prediction and comparison with the Cramer-Rao bound will be published in the near future.

5. REFERENCES

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