

Mobile Location Using Signal Strength Measurements in a Cellular System

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Abstract—In a high-capacity mobile telephone system using cellular technology, a mobile location technique which determines the radio zone in which a moving vehicle exists is one of the most important techniques for the system control. A mobile location technique using a signal strength measurement scheme is described. The probability that a mobile is judged to exist in a certain zone (zone selection rate) is first defined. Dependency of this rate on land mobile propagation characteristics is then discussed in detail. A field test for the justification of this location technique was carried out in the Tokyo metropolitan area in which the field test results agreed well with the estimated values.

I. INTRODUCTION

IN A HIGH-CAPACITY cellular mobile telephone system, a location technique which detects the zone containing the moving vehicle of interest is necessary for the purpose of channel setup and handoff. Various location techniques, such as signal strength measurement [1], angle detection [2], [3], and arrival time measurement [4], have been proposed. The signal strength measurement method, which allows the detection of a better radio zone, is considered to be the most useful one for vehicle location in the cellular mobile telephone system because the high-quality radio channel should be assigned to the moving vehicle anywhere in the service area. Although this method has been treated by many authors [5], [6], there is no report which presents the relation between the zone detection accuracy and the various factors including propagation characteristics and detection time in detail.

This paper presents an estimation method of zone detection error in the signal strength measurement. First, the sequence of zone detection in cellular mobile telephone systems in Japan and the land mobile propagation characteristics are described. After the dependency of the measurement accuracy of the received signal strength level on the detection time is analyzed, the probability that a mobile is judged to exist in a certain zone (zone selection rate (ZSR)) is theoretically given by a simple closed form. The theoretical results are verified by the field tests carried out in the Tokyo metropolitan area in May 1977.

II. MOBILE LOCATION AND LAND MOBILE PROPAGATION

A. Mobile Location

The Japanese high-capacity cellular mobile telephone system, which has been researched and developed by Nippon

Telegraph and Telephone, possesses the following six fundamental control functions: land-originated call, mobile-originated call, disconnection, toll charges, handoff, and registration [7], [8]. Among them, land-originated call, mobile-originated call, and handoff need mobile location techniques, which are used as follows.

1) *Land-Originated Call*: A mobile unit in a moving vehicle of interest is always tuning its radio channel to the forward (base-to-mobile) paging channel in idle state. If the vehicle detects its own identification code in the received paging signals, it replies with a paging response via the backward (mobile-to-base) paging channel. Since a common frequency radio channel is assigned for the paging channel in a group of neighboring radio zones, the paging response burst-signal transmitted from the vehicle is simultaneously received at several base stations, located at the respective centers of the neighboring radio zones. Each base station detects the received burst-signal and then sends the detected baseband signal to the control station via the land-line data channel. In this case, the median value of the received burst-signal level is also detected, coded to a 4-bit decimal code, and sent to the control station via the land-line data channel. At the control station, signal level comparisons are made based on coded median values, and the moving vehicle is judged to exist in the radio zone corresponding to the highest received signal level. An idle speech channel in the radio zone is then assigned to the vehicle. Several tens of speech channels are preassigned to each radio zone according to the traffic condition. The channel assignment signal is sent via the forward paging channel. After that, the vehicle switches its radio channel from the paging channel to the assigned speech channel and the call is set up for ringing.

2) *Mobile-Originated Call*: If a customer of interest goes off-hook in a moving vehicle, the mobile unit automatically tunes its radio channel from the paging channel to the access channel. A common frequency radio channel is also assigned for the access channel in a group of neighboring radio zones. The access burst-signal is transmitted from the mobile unit via the forward (mobile-to-base) access channel. In the same manner as land-originated calls, the control station judges the mobile to exist in the radio zone corresponding to the highest received signal level and assigns an idle speech channel in the radio zone to the vehicle. The channel assignment signal is sent via the backward (base-to-mobile) access channel. Then the vehicle switches its radio channel from the access channel to the assigned speech channel and the call is set up for dialing.

3) *Handoff*: If a vehicle of interest moves to a zone boundary during a call, the base station of the radio zone detects the deterioration of signal-to-noise ratio (SNR) in the speech chan-

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nel and requests from the control station a tracking exchange of the speech channel. The control station then orders the SNR check of the speech channel to the neighboring base stations. Comparing the respective signal levels of the neighboring base stations with that of the original base station, the control station selects a new radio zone corresponding to the highest received signal level and assigns an idle speech channel in the new radio zone to the vehicle. If there does not exist any neighboring base station having a higher signal level than the original one or having an idle speech channel, the call will continue via the deteriorated speech channel until the SNR becomes less than the service threshold level.

In the above described sequences it is found that each location needs a relative, rather than absolute, value of the received signal level. It is important to detect the radio zone in which a vehicle can expect a better quality radio channel than the other radio zones, but not to detect the accurate location of a vehicle. Furthermore, it is found that these sequences, especially in the call setup, should obviously be fast and automatic. Namely, the signal strength is measured in a short time. However, the signal transmission between a moving vehicle and a fixed base station usually attends the multipath phenomenon because of the scatterers, the terrain, and the effect of other obstacles. The received wave has the fluctuation as shown in Fig. 1. When we measure the signal strength in a short time, this fluctuation results in a detection error. Thus it is necessary to consider the relation between the accuracy of the detected values and the propagation characteristics.

B. Land Mobile Propagation

Land mobile radio propagation is characterized by three components: the instantaneous variation, the narrow area median, and the wide area median signal strength (long term median) [9]. The instantaneous variation is caused by the traversing of a mobile through a standing wave pattern which is produced by the summing of the multipath waves and is Rayleigh distributed about the narrow area median. The narrow area median is attributed to shadowing caused by structures and terrain variations and is lognormally distributed about the long term median. The variation of the narrow area median is independent of the instantaneous variation. The instantaneous variation fluctuates at a rate several hundred times as rapid as the narrow area median. The long term median is a median level of the received signal strength within about 1-km running distance. In general, the long term median decreases as the distance from the mobile to the base station increases.

Therefore, when the base station measures the signal strength and estimates the long term median as shown in Fig. 1, the accuracy of the measured median depends upon the deviation of the narrow area median and the instantaneous variation.

III. DEVIATION OF THE MEASURED VALUE

A. Deviation for Instantaneous Variation

We consider the variation of the measured mean. For a stationary random process $\xi(t)$, we define the mean of a T -second

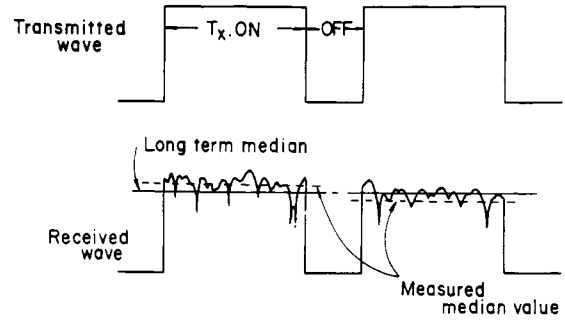


Fig. 1. Deviation of measured median.

interval (T -second mean) as

$$m_T = \frac{1}{T} \int_0^T \xi(t) dt. \quad (1)$$

Then, if the limit exists,

$$\lim_{T \rightarrow \infty} m_T = \langle \xi(t) \rangle = m. \quad (2)$$

For a fixed T , m_T is distributed about m , and its variance is given by

$$\sigma_T^2 = \frac{1}{T} \int_{-T}^T \left(1 - \frac{|\tau|}{T}\right) [R_\xi(\tau) - m^2] d\tau, \quad (3)$$

where

$$R_\xi(\tau) = \langle \xi(t+\tau) \xi(t) \rangle \quad (4)$$

is the autocorrelation function [10]. Therefore, we can determine the variance of the T -second mean when we obtain $R_\xi(\tau)$ and m .

In a usual land mobile environment the received signal possesses an envelope $X(t)$ which is Rayleigh distributed and can, therefore, be treated as a narrow-band Gaussian random process. The signal strength detector must measure a wide range of signal levels and thus generally employs a logarithmic compression circuit. Therefore, it is necessary to learn the autocorrelation function after logarithmic compression. Suppose that this logarithmic transformation is defined as

$$y = f(X) = 2a \log_e X + b \quad (5)$$

where a and b are constants. By calculation as shown in the Appendix, we get the variance of the T -second mean as

$$\sigma_{MT}^2 = \frac{a^2}{T} \int_{-T}^T \left(1 - \frac{|\tau|}{T}\right) \{ \rho^2(\tau) + 2.498 \times 10^{-1} \times \rho^4(\tau) + 1.108 \times 10^{-1} \times \rho^6(\tau) + \dots \} d\tau. \quad (6)$$

The function $\rho(\tau)$ is given by

$$\rho(\tau) = J_0(2\pi f_D \tau), \quad (7)$$

where $J_0(\cdot)$ is the zero-order Bessel function and f_D is the maximum Doppler frequency [11]. Since $|\rho(\tau)| < 1$, an approximate expression for σ_{MT}^2 can be obtained:

$$\sigma_{MT}^2 = \frac{a'^2}{T} \int_{-T}^T \left(1 - \frac{|\tau|}{T}\right) J_0^2(2\pi f_D \tau) d\tau, \quad (8)$$

where a' is the correction factor for the approximation. This equation shows that σ_{MT}^2 does not depend on the mean. If we do not make the logarithmic transformation, σ_{MT}^2 depends on the mean [12]. This fact is very interesting.

At $T = 0$, the standard deviation $\sigma_{MT}(T = 0)$ can be obtained precisely. For the transformation of $y = 20 \log_{10} X$, it becomes

$$\sigma_{MT}(T = 0) = \sqrt{\langle (y - \alpha_0)^2 \rangle} = 5.57. \quad (9)$$

On the other hand, if we set $T = 0$ in (8), then $\sigma_{MT}(T = 0) = a'$. When we take $a' = 5.57$, (8) agrees closely with (6) for $f_D \cdot T > 0.3$.

In the signal detection process for a land mobile system, the T -second median is used practically instead of the T -second mean. Let us consider their standard deviations. If $T \rightarrow \infty$, they converge to zero apparently. For $T \rightarrow 0$, we calculate the standard deviation of the median σ_{m0} . We define X_0 as the median when X is Rayleigh distributed. For the transformation of $y = 20 \log_{10} X$ and $y_0 = 20 \log_{10} X_0$, we get

$$\sigma_{m0} = \sqrt{\langle (y - y_0)^2 \rangle} = 5.61. \quad (10)$$

From (9) and (10), their difference is only 0.04 dB. Fig. 2 shows the standard deviations of the T -second mean and the T -second median. The solid lines are the theoretical values for the T -second mean, and the plotted points are the simulation results for the T -second median. The manner of simulation is described later in Section III-C. Except for $T < 0.05$ s, they show good agreement, and therefore (8) can be used practically for the estimation of the deviation of the T -second median.

B. Deviation of the Narrow Area Median

As the analysis model, we consider the variance of the measured value when the long term median is modulated lognormally. Let m_0 dB be the mean of the signal strength before the modulation. If $X_1(t)$ dB is the median superimposed by the modulation, the composite output signal is expressed as

$$X_2(t) = m_0 + X_1(t), \quad (11)$$

where we suppose that $X_1(t)$ is the Gaussian noise with mean

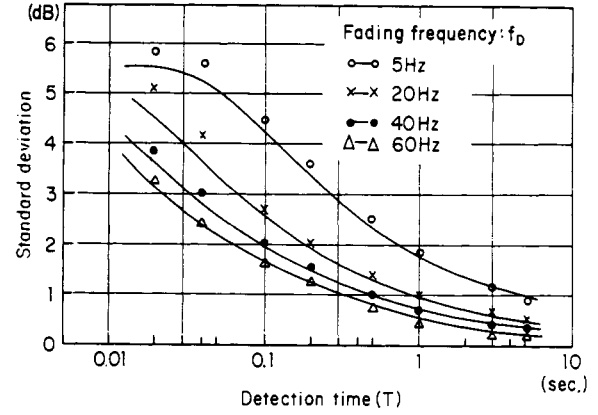


Fig. 2. Standard deviation of T -second mean and T -second median.

$\langle X_1(t) \rangle = 0$, variance σ_S^2 , and spectral density $S_{X_1}(f)$ as

$$S_{X_1}(f) = \begin{cases} \sigma_S^2 / (2 \cdot f_m), & |f| \leq f_m \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

where f_m is the maximum frequency of the spectral density. Because the narrow area median varies much more slowly than the instantaneous variation, f_m is much smaller than f_D . The autocorrelation function $R_{X_2}(\tau)$ of $X_2(t)$ is given by

$$R_{X_2}(\tau) = \langle X_2(t) \cdot X_2(t + \tau) \rangle = R_{X_1}(\tau) + m_0^2 \quad (13)$$

where $R_{X_1}(\tau)$ is the autocorrelation function of $X_1(t)$ and is given by

$$R_{X_1}(\tau) = \int_{-\infty}^{\infty} S_{X_1}(f) e^{j2\pi f\tau} df = \sigma_S^2 \frac{\sin(2\pi f_m \tau)}{2\pi f_m \tau}. \quad (14)$$

Therefore, the variance of the T -second median σ_{ST}^2 is obtained by using (3):

$$\sigma_{ST}^2 = \frac{\sigma_S^2}{T} \int_{-T}^T \left(1 - \frac{|\tau|}{T}\right) \frac{\sin(2\pi f_m \tau)}{2\pi f_m \tau} d\tau. \quad (15)$$

The practically measured deviation is composed of the deviations of instantaneous variation and narrow area median. The T -second medians of the instantaneous variation and the narrow area median are independent. If the distribution of the T -second median of the instantaneous variation is lognormal, the variance of the measured value σ_T^2 can be written

$$\sigma_T^2 = \sigma_{MT}^2 + \sigma_{ST}^2, \quad (16)$$

where σ_{MT}^2 and σ_{ST}^2 are given by (8) and (15), respectively.

C. Simulation Results

The simulation block diagram is shown in Fig. 3. The propagation path is composed of an attenuator and a fading simula-

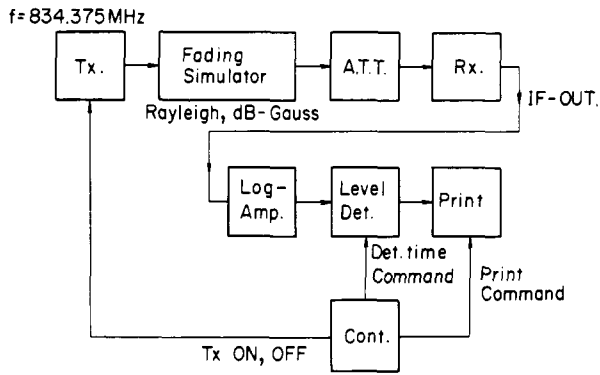


Fig. 3. Simulation block diagram.

tor which can simulate Rayleigh and lognormal fading [13]. The transmitter T_x sends a continuous wave on 834.375 MHz. After fading, this wave is received by the receiver R_x . The IF output signal of the R_x is compressed by a logarithmic circuit, log-amp, and then becomes the input signal to the level detection circuit. The level detection circuit consists of a high-speed sampler, a level detector, and counters. The clock speed of the sampler is $10^6 \times 1/T$ (Hz), where T is the detection time. The level detector detects the input signal level in $32 (= 2^5)$ steps and operates the counter corresponding to the detected step. At the end of the detection time, the counter's values are cumulated by another counter, and the T -second median value is coded to a 5-bit decimal code. The control unit makes the level detection circuit repeat the above action about 10^4 times. When the time is over, the control unit calculates the standard deviation of the detected T -second median values. The results are printed out in accordance with the orders of the control unit.

The simulation results are shown in Fig. 4. The solid lines are obtained by using (16). This figure shows that (16) can be applied to the estimation of the T -second variance for $T > 0.02$ s.

IV. THE ZSR IN A CELLULAR SYSTEM

A. Zone Selection Rate

The zone detection error in using signal strength measurements is due to the deviation of the measured T -second median. To reduce this error it is necessary to keep the deviation σ_T as small as possible by increasing the detection time T . However, to increase the efficiency of the mobile location system, it is necessary to reduce the time T .

As a simple example we take a zone detection error in a two radio zone (A and B zone) system. The experiment for this model was carried out by using the simulator equipment. To estimate the maximum error we supposed that the received signals were uncorrelated. The ZSR of the A zone P_A is shown in Fig. 5. P_A is a function of the variances σ_A^2 and σ_B^2 and the value ΔE which is the difference between the long term median values received at the base stations. This figure shows that the smaller the variances σ_A^2 and σ_B^2 are, the smaller the error will be. If $\sigma_A = \sigma_B = 0$, then, $P_A = 100$ percent for $\Delta E \geq 0$ and $P_A = \text{zero percent}$ for $\Delta E \leq 0$. For

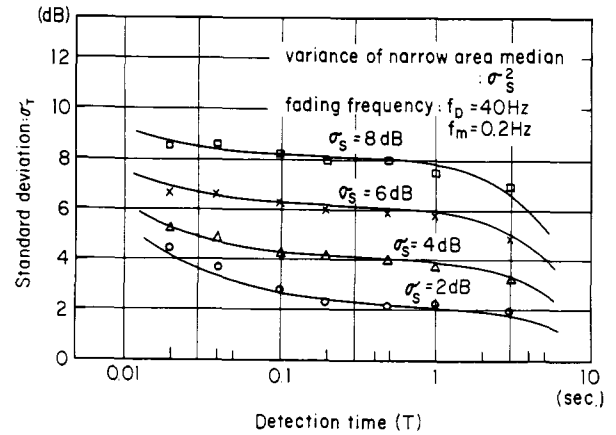
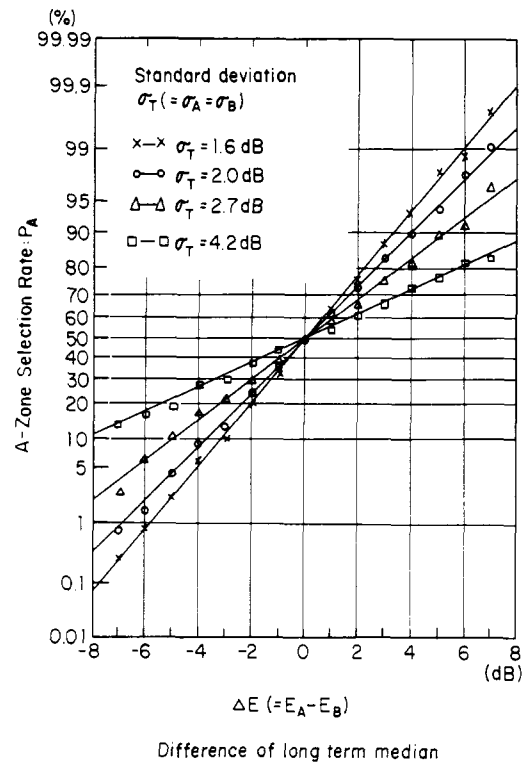


Fig. 4. Standard deviation of detected value.

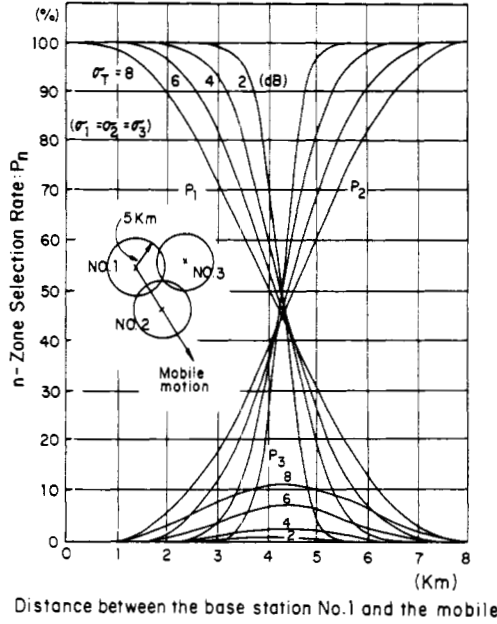
Fig. 5. A-ZSR: P_A .

example, if $\Delta E = 4$ dB and $\sigma_T = 2$ dB, P_A is 90 percent. This means that ten percent of the mobiles in the A zone are judged to exist in the B zone. Therefore, ZSR can be used to estimate the zone detection accuracy.

B. Application to the Cellular System

Suppose that there are N base stations and a mobile exists at a point X . The n -ZSR $P_n(X)$ is obtained by considering whether the median signal strength at base station n is larger than the medians at other stations. Thus if the received signals are uncorrelated, then

$$P_n(X) = \int_{-\infty}^{\infty} p_n(E) \prod_{\substack{m=0 \\ m \neq n}}^N \left[\int_{-\infty}^E p_m(E_X) dE_X \right] dE, \quad (17)$$

Fig. 6. N -ZSR: P_n .

where $p_k(E)$ is the probability density function of the median received at the k base station. Since the measured median distributes lognormally, $p_k(E)$ can be written as

$$p_k(E) = \frac{1}{\sqrt{2\pi}\sigma_k} \exp \left\{ -\frac{(E_k(X_k) - E)^2}{2\sigma_k^2} \right\} \quad (18)$$

where

- X_k distance between the mobile and the k base station,
- $E_k(X_k)$ long term median received at the k base station,
- σ_k standard deviation about the $E_k(X_k)$, given by (16).

Substituting (18) into (17) we get

$$P_n(X) = \frac{1}{2^{N-1}\sqrt{\pi}} \int_{-\infty}^{\infty} \prod_{\substack{m=0 \\ m \neq n}}^N \left[1 \pm \operatorname{erf} \left(\frac{\sigma_n}{\sigma_m} \left| y - \frac{E_m(X_m) - E_n(X_n)}{\sqrt{2}\sigma_n} \right| \right) \right] e^{-y^2} dy \quad (19)$$

where \pm correspond to $y \geq (E_m - E_n)/\sqrt{2}\sigma_n$, respectively, and $\operatorname{erf}(X)$ is defined by

$$\operatorname{erf}(X) = \frac{2}{\sqrt{\pi}} \int_0^X e^{-t^2} dt.$$

From (19), therefore, the estimated value of ZSR can be calculated by using the propagation curve which gives the long term median and σ_k , which is given by (16). As a simple example, Fig. 6 shows ZSR for a three-base-station model calculated by using the standard urban area propagation curves [9]. On the other hand, if the actually measured propagation data were given, we could estimate ZSR by measuring the long term median and the deviation σ_S of the narrow area median.

V. MOBILE LOCATION EXPERIMENT

In order to confirm our estimation of ZSR we examined the propagation characteristics obtained from a field test in the Tokyo metropolitan area in May 1977, and simulated mobile location using these propagation data.

A. Field Test

Three base stations and a mobile were used in the field test. The arrangement of the base stations and the running courses are shown in Fig. 7. The field test block diagram is shown in Fig. 8. The base stations (Chiyoda, Ekoda, and Karagasaki) are about 10 km away from each other in an urban area. Each base station consists of an omnidirectional antenna (gain = 12 dB) and a signal strength receiver. The received signals are sent to the Musashino Mobile Center using an analog data transmission network. The antenna heights of Chiyoda, Ekoda, and Karagasaki are 140 m, 45 m, and 90 m, respectively. A 5-W mobile with a 0-dB vertical monopole antenna continuously transmits an unmodulated carrier wave at 834.425 MHz.

B. Data Processing and the Experiment

The running courses were divided into 37 segments of 1-km length in order to obtain the propagation characteristic factors. For the short thick lines of Fig. 7, we measured the long term median E_L , the median of about 20 m for the narrow area median, and the deviation σ_S and the correlation coefficient ρ_{mn} for each segment.

To calculate ZSR by using (19) the detection time T and the deviation σ_T must be determined. The total time for the location sequence of the cellular system in Japan is about 0.5 s, and the detection time is about 0.1 s [14]. Therefore, we used the deviation $\sigma_{0.1}$.

The mobile location experiment was carried out for a 0.1-s detection time using the location simulator which executed the location sequence and calculated ZSR. About 400 data of zone selection were obtained for each segment, and ZSR was calculated for three base stations.

C. Results

The examples of the propagation factors are shown in Table I corresponding to the segment number in Fig. 7. The deviation σ_S obtained in our field test was near the value of 4.5 dB reported by Kozono [15]. Since the received signals were uncorrelated, we used (19) for the estimation of ZSR. The experimental and estimated values of ZSR are shown in Table II and Fig. 9.

In this figure we regard for convenience that the mobile run between two base stations is as shown by the dashed lines in Fig. 7. The value of the transverse axis shows the received signal level at the base station from which the mobile started. Therefore, the values for routes A, B, and C are the signal levels at Chiyoda, Ekoda, and Karagasaki, respectively. For example, when we take route A, the mobile runs from the Chiyoda zone to the Ekoda zone. Accordingly, as the mobile moves, the signal level at Chiyoda becomes small and the signal level at Ekoda becomes large. In this case, Chiyoda's rate

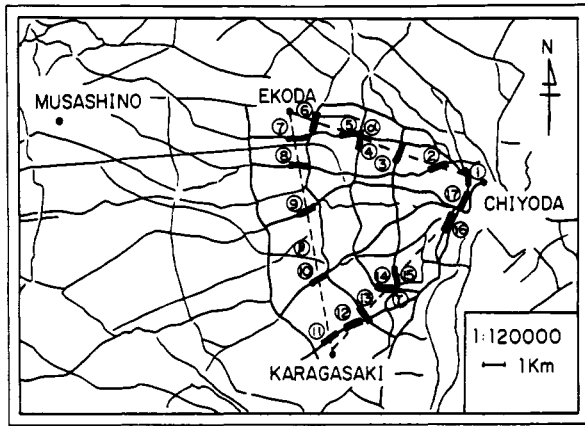


Fig. 7. Arrangement of base stations.

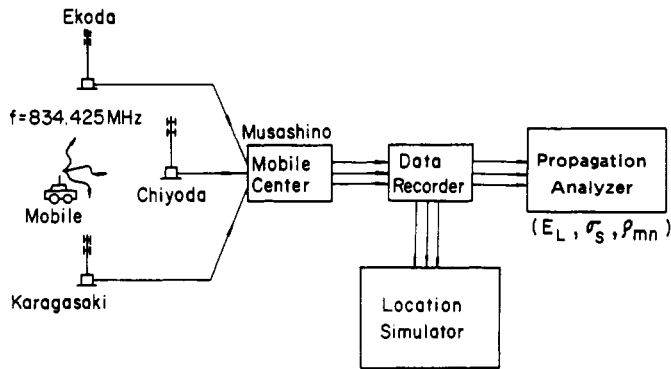


Fig. 8. Block diagram of field test.

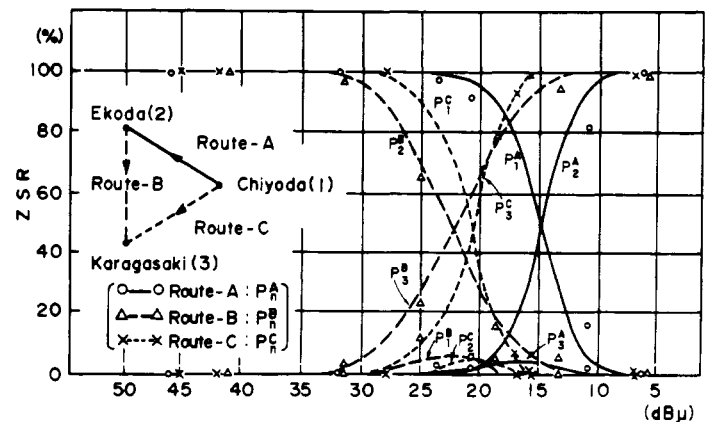
TABLE I
PROPAGATION FACTORS

Seg- ment No.	Chiyoda			Ekoda			Karagasaki		
	E_{L1} (dBμ)	σ_S (dB)	σ_{01} (dB)	E_{L2} (dBμ)	σ_S (dB)	σ_{01} (dB)	E_{L3} (dBμ)	σ_S (dB)	σ_{01} (dB)
1	45.9	5.9	6.3	3.0	4.5	5.0	14.7	5.4	5.7
2	32.1	3.1	3.6	8.2	3.8	4.2	4.1	3.5	4.0
3	23.8	4.8	5.2	8.9	5.9	6.3	1.3	2.0	2.6
4	20.4	4.9	5.3	6.9	2.0	2.6	6.2	4.8	5.2
5	10.9	4.1	4.6	23.9	7.2	7.5	0*	0*	0*
6	6.2	3.6	4.1	37.0	4.0	4.5	1.7	1.8	2.4
7	12.6	4.9	5.3	41.5	4.3	4.8	4.5	3.9	4.4
8	23.0	2.4	3.0	32.5	3.1	3.6	17.8	2.5	3.6
9	16.6	3.6	4.1	24.8	4.5	5.0	21.5	5.1	5.4
10	10.3	5.9	6.3	18.6	5.6	6.0	25.8	6.8	7.2
11	10.4	5.2	5.5	13.5	1.5	2.1	30.0	4.7	5.1
12	7.0	5.4	5.7	5.8	2.5	3.1	30.3	5.4	5.7
13	15.8	3.0	3.5	10.8	3.1	3.5	29.6	1.4	2.0
14	17.2	2.4	3.0	0*	0*	0*	25.0	3.1	3.6
15	28.1	2.2	2.9	0*	0*	0*	20.3	2.9	3.5
16	42.0	3.7	4.1	10.1	6.5	6.9	11.5	3.8	4.2
17	45.2	6.2	6.6	5.4	6.0	6.4	17.6	4.2	4.8

* received signal level was below the noise level

TABLE II
ZSR

Seg- ment NO	Chiyoda (%)		Ekoda (%)		Karagasaki (%)	
	experiment	estimation	experiment	estimation	experiment	estimation
1	100	100	0	0	0	0
2	100	100	0	0	0	0
3	98	99	2	1	0	0
4	92	97	6	1	2	1
5	17	5	81	95	2	0
6	0	0	100	100	0	0
7	0	0	100	100	0	0
8	3	2	97	98	0	0
9	12	4	65	68	23	28
10	5	4	16	18	79	78
11	0	0	5	1	95	99
12	1	0	0	0	99	100
13	1	0	0	0	99	100
14	7	6	0	1	93	93
15	100	98	0	0	0	2
16	100	100	0	0	0	0
17	100	100	0	0	0	0



Signal strength of the base station which the mobile started

Fig. 9. ZSR for field test data.

P_1^A , Ekoda's rate P_2^A , and Karagasaki's rate P_3^A are plotted as circles. The estimated values are shown as lines connecting the values at each segment. The crossing points of the lines, where ZSR is near 50 percent, were estimated by using the propagation factors at each segment. For example, take segments 4 and 5 adjoining each other. At segment 4, from Table I, Chiyoda's level E_{L1} is larger than Ekoda's level E_{L2} , but at segment 5 this relation turned upside down. Each station's mean value of deviation between these segments is about 5 dB. Therefore, we suppose that P_1^A and P_2^A will become the same value at the middle level between E_{L1} (value of segment 4) and E_{L2} (value of segment 5), and Karagasaki's level E_{L3} will be the mean value between at each segment. The points α , β , and γ in Fig. 7 correspond to the crossing points of the lines in Fig. 9.

Fig. 9 or Table II shows that the difference between the estimated ZSR given by (19) and the experimental ZSR is

about eight percent. Therefore, we have a good estimation of the zone detection accuracy, namely ZSR, by using (19) and the propagation data at the point of interest.

VI. CONCLUSION

A mobile location technique is necessary for a cellular mobile telephone system to be operated efficiently. We considered the signal strength measurement method of this location technique and showed that ZSR, which is available to a system design, can be calculated by using the long term median E_L and the deviation σ_S of narrow area median in the mobile radio propagation characteristics. For the estimation of E_L and σ_S , the methods of Okumura [9] and Kozono [15] are practicable in this location technique.

In the design of a cellular system, ZSR is used to estimate

- a) the probability that a base station is selected for a channel set up for a call,
- b) speech channel traffic of a base station,
- c) average speech quality at a base station and speech quality of a call.

Furthermore, if the probability density function of the deterioration frequency for the SNR of a speech channel was given, the handoff frequency could be estimated by using ZSR.

APPENDIX

The autocorrelation function $R_y(\tau)$ is given by

$$R_y(\tau) = \langle y(t+\tau) \cdot y(t) \rangle \\ = \int_0^\infty \int_0^\infty f(x_1)f(x_2)p(x_1, x_2) dx_1 dx_2 \quad (A1)$$

where $x_1 = x(t)$, $x_2 = x(t + \tau)$, and $p(x_1, x_2)$ is their joint probability density function. Since $x(t)$ is Rayleigh distributed, we get $p(x_1, x_2)$ by using the orthogonal expansion [16]:

$$p(x_1, x_2) = \frac{x_1 x_2}{\sigma^4(1-\rho^2)} \\ \exp \left\{ -\frac{x_1^2 + x_2^2}{2\sigma^2(1-\rho^2)} \right\} I_0 \left\{ \frac{x_1 x_2}{\sigma^2(1-\rho^2)} \right\} \\ = p(x_1)p(x_2) \sum_{m=0}^{\infty} L_m \left(\frac{x_1^2}{2\sigma^2} \right) L_m \left(\frac{x_2^2}{2\sigma^2} \right) \rho^{2m} \quad (A2)$$

where

- ρ normalized autocorrelation function of narrow-band Gaussian noise,
- σ^2 variance of the narrow-band Gaussian noise,
- I_0 zero-order modified Bessel function of the first kind,
- L_m zero-order Laguerre function,

$$p(x) = \frac{x}{\sigma^2} \exp \left(-\frac{x^2}{2\sigma^2} \right).$$

Then

$$R_y(\tau) = \int_0^\infty \int_0^\infty f(x_1)f(x_2)p(x_1)p(x_2) \sum_{m=0}^{\infty} L_m \left(\frac{x_1^2}{2\sigma^2} \right) \\ \times L_m \left(\frac{x_2^2}{2\sigma^2} \right) \rho^{2m} dx_1 dx_2 = \sum_{m=0}^{\infty} \alpha_m^2 \rho^{2m} \quad (A3)$$

$$\alpha_m = \int_0^\infty f(x) \frac{x}{\sigma^2} e^{-x^2/2\sigma^2} L_m \left(\frac{x^2}{2\sigma^2} \right) dx. \quad (A4)$$

Using (5), the coefficients are given as follows:

$$\alpha_0 = -a\gamma + a \log_e (2\sigma^2) + b \quad (A5)$$

$$\alpha_m = a \int_0^\infty (\log_e x) e^{-x} L_m(x) dx, \quad (m \neq 0) \quad (A6)$$

where γ is Euler's constant. Substituting this result into (A3) we get

$$R_y(\tau) = \alpha_0^2 + a^2(\rho^2(\tau) + 2.498 \times 10^{-1} \times \rho^4(\tau) \\ + 1.108 \times 10^{-1} \times \rho^6(\tau) + \dots) \quad (A7)$$

where α_0 is equal to the mean after the transformation of a random variable. Therefore, using (3), we get (6).

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REFERENCES

- [1] W. G. Figel, N. H. Shepherd, and W. F. Trammel, "Vehicle location by a signal attenuation method," *IEEE Trans. Veh. Technol.*, vol. VT-18, Nov. 1969.
- [2] P. T. Porter, "Supervision and control features of a small-zone radio telephone system," *IEEE Trans. Veh. Technol.*, vol. VT-20, Aug. 1971.
- [3] G. D. Ott, "Vehicle location in cellular mobile radio systems," *IEEE Trans. Veh. Technol.*, vol. VT-26, Feb. 1977.
- [4] H. Staras and S. N. Honickman, "The accuracy of vehicle location by trilateration in a dense urban environment," *IEEE Trans. Veh. Technol.*, vol. VT-21, Feb. 1972.
- [5] T. Nagatsu, "Vehicle locating by field detection in the small-cell land mobile system," *Rev. E.C.L., NTT, Japan*, vol. 21, 1973.
- [6] T. Nomura and N. Yoshikawa, "Multiple radio plans in mobile radio systems," *IEEE Trans. Veh. Technol.*, vol. VT-25, Aug. 1976.
- [7] N. Yoshikawa and T. Nomura, "On the design of a small zone land mobile radio system in UHF band," *IEEE Trans. Veh. Technol.*, vol. VT-25, Aug. 1976.
- [8] F. Ikegami, "Mobile radio communication in Japan," *IEEE Trans. Commun.*, vol. COM-20, 1972.
- [9] Y. Okumura *et al.*, "Field strength and its variability in VHF and UHF land mobile radio service," *Rev. E.C.L., NTT, Japan*, vol. 16, 1968.
- [10] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*. New York: McGraw-Hill, 1965.
- [11] M. J. Gans, "A power-spectral theory of propagation in the mobile-radio environment," *IEEE Trans. Veh. Technol.*, vol. VT-21, Feb. 1972.
- [12] W. C. Y. Lee and Y. S. Yeh, "On the estimation of the second-order statistics of log normal fading in mobile radio environment," *IEEE Trans. Commun.*, COM-22, June 1974.
- [13] K. Hirade *et al.*, "Fading simulator for land mobile radio communication," *Trans. IECE Japan*, vol. 58-B, Sept. 1975.

- [14] T. Nagatsu *et al.*, "Base station radio equipment for 800 MHz band land mobile telephone system," *Rev. E.C.L.*, NTT, Japan, vol. 25, 1977.
- [15] S. Kozono and K. Watanabe, "Influence of environmental buildings on UHF land mobile radio propagation," *IEEE Trans. Commun.*, vol. COM-25, Oct. 1977.
- [16] W. Magnus *et al.*, *Formulas and Theorems for the Special Functions of Mathematical Physics*. Springer-Verlag, 1966, p. 242.



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Studies of Base-Station Antenna Height Effects on Mobile Radio

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Abstract—As is well known, a base-station antenna height gain factor of 6 dB/octave has been predicted theoretically for signal path loss over flat ground and has been verified by measured data. However, the 6-dB/octave rule for antenna height effect cannot be used to predict signal strength for terrain contours if the terrain is not flat. A model has been developed for waves propagating over a nonflat ground which allows the antenna height effect to be predicted in different types of actual terrain contours. In the model, the actual terrain profile is classified as one of two different kinds of general terrain types. The relative received power due to the actual terrain path contour is predicted by considering the reflection points of the waves along the path. Experimental data have been used to verify the theoretically estimated results and they show good agreement.

I. INTRODUCTION

AS IS WELL KNOWN, a base-station antenna height gain factor of 6 dB/octave (i.e., doubling the antenna height increases signal level by 6 dB) has been predicted theoretically for path loss over flat ground [1]–[3]. The measurements [4], [5] in flat suburban and urban areas have generally agreed with this fact. It is observed from the measured data collected in hilly areas, however, the 6-dB/octave rule for antenna height effect cannot be used to predict signal strength

for terrain contour if the terrain is not flat. Since the model [6]–[8] used to obtain 6 dB/octave for the antenna height effect at the base station is generally workable for terrestrial propagations, we have only to check if this model can be applied to a mobile radio environment.

II. DESCRIPTION OF AN EXISTING MODEL FOR FLAT TERRAIN

In this section an existing model [8], [9] used for flat terrestrial propagation is examined for mobile radio reception. Assume that a base-station transmitter and a mobile receiver are separated by a large distance d , and the terrain between the two sites is flat as shown in Fig. 1. Three possible kinds of waves may occur at the mobile receiver: a direct wave, a reflected wave, and a surface wave. The resultant received signal power [8], [9] is then

$$P_r = P_t g_1 g_2 \left(\frac{\lambda}{4\pi d} \right)^2 \cdot \underbrace{|1 + \rho e^{j\Delta}|^2}_{\text{reflected wave}} + \underbrace{|\eta|^2}_{\text{surface wave}} \quad (1)$$

where

P_t transmitting power into the antenna,
 g_1 gain of the base antenna,

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