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# PASSIVE LOCALIZATION FROM DOPPLER SHIFTED FREQUENCY MEASUREMENTS

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## ABSTRACT

The position and speed of a moving source that radiates a pure tone can be determined by measurements of Doppler-shifted frequencies at five or more different locations. The equations relating the measured frequencies to the localization parameters (i.e., the five unknowns of $x-y$ position and speed and rest tone frequency) are nonlinear and there is no closed-form solution. Besides iterative methods, non-linear equations can also be solved by a grid search, whereby all possible combination of source parameters are entered into the equation to see if they constitute a solution. This would require a five-dimensional search. This paper introduces a more efficient search scheme by introducing intermediate variables which are products of some of the unknowns. The search dimension is only two, in $x-y$ position, instead of five. The resultant intermediate equations become linear and are easier to solve; the major calculation at each grid point requires only an inversion of a 3x3 matrix. The Cramer-Rao bound for localization is also given, together with simulation results that verify the method.

## I INTRODUCTION

When an object radiates a constant frequency tone, it is possible to determine its position and velocity through a measurement of its Doppler-shifted tone. An application example is in the passive localization of an underwater radiating source by sonobuoys (sensors). These sensors intercept the acoustic radiations and measure the Doppler-shifted frequency (DSF), as illustrated in Figure 1. For simultaneous localization, meaning localization based on a single DSF measurement for each sensor at the same instant, at least 5 separate sensors [1] are required to provide 5 independent measurements to solve for the 5 parameters of $x, y, \dot{x}, \dot{y}, f_0$, denoting respectively the source's position and velocity in rectangular coordinates and the rest tone frequency. If the number of sensors in simultaneous contact with the source is less than 5, then localization has to be performed over several measurement instants, assuming the source is on a non-maneuvering course [2].

Due to the nonlinear relationship (see equation (1) in the next section) between the unknowns and measurements, no closed form solution exists for this problem. Most solution methods [2], [3], [4] are iterative, choosing successive values of the unknowns to minimize a cost function. When the time differences of arrival (TDOA) of a signal at the sensors are known, together with the DSF, a closed form solution is available [5]. However, in the case of a source radiating a pure tone, TDOA measurements are highly ambiguous and cannot be used. If $f_0$ is known, [6] gives an exact solution for a single sensor measuring three DSF over time and the determining the source parameters of range, velocity, and the angle between them, but not the relative bearing between source and sensor. The source is assumed to be non-maneuvering.

In [7], a grid search technique is proposed. By introducing the variable of frequency rate into the localization equation, the grid search dimension is only three, in $x, y$, and $f_0$. The present work gives another grid search solution to the simultaneous localization problem, i.e., at least 5 simultaneous DSF measurements. The novelty of the method lies in treating the products $f_0\dot{x}$ and $f_0\dot{y}$ as intermediate unknowns, thereby resulting in a quick and simple solution to the intermediate equations. The number of grid search parameters is also reduced to two of $x$ and $y$. In the remainder of this paper, section II gives a description of localization by DSF measurements and the proposed grid search scheme. The Cramer-Rao lower bound (CRLB) for DSF localization is also derived. Simulation results that verify the theoretical development are provided in section III. The conclusions are drawn in section IV.

## II PROBLEM FORMULATION AND SOLUTION

### A. The Grid Search Technique

Referring to Figure 1, a source moving at velocity $V$ is radiating a pure tone frequency $f_0$. There are $N$ sensors placed at random but known positions $(x_i, y_i), i = 1, \ldots, N$. Each sensor intercepts this tone and measures a DSF:

$$f_i = f_0\left(1 + \frac{V\cos\alpha_i}{c}\right) + \epsilon_i \tag{1}$$

In (1), $f_i$ is the DSF measured at the ith sensor, $c$ is the speed of travel of the signal, $\alpha_i$ is the angle between the range and velocity vectors, and $\epsilon_i$ is the random variable denoting the errors in the DSF measurement. The problem is to determine the source parameters $x, y, \dot{x}, \dot{y}, f_0$ from $f_i$. Rewritten in rectangular coordinates, (1) becomes:

$$f_i = f_0(1 + \dot{x}K_i + \dot{y}M_i) + \epsilon_i \tag{2}$$

where:

$$K_i = \frac{x - x_i}{cr_i} \tag{3}$$

$$M_i = \frac{y - y_i}{cr_i} \tag{4}$$

$$r_i = [(x - x_i)^2 + (y - y_i)^2]^{1/2} \tag{5}$$

Although $f_0$ is not required for localization, it is nevertheless an unknown in (2) and therefore at least five $f_i$ from five sensors are needed to ensure observability, i.e., a determined solution.

The measurement of $f_i$ is not addressed here. Typically the DSF is obtained from a frequency line tracker [8]–[11] which performs a spectral analysis on the received signal. With the additional assumption that the source is non-maneuvering, a line tracker can give a filtered estimate of DSF through, for example, a Kalman filtering of the DSF measurements.



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Given $f_i$, there is no closed form solution for the source parameters from (2). Using nonlinear minimization algorithms is not simple since there are five unknowns and there is no guarantee of convergence to the global minimum. Many initial conditions need to be tried. Grid searching, whereby all possible values of the unknowns, within a chosen grid, are checked to see if they satisfy (2), is an alternative that circumvents the local minimum difficulty. The computations may be impractical if the search is in five dimensions. However, as will be demonstrated next, by considering $f_0\dot{x}$ and $f_0\dot{y}$ as intermediate unknowns, the search dimension becomes only two, in $x$ and $y$.

For $i = 1, \ldots, N$ and for a given grid point $(x, y)$ as a possible source position, (2) appears in matrix form as:

$$\zeta = Ag + \theta \tag{6}$$

where the $N$x3 matrix:

$$A = \begin{bmatrix} K_1 & M_1 & 1 \\ \vdots & \vdots & \vdots \\ K_i & M_i & 1 \\ \vdots & \vdots & \vdots \\ K_N & K_N & 1 \end{bmatrix} \tag{7}$$

the $N$x1 measurement vector is:

$$\zeta^T = [f_1, \ldots, f_i, \ldots, f_N] \tag{8}$$

the 3x1 unknown vector is:

$$g^T = [f_0\dot{x}, f_0\dot{y}, f_0] \tag{9}$$

and the $N$x1 error vector is:

$$\theta^T = [\epsilon_1, \ldots, \epsilon_i, \ldots, \epsilon_n] \tag{10}$$

The generalized inverse [12] solution to (6) is:

$$g = (A^TA)^{-1}A^T\zeta \tag{11}$$

Hence we obtain, for a given grid point $(x, y)$ and the measured $\zeta$, the values of $f_0$, $\dot{x}$, $\dot{y}$ that best fit (2) in the least square sense [12].

Let

$$J = J(x, y) = \| Ag - \zeta \|^2 \tag{12}$$

be the sum of the mean square fitting errors of (6) for a given $(x, y)$. Since (6) is an overdetermined equation ($N \geq 5$), $J$ in general is not zero, even if $\theta = 0$. Indeed, if $\theta = 0$ and $J = 0$, then that $(x, y)$ is either the exact location of the source or is one of the non-unique (or ghost) solutions of the nonlinear equation. The basis for the grid search is now clear. We compute (12) for all points $(x, y)$ on a grid and select a set, say five or six of $(x, y)$ whose $J(x, y)$ are the lowest, as possible solutions. The reasons for not selecting the $(x, y)$ with lowest $J$ are that (i), there are generally ghost solutions and (ii), due to noisy conditions or coarse grid spacing, the $(x, y)$ closest to the source does not necessarily have the lowest $J$.

The desired solution can be determined from this set either from *a priori* information on bounds of source location, or from addition measurements such as bearing (available in some sensors); or if a track has been established, from the proximity to the predicted source position. Failing all these, the only solution is to keep the solution sets over several localization periods. Then the correct solutions can be determined from the ghost solutions by a track sort algorithm [7] that is based on a non-maneuvering source assumption. Certainly, as will be seen in Section III, increasing the number of sensors (though this may be difficult to realize in practice) will also reduce ghost solutions. For this will then reduce the degree of redundancy in (1).

It is noted that the grid search requirement is rather modest; the major calculation being the inversion of a 3x3 matrix in (11) at each grid point. Applying known physical constraints will also help to reduce the computations. For example, in sonar, the grid size is at most a few kilometers on a side at the beginning of a track, and as the track develops, the search area will decrease to a region as determined by the variance of the tracker. The grid resolution, which determines the number of points to be evaluated on the grid, need not be much finer than the CRLB of the estimator or the tolerance limits on the sensor positions. Values of $f_0$, $\dot{x}$, $\dot{y}$ obtained from (11) must also be within some physical constraints; otherwise there is no need to proceed further to evaluate (12) for that particular grid point. To establish these limits, let $V_{max}$ be the expected maximum source speed. Then $\dot{x}$ and $\dot{y}$ must satisfy:

$$\dot{x}^2 + \dot{y}^2 \leq V_{max}^2 \tag{13}$$

Next, let $f_{min}$ and $f_{max}$ be the minimum and maximum frequencies among the $f_i$. Then from (1) the limits for $f_0$ are at $a_i = \pm 180°$:

$$\frac{f_{min}}{1 - V_{max}/c} \leq f_0 \leq \frac{f_{max}}{1 + V_{max}/c} \tag{14}$$

If a grid point $(x, y)$ produces from (11) values of $f_0$, $\dot{x}$, $\dot{y}$ that violate (13) or (14), that point is not a valid solution.

## B. The CRLB

It is rather straightforward to find the CRLB for the estimation of:

$$[x, \dot{x}, y, \dot{y}, f_o] = [\theta_1, \ldots, \theta_5] \tag{15}$$

In (2), let the $\epsilon_i$ be independent zero-mean Gaussian random variables of variance $\sigma^2$. Then the joint probability density function of the $f_i$ is then:

$$p(f_1, \ldots, f_N) = \frac{1}{2\pi\sigma^2} e^{\frac{\sum (f_i - \bar{f}_i)^2}{2\sigma^2}} \tag{16}$$

where:

$$\bar{f}_i = E[f_i] = f_0(1 + \dot{x}K_i + \dot{y}M_i) \tag{17}$$

and for notational convenience the summation in (16) and the sequel is understood to be from $i = 1$ to $N$. Taking the logarithm of (16) yields:

$$\ln(p) = -\frac{\sum f_i - \bar{f}_i}{2\sigma^2} - \frac{N}{2}\ln(2\sigma^2) \tag{18}$$

Taking the partial of (18) with respect to $\theta_j$ gives:

$$\frac{\partial \ln(p)}{\partial \sigma_j} = -\frac{\sum f_i - \bar{f}_i}{\sigma^2} \frac{\partial f}{\partial \sigma_j} \tag{19}$$

The $jk^{th}$ element of the Fisher Information Matrix (FIM) [13] is then:

$$FIM_{jk} = E\left\{ \frac{1}{\sigma^2} \sum_i (f_i - \bar{f}_i) \frac{\partial f_i}{\partial \theta_j} \frac{1}{\sigma^2} \sum_m (f_m - \bar{f}_m) \frac{\partial f_i}{\partial \theta_k} \right\} \tag{20}$$

On simplifying using the independent $\epsilon_i$ assumption, (20) simplifies to

$$FIM_{jk} = \frac{1}{\sigma^2} \sum_i \frac{\partial f_i}{\partial \theta_j} \frac{\partial f_i}{\partial \theta_k} \tag{21}$$

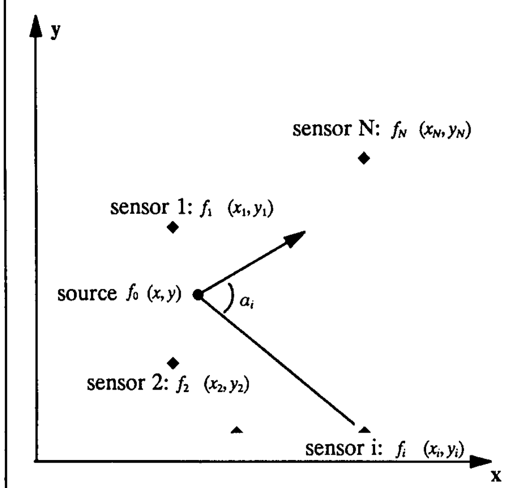


Figure 1- **Passive Localization**

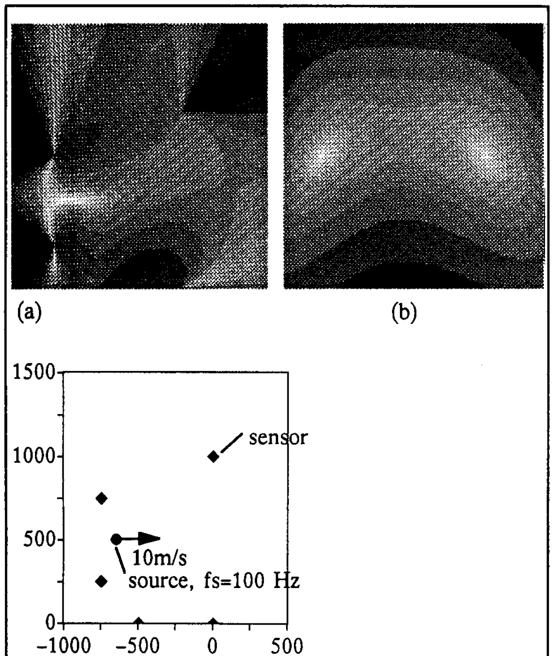


Figure 2- **Plot of J for 5 Sensors**

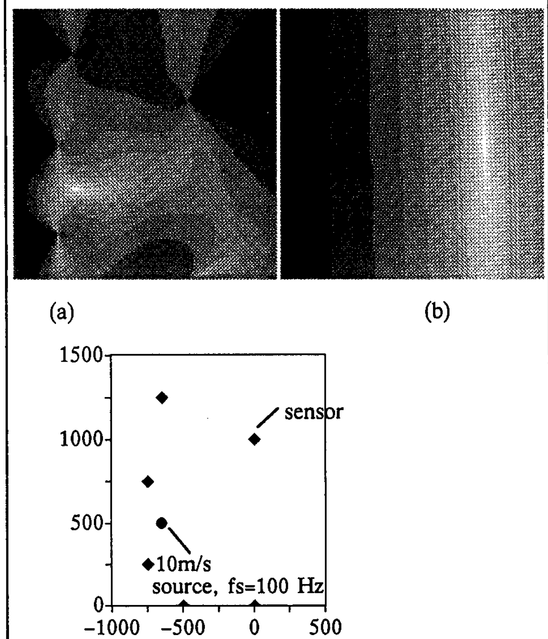


Figure 3- **Plot of J for 6 Sensors**

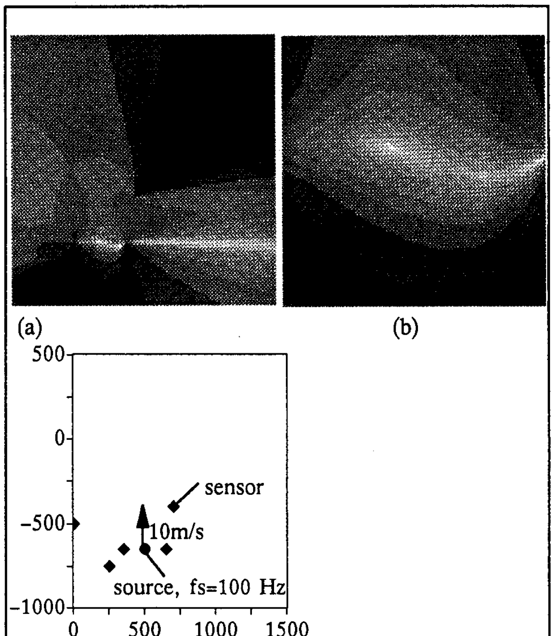


Figure 4- **Plot of J used in Simulations**

which can be evaluated from (2), given the $\theta_i$. The CRLB for $\theta_i$ is then $(\mathrm{FIM}^{-1})_{ii}$.

## III SIMULATION RESULTS

We have performed several experiments to verify the grid search technique and to study the properties of passive localization by DSF measurements.

In the first experiment, five sensors and a source are placed randomly as shown in Figure 2a. The source radiates an $f_0 = 100\mathrm{Hz}$ and travels at $\dot{x} = 10\mathrm{m/s}$, $\dot{y} = 0\mathrm{m/s}$. The grid area is 1500m x 1500m and is divided into 120 x 120 grid points with spacing 1500/119 m. The DSF, $f_i$, are computed according to (1), with $\epsilon_i = 0$. The sum of the squared errors, $J$ in (11), is represented by a 16 level gray scale with the higher $J$ values given a darker tone. It is seen in Figure 2a that there are several grid points whose $J \approx 0$, including of course the point at the source position. It turns out that, except for the points to the immediate left of the source (these are ghost solutions), the other $J \approx 0$ points produce answers that violate (13) or (14). An enlargement (the $x - y$ scales are different) of the region near the source is given in Figure 2b. To see the effects of having an additional sensor, six sensors are used in the second experiment. The plot of $J$ is in Figure 3a where it is seen that the previous (the five sensor case) ghost solution in Figure 2b has now disappeared. Again an enlargement of the region near the source is given in Figure 3b.

The third experiment compares the grid search localization errors with the CRLB. Since the comparison is meaningful only under small error conditions, the five sensor placement pattern is purposefully chosen so that it will not give rise to ghost solutions within the search area. Figure 4 is a plot of the source-sensor configuration and $J$. To increase the solution resolution, the grid point with the smallest $J$ is used as the starting point for a Levenberg-Marquard algorithm [14] which performs a nonlinear minimization of (12). The algorithm is arbitrarily stopped after 50 iterations and the $\theta_i$ are taken as the solution at that instant. From 300 independent runs, and various $\sigma$ (in Hz), the root mean square errors (RMSE) are computed and listed in Table 1, which also contains the CRLB standard deviations. It is seen that the grid search errors are very close to the CRLB when the noise levels are low.

## IV CONCLUSIONS

This paper has presented a new grid search technique for passive localization from DSF measurements only. Even though there are 5 unknown source parameters, the search is only in 2 dimensions in the $x$ and $y$ coordinates. This reduction in search dimensions is a consequence of treating two products of unknowns as intermediate variables. The computations for evaluation of fitting errors at each grid point are simple and can be further reduced through eliminating points that are outside physical bounds. Simulation results have verified the validity of the method and its performance at low noise levels is close to the CRLB.


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