



Perturbations to the Cosmic Microwave Background (<http://lambda.gsfc.nasa.gov>)

$$\Theta'_0 = -\frac{ck}{\mathcal{H}}\Theta_1 - \Phi'$$

$$\Theta'_1 = \frac{ck}{3\mathcal{H}}\Theta_0 - \frac{2ck}{3\mathcal{H}}\Theta_2 + \frac{ck}{3\mathcal{H}}\Psi + \tau' \left[\Theta_1 + \frac{1}{3}v_b \right],$$

$$\Theta'_\ell = \frac{\ell ck}{(2\ell+1)\mathcal{H}}\Theta_{\ell-1} - \frac{(\ell+1)ck}{(2\ell+1)\mathcal{H}}\Theta_{\ell+1} + \tau' \left[\Theta_\ell - \frac{1}{10}\Theta_\ell\delta_{\ell,2} \right], \quad \ell \geq 2$$

$$\delta' = \frac{ck}{\mathcal{H}}v - 3\Phi'$$

$$v' = -v - \frac{ck}{\mathcal{H}}\Psi$$

$$\delta'_b = \frac{ck}{\mathcal{H}}v_b - 3\Phi'$$

$$v'_b = -v_b - \frac{ck}{\mathcal{H}}\Psi + \tau'R(3\Theta_1 + v_b)$$

$$\Phi' = \Psi - \frac{c^2k^2}{3\mathcal{H}^2}\Phi + \frac{H_0^2}{2\mathcal{H}^2} \left[\Omega_m a^{-1}\delta + \Omega_b a^{-1}\delta_b + 4\Omega_r a^{-2}\Theta_0 \right]$$

$$\Psi = -\Phi - \frac{12H_0^2}{c^2k^2a^2}\Omega_r\Theta_2, \quad R = \frac{4\Omega_r}{3\Omega_b a}$$