Representation Theory in Quantum Computing

A Thesis

Presented to

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Bachelor of Arts

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Approved for the Committee
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I want to thank a few people.

Preface

This is an example of a thesis setup to use the reed thesis document class.

List of Abbreviations

You can always change the way your abbreviations are formatted. Play around with it yourself, use tables, or come to CUS if you'd like to change the way it looks. You can also completely remove this chapter if you have no need for a list of abbreviations. Here is an example of what this could look like:

ABC	American Broadcasting Company
CBS	Columbia Broadcasting System
CDC	Center for Disease Control
CIA	Central Intelligence Agency
CLBR	Center for Life Beyond Reed
CUS	Computer User Services
FBI	Federal Bureau of Investigation
NBC	National Broadcasting Corporation

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Abstract

The preface pretty much says it all.

The introduction, yay!

Introduction

Background

We first introduce quantum computing and representation theory.

0.1 Quantum Computing

Quantum computing exploits the nonclassical nature of quantum mechanics to achieve asymptotic improvements in runtime over classical algorithms. Not all problems admit a quantum speedup; we will be more specific about our specific computational framework after the neccesary background. To illuminate the physical-mathematical framework in which quantum computing takes place, we will expound some neccesary linear algebra and the postulates of quantum mechanics from [**?**][Chapter 2].

0.1.1 Quantum Mechanics

Postulate 0.1.1 ([**?**][2.2.1)**.**] Associated to any isolated physical system is a complex vector space with inner product (that is, a Hilbert space) known as the *state space* of the system. The system is completely described by its *state vector*, which is a unit vector in the system's state space.

Throughout this section we use the "bra-ket" notation for vectors of vector spaces, which we inherit from physics, which uses the symbols *ket·* called ket, *bra·* called bra, and *braket·|·* called braket (as we have bracketed the symbols);

$$|\psi\rangle$$

is a vector we have labeled ψ.

Definition 0.1.1 (Inner product, [**?**][2.1.4)**.**] A function $(\cdot, \cdot) : V \times V \to \mathbb{C}$ is a (complex) inner product if it satisfies the following requirements:

1. $(\cdot, \cdot)$ is linear in the second argument:

$$\left(|v\rangle, \sum_i \lambda_i |w_i\rangle \right) = \sum_i \lambda_i \left(|v\rangle, |w_i\rangle \right).$$

2. $(|v\rangle, |w\rangle) = (|v\rangle, |w\rangle)^*$.

3. $(|v\rangle, |v\rangle) \geq 0$ with equality if and and only if $|v\rangle = 0$.

The intuition of braket notation is that $\langle \phi|$ is the dual of the vector $|\phi\rangle$, which we notate in matrices turning a column vector into a row vector. Therefore taking the standard matrix product

$$\langle \phi| \, |\psi\rangle$$

is equivalent to the standard inner product $(|\phi\rangle, |psi\rangle) = \sum_{k=1}^{n} \phi_k^* \psi_k$, which we notate as

$$\langle \phi|\psi\rangle.$$

The elegance of braket notation, apart from saving space, is that we can easily tell the output of complicated linear expressions; it is immediate that $\langle \phi|\psi\rangle \langle x|y\rangle$ outputs a scalar, while it takes a little thinking to see that

$$x \quad \overset{x}{\underset{y}{y \, x}} \quad \overset{x}{\underset{y}{y}}$$

outputs a scalar.

In our quantum computing model we regard these physical systems as memory and perform operations on this memory through the appropriate mathematical morphism, which in this case are linear operations. A minor detail is that Hilbert spaces have more properties when they are infinite dimensional, but in our case we will only ever use finite memory and so our hilbert spaces are always finite dimensional.

Results

The First

This is the first page of the first chapter. You may delete the contents of this chapter so you can add your own text; it's just here to show you some examples.

0.2 References, Labels, Custom Commands and Footnotes

It is easy to refer to anything within your document using the `label` and `ref` tags. Labels must be unique and shouldn't use any odd characters; generally sticking to letters and numbers (no spaces) should be fine. Put the label on whatever you want to refer to, and put the reference where you want the reference. LaTeX will keep track of the chapter, section, and figure or table numbers for you.

0.2.1 References and Labels

Sometimes you'd like to refer to a table or figure, e.g. you can see in Figure 2 that you can rotate figures . Start by labeling your figure or table with the label command (`\label{labelvariable}`) below the caption (see the chapter on graphics and tables for examples). Then when you would like to refer to the table or figure, use the ref command (`\ref{labelvariable}`). Make sure your label variables are unique; you can't have two elements named "default." Also, since the reference command only puts the figure or table number, you will have to put "Table" or "Figure" as appropriate, as seen in the following examples:

As I showed in Table 1 many factors can be assumed to follow from inheritance. Also see the Figure 1 for an illustration.

0.2.2 Custom Commands

Are you sick of writing the same complex equation or phrase over and over?

The custom commands should be placed in the preamble, or at least prior to the first usage of the command. The structure of the `\newcommand` consists of the name of the new command in curly braces, the number of arguments to be made in square brackets and then, inside a new set of curly braces, the command(s) that make up the new command. The whole thing is sandwiched inside a larger set of curly braces.

In other words, if you want to make a shorthand for H_2SO_4, which doesn't include an argument, you would write: `\newcommand{\hydro}{H$_2$SO$_4$}` and then when you needed to use the command you would type `\hydro`. (sans verb and the equals sign brackets, if you're looking at the .tex version). For example: H_2SO_4

0.2.3   Footnotes and Endnotes

You might want to footnote something.[1] Be sure to leave no spaces between the word immediately preceding the footnote command and the command itself. The footnote will be in a smaller font and placed appropriately. Endnotes work in much the same way. More information can be found about both on the CUS site.

0.3   Bibliographies

Of course you will need to cite things, and you will probably accumulate an armful of sources. This is why BibTeX was created. For more information about BibTeX and bibliographies, see our CUS site (`web.reed.edu/cis/help/latex/index.html`). There are three pages on this topic: *bibtex* (which talks about using BibTeX, at `/latex/bibtex.html`), *bibtexstyles* (about how to find and use the bibliography style that best suits your needs, at `/latex/bibtexstyles.html`) and *bibman* (which covers how to make and maintain a bibliography by hand, without BibTeX, at at `/latex/bibman.html`). The last page will not be useful unless you have only a few sources. There used to be APA stuff here, but we don't need it since I've fixed this with my apa-good natbib style file.

0.3.1   Tips for Bibliographies

1. Like with thesis formatting, the sooner you start compiling your bibliography for something as large as thesis, the better. Typing in source after source is mind-numbing enough; do you really want to do it for hours on end in late April? Think of it as procrastination.

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[1]footnote text

2. The cite key (a citation's label) needs to be unique from the other entries.

3. When you have more than one author or editor, you need to separate each author's name by the word "and" e.g.
   `Author = {Noble, Sam and Youngberg, Jessica},`.

4. Bibliographies made using BibTeX (whether manually or using a manager) accept LaTeX markup, so you can italicize and add symbols as necessary.

5. To force capitalization in an article title or where all lowercase is generally used, bracket the capital letter in curly braces.

6. You can add a Reed Thesis citation option. The best way to do this is to use the phdthesis type of citation, and use the optional "type" field to enter "Reed thesis" or "Undergraduate thesis". Here's a test of Chicago, showing the second cite in a row being different. Also the second time not in a row should be different. Of course in other styles they'll all look the same.

0.4   Anything else?

If you'd like to see examples of other things in this template, please contact CUS (email cus@reed.edu) with your suggestions. We love to see people using LaTeX for their theses, and are happy to help.

Mathematics and Science

0.5   Math

T_EX is the best way to typeset mathematics. Donald Knuth designed T_EX when he got frustrated at how long it was taking the typesetters to finish his book, which contained a lot of mathematics.

If you are doing a thesis that will involve lots of math, you will want to read the following section which has been commented out. If you're not going to use math, skip over this next big red section. (It's red in the .tex file but does not show up in the .pdf.)

Tables and Graphics

0.6   Tables

The following section contains examples of tables, most of which have been commented out for brevity. (They will show up in the .tex document in red, but not at all in the .pdf). For more help in constructing a table (or anything else in this document), please see the LaTeX pages on the CUS site.

Table 1: Correlation of Inheritance Factors between Parents and Child

Factors	Correlation between Parents & Child	Inherited
Education	-0.49	Yes
Socio-Economic Status	0.28	Slight
Income	0.08	No
Family Size	0.19	Slight
Occupational Prestige	0.21	Slight

If you want to make a table that is longer than a page, you will want to use the longtable environment. Uncomment the table below to see an example, or see our online documentation.

0.7 Figures

If your thesis has a lot of figures, LaTeX might behave better for you than that other word processor. One thing that may be annoying is the way it handles "floats" like tables and figures. LaTeX will try to find the best place to put your object based on the text around it and until you're really, truly done writing you should just leave it where it lies. There are some optional arguments to the figure and table environments to specify where you want it to appear; see the comments in the first figure.

If you need a graphic or tabular material to be part of the text, you can just put it inline. If you need it to appear in the list of figures or tables, it should be placed in the floating environment.

To get a figure from StatView, JMP, SPSS or other statistics program into a figure, you can print to pdf or save the image as a jpg or png. Precisely how you will do this depends on the program: you may need to copy-paste figures into Photoshop or other graphic program, then save in the appropriate format.

Below we have put a few examples of figures. For more help using graphics and the float environment, see our online documentation.

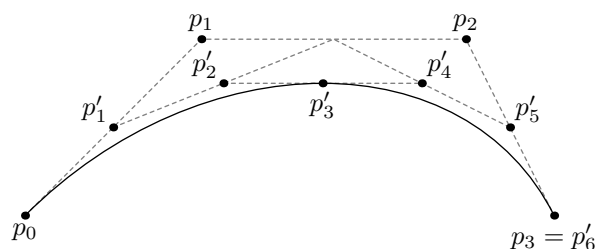And this is how you add a figure with a graphic:

Figure 1: A Figure

0.8 More Figure Stuff

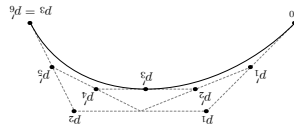You can also scale and rotate figures.

Figure 2: A Smaller Figure, Flipped Upside Down

0.9 Even More Figure Stuff

With some clever work you can crop a figure, which is handy if (for instance) your EPS or PDF is a little graphic on a whole sheet of paper. The viewport arguments are the lower-left and upper-right coordinates for the area you want to crop.
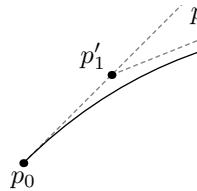
Figure 3: A Cropped Figure

0.9.1 Common Modifications

The following figure features the more popular changes thesis students want to their figures. This information is also on the web at `web.reed.edu/cis/help/latex/graphics.html`.
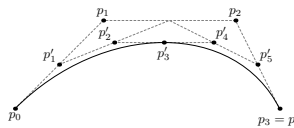
Figure 4: Subdivision of arc segments. You can see that $p_3 = p_6'$.

Conclusion

Here's a conclusion, demonstrating the use of all that manual incrementing and table of contents adding that has to happen if you use the starred form of the chapter command. The deal is, the chapter command in LaTeX does a lot of things: it increments the chapter counter, it resets the section counter to zero, it puts the name of the chapter into the table of contents and the running headers, and probably some other stuff.

So, if you remove all that stuff because you don't like it to say "Chapter 4: Conclusion", then you have to manually add all the things LaTeX would normally do for you. Maybe someday we'll write a new chapter macro that doesn't add "Chapter X" to the beginning of every chapter title.

4.1 More info

And here's some other random info: the first paragraph after a chapter title or section head *shouldn't be* indented, because indents are to tell the reader that you're starting a new paragraph. Since that's obvious after a chapter or section title, proper typesetting doesn't add an indent there.

References

[BMMN98] Meenaxi Bhattacharjee, Dugald Macpherson, Rögnvaldur G. Möller, and Peter M. Neumann. Wreath products. In *Notes on Infinite Permutation Groups*, pages 67–76. Springer, Berlin, Heidelberg, 1998.

[CP21] Daniel Copeland and Jamie Pommersheim. Quantum query complexity of symmetric oracle problems, March 2021.

[dV24] Coen del Valle. A character theoretic formula for base size, September 2024.

[JL01] Gordon James and Martin Liebeck. *Representations and Characters of Groups*. Cambridge University Press, Cambridge, 2 edition, 2001.

[NC10] Michael A. Nielsen and Isaac L. Chuang. *Quantum Computation and Quantum Information: 10th Anniversary Edition*. Cambridge University Press, December 2010.

[Zha15] Mark Zhandry. Quantum Oracle Classification - The Case of Group Structure, October 2015.