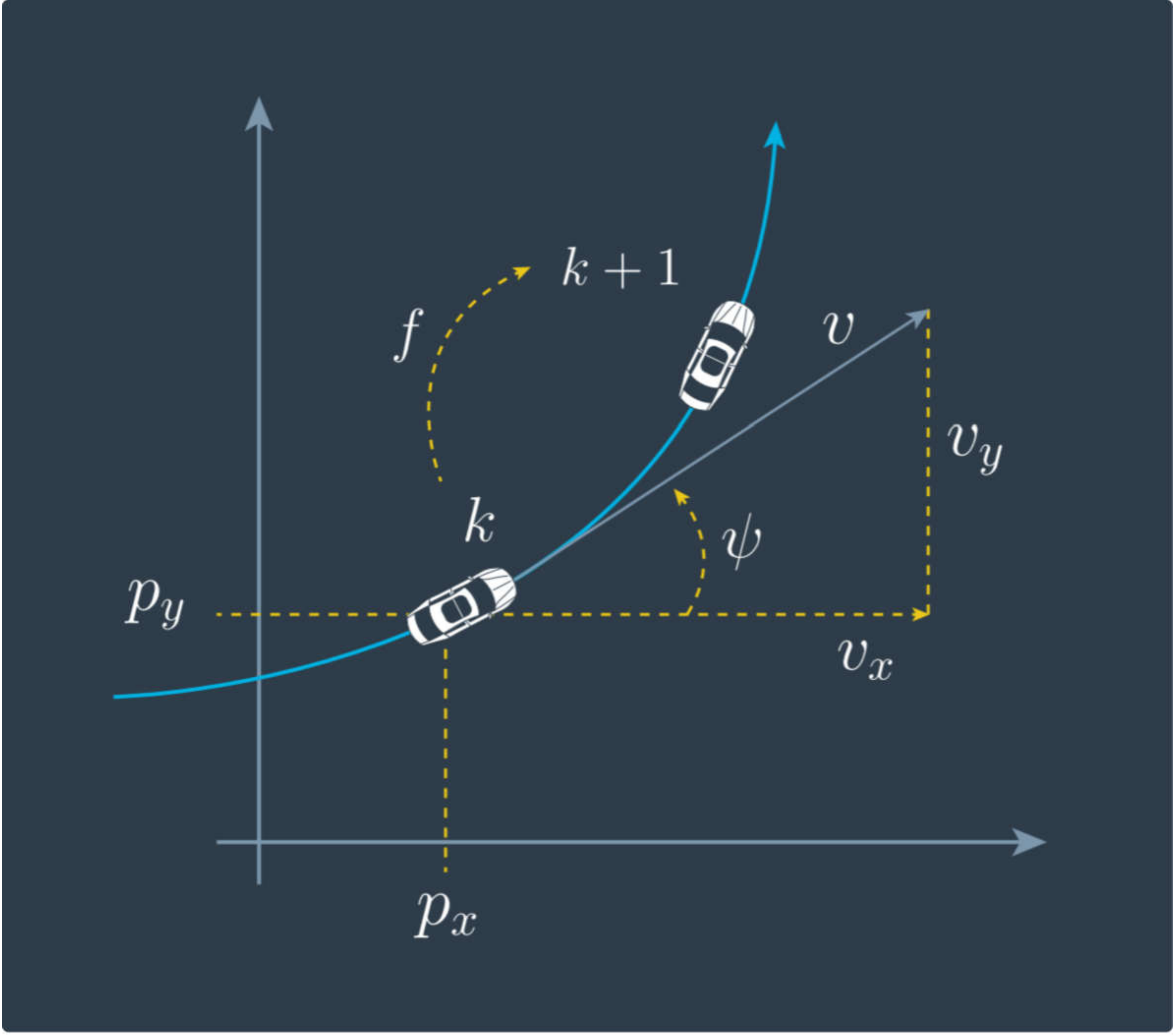
CTRV Model

A mathematical model to describe the motion of a mobile robot.

1 minute read

Introduction

The constant turn rate and constant velocity model (in short CTRV) is used to model vehicles. Using the function $x_{k|k} = f(x_k, \nu_k)$ the model predicts the new state $x_{k|k}$ of the vehicle at time $k+1$ from the state x_k and the noise vector ν_k at the current time step k. In addition to a constant velocity the model assumes also a constant turn rate which makes it more accurate than the constant velocity model, especially in curves.



The CTRV Model (Source: [Udacity self driving car ND (https://classroom.udacity.com/nanodegrees/nd013/parts/40f38239-66b6-46ec-ae68-03afd8a601c8/modules/0949fca6-b379-42af-a919-ee50aa304e6a/lessons/daf3dee8-7117-48e8-a27a-fc4769d2b954/concepts/ec188154-36ef-4f3c-bdd4-6eccf48181bf)])

State Vector and Process Model

The state vector of the ctrv model is given as

$$x = \begin{bmatrix} p_x & p_y & v & \psi & \dot{\psi} \end{bmatrix}^T \tag{1}$$

To derive the process model we investigate in the change rate of the state x, which is called $\dot{x}$. From the geometric relations, shown in the image above, we find how the change rate $\dot{x}$ depends on the state x, which is a differential equation $\dot{x} = g(x)$. To derive this differential equation the goal is to express the five time derivatives of the state, in dependency of any of the state elements.

$$\dot{x} = \begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{v} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} v \cdot \cos\psi \\ v \cdot \cos\psi \\ 0 \\ \dot{\psi} \\ 0 \end{bmatrix} \tag{2}$$

Obviously the change in velocity and turn rate is zero because this is the underlying assumption of the ctrv model. A constant velocity v and a constant turn rate $\dot{\psi}$ is not changing. Put in mathematical terms, the derivative of a constants is zero.

Discrete State Prediction

The discrete time step k relates to the continuous time value t_k. To get from the discrete time step $k := t_k$ to $k+1 := t_{k+1}$ we make use of the time difference $\Delta t = t_{k+1} - t_k$ and integrate the change rate $\dot{x}$ of the state x over this time period. The result of this integral is added to the current state x_k.

$$x_{k+1} = x_k + \int_{t_k}^{t_{k+1}} \begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{v} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} \mathrm{d}t \tag{3}$$

To solve this integral, every row of the change rate vector can be integrated.

$$\int_{t_k}^{t_{k+1}} \begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{v} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} \mathrm{d}t = \begin{bmatrix} \int_{t_k}^{t_{k+1}} \dot{p}_x \mathrm{d}t \\ \int_{t_k}^{t_{k+1}} \dot{p}_y \mathrm{d}t \\ \int_{t_k}^{t_{k+1}} \dot{v} \mathrm{d}t \\ \int_{t_k}^{t_{k+1}} \dot{\psi} \mathrm{d}t \\ \int_{t_k}^{t_{k+1}} \ddot{\psi} \mathrm{d}t \end{bmatrix} = \begin{bmatrix} \int_{t_k}^{t_{k+1}} v \cdot \cos\psi \mathrm{d}t \\ \int_{t_k}^{t_{k+1}} v \cdot \sin\psi \mathrm{d}t \\ \int_{t_k}^{t_{k+1}} \dot{v} \mathrm{d}t \\ \int_{t_k}^{t_{k+1}} \dot{\psi} \mathrm{d}t \\ \int_{t_k}^{t_{k+1}} \ddot{\psi} \mathrm{d}t \end{bmatrix} \tag{4}$$

$$= \begin{bmatrix} \int_{t_k}^{t_{k+1}} v \cdot \cos\psi \mathrm{d}t \\ \int_{t_k}^{t_{k+1}} v \cdot \sin\psi \mathrm{d}t \\ \int_{t_k}^{t_{k+1}} 0 \mathrm{d}t \\ \int_{t_k}^{t_{k+1}} \dot{\psi} \mathrm{d}t \\ \int_{t_k}^{t_{k+1}} 0 \mathrm{d}t \end{bmatrix} = \begin{bmatrix} \int_{t_k}^{t_{k+1}} v \cdot \cos\psi \mathrm{d}t \\ \int_{t_k}^{t_{k+1}} v \cdot \sin\psi \mathrm{d}t \\ 0 \\ \int_{t_k}^{t_{k+1}} \dot{\psi} \mathrm{d}t \\ 0 \end{bmatrix} \tag{5}$$

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Updated: October 17, 2017

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