Program Structure & Algorithms 2021 Fall

Assignment No.1

Jing Dai NUID 001569042

1、Task

Imagine a drunken man who, starting out leaning against a lamp post in the middle of an open space, takes a series of steps of the same length: 1 meter . The direction of these steps is randomly chosen from North, South, East or West. **After n steps, how far (*d*), generally speaking, is the man from the lamp post?** Note that *d* is the Euclidean distance of the man from the lamp-post. **Deduce the relationship**.

Please find the code on

https://github.com/Dalek371/INFO6205/tree/Fall2021/src/main/java/edu/neu/coe/info6205/randomwalk

2、Relationship Conclusion

$$d = \sqrt{n}$$

Just a approximation of the result.

3、Evidence

1) Output & Graphical Representation

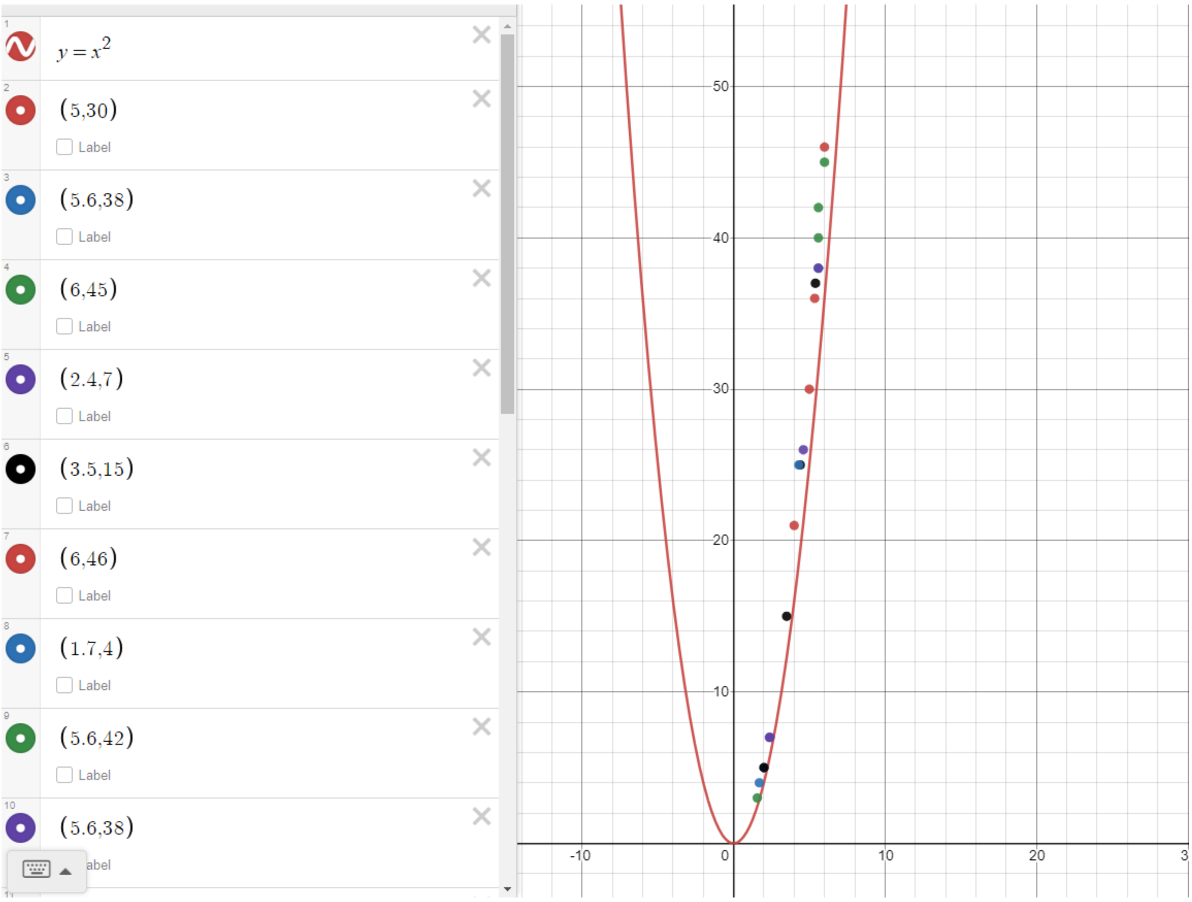
1.1 Output Screenshot

```
Run:    RandomWalk
D:\App\Code\JavaSdk\bin\java.exe ...
38 steps: 5.592512320193489 over 1000 experiments
45 steps: 5.955367073853146 over 1000 experiments
30 steps: 4.941399211869247 over 1000 experiments
7 steps: 2.416809573681239 over 1000 experiments
15 steps: 3.49501879883062 over 1000 experiments
46 steps: 5.970930278015102 over 1000 experiments
4 steps: 1.703447040456802 over 1000 experiments
42 steps: 5.58585166379931 over 1000 experiments
38 steps: 5.595143618300187 over 1000 experiments
25 steps: 4.3822827174075805 over 1000 experiments

Process finished with exit code 0
```

```
Run:    RandomWalk
D:\App\Code\JavaSdk\bin\java.exe ...
36 steps: 5.358187622661847 over 1000 experiments
5 steps: 2.044390871091556 over 1000 experiments
3 steps: 1.5680401999899196 over 1000 experiments
7 steps: 2.361545900953882 over 1000 experiments
37 steps: 5.404052067859474 over 1000 experiments
21 steps: 4.047621703932246 over 1000 experiments
25 steps: 4.299438286752818 over 1000 experiments
40 steps: 5.612458927298704 over 1000 experiments
26 steps: 4.582277240475909 over 1000 experiments
5 steps: 2.0113895863126894 over 1000 experiments

Process finished with exit code 0
```

Then we put all the points into the coordinate system with (n,d) as (x,y)

We will easily get a set of points that is very similar and close to the function of **$y=x^2$** as the following graph



2) Mathematics deduction

To get to the conclusion of

$$d = \sqrt{n}$$

We consider the drunken man walking in a coordinate system and the lamp spot as the origin, then we will get his position as (x,y)

and the distance will be

$$d = \sqrt{x^2 + y^2}$$

And we assume him walking on

West-East direction (x axis) for **i** steps

North-South direction (y axis) for **k** steps

$$n = i + k$$

Then we will have

$$\begin{cases} X = X1 + X2 + X3 ... + Xi \\ Y = Y1 + Y2 + Y3 ... + Yk \end{cases}$$

If we see Xa/Ya represent the steps as -1/1 for opposite direction.

$$
\begin{aligned}
X^2 &= (X_1 + X_2 + X_3 ... + X_i)^2 \\
&= X_1^2 + X_1X_2 + X_1X_3 + ... + X_1X_i \\
&\quad + X_2^2 + X_1X_2 + X_2X_3 + ... + X_2X_i \\
&\quad + X_3^2 + X_1X_3 + X_2X_3 + ... + X_3X_i \\
&\quad ... \\
&\quad + Xi^2 + X_1X_i + X_2X_i + ... + X_{i-1}X_i \\
&= (X1^2 + X2^2 + X3^2 ... + Xi^2) + 2(X_1X_2 + X_1X_3 + X_1X_4 ... + X_{i-1}X_i)
\end{aligned}
$$

$$
\begin{gathered}
\because Xa = -1 \ or \ 1, \\
Xa^2 = 1 \\
\therefore (X1^2 + X2^2 + X3^2 ... + Xi^2) = 1 * i = i
\end{gathered}
$$

Each XaXa pair will be within the following types:

$$
\begin{cases}
1, & -1 \ = -1 \\
1, & \ \ 1 \ \ = 1 \\
-1, & \ \ 1 \ \ = -1 \\
-1, & -1 \ = 1
\end{cases}
$$

and the probability of these pairs will be the same because it's **Random**

On average will be 0,

$$\therefore 2(X_1X_2 + X_1X_3 + X_1X_4 ... + X_{i-1}X_i) = 0$$

Therefore,

$$X^2 = (X1^2 + X2^2 + X3^2 ... + Xi^2) + 2(X_1X_2 + X_1X_3 + X_1X_4 ... + X_{i-1}X_i) = i + 0 = i$$

the same procedure may be easily adapted to Y^2

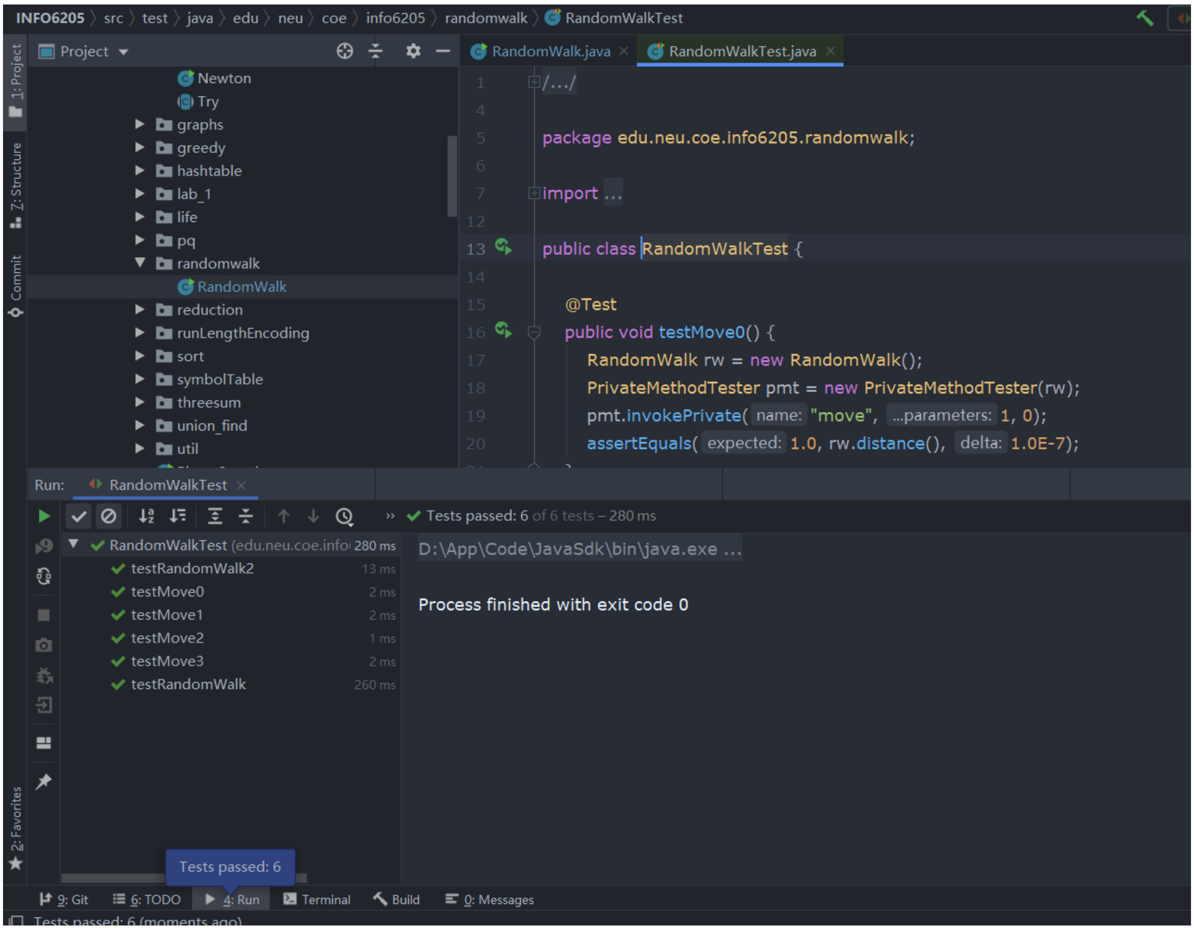
$$Y^2 = (Y1^2 + Y2^2 + Y3^2 ... + Yk^2) + 2(Y_1Y_2 + Y_1Y_3 + Y_1Y_4 ... + Y_{k-1}Y_k) = k + 0 = k$$

So, we can approximately deduce that

$$d = \sqrt{X^2 + Y^2} = \sqrt{i + k} = \sqrt{n}$$

QED

4、Unit tests result


INFO6205 › src › test › java › edu › neu › coe › info6205 › randomwalk › RandomWalkTest
Project
Newton
Try
graphs
greedy
hashtable
lab_1
life
pq
randomwalk
RandomWalk
reduction
runLengthEncoding
sort
symbolTable
threesum
union_find
util
RandomWalk.java
RandomWalkTest.java
/.../
package edu.neu.coe.info6205.randomwalk;
import ...
public class RandomWalkTest {
@Test
public void testMove0() {
RandomWalk rw = new RandomWalk();
PrivateMethodTester pmt = new PrivateMethodTester(rw);
pmt.invokePrivate(name: "move", ...parameters: 1, 0);
assertEquals(expected: 1.0, rw.distance(), delta: 1.0E-7);
Run: RandomWalkTest
Tests passed: 6 of 6 tests – 280 ms
RandomWalkTest (edu.neu.coe.info 280 ms
testRandomWalk2 13 ms
testMove0 2 ms
testMove1 2 ms
testMove2 1 ms
testMove3 2 ms
testRandomWalk 260 ms
D:\App\Code\JavaSdk\bin\java.exe ...
Process finished with exit code 0
Tests passed: 6
9: Git
6: TODO
4: Run
Terminal
Build
0: Messages
Tests passed: 6 (moments ago)