Materials used: http://modelai.gettysburg.edu/2011/clue/clue.pdf

Part 2

A = Amy says truth

B = Bob says truth

C = Cal says truth

1. First, let's form logic from the sentences given in the problem:

$A \Leftrightarrow A \wedge C$

$B \Leftrightarrow \sim C$

$C \Leftrightarrow B \vee \sim A$

2. From the statements above, we can do the following:

$\{A \Leftrightarrow A \wedge C, B \Leftrightarrow \sim C, C \Leftrightarrow B \vee \sim A\}$, then

$\{A \Rightarrow A \wedge C, A \wedge C \Rightarrow A, B \Rightarrow \sim C, \sim C \Rightarrow B, C \Rightarrow B \vee \sim A, B \vee \sim A \Rightarrow C\}$, then

$\{\sim A \vee A \wedge C, \sim(A \wedge C) \vee A, \sim B \vee \sim C, C \vee B, \sim C \vee B \vee \sim A, \sim(B \vee \sim A) \vee C\}$, (Since A => B is ~A v B)

$\{\sim A \vee C, \sim B \vee \sim C, C \vee B, \sim C \vee B \vee \sim A, \sim(B \vee \sim A) \vee C\}$

$\{\sim A \vee C, \sim B \vee \sim C, C \vee B, \sim C \vee B \vee \sim A, (A \vee C) \wedge (\sim B \vee C)\}$

3. After simplification, we should form the knowledge base to help us make appropriate assumptions using resolution techniques. Our initial Knowledge Base looks like this:

1. ~A∨C
2. ~B ∨~C
3. C ∨ B
4. ~C ∨B ∨ ~A
5. (A∨C) ∧(~B∨C)

4. We can now expand our knowledge base with assumptions. Let's start with Amy (A).

6. A (Assume the negation)
7. (From 1 and 6) C
8. (From 2 and 7) ~B
9. (From 4 and 8) ~A ∨ ~C
10. (From 1 and 9) ~A
11. (From 6 and 10) {} (turns out to be a contradiction.)

5. Let's continue with Cal (C) using our new knowledge base.

1. ~A∨C
2. ~B ∨~C
3. C ∨ B
4. ~C ∨B ∨ ~A
5. (A∨C) ∧ (~B∨C)

6. ~A

Using resolution, we can assume...

7. ~C (Let's assume the truth)
8. (From 6 and 7) ~A ∨ ~C
9. (From 3, 5 and 8) {} (we get contradiction)

6. Let's continue with Bob (B) telling truth using our new knowledge base.

1. ~A∨C
2. ~B ∨~C
3. C ∨ B
4. ~C ∨B ∨ ~A
5. (A∨C) ∧ (~B∨C)
6. ~A
7. C

Using resolution, we can assume...

8. B (Let's assume the negation)
9. (From 2 and 8) ~C
10. (From 7 and 9) {} (We get contradiction)

After forming assumptions for every person, we can form the following Knowledge Base:

1. ~A∨C

2. ~B ⋁~C
3. C ⋁ B
4. ~C ⋁B ⋁ ~A
5. (A⋁C) ⋀ (~B⋁C)
6. ~A
7. C
8. ~B

From the Knowledge Base shown above, we can conclude that **Cal is the only truth teller.**