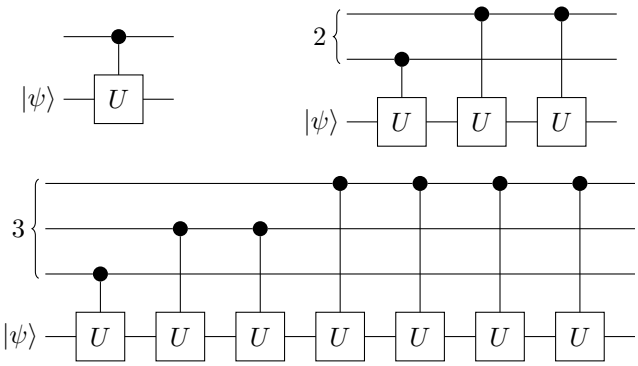
A Counting Phase

- **Learning Outcome.** Relate phase accumulation to binary encodings.
- **Description.** Later in this challenge, we will study quantum phase estimation which is used to determine the phase (i.e., angle of rotation) induced by applying a gate U to a state $|\psi\rangle$. An important step in quantum phase estimation is the phase accumulation stage. This step is performed on the state $|\psi\rangle$, together with some number of qubits related to the accuracy of the phase estimation. The phase accumulation circuits for one, two, and three qubits are shown below.

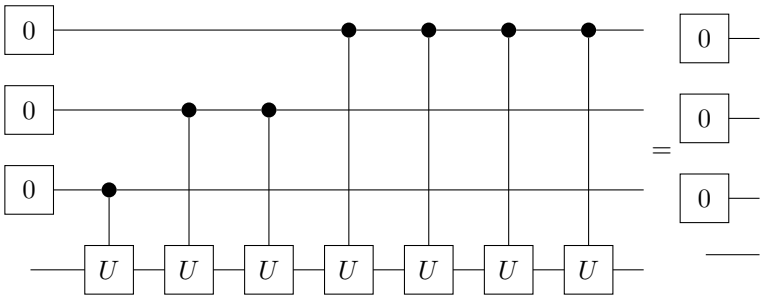


Note that on the first wire we started out with a single control. On the second wire, we double the number of controls, ending up with two controls on the second wire. Continuing in a similar fashion, we double the number of controls again, ending up with 4 controls on the third wire. Those familiar with binary, might notice a similarity between this doubling and the way we write numbers in binary. Below is the binary encoding for all numbers up to 7.

$0 \mapsto \|000\rangle$	$1 \mapsto \|001\rangle$	$2 \mapsto \|010\rangle$	$3 \mapsto \|011\rangle$
$4 \mapsto \|100\rangle$	$5 \mapsto \|101\rangle$	$6 \mapsto \|110\rangle$	$7 \mapsto \|111\rangle$

It turns out that if we view the first three qubits as a binary encoding, then it is easy to predict that will happen to the fourth qubit. In this activity, you will pick three numbers between 1 and 7, and then check what this circuit does when the first three qubits encode your chosen number. As

an example, we have provided the solution for $|000\rangle$.



In your solutions, you should explain your work. Can you guess the pattern from these four examples?

- **Submission.** Three circuit equations, and a sentence stating the pattern which allows you predict each output from each input.