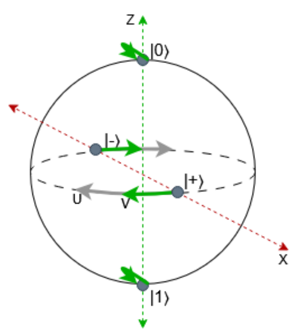
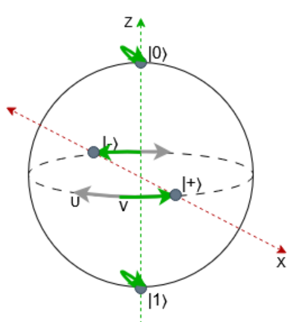
Controlling a Gate

- **Description.** As the NV center is exposed to the atmospheric pressure, the state of the qubit stored within the diamond will rotate around some axis. In practice, this rotation could be around some axis other than the Z- or X-axis, but for the purposes of this challenge let's assume that the rotation is around the Z-axis. We can represent the rotation by a spider $-\!\!\left(\theta\right)\!\!-$ where θ is how far the qubit has rotated after one second. In the quantum phase estimation algorithm we will need to be able to control this rotation, so that we can accumulate the angles of rotation (i.e., the phases). For this reason, let's call this rotation U. Since the atmosphere is out of our control, we cannot control U directly. However, there is a clever solution to this problem. Let's pretend that we also have access to a gate called V defined to be $-\!\!\left(\frac{\theta}{2}\right)\!\!-$. Intuitively, the V gate will rotate the qubit half as far as U, as illustrated below.

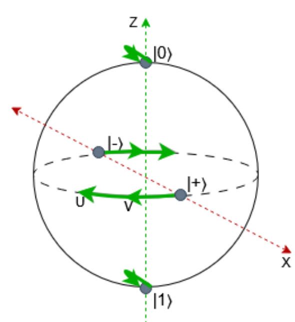


Our goal is to implement the controlled U gate using V gates. We can start by performing a single V gate. This will bring us half way to performing the V gate. Now there are two cases to consider. In the first case, the control qubit will be in state $|0\rangle$, so we would like the end result to be that no rotation is performed. We can achieve this by performing a $-\!\!\left(\frac{-\theta}{2}\right)\!\!-$ which will cancel out with the V rotation. In the second case, the control qubit will be in the state $|1\rangle$, so we can simply apply V a second time, to complete the rotation. Both cases These two cases are illustrated below.

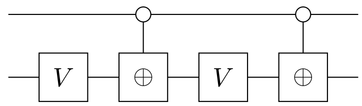
$|0\rangle$:



$|1\rangle$:



Since we know that NOT gates flip the signs of Z-rotations, then we might hope that the circuit looks as follows.



Your goal is to convert this circuit into a ZX-diagram, and then use the ZX-calculus to verify that this circuit is the same as a controlled U-gate. Remember that we can check this by seeing what happens in the case where the control qubit is $|0\rangle$, and the case where the control qubit is $|1\rangle$.

- **Submission.** One or more equations of ZX-diagrams, and an explanation for why these equations verify the circuit.