Superdense Coding (Advanced)

• **Description.** In this activity, you will use the ZX-calculus to verify that superdense coding actually works. Intuitively, the fact that Alice and Bob share a Bell pair allows for Bob to encode his message across the pair of qubits, despite only having direct access to one of the qubits. Of course, this intuition seems to be at odds with our covnentional understanding of the world, so we will need some more evidence to back it up. To do this, you will start by converting your superdense coding circuit into a ZX-diagram. You should end up with the following ZX-diagram.



Note that Bob's encoded message is $|ab\rangle$ and Alice's decoded message is $|cd\rangle$, where a, b, c, and d are each either 0 or 1. Since superdense coding is supposed to send Bob's message to Alice, we would like to show that a=c and b=d. Your goal is to show that this is true using the ZX-calculus.

• **Hint.** In superdense coding, Bob sets up a state and then Alice measures the state. This means that we should be able to rewrite the big ZX-diagram as two measurements. Try to do this. Eventually, you should end up with a diagram that looks as follows.



This diagram says that the only possible measurement outcomes will have a = c and b = d. Can you explain why this is true?

• Submission. A equation of ZX-diagrams with justification.