Geometric Proof for Pythagoras' Theorem

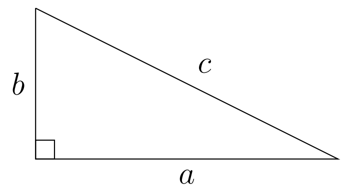
You might be familiar with Pythagoras' theorem for right angled triangles. where for a right angled triangle with sides a, b, and c, where c is the hypotenuse we know that

$$c^2 = a^2 + b^2.$$

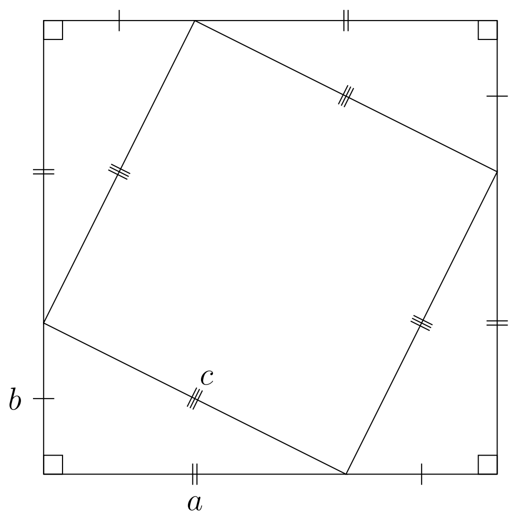
But can we be convinced that this is in fact true?

Proof

First let there be a generic right angled triangle with side lengths a, b, and c wwith c as the hypotenuse.



From this four of these triangles we can construct a square.



Each side of this square is the $a + b$, this means that the area of the whole square (A) is given by

$$A = (a + b) \times (a + b),$$

or

$$A = (a + b)^2.$$

We can also calculate the area of the large square using the individual parts. The formula for the area of a triangle is $\frac{1}{2}bh$ so the area of each of the four right angled triangles is,

$$A_t = \frac{1}{2}ab.$$

The other component of the larger square, is the area of the smaller square within (we should probably prove this is a square, but that can be an exercise for the reader), which is

$$A_s = c^2,$$

This means that the total area is also given by

$$\begin{aligned}
A &= 4 \times A_t + A_s, \\
A &= 4 \times \frac{1}{2}ab + c^2, \\
A &= 2ab + c^2.
\end{aligned}$$

We can equate these two different calculations of the area

$$(a+b)^2 = 2ab + c^2,$$

then simplify each side of the equation. First expand the LHS

$$a^2 + 2ab + b^2 = 2ab + c^2,$$

then we can subtract $2ab$ from both sides,

$$\begin{gathered}
a^2 + 2ab + b^2 - 2ab = 2ab + c^2 - 2ab, \\
a^2 + b^2 = c^2.
\end{gathered}$$

As such, we are done, we have shown that any generic right angled triangle must adhere to Pythagoras' formula.