Sheet Title

Question Bank

1. The core of derivatives is the idea of calculating the gradient of extremely small sections of a curve, and this is captured by the equation

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

As an example lets take the derivative of the function

$$f(x) = x^3 + 5x.$$

Expanding the function applications in the limit gives us

$$f'(x) = \lim_{h \to 0} \frac{((x+h)^3 + 5(x+h)) - (x^3 + 5x)}{h},$$

the initial problem with this is that we are dividing by h, and this means we cannot substitute $h = 0$ to simplify our limit—since dividing by 0 is undefined. First let's expand and simplify the expression in the numerator,

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{x^3 + 3hx^2 + 3h^2x + h^3 + 5x + 5h - x^3 - 5x}{h}, \\ &= \lim_{h \to 0} \frac{3hx^2 + 3h^2x + h^3 + 5h}{h}, \\ &= \lim_{h \to 0} \frac{h\left(3x^2 + 3hx + h^2 + 5\right)}{h}. \end{aligned}$$

By factoring out h in the numerator, and armed with the knowledge that h never actually equals 0, we can simplify it from the top and the bottom,

$$f'(x) = \lim_{h \to 0} \left(3x^2 + 3hx + h^2 + 5\right).$$

Finally, since we are no longer dividing by h we can evaluate the limit by substituting $h = 0$,

$$\begin{aligned} f'(x) &= 3x^2 + 3x \cdot 0 + 0^2 + 5, \\ &= 3x^2 + 5. \end{aligned}$$

For the following functions, find their derivatives with respect to x, by applying the first principles formula.

a) $f(x) = x^3$

b) $f(x) = 4x$

c) $f(x) = 2x^2 - 3x$

d) $f(x) = x(2x+1)$

e) $f(x) = (x^2 + 2)(x - 4)$

f) $f(x) = \dfrac{x+2}{x}$

2. Since finding a derivative from first principles is time consuming and annoying, we will learn some rules that we can apply with impunity. First we will cover the power rule, which often looks like

$$\text{if } y = x^n \text{ then } \frac{dy}{dx} = nx^{n-1}.$$

So for a function such as

$$f(x) = x^4, \quad f'(x) = 4x^3,$$

and similarly,

$$f(x) = \frac{2}{x^3} = 2x^{-3}, \quad f'(x) = -6x^{-4} = -\frac{6}{x^4}.$$

For each of the following functions, find their derivatives by applying this power rule.

a) $f(x) = x^5$

b) $f(x) = 3x^2$

c) $f(x) = -5x^4$

d) $f(x) = -7x^{10}$

e) $f(x) = \dfrac{x^3}{18}$

f) $f(x) = \dfrac{2x^6}{9}$

g) $f(x) = x^{-2}$

h) $f(x) = -2x^{-4}$

i) $f(x) = \dfrac{1}{x^3}$

j) $f(x) = \dfrac{5}{4x^4}$

k) $f(x) = 3x^2(2x^3 + x^2)$

l) $f(x) = 4x^3\left(x^2 - 2x + \dfrac{3}{2}\right)$

m) $f(x) = x^{\frac{1}{2}}$

n) $f(x) = \dfrac{3}{4}x^{\frac{2}{3}}$

o) $f(x) = x^{-\frac{1}{2}}$

p) $f(x) = 5\sqrt{x}$

q) $f(x) = 4\sqrt{x^3}$

r) $f(x) = \dfrac{8}{3}x\sqrt{x}$

s) $f(x) = \dfrac{2}{\sqrt{x}}$

t) $f(x) = \dfrac{5}{\sqrt[3]{x^2}}$

3. hello

Answers

1. a) $f'(x) = 3x^2$

   b) $f'(x) = 4$

   c) $f'(x) = 4x - 3$

   d) $f'(x) = 4x + 1$

   e) $f'(x) = 3x^2 - 8x + 2$

   f) $f'(x) = -\dfrac{2}{x^2}$

2. a) $f'(x) = 5x^4$

   b) $f'(x) = 6x$

   c) $f'(x) = -20x^3$

   d) $f'(x) = -70x^9$

   e) $f'(x) = \dfrac{x^2}{6}$

   f) $f'(x) = \dfrac{4x^5}{3}$

   g) $f'(x) = -2x^{-3}$ or $\dfrac{-2}{x^3}$

   h) $f'(x) = 8x^{-5}$ or $\dfrac{8}{x^5}$

   i) $f'(x) = \dfrac{-3}{x^4}$

   j) $f'(x) = \dfrac{-5}{x^5}$

   k) $f'(x) = 30x^4+12x^3$ or $6x^3(5x+2)$

   l) $f'(x) = 20x^4 - 8x^3 + 18x^2$ or $2x^2(10x^2 - 4x + 9)$

   m) $f'(x) = \dfrac{1}{2}x^{-\frac{1}{2}}$ or $\dfrac{1}{2\sqrt{x}}$

   n) $f'(x) = \dfrac{1}{2}x^{-\frac{1}{3}}$ or $\dfrac{1}{2\sqrt[3]{x}}$

   o) $f'(x) = -\dfrac{1}{2}x^{-\frac{3}{2}}$ or $-\dfrac{1}{2\sqrt{x^3}}$

   p) $f'(x) = \dfrac{5}{2\sqrt{x}}$

   q) $f'(x) = 6\sqrt{x}$

   r) $f'(x) = 4\sqrt{x}$

   s) $f'(x) = -\dfrac{1}{\sqrt{x^3}}$

   t) $f'(x) = -\dfrac{10}{3\sqrt[3]{x^5}}$