Derivative Helper

Note on Notation

There are a few ways that you are likely to see derivatives notated in NSW HSC maths textbooks and exam papers. Just remember that these are not hard rules and you should use the notation that fits with a question or feels most comfortable to you while communicating your intentions clearly.

When a situation involves functions you are likely to see derivatives as $f'(x)$, $g'(x)$ and $f''(x)$, where the derivative is with respect to the input variable—usually x. This apostrophe mark may also be used with variables such as $y = 3x^2$ becomes $y' = 6x$. This mark may be pronounced "prime", so you might say "y prime" to mean the first derivative of y.

The other main form of notation is $\frac{dy}{dx}$, which represents the derivative of y with respect to x. Usually this is seen when equations use y or some other variable such as $u = 3x$ gives $\frac{du}{dx} = 3$. Multiple derivatives are represented with index notation as $\frac{d^2u}{dx^2}$ would be the second derivative of u with respect to x. A rarer use of this notation is $\frac{d}{dx}$ which is an operator that roughly means "take the derivative of [expression] with respect to x". For example, if we wanted the derivative of $2x^4$ we might write $\frac{d}{dx}(2x^4) = 8x$.

Constants and Coefficients

The derivative of a constant expression is 0. For example, if $u = 10$ then $\frac{du}{dx} = 0$. More generally for any constant C, $\frac{d}{dx}(C) = 0$.

Constant coefficients are a bit different, consider a function, f, that is another function, g, multiplied by a constant, C. The derivative of this function, $f(x) = Cg(x)$, would be

$$f'(x) = \frac{d}{dx}(Cg'(x)) = C\frac{d}{dx}(g(x)) = Cg'(x).$$

Notice that the constant coefficient doesn't change, I like to imagine it's just along for the ride. A more concrete example might be the function $f(x) = 4x^3$, in this case 4 is the constant coefficient of the function $g(x) = x^3$. So when we want to find $f'(x)$ we can apply the power rule to x^3 then multiply by 4, which might look like

$$f'(x) = 4 \cdot \frac{d}{dx}(x^3) = 4 \cdot 3x^2 = 12x^2.$$

Sums and Subtraction

When a function is made up of multiple expressions separated by addition or subtraction—which is just funky addition—you can find the derivative of the function, by finding the derivative of each of the smaller parts. For example, if $f(x) = g_1(x) + g_2(x) - g_3(x)$ then $f'(x) = g_1'(x) + g_2'(x) - g_3'(x)$. For a more concrete example consider $f(x) = x^2 + 3x + 9$, to find the derivative we will find the derivative of each part.

$$\begin{aligned} f'(x) &= \frac{d}{dx}(x^2) + 3 \cdot \frac{d}{dx}(x) - \frac{d}{dx}(9), \\ &= 2x + 3 \cdot 1 - 0, \\ &= 2x + 3. \end{aligned}$$

Formulas

Power Rule

The power rule is used for expressions of a variable that have a constant exponent. For simple powers of x use the rule

$$y = x^n \quad \frac{dy}{dx} = nx^{n-1}.$$

The more general formula uses the chain rule, and is

$$y = (f(x))^n \quad \frac{dy}{dx} = nf'(x)[f(x)]^{n-1}.$$

Product Rule

The product rule is used when a function is a product of two expressions in terms of x. The common formula is for when $y = uv$,

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dv}.$$