Gradient Intercept and General Form Conversion

Question Bank

NOTE: For answers that are fractions, leave them as fractions but simplify them if possible.

1. Equations of the form $y = mx + c$ are in gradient-intercept form. So called, because m (the coefficient of x) tells you the gradient of the line and c tells you the y-intercept of the line. As a warm up, for each of the following equations determine i) their gradient, and ii) their y-intercept.

   a) $y = 2x + 5$

   b) $y = 3x - 2$

   c) $y = -x + 3$

   d) $y = -3x + 13$

   e) $y = 155x + 240$

   f) $y = \frac{1}{3}x - 3$

   g) $y = \frac{x}{2} + 23$

   h) $y = -\frac{3}{7}x - \frac{4}{9}$

   i) $y = -\frac{2x}{5} + \frac{6}{5}$

2. An often under-stressed feature of the gradient-intercept form is that y **must have a coefficient of 1**, hence an equation such as $3y = 6x - 5$ is not in gradient-intercept from and we **cannot** say 6 is the gradient and -5 is the y-intercept. We must divide both sides by the current coefficient of y (3) to make y's coefficient 1,

$$\begin{aligned} \frac{3y}{3} &= \frac{6x - 5}{3}, \\ y &= \frac{6x}{3} - \frac{5}{3}, \\ y &= 2x - \frac{5}{3}. \end{aligned}$$

   For each of the following equations, i) convert them into gradient-intercept form, ii) determine their gradient and iii) determine their y-intercept.

   a) $2y = 4x - 6$

   b) $3y = -6x + 12$

   c) $-5y = 5x + 25$

   d) $-7y = -21x - 28$

e) $2y = 5x + 6$

f) $-5y = 15x - 17$

g) $-6y = -15x + 42$

h) $3y = -9x + 13$

i) $-3y = -5x - 8$

j) $8y = 12x - 4$

k) $4y = 3x + 18$

l) $-9y = 21x + 39.$

m) $\dfrac{y}{2} = 2x - \dfrac{1}{2}$

n) $\dfrac{y}{3} = -\dfrac{4}{3}x + 12$

o) $-\dfrac{y}{5} = 2x - 8$

p) $-\dfrac{y}{2} = -\dfrac{3x}{2} - \dfrac{7}{6}$

q) $\dfrac{3y}{2} = 6x + 15$

r) $\dfrac{5y}{3} = -x - 15$

s) $-\dfrac{2y}{7} = \dfrac{3x}{14} + 4$

t) $-\dfrac{9}{4}y = -\dfrac{27}{2}x + \dfrac{7}{4}$

3. The general form of an equation is $ax + by + c = 0$. In this form c **is not the same** as the c in the gradient-intercept form, and **does not** represent the y-intercept. Since this form does not tell us much information we usually convert from general form into gradient-intercept form if we plan to draw the line. To do this first we need to isolate y, consider the equation

$$4x - 2y - 5 = 0,$$

first we can subtract $4x$ from then add 5 to both sides,

$$\begin{aligned} 4x - 2y - 5 &= 0, \\ 4x - 2y - 5 - 4x &= 0 - 4x, \\ -2y - 5 &= -4x, \\ -2y - 5 + 5 &= -4x + 5, \\ -2y &= -4x + 5. \end{aligned}$$

Then we can divide both sides by -2,

$$\begin{aligned} \frac{-2y}{-2} &= \frac{-4x + 5}{-2}, \\ y &= \frac{-4x}{-2} + \frac{5}{-2}, \\ y &= 2x - \frac{5}{2}. \end{aligned}$$

For each of the following general form equations, rearrange them into gradient-intercept from.

a) $x + y - 4 = 0$
b) $2x - y + 3 = 0$
c) $x + 2y + 6 = 0$
d) $2x - 3y - 7 = 0$
e) $-5x - 3y = 0$
f) $6x + 9y + 24 = 0$
g) $-x + 2y - 12 = 0$
h) $12x - 8y + 14 = 0$
i) $2x + \frac{3}{4}y + 6 = 0$
j) $-7x - \frac{14}{3}y - \frac{7}{2} = 0$
k) $-\frac{2}{5}x - 3y + 4 = 0$
l) $\frac{3}{13}x + 12y + \frac{9}{26} = 0$
m) $\frac{12}{5}x - \frac{1}{7}y + 2 = 0$
n) $-\frac{8}{21}x + \frac{42}{25}y - \frac{2}{7} = 0$

4. To convert from gradient-intercept form into genera form, we need to "move" all the terms onto one side so that one side of the equation is 0. Consider the equation

$$y = -3x + \frac{3}{4},$$

we would add $3x$ to, then subtract $\frac{3}{4}$ from, both sides of the equation,

$$\begin{aligned} y + 3x &= -3x + \frac{3}{4} + 3x, \\ y + 3x &= \frac{3}{4}, \\ y + 3x - \frac{3}{4} &= \frac{3}{4} - \frac{3}{4}, \\ y + 3x - \frac{3}{4} &= 0. \end{aligned}$$

Then to make the equation look nicer, we might move the terms such that it looks more like the form $ax + by + c = 0$,

$$3x + y - \frac{3}{4} = 0.$$

An optional extra that is often nice, is to remove fractions from the

equation. Since ours has $\frac{3}{4}$ we might multiply both sides by 4

$$4\left(3x + y - \frac{3}{4}\right) = 0 \times 4,$$
$$(4 \times 3x) + (4 \times y) + \left(4 \times -\frac{3}{4}\right) = 0,$$
$$12x + 4y - 3 = 0.$$

For each of the following equations in gradient-intercept form, convert them into general form and simplify where possible. *(Your answers to these questions might not exactly match the provided solutions because there are many valid general forms for each line, if this happens try to multiply or divide your answer and see if you can match the provided solution. The solutions tend to have no fractions and a positive coefficient of x)*

a) $y = x - 4$

b) $y = -2x + 5$

c) $y = -3x - 2$

d) $y = 2x + 4$

e) $y = 7x - 10$

f) $y = 5x + 20$

g) $y = \frac{1}{3}x + 5$

h) $y = -\frac{1}{7}x + 2$

i) $y = -7x + \frac{49}{6}$

j) $y = 4x - \frac{9}{10}$

k) $y = \frac{12}{5}x - \frac{10}{3}$

l) $y = -\frac{4}{5}x - \frac{3}{4}$

m) $y = -\frac{3}{4}x + \frac{7}{2}$

n) $y = \frac{5}{12}x - \frac{5}{18}$

Answers

1. a) i) 2
   ii) 5
   b) i) 3
   ii) -2
   c) i) -1
   ii) 3
   d) i) -3
   ii) 13
   e) i) 155
   ii) 240
   f) i) $\frac{1}{3}$
   ii) -3
   g) i) $\frac{1}{2}$
   ii) 23
   h) i) $-\frac{3}{7}$
   ii) $-\frac{4}{9}$
   i) i) $-\frac{2}{5}$
   ii) $\frac{6}{5}$

2. a) i) $y = 2x - 3$
   ii) 2
   iii) -3
   b) i) $y = -2x + 4$
   ii) -2
   iii) 4
   c) i) $y = -x - 5$
   ii) -1
   iii) -5
   d) i) $y = 3x + 4$
   ii) 3
   iii) 4
   e) i) $y = \frac{5}{2}x + 3$
   ii) $\frac{5}{2}$
   iii) 3
   f) i) $y = -3x + \frac{17}{5}$
   ii) -3
   iii) $\frac{17}{5}$
   g) i) $y = \frac{5}{2}x - 7$
   ii) $\frac{5}{2}$
   iii) -7
   h) i) $y = -3x + \frac{13}{3}$
   ii) -3
   iii) $\frac{13}{3}$
   i) i) $y = \frac{5}{3}x + \frac{8}{3}$
   ii) $\frac{5}{3}$
   iii) $\frac{8}{3}$
   j) i) $y = \frac{3}{2}x - \frac{1}{2}$
   ii) $\frac{3}{2}$
   iii) $-\frac{1}{2}$
   k) i) $y = \frac{3}{4}x + \frac{9}{2}$

ii) $\frac{3}{4}$

iii) $\frac{9}{2}$

l) i) $y = -\frac{7}{3}x - \frac{13}{3}$

ii) $-\frac{7}{3}$

iii) $-\frac{13}{3}$

m) i) $y = 4x - 1$

ii) 4

iii) -1

n) i) $y = -4x + 36$

ii) -4

iii) 36

o) i) $y = -10x + 40$

ii) -10

iii) 40

p) i) $y = 3x + \frac{7}{3}$

ii) 3

iii) $\frac{7}{3}$

q) i) $y = 4x + 10$

ii) 4

iii) 10

r) i) $y = -\frac{3}{5}x - 9$

ii) $-\frac{3}{5}$

iii) -9

s) i) $y = -\frac{3}{4}x - 14$

ii) $-\frac{3}{4}$

iii) -14

t) i) $y = 6x - \frac{7}{9}$

ii) 6

iii) $-\frac{7}{9}$

3. a) $y = -x + 4$

b) $y = 2x + 3$

c) $y = -\frac{1}{2}x - 3$

d) $y = \frac{2}{3}x - \frac{7}{3}$

e) $y = -\frac{5}{3}x$

f) $y = -\frac{2}{3}x - \frac{8}{3}$

g) $y = \frac{1}{2}x + 6$

h) $y = \frac{3}{2}x + \frac{7}{4}$

i) $y = -\frac{8}{3}x - 8$

j) $y = -\frac{3}{2}x - \frac{3}{4}$

k) $y = -\frac{2}{15}x + \frac{4}{3}$

l) $y = -\frac{1}{52}x - \frac{3}{104}$

m) $y = \frac{84}{5}x + 14$

n) $y = \frac{100}{441}x + \frac{25}{147}$

4. a) $x - y - 4 = 0$

b) $2x + y - 5 = 0$

c) $3x + y + 2 = 0$

d) $2x - y + 4 = 0$

e) $7x - y - 10 = 0$

f) $5x - y + 20 = 0$

g) $x - 3y + 15 = 0$

h) $x + 7y - 14 = 0$

i) $42x + 6y - 49 = 0$

j) $40x - 10y - 9 = 0$

k) $36x - 15y - 50 = 0$

l) $16x + 20y + 15 = 0$

m) $3x + 4y - 14 = 0$

n) $15x - 36y - 10 = 0$