Simple Quadratic Equations

Example Problems

EXAMPLE 1: Solve the following equation for x,

$$x^2 = 49.$$

SOLUTION: To find the answer to the equation we need to find the square root of both sides of the equation,

$$\sqrt{x^2} = \sqrt{49},$$

the LHS simplifies and for the RHS we can evaluate the square root,

$$x = \pm 7.$$

Note here that the $\pm$ is important since $7^2 = 49$ and $(-7)^2 = 49$ so we use it to indicate that $x = -7$ or 7.

EXAMPLE 2: Solve the following equation for x,

$$3x^2 - 19 = 56.$$

SOLUTION: In order to simplify the LHS, we first will remove the -19 since it is the last operation performed on x. We can do this by adding 19 to both sides

$$\begin{aligned} 3x^2 - 19 + 19 &= 56 + 19, \\ 3x^2 &= 75. \end{aligned}$$

Next we will divide both sides by 3,

$$\begin{aligned} 3x^2 \div 3 &= 75 \div 3, \\ x^2 &= 25, \end{aligned}$$

and finally square root both sides,

$$\begin{aligned} \sqrt{x^2} &= \sqrt{25}, \\ x &= \pm 5. \end{aligned}$$

EXAMPLE 3: Solve the following equation for x,

$$\frac{x^2 - 12}{4} = 13.$$

SOLUTION: To simplify the LHS first we need to notice that the numerator of the fraction has implicit brackets,

$$\frac{(x^2 - 12)}{4} = 13,$$

which means that we first need to multiply both sides by 4,

$$\begin{aligned} \frac{x^2 - 12}{4} \times 4 &= 13 \times 4, \\ x^2 - 12 &= 52, \end{aligned}$$

then we add 12 and square root

$$\begin{aligned} x^2 - 12 + 12 &= 52 + 12, \\ x^2 &= 64, \\ \sqrt{x^2} &= \sqrt{64}, \\ x &= \pm 8. \end{aligned}$$

Question Bank

NOTE: Any questions where you get a decimal as an answer can be rounded to 2 decimal places.

1. Solve the following equations.

   a) $x^2 = 9$
   b) $x^2 = 1$
   c) $x^2 = 441$
   d) $x^2 = 169$
   e) $x^2 = 0$
   f) $x^2 = 200$

2. Solve the following equations.

   a) $5x^2 = 45$
   b) $13x^2 = 637$
   c) $-3x^2 = -27$
   d) $x^2 + 14 = 95$
   e) $x^2 - 17 = 47$
   f) $x^2 - 256 = -87$
   g) $x^2 - 36 = 0$
   h) $\frac{x^2}{3} = 27$
   i) $\frac{x^2}{4} = 25$
   j) $\frac{x^2}{28} = \frac{9}{7}$
   k) $\frac{x^2}{52} = 3.25$
   l) $\frac{-x^2}{4} = -16$

3. Solve the following equations for x.

   a) $5x^2 + 14 = 59$
   b) $3x^2 - 270 = 162$
   c) $7x^2 - 53 = -25$
   d) $2x^2 + 42 = 140$
   e) $\frac{x^2}{8} - 13 = -11$
   f) $\frac{x^2}{6} + 25 = 49$
   g) $\frac{x^2}{12} + 15.25 = 22$
   h) $\frac{x^2}{64} + \frac{15}{4} = 6$

4. Solve the following equations for x.

   a) $-x^2 = -25$
   b) $-3x^2 = -363$
   c) $\frac{-x^2}{5} = -20$
   d) $\frac{-3x^2}{4} = -27$
   e) $-x^2 + 9 = -40$
   f) $-x^2 - 17 = -33$

g) $45 - x^2 = 9$

h) $23 - x^2 = -58$

i) $14 - 2x^2 = -36$

j) $48 - 3x^2 = 11$

5. Solve the following equations for x.

a) $3(x^2 + 9) = 102$

b) $8(x^2 - 36) = -160$

c) $2(3x^2 - 10) = 4$

d) $11(5x^2 + 21) = 1111$

e) $5(100 - x^2) = 95$

f) $7(83 - x^2) = -427$

g) $-6(2x^2 - 100) = 12$

h) $3(3 - 3x^2) = -216$

6. Solve the following equations for x.

a) $(x^2 + 3) \div 6 = 14$

b) $(x^2 - 9) \div 10 = 4$

c) $\dfrac{x^2 + 39}{4} = 12$

d) $\dfrac{x^2 - 72}{7} = 49$

e) $\dfrac{8 - x^2}{6} = 2$

f) $\dfrac{24 - 2x^2}{12} = -4$

7. Oliver wants to cut a square sheet of paper with an area of 25cm^2. How long should the sides of the square be?

8. The library wants to build 7 identical square study rooms. If the total area of the space will be 63m^2, what will be the dimensions of the rooms?

9. Zena is planning to fashion a sphere out of some sheet metal. If the area of sheet metal she can use is $3{,}217\text{cm}^2$, what will be the radius of her sphere? (The surface area (A) of a sphere is given by its radius (r) with the formula $A = 4\pi r^2$).

10. Layla wants to expand her square paddock so that each side is 3m longer. If the new area of the paddock will be 169m^2 what was the original area of the paddock?

11. Cooper cuts 12 identical squares out of a sheet of A4 paper. The paper has dimensions 297mm by 210mm, if the remaining piece of paper has an area of $41{,}202\text{mm}^2$ what is the side length of each square?

12. Natalie drops a ball from the top of a 100m tall building. The height of her ball above the ground (h meters) after t seconds of travel time is modelled by the equation $h = 100 - 10t^2$.

    a) How high will the ball be after 1.5 seconds?

    b) After how many seconds will the ball be 10m above the ground.

    c) How many seconds will it take for the ball to hit the ground.

13. Dylan is being launched into space on a rocket. The computer modelling predicts that his height above sea level (h meters), after t seconds, should be modelled by the equation $h = 150 + 45t^2$.

    a) How high above sea level is Dylan before the rocket launches?

    b) How long should it take for Dylan to be 1km above sea level?

    c) The model is wrong, and Dylan is only ever one quarter the height that the model predicts. How high will he actually be after 7 seconds?

    d) With this new reality in mind, how many seconds will it actually take for Dylan to be 1km above sea level?

14. A new water bottle is a rectangular prism with a square base. Four of these water bottles are each filled until the water in each bottle reaches 12cm. The total water poured into the bottles is 1,452ml. (Remember that $1\text{cm}^3 = 1\text{ml}$)

    a) How long are the sides of the square bases on each water bottle?

    b) If each water bottle is 20cm tall, how much water would all four bottles hold together?

15. Explain why the equation $x^2 + 36 = 0$ has no real solutions.

Answers

1. a) $x = \pm 3$
   b) $x = \pm 1$
   c) $x = \pm 21$
   d) $x = \pm 13$
   e) $x = 0$
   f) $x = \pm 14.14$ ($x = \pm 10\sqrt{2}$ as an exact value)

2. a) $x = \pm 3$
   b) $x = \pm 7$
   c) $x = \pm 3$
   d) $x = \pm 9$
   e) $x = \pm 8$
   f) $x = \pm 13$
   g) $x = \pm 6$
   h) $x = \pm 9$
   i) $x = \pm 10$
   j) $x = \pm 6$
   k) $x = \pm 13$
   l) $x = \pm 8$

3. a) $x = \pm 3$
   b) $x = \pm 12$
   c) $x = \pm 2$
   d) $x = \pm 7$
   e) $x = \pm 4$
   f) $x = \pm 12$
   g) $x = \pm 9$
   h) $x = \pm 12$

4. a) $x = \pm 5$
   b) $x = \pm 11$
   c) $x = \pm 10$
   d) $x = \pm 6$
   e) $x = \pm 7$
   f) $x = \pm 4$
   g) $x = \pm 6$
   h) $x = \pm 9$
   i) $x = \pm 5$
   j) $x = \pm 3$

5. a) $x = \pm 5$
   b) $x = \pm 4$
   c) $x = \pm 2$
   d) $x = \pm 4$
   e) $x = \pm 9$
   f) $x = \pm 12$
   g) $x = \pm 7$
   h) $x = \pm 5$

6. a) $x = \pm 9$
   b) $x = \pm 7$
   c) $x = \pm 3$
   d) $x = \pm 11$
   e) $x = \pm 4$
   f) $x = \pm 6$

7. 5cm

8. 3m $\times$ 3m
9. 16.00cm
10. 100m^2
11. 42mm
12. a) 77.5m
    b) 3 seconds
    c) 3.16 seconds or $\sqrt{10}$ in exact form.
13. a) 150m
    b) 4.35 seconds
    c) 588.75m
    d) 9.25 seconds
14. a) 5.5cm
    b) 2,420ml
15. If we simplify the equation we get $x^2 = -36$ and since the square of any number (positive or negative) is positive, it is impossible for the square of a real number to equal -36.