Tips for Solving Equations

Solving an equation is a series of steps where you can apply an operation to both sides of an equality until you have a equation that equates a variable with a value. It is the process of taking a equation such as

$$4(x+5) = 48,$$

and from this equation, determining that

$$x = 7.$$

Adding to Both Sides

Example One

The first tool in our toolbox is adding a number to both sides. Consider the equation

$$x + 10 = 15,$$

we can add 5 to both sides,

$$x + 10 + 5 = 15 + 5,$$

since we can simplify $x+10+5$ into $x+15$ and $15+5$ into 20, the equation becomes

$$x + 15 = 20.$$

While this example isn't as interesting there are others where this tool is useful for simplifying an equation.

Example Two

Consider the equation

$$x - 15 = 7,$$

we might try to add 15 to both sides and see what happens,

$$x - 15 + 15 = 7 + 15,$$

in this case we can simplify the $x - 15 + 15$ into just x. This gives us the new equation

$$x = 22,$$

and we have solved it!

Example Three

This can be used in more complex equations when there is a "lonely subtraction" so to speak. For example something a bit crazier such as

$$5\left(\frac{x}{2}-12\right)^2-22=13,$$

can be simplified by adding 22 to both sides.

$$5\left(\frac{x}{2}-12\right)^2-22+22=13+22,$$
$$5\left(\frac{x}{2}-12\right)^2=35.$$

Example Four

There are times when adding to both sides is not as useful. Consider the equation

$$4(x-8)=12,$$

we might try to add 8 to both sides,

$$4(x-8)+8=12+8,$$

but unfortunately the brackets protect the -8 and $+8$ from being simplified. If you expand the LHS into,

$$4x-32=12,$$

you might notice that we could instead expand then do the addition.

Example Five

Constant numbers aren't the only number we can add to both sides of an equation. Often it might be helpful to add something such as $5x$ or u^2 to both sides of an equation. Take the equation

$$5x+15=-3x+12,$$

lets see what happens when we add $3x$ to both sides,

$$5x+15+3x=-3x+12+3x,$$

we can collect like terms to get

$$8x+15=12.$$

Example Six (Complex)

Sometimes you can add to both sides of an equation to simplify, even though it might not feel like you should. These situations are often better handled by other techniques but often there is no best method. Consider

$$\frac{x^2 - 4x - 12}{x + 2} = 10,$$

we can add 4 to both sides

$$\begin{aligned}
\frac{x^2 - 4x - 12}{x + 2} + 4 &= 10 + 4, \\
\frac{x^2 - 4x - 12}{x + 2} + \frac{4}{1} &= 14, \\
\frac{x^2 - 4x - 12}{x + 2} + \frac{4 \times (x + 2)}{1 \times (x + 2)} &= 14, \\
\frac{x^2 - 4x - 12}{x + 2} + \frac{4(x + 2)}{x + 2} &= 14, \\
\frac{x^2 - 4x - 12}{x + 2} + \frac{4x + 8}{x + 2} &= 14, \\
\frac{x^2 - 4x - 12 + 4x + 8}{x + 2} &= 14, \\
\frac{x^2 - 4}{x + 2} &= 14.
\end{aligned}$$

As you can see we have in a way made it simpler, you will have to take my word for it a little bit.

Subtracting from Both Sides

Example One

Similar to when you are adding to both sides of an equation, for subtraction you might look for "lonely addition". Take the following equation,

$$3x + 17 = 32,$$

lets try subtract 17 from both sides,

$$3x + 17 - 17 = 32 - 17,$$

once we simplify both sides we should have

$$3x = 15.$$