Graphing Quadratics using Roots

Example Problems

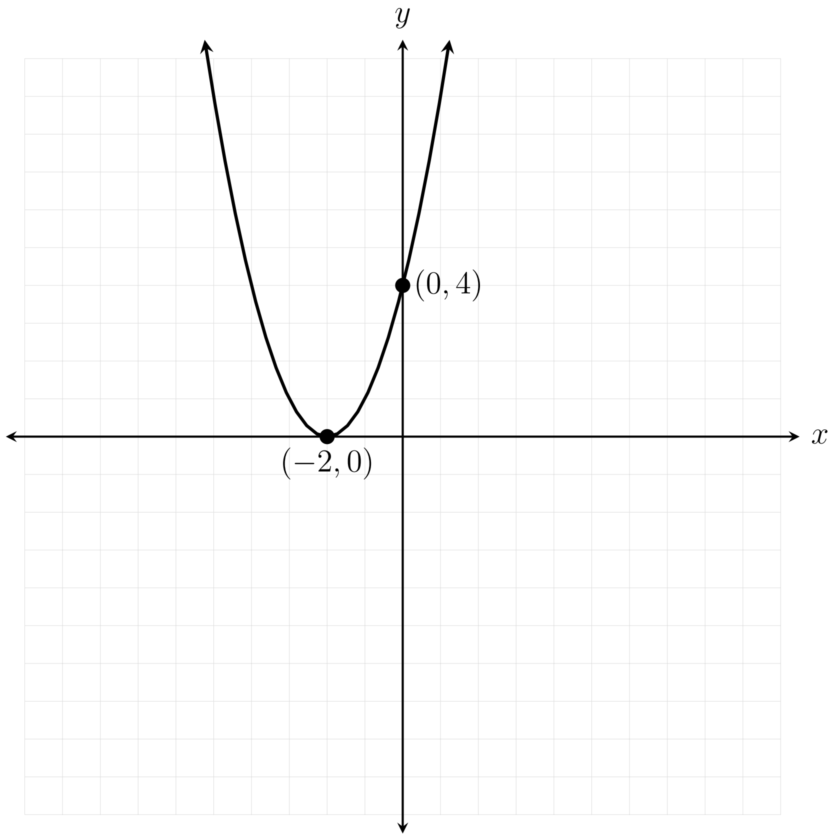
EXAMPLE 1: Draw a graph of the quadratic equation $y = (x+2)^2$, and mark all important features.
SOLUTION: First let's find the roots of the graph—when the graph crosses the x axis—by making $y = 0$,

$$0 = (x+2)^2,$$

this will be true when $x = -2$. This means our graph will cross the x axis only once, and that for our quadratic this must be the turning point. Then we want to find the y-intercept, this will occur when $x = 0$,

$$\begin{aligned} y &= (0+2)^2, \\ y &= 2^2, \\ y &= 4. \end{aligned}$$

With this information we can plot the key points and sketch the graph,



EXAMPLE 2: Draw a graph of the quadratic equation $y = x^2 + 2x - 8$ and label all important features.
SOLUTION: First we must determine where the roots of the graph will be, and to do this we will factorise the quadratic expression. This gives

$$y = (x-2)(x+4),$$

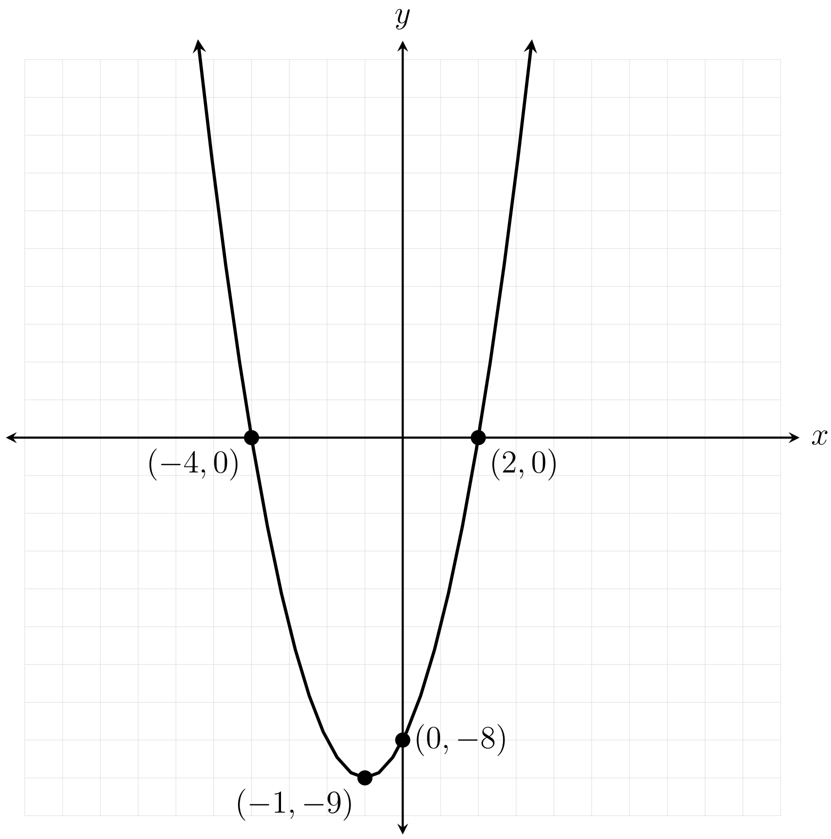
which tells us that the roots will occur at $x = 2$ and $x = -4$. Further we can find the y-intercept by substituting $x = 0$ to get

$$\begin{aligned} y &= 0^2 + 2 \times 0 - 8 \\ y &= -8. \end{aligned}$$

Finally we will find the turning point. The turning point of a parabola always lies on its axis of symmetry. The axis of symmetry can be calculated either by finding the halfway point between the roots -4 and 2—which would be -1—or by using the formula $\frac{-b}{2}$ which is also -1. We can then substitute the axis of symmetry as x to find the y coordinate of the turning point.

$$\begin{aligned} y &= (-1)^2 + 2 \times -1 - 8, \\ y &= 1 - 2 - 8, \\ y &= -9. \end{aligned}$$

Then we can plot these points and sketch the graph.



Example 3: Draw a graph of the quadratic equation $y = -x^2 + x + 6$ and label all important features.
Solution: First we might notice that the coefficient of x^2 is -1, which means our parabola will be concave down—the arms will point downwards. To factorise the quadratic expression we will factor -1 out of the trinomial expression then factorise

it like any other quadratic,

$$y = -(x^2 - x - 6),$$
$$y = -(x - 3)(x + 2).$$

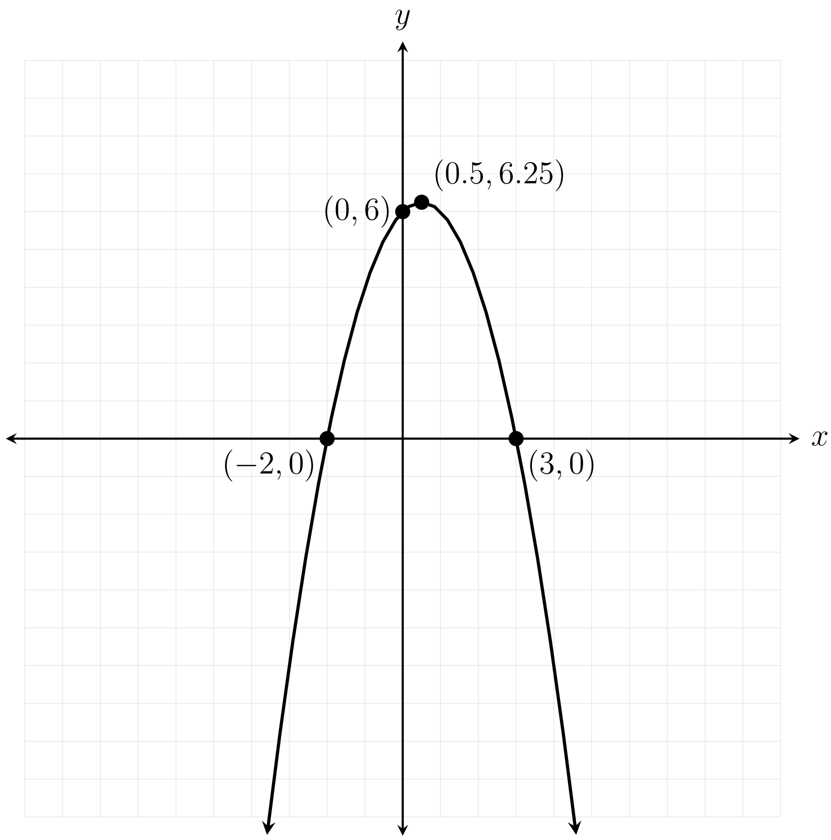
From this we can determine that the roots will be $x = -2$ and $x = 3$. We can find the y-intercept by substituting $x = 0$,

$$y = -0^2 + 0 + 6,$$
$$y = 6.$$

Finally, the x coordinate of the axis of symmetry will be 0.5, so we can find the y coordinate of the turning point to be

$$y = -0.5^2 + 0.5 + 6,$$
$$y = 6.25.$$

Then we can plot the key points and sketch the graph.



Question Bank

1. For each of the following quadratic equations find their roots, y-intercept, and turning point. Then graph the equation and label all the important points.

   a) $y = x^2$
   b) $y = x^2 + 3x + 2$
   c) $y = x^2 + 2x + 1$
   d) $y = x^2 - 4x + 3$
   e) $y = x^2 - 6x + 9$
   f) $y = x^2 + 11x + 28$
   g) $y = x^2 - 14x + 45$
   h) $y = x^2 - 3x - 4$
   i) $y = x^2 + 2x - 24$
   j) $y = x^2 - x - 56$
   k) $y = -x^2 - 9x - 20$
   l) $y = -x^2 - 2x - 1$
   m) $y = -x^2 + 11x - 24$
   n) $y = -x^2 + 8x - 16$
   o) $y = -x^2 + 2x + 35$
   p) $y = -x^2 - 3x + 10$