
Onsager Documentation

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CONTENTS

1	Onsager	3
2	References	5
3	Contributors	7
4	Support	9
5	Crystal	11
6	CrystalStars	21
7	Supercell	29
8	PowerExpansion	33
9	GFcalc	41
10	OnsagerCalc	45
11	Automator	53
12	Notebooks	57
12.1	Fe-C diffusion and elastodiffusivity	57
12.2	Garnet correlation coefficients	62
12.3	Convergence of Green function calculation	69
12.4	Tracer correlation coefficients	74
13	Indices and tables	81
	Python Module Index	83
	Index	85

Documentation for Onsager python module for automated computation of diffusivity for interstitial and vacancy mediated diffusion.

Contents:

ONSAGER

Documentation now available at the [Onsager github page](<http://dallastrinkle.github.io/Onsager/>). Please cite as `[[DOI](https://zenodo.org/badge/14172/DallasTrinkle/Onsager.svg)]](https://zenodo.org/badge/latestdoi/14172/DallasTrinkle/Onsager)` or see [Onsager github](#) for current version doi information.

The Onsager package provides routines for the general calculation of transport coefficients in vacancy-mediated diffusion and interstitial diffusion. It does this using a Green function approach, combined with point group symmetry reduction for maximum efficiency.

Typical usage looks like:

```
#!/usr/bin/env python

from onsager import crystal
from onsager import OnsagerCalc

...
```

Many of the subpackages within Onsager are support for the main attraction, which is in `OnsagerCalc`. Interstitial calculation examples are available in `bin`, including three YAML input files, as well as a interstitial diffuser. An example of vacancy-mediated diffusion is shown in `bin/fivefreq.py`, which computes the well-known five-frequency model for substitutional solute transport in an FCC lattice.

The tests for the package are include in `test`; `tests.py` will run all of the tests in the directory with verbosity level 2. This can be time-consuming (on the order of several of minutes) to run all tests; coverage is currently >90%.

The code uses YAML files for input/output of diffusion data for the interstitial calculator. The vacancy-mediated calculator requires much more data, and uses HDF5 format to save/reload as needed. The vacancy-mediated calculator uses tags (unique human-readable-ish strings) to identify all (symmetry-unique) vacancy, solute, and complex states, and transitions between them.

Releases:

- 0.9. Full release of Interstitial calculator, along with theory paper (see References below).
- 0.9.1. Added spin degrees of freedom to `crystal` for symmetry purposes; added `supercell` class to aid in automated setup of calculation.
- 1.0 Now including automator for supercell calculations.
- 1.1 Automator update with Makefile; corrections for possible overflow error when `omega2` gets very large.
- 1.2 Combined “large `omega2`” and “non-zero bias” algorithms; notebook for Fe-C added to documentation; cleanup of code and improved testing.

REFERENCES

- Dallas R. Trinkle, “Diffusivity and derivatives for interstitial solutes: Activation energy, volume, and elastodiffusion tensors.” *Philos. Mag.* (2016) [doi:10.1080/14786435.2016.1212175](http://dx.doi.org/10.1080/14786435.2016.1212175) [arXiv:1605.03623](http://arxiv.org/abs/1605.03623)

CONTRIBUTORS

- Dallas R. Trinkle, initial design, derivation, and implementation.
- Ravi Agarwal, testing of HCP interstitial calculations; testing of HCP vacancy-mediated diffusion calculations
- Abhinav Jain, testing of HCP vacancy-mediated diffusion calculations.

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CRYSTAL

Crystal:

The crystal module defines the `crystal` class, and `GroupOp` for group operations. `Crystal` class

Class to store definition of a crystal, along with some analysis 1. geometric analysis (nearest neighbor displacements) 2. space group operations 3. point group operations for each basis position 4. Wyckoff position generation (for interstitials)

`crystal.CombineTensorBasis (b1, b2, symmetric=True)`

Combines (intersects) two tensor spaces into one; uses SVD to compute null space.

Parameters

- **b1** – list of tensors
- **b2** – list of tensors

Return tensorbasis list of 2nd-rank symmetric tensors making up the basis

`crystal.CombineVectorBasis (b1, b2)`

Combines (intersects) two vector spaces into one.

Parameters

- **b1** – (dim, vect) – dimensionality (0..3), vector defining line direction (1) or plane normal (2)
- **b2** – (dim, vect)

Return dim dimensionality, 0..3

Return vect vector defining line direction (1) or plane normal (2)

`class crystal.Crystal (lattice, basis, chemistry=None, spins=None, NOSYM=False, noreduce=False)`

A class that defines a crystal, as well as the symmetry analysis that goes along with it. Now includes optional spins. These can be vectors or “scalar” spins, for which we need to consider a phase factor. In general, they can be complex. Ideally, they should have magnitude either 0 or 1.

Specified by a lattice (3 vectors), a basis (list of lists of positions in direct coordinates). Can also name the elements (chemistry), and specify spin degrees of freedom.

`classmethod BCC (a0, chemistry=None)`

Create a body-centered cubic crystal with lattice constant a0

Parameters **a0** – lattice constant

Return BCC crystal

`classmethod FCC (a0, chemistry=None)`

Create a face-centered cubic crystal with lattice constant a0

Parameters **a0** – lattice constant

Return FCC crystal

FullVectorBasis (*chem=None*)

Generate our full vector basis, using the information from our crystal

Parameters **chem** – (optional) chemical index to consider; otherwise return a list of such

Return **VBfunctions** (list) of our unique vector basis lattice functions, normalized; each is an array (NVbasis x Nsites x 3)

Return **VVouter** (list) of our VV “outer” expansion (NVbasis x NVbasis for each chemistry)

classmethod **HCP** (*a0, c_a=1.6329931618554521, chemistry=None*)

Create a hexagonal closed packed crystal with lattice constant a0, c/a ratio c_a

Parameters

- **a0** – lattice constant
- **c_a** – (optional) c/a ratio, default=ideal $\sqrt{8/3}$

Return HCP crystal

SymmTensorBasis (*ind*)

Generates the symmetric tensor basis corresponding to an atomic site

Parameters **ind** – tuple index for atom

Return **tensorbasis** list of 2nd-rank symmetric tensors making up the basis

VectorBasis (*ind*)

Generates the vector basis corresponding to an atomic site

Parameters **ind** – tuple index for atom

Return **dim** dimensionality, 0..3

Return **vect** vector defining line direction (1) or plane normal (2)

Wyckoffpos (*uvec*)

Generates all the equivalent Wyckoff positions for a unit cell vector.

Parameters **uvec** – 3-vector (float) vector in direct coordinates

Return **Wyckofflist** list of equivalent Wyckoff positions

__init__ (*lattice, basis, chemistry=None, spins=None, NOSYM=False, noreduce=False*)

Initialization; starts off with the lattice vector definition and the basis vectors. While it does not explicitly store the specific chemical elements involved, it does store that there are different elements.

Parameters

- **lattice** – array[3,3] or list of array[3] lattice vectors; if [3,3] array, then the vectors need to be in *column* format so that the first lattice vector is lattice[:,0]
- **basis** – list of array[3] or list of list of array[3] crystalline basis vectors, in unit cell coordinates. If a list of lists, then there are multiple chemical elements, with each list corresponding to a unique element
- **chemistry** – (optional) list of names of chemical elements

- **spins** – (optional) list of numbers (complex) / vectors or list of list of same spins for individual atoms; if not None, needs to match the basis. Can either be scalars or vectors, corresponding to collinear or non-collinear magnetism
- **NOSYM** – turn off all symmetry finding (except identity)
- **noreduce** – do not attempt to reduce the atomic basis

__repr__ ()

String representation of crystal (lattice + basis)

__str__ ()

Human-readable version of crystal (lattice + basis)

__weakref__

list of weak references to the object (if defined)

addbasis (*basis*, *chemistry=None*, *spins=None*)

Returns a new Crystal object that contains additional sites (assumed to be new chemistry). This is intended to “add in” interstitial sites. Note: if the symmetry is to be maintained, should be the output from Wyckoffpos().

Parameters

- **basis** – list (or list of lists) of new sites
- **chemistry** – (optional) list of chemistry names
- **spins** – (optional) list of spins

Return Crystal new crystal object, with additional sites

calcmetric ()

Computes the volume of the cell and the metric tensor

Return volume cell volume

Return metric tensor 3x3

cart2pos (*v*)

Return the lattvec and index corresponding to an atomic position in cartesian coord.

Parameters **v** – 3-vector (float) position in Cartesian coordinates

Return lattvec 3-vector (integer) lattice vector in direct coordinates,

Return (c,i) tuple of matching basis atom; None if no match

cart2unit (*v*)

Return the lattvec and unit cell coord. corresponding to a position in cartesian coord.

Parameters **v** – 3-vector (float) position in Cartesian coordinates

Return lattvec 3-vector (integer) lattice vector in direct coordinates,

Return uvec 3-vector (float) inside unit cell, in direct coordinates

center ()

Center the atoms in the cell if there is an inversion operation present.

chemindex (*chemistry*)

Return index corresponding to chemistry; None if not present.

Parameters **chemistry** – value to check

Return index corresponding to chemistry

classmethod fromdict (*yamldict*)

Creates a Crystal object from a *very simple* YAML-created dictionary

Parameters **yamldict** – dictionary; must contain ‘lattice’ (using *row* vectors!) and ‘basis’; can contain optional ‘lattice_constant’

Return **Crystal(lattice.T, basis)** new crystal object

fullkptmesh (*Nmesh*)

Creates a k-point mesh of density given by Nmesh; does not symmetrize but does put the k-points inside the BZ. Does not return any *weights* as every point is equally weighted.

Parameters **Nmesh** – mesh divisions Nmesh[0] x Nmesh[1] x Nmesh[2]

Return **kpt** array[Nkpt][3] of kpoints

g_cart (*g, x*)

Apply a space group operation to a (Cartesian) vector position

Parameters

- **g** – group operation (GroupOp)
- **x** – 3-vector position in space

Return **gx** 3-vector position in space (Cartesian coordinates)

static g_direct (*g, direc*)

Apply a space group operation to a direction

Parameters

- **g** – group operation (GroupOp)
- **direc** – 3-vector direction

Return **gdirec** 3-vector direction

g_direct_equivalent (*d1, d2, threshold=1e-08*)

Tells us if two directions are equivalent by according to the space group

Parameters

- **d1** – direction one (array[3])
- **d2** – direction two (array[3])
- **threshold** – threshold for equality

Return **equivalent** True if equivalent by a point group operation

g_pos (*g, lattvec, ind*)

Apply a space group operation to an atom position specified by its lattice and index

Parameters

- **g** – group operation (GroupOp)
- **lattvec** – 3-vector (integer) lattice vector in direct coordinates
- **ind** – two-tuple index specifying the atom: (atomtype, atomindex)

Return **glatt** 3-vector (integer) lattice vector in direct coordinates

Return **gindex** tuple of new basis atom

static g_tensor (*g, tensor*)

Apply a space group operation to a 2nd-rank tensor

Parameters

- **g** – group operation (GroupOp)
- **tensor** – 2nd-rank tensor

Return **gtensor** 2nd-rank tensor

static g_vect (*g, lattvec, uvec*)

Apply a space group operation to a vector position specified by its lattice and a location in the unit cell in direct coordinates

Parameters

- **g** – group operation (GroupOp)
- **lattvec** – 3-vector (integer) lattice vector in direct coordinates
- **uvec** – 3-vector (float) vector in direct coordinates
- **guvec** – 3-vector (float) vector in direct coordinates

Return **glatt** 3-vector (integer) lattice vector in direct coordinates

genBZG ()

Generates the reciprocal lattice G points that define the Brillouin zone.

Return **Garray** array of G vectors that define the BZ, in Cartesian coordinates

genWyckoffsets ()

Generate our Wyckoff sets.

Return **Wyckoffsets** set of sets of tuples of positions that correspond to identical Wyckoff positions

gengroup ()

Generate all of the space group operations. Now handles spins! Doesn't store spin phase factors for each group operation, though.

Return **Gset** frozenset of group operations

genpoint ()

Generate our point group indices. Done with crazy list comprehension due to the structure of our basis.

Return **Gpointlists** list of lists of frozensets of point group operations that leave a site unchanged

inBZ (*vec, BZG=None, threshold=1e-05*)

Tells us if vec is inside our set of defining points.

Parameters

- **vec** – array [3], vector to be tested
- **BZG** – array [:3], optional (default = self.BZG), array of vectors that define the BZ
- **threshold** – double, optional, threshold to use for "equality"

Return **inBZ** False if outside the BZ, True otherwise

jumpnetwork (*chem, cutoff, closestdistance=0*)

Generate the full jump network for a specific chemical index, out to a cutoff. Organized by symmetry-unique transitions. Note that i->j and j->i are always related to one-another, but by equivalence of transition state, not symmetry. Now updated with closest-distance parameter.

Parameters

- **chem** – index corresponding to the chemistry to consider
- **cutoff** – distance cutoff
- **closestdistance** – closest distance allowed in transition (can be a list)

Return jumpnetwork list of symmetry-unique transitions; each is a list of tuples: $((i, j), \delta x)$ corresponding to jump from $i \rightarrow j$ with vector δx

jumpnetwork2lattice (*chem, jumpnetwork*)

Convert a “standard” jumpnetwork (that specifies displacement vectors δx) into a lattice representation, where we replace δx with the lattice vector from i to j .

Parameters

- **chem** – index corresponding to the chemistry to consider
- **jumpnetwork** – list of symmetry-unique transitions; each is a list of tuples: $((i, j), \delta x)$ corresponding to jump from $i \rightarrow j$ with vector δx

Return jumplattice list of symmetry-unique transitions; each is a list of tuples: $((i, j), R)$ corresponding to jump from i in unit cell 0 $\rightarrow j$ in unit cell R

minlattice ()

Try to find the optimal lattice vector definition for a crystal. Our definition of optimal is (a) length of each lattice vector is minimal; (b) the vectors are ordered from shortest to longest; (c) the vectors have minimal dot product; (d) the basis is right-handed.

Works recursively, and in-place.

nnlist (*ind, cutoff*)

Generate the nearest neighbor list for a given cutoff. Only consider neighbor vectors for atoms of the same type. Returns a list of cartesian vectors.

Parameters

- **ind** – tuple index for atom
- **cutoff** – distance cutoff

Return nnlist list of nearest neighbor vectors

pos2cart (*lattvec, ind*)

Return the cartesian coordinates of an atom specified by its lattice and index

Parameters

- **lattvec** – 3-vector (integer) lattice vector in direct coordinates
- **ind** – two-tuple index specifying the atom: (atomtype, atomindex)

Return v 3-vector (float) in Cartesian coordinates

reduce (*threshold=1e-08*)

Reduces the lattice and basis, if needed. Works (tail) recursively.

reducekptmesh (*kptfull, threshold=1e-08*)

Takes a fully expanded mesh, and reduces it by symmetry. Assumes every point is equally weighted. We would need a different (more complicated) algorithm if not true...

Parameters

- **kptfull** – array[Nkpt][3] of kpoints
- **threshold** – threshold for symmetry equality

Return kptsymm array[Nsymm][3] of kpoints

Return weight array[Nsymm] of weights (integrates to 1)

remapbasis (*supercell*)

Takes the basis definition, and using a supercell definition, returns a new basis

Parameters **supercell** – integer array[3,3]

Return atomic basis list of list of positions

simpleYAML (*a0=1.0*)

Creates a simplified YAML dump, in case we don't want to output the full symmetry analysis

Return YAML string dump

sitelist (*chem*)

Return a list of lists of Wyckoff-related sites for a given chemistry. Done with a single list comprehension–useful as input for diffusion calculation

Parameters **chem** – index corresponding to chemistry to consider

Return symmequivsites list of lists of indices that are equivalent by symmetry

strain (*eps*)

Returns a new Crystal object that is a strained version of the current.

Parameters **eps** – strain tensor

Return Crystal new crystal object, strained

unit2cart (*lattvec, uvec*)

Return the cartesian coordinates of a position specified by its lattice and unit cell coordinates

Parameters

- **lattvec** – 3-vector (integer) lattice vector in direct coordinates
- **uvec** – 3-vector (float) unit cell vector in direct coordinates

Return v 3-vector (float) in Cartesian coordinates

static vectlist (*vb*)

Returns a list of orthonormal vectors corresponding to our vector basis.

Parameters **vb** – (dim, v)

Return vlist list of vectors

class crystal. GroupOp

A class corresponding to a group operation. Based on namedtuple, so it is immutable.

Intended to be used in combination with Crystal, we have a few operations that can be defined out-of-the-box.

Parameters

- **rot** – np.array(3,3) integer idempotent matrix
- **trans** – np.array(3) real vector
- **cartrot** – np.array(3,3) real unitary matrix
- **indexmap** – tuples of tuples, containing the atom mapping

static GroupOp_constructor (*loader, node*)

Construct a GroupOp from YAML

static GroupOp_representer (*dumper, data*)

Output a GroupOp

__add__ (*other*)

Add a translation to our group operation

__eq__ (*other*)

Test for equality—we use `numpy.isclose` for comparison, since that’s what we usually care about

__hash__ ()

Hash, so that we can make sets of group operations

__mul__ (*other*)

Multiply two group operations to produce a new group operation

__ne__ (*other*)

Inequality == not **__eq__**

__sane__ ()

Return true if the cartrot and rot are consistent and ‘sane’

__str__ ()

Human-readable version of `groupop`

__sub__ (*other*)

Add a (negative) translation to our group operation

eigen ()

Returns the type of group operation (single integer) and eigenvectors. 1 = identity 2, 3, 4, 6 = n- fold rotation around an axis negative = rotation + mirror operation, perpendicular to axis “special cases”: -1 = mirror, -2 = inversion

`eigenvect[0]` = axis of rotation / mirror `eigenvect[1]`, `eigenvect[2]` = orthonormal vectors to define the plane giving a right-handed coordinate system and where rotation around [0] is positive, and the positive imaginary eigenvector for the complex eigenvalue is [1] + i [2].

Return type integer

Return eigenvectors list of [ev0, ev1, ev2]

classmethod ident (*basis*)

Return a group operation corresponding to identity for a given basis

incell ()

Return a version of `groupop` where the translation is in the unit cell

inhalf ()

Return a version of `groupop` where the translation is in the centered unit cell

inv ()

Construct and return the inverse of the group operation

static optype (*rot*)

Returns the type of group operation (single integer) and eigenvectors. 1 = identity 2, 3, 4, 6 = n- fold rotation around an axis negative = rotation + mirror operation, perpendicular to axis “special cases”: -1 = mirror, -2 = inversion

Parameters **rot** – rotation matrix (can be the integer rot)

Return type integer

crystal. ProjectTensorBasis (*tensor, basis*)

Given a tensor, project it onto the basis.

Parameters

- **tensor** – tensor

- **basis** – list consisting of an orthonormal basis

Return tensor tensor, projected

crystal. **SymmTensorBasis** (*rotype, eigenvect*)

Returns a symmetric second-rank tensor basis corresponding to the otype and eigenvectors for a GroupOp

Parameters

- **rotype** – output from eigen()
- **eigenvect** – eigenvectors

Return tensorbasis list of 2nd-rank symmetric tensors making up the basis

crystal. **VectorBasis** (*rotype, eigenvect*)

Returns a vector basis corresponding to the otype and eigenvectors for a GroupOp

Parameters

- **rotype** – output from eigen()
- **eigenvect** – eigenvectors

Return dim dimensionality, 0..3

Return vect vector defining line direction (1) or plane normal (2)

crystal. **Voigtstrain** (*e1, e2, e3, e4, e5, e6*)

Returns a symmetric strain tensor from the Voigt reduced strain values.

Parameters

- **e1** – xx
- **e2** – yy
- **e3** – zz
- **e4** – yz + zx
- **e5** – zx + xz
- **e6** – xy + yx

Return strain symmetric strain tensor

crystal. **incell** (*vec*)

Returns the vector inside the unit cell (in [0,1]**3)

Parameters **vec** – 3-vector (unit coord)

Returns 3-vector

crystal. **inhalf** (*vec*)

Returns the vector inside the centered cell (in [-0.5,0.5]**3)

Parameters **vec** – 3-vector (unit coord)

Returns 3-vector

crystal. **maptranslation** (*oldpos, newpos, oldspins=None, newspins=None, threshold=1e-08*)

Given a list of transformed positions, identify if there's a translation vector that maps from the current positions to the new position.

The mapping specifies the index that the *translated* atom corresponds to in the original position set. If unable to construct a mapping, the mapping return is None; the translation vector will be meaningless.

If old/newspins are given then ONLY mappings that maintain spin are considered. This means that a loop is needed to consider possible spin phase factors.

Parameters

- **oldpos** – list of list of array[3]
- **newpos** – list of list of array[3], same layout as oldpos
- **oldspins** – (optional) list of list of numbers/arrays
- **newspins** – (optional) list of list of numbers/arrays

Return translation array[3]

Return mapping list of list of indices

`crystal.ndarray_representer (dumper, data)`

Output a numpy array

CRYSTALSTARS

CrystalStars:

The crystalStars module defines the classes corresponding to stars (in this case, for solute-vacancy complexes that are equivalent by space group symmetry), and vector stars (the inclusion of a vector basis on the stars). These modules are primarily responsible for all the symmetry analysis, and converting that into matrix forms for rapid numerical evaluation as needed. Stars module, modified to work with crystal class

Classes to generate star sets, double star sets, and vector star sets; a lot of indexing functionality.

NOTE: The naming follows that of stars; the functionality is extremely similar, and this code was modified as little as possible to translate that functionality to *crystals* which possess a basis. In the case of a single atom basis, this should reduce to the stars object functionality.

The big changes are:

- Replacing NNvect star (which represents the jumps) with the jumpnetwork type found in crystal
- Using the jumpnetwork_latt representation from crystal
- Representing a “point” as a solute + vacancy. In this case, it is a tuple (s,v) of unit cell indices and a vector dx or dR (dx = Cartesian vector pointing from solute to vacancy; dR = lattice vector pointing from unit cell of solute to unit cell of vacancy). This is equivalent to our old representation if the tuple (s,v) = (0,0) for all sites. Due to translational invariance, the solute always stays inside the unit cell
- Using indices into the point list rather than just making lists of the vectors themselves. This is because the “points” now have a more complex representation (see above).

crystalStars. **PSlist2array** (*PSlist*)

Take in a list of pair states; return arrays that can be stored in HDF5 format

Parameters **PSlist** – list of pair states

Return **ij** int_array[N][2] = (i,j)

Return **R** int[N][3]

Return **dx** float[N][3]

class crystalStars. **PairState**

A class corresponding to a “pair” state; in this case, a solute-vacancy pair, but can also be a transition state pair. The solute (or initial state) is in unit cell 0, in position indexed i; the vacancy (or final state) is in unit cell R, in position indexed j. The cartesian vector dx connects them. We can add and subtract, negate, and “endpoint” subtract (useful for determining what Green function entry to use)

Parameters

- **i** – index of the first member of the pair (solute)
- **j** – index of the second member of the pair (vacancy)

- **R** – lattice vector pointing from unit cell of *i* to unit cell of *j*
- **dx** – Cartesian vector pointing from first to second member of pair

static PairState_constructor (*loader, node*)

Construct a GroupOp from YAML

static PairState_representer (*dumper, data*)

Output a PairState

__add__ (*other*)

Add two states: works if and only if $\text{self.j} == \text{other.i}$ (*i,j*) $R + (j,k) R' = (i,k) R + R'$: works for thinking about transitions... Note: $a + b \neq b + a$, and may be that only one of those is even defined

__eq__ (*other*)

Test for equality—we don't bother checking *dx*

__hash__ ()

Hash, so that we can make sets of states

__ne__ (*other*)

Inequality $==$ not **__eq__**

__neg__ ()

Negation of state (swap members of pair) - (*i,j*) $R = (j,i) -R$ Note: $a + (-a) == (-a) + a == 0$ because we define what "zero" is.

__sane__ (*crys, chem*)

Determine if the *dx* value makes sense given everything else...

__str__ ()

Human readable version

__sub__ (*other*)

Add a negative: $a-b$ points from initial of *a* to initial of *b* if same final state (*i,j*) $R - (k,j) R' = (i,k) R - R'$ Note: this means that $(a-b) + b = a$, but $b + (a-b)$ is an error. $(b-a) + a = b$

__xor__ (*other*)

Subtraction on the endpoints (sort of the "opposite" of $a-b$): $a \wedge b$ points from final of *b* to final of *a* if same initial state (*i,j*) $R \wedge (i,k) R' = (k,j) R - R'$ Note: $b + (a \wedge b) = a$ but $(a \wedge b) + b$ is an error. $a + (b \wedge a) = b$

classmethod fromcrys (*crys, chem, ij, dx*)

Convert (*ij*), *dx* into PairState

classmethod fromcrys_latt (*crys, chem, ij, R*)

Convert (*ij*), *R* into PairState

g (*crys, chem, g*)

Apply group operation.

Parameters

- **crys** – crystal
- **chem** – chemical index
- **g** – group operation (from *crys*)

Return $g * \text{PairState}$ corresponding to group operation applied to self

iszero ()

Quicker than $\text{self} == \text{PairState.zero}()$

classmethod zero (*n=0*)

Return a “zero” state

class crystalStars. StarSet (*jumpnetwork, crys, chem, Nshells=0, originstates=False, lattice=False*)

A class to construct crystal stars, and be able to efficiently index.

Takes in a jumpnetwork, which is used to construct the corresponding stars, a crystal object with which to operate, a specification of the chemical index for the atom moving (needs to be consistent with jumpnetwork and crys), and then the number of shells.

In this case, shells = number of successive “jumps” from a state. As an example, in FCC, 1 shell = 1st neighbor, 2 shell = 1-4th neighbors.

__add__ (*other*)

Add two StarSets together; done by making a copy of one, and iadding

__contains__ (*PS*)

Return true if PS is in the star

__iadd__ (*other*)

Add another StarSet to this one; very similar to generate()

__init__ (*jumpnetwork, crys, chem, Nshells=0, originstates=False, lattice=False*)

Initiates a star set generator for a given jumpnetwork, crystal, and specified chemical index. Does not include “origin states” by default; these are PairStates that iszero() is True; they are only needed if crystal has a nonzero VectorBasis.

Parameters

- **jumpnetwork** – list of symmetry unique jumps, as a list of list of tuples; either ((i, j), dx) for jump from i to j with displacement dx, or ((i, j), R) for jump from i in unit cell 0 -> j in unit cell R
- **crys** – crystal where jumps take place
- **chem** – chemical index of atom to consider jumps
- **Nshells** – number of shells to generate
- **originstates** – include origin states in generate?
- **lattice** – which form does the jumpnetwork take?

__str__ ()

Human readable version

__weakref__

list of weak references to the object (if defined)

addhdf5 (*HDF5group*)

Adds an HDF5 representation of object into an HDF5group (needs to already exist).

Example: if f is an open HDF5, then StarSet.addhdf5(f.create_group('StarSet')) will (1) create the group named 'StarSet', and then (2) put the StarSet representation in that group.

Parameters HDF5group – HDF5 group

copy (*empty=False*)

Return a copy of the StarSet; done as efficiently as possible; empty means skip the shells, etc.

diffgenerate (*S1, S2, threshold=1e-08*)

Construct a starSet using endpoint subtraction from starset S1 to starset S2. Will include zero. Points from vacancy states of S1 to vacancy states of S2.

Parameters

- **S1** – starSet for start
- **S2** – starSet for final
- **threshold** – threshold for sorting magnitudes (can influence symmetry efficiency)

generate (*Nshells, threshold=1e-08, originstates=False*)

Construct the points and the stars in the set. Does not include “origin states” by default; these are PairStates that iszero() is True; they are only needed if crystal has a nonzero VectorBasis.

Parameters

- **Nshells** – number of shells to generate; this is interpreted as subsequent “sums” of jumplist (as we need the solute to be connected to the vacancy by at least one jump)
- **threshold** – threshold for determining equality with symmetry
- **originstates** – include origin states in generate?

jumpnetwork_omega1 ()

Generate a jumpnetwork corresponding to vacancy jumping while the solute remains fixed.

Return jumpnetwork list of symmetry unique jumps; list of list of tuples (i,f), dx where i,f index into states for the initial and final states, and dx = displacement of vacancy in Cartesian coordinates. Note: if (i,f), dx is present, so if (f,i), -dx

Return jumptype list of indices corresponding to the (original) jump type for each symmetry unique jump; useful for constructing a LIMB approximation, and needed to construct delta_omega

Return starpair list of tuples of the star indices of the i and f states for each symmetry unique jump

jumpnetwork_omega2 ()

Generate a jumpnetwork corresponding to vacancy exchanging with a solute.

Return jumpnetwork list of symmetry unique jumps; list of list of tuples (i,f), dx where i,f index into states for the initial and final states, and dx = displacement of vacancy in Cartesian coordinates. Note: if (i,f), dx is present, so if (f,i), -dx

Return jumptype list of indices corresponding to the (original) jump type for each symmetry unique jump; useful for constructing a LIMB approximation, and needed to construct delta_omega

Return starpair list of tuples of the star indices of the i and f states for each symmetry unique jump

classmethod loadhdf5 (*crys, HDF5group*)

Creates a new StarSet from an HDF5 group.

Parameters

- **crys** – crystal object–MUST BE PASSED IN as it is not stored with the StarSet
- **HDFgroup** – HDF5 group

Return StarSet new StarSet object

starindex (*PS*)

Return the index for the star to which pair state PS belongs; None if not found

stateindex (*PS*)

Return the index of pair state PS; None if not found

symmatch (*PS1, PS2*)

True if there exists a group operation that makes $PS1 == PS2$.

symmequivjumplist (*i, f, dx*)

Returns a list of tuples of symmetry equivalent jumps

Parameters

- **i** – index of initial state
- **f** – index of final state
- **dx** – displacement vector

Return symmjumplist list of tuples of ((gi, gf), gdx) for every group op

class crystalStars. **VectorStarSet** (*starset=None*)

A class to construct vector star sets, and be able to efficiently index.

All based on a StarSet

GFexpansion ()

Construct the GF matrix expansion in terms of the star vectors, and indexed to GFstarset.

Return GFexpansion array[Nsv, Nsv, NGFstars] the GF matrix[i, j] =
sum(GFexpansion[i, j, k] * GF(starGF[k]))

Return GFstarset starSet corresponding to the GF

__init__ (*starset=None*)

Initiates a vector-star generator; work with a given star.

Parameters starset – StarSet, from which we pull nearly all of the info that we need

__weakref__

list of weak references to the object (if defined)

addhdf5 (*HDF5group*)

Adds an HDF5 representation of object into an HDF5group (needs to already exist).

Example: if f is an open HDF5, then StarSet.addhdf5(f.create_group('VectorStarSet')) will

(1) create the group named 'VectorStarSet', and then (2) put the VectorStarSet representation in that group.

Parameters HDF5group – HDF5 group

bareexpansions (*jumpnetwork, jumptype*)

Construct the bare diffusivity expansion in terms of the jumpnetwork. We return the reference (0) contribution so that the change can be determined; this is useful for the vacancy contributions. This saves us from having to deal with issues with our outer shell where we only have a fraction of the escapes, but as long as the kinetic shell is one more than the thermodynamics (so that the interaction energy is 0, hence no change in probability), this will work. The PS (pair stars) is useful for including the probability factor for the endpoint of the jump; we just call it the 'probfactor' below.

Note also: this *currently assumes* that the displacement vector *does not change* between omega0 and omega(1/2).

Parameters

- **jumpnetwork** – jumpnetwork of symmetry unique omega1-type jumps, corresponding to our starset. List of lists of (IS, FS), dx tuples, where IS and FS are indices corresponding to states in our starset.

- **jumptype** – specific omega0 jump type that the jump corresponds to

Return D0expansion array[3,3, Njump_omega0] the $D0[a,b,jt] = \text{sum}(D0\text{expansion}[a,b,jt] * \text{sqrt}(\text{probfactor0}[PS[jt][0]] * \text{probfactor0}[PS[jt][1]) * \text{omega0}[jt])$

Return D1expansion array[3,3, Njump_omega1] the $D1[a,b,k] = \text{sum}(D1\text{expansion}[a,b,k] * \text{sqrt}(\text{probfactor}[PS[k][0]] * \text{probfactor}[PS[k][1]) * \text{omega}[k])$

biasexpansions (*jumpnetwork, jumptype, omega2=False*)

Construct the bias1 and bias0 vector expansion in terms of the jumpnetwork. We return the bias0 contribution so that the $db = \text{bias1} - \text{bias0}$ can be determined. This saves us from having to deal with issues with our outer shell where we only have a fraction of the escapes, but as long as the kinetic shell is one more than the thermodynamics (so that the interaction energy is 0, hence no change in probability), this will work. The PS (pair stars) is useful for including the probability factor for the endpoint of the jump; we just call it the ‘probfactor’ below. *Note:* this used to be separated into bias1expansion, and bias2expansion, and had terms that are now in rateexpansions. Note also that if jumpnetwork_omega2 is passed, it also works for that. However, in that case we have a different approach for the calculation of bias1expansion: if there are origin states, they get the negative summed bias of the others.

Parameters

- **jumpnetwork** – jumpnetwork of symmetry unique omega1-type jumps, corresponding to our starset. List of lists of (IS, FS), dx tuples, where IS and FS are indices corresponding to states in our starset.
- **jumptype** – specific omega0 jump type that the jump corresponds to
- **omega2** – (optional) are we dealing with the omega2 list, so we need to remove origin states? (default=False)

Return bias0expansion array[Nsv, Njump_omega0] the $\text{gen0 vector}[i] = \text{sum}(\text{bias0expansion}[i, k] * \text{sqrt}(\text{probfactor0}[PS[k]]) * \text{omega0}[k])$

Return bias1expansion array[Nsv, Njump_omega1] the $\text{gen1 vector}[i] = \text{sum}(\text{bias1expansion}[i, k] * \text{sqrt}(\text{probfactor}[PS[k]] * \text{omega1}[k])$

generate (*starset, threshold=1e-08*)

Construct the actual vectors stars

Parameters **starset** – StarSet, from which we pull nearly all of the info that we need

generateouter ()

Generate our outer products for our star-vectors.

Return outer array [3, 3, Nvstars, Nvstars] $\text{outer}[:, :, i, j]$ is the 3x3 tensor outer product for two vector-stars $vs[i]$ and $vs[j]$

classmethod loadhdf5 (*SSet, HDF5group*)

Creates a new VectorStarSet from an HDF5 group.

Parameters

- **SSet** – StarSet–MUST BE PASSED IN as it is not stored with the VectorStarSet
- **HDFgroup** – HDF5 group

Return VectorStarSet new VectorStarSet object

originstateVectorBasisfolddown (*elemtype='solute'*)

Construct the expansion to “fold down” from vector stars to origin states.

Parameters **elemtype** – ‘solute’ of ‘vacancy’, depending on which site we need to reduce

Return OSIndices list of indices corresponding to origin states

Return foldddown [NOS, Nvstars] to map vector stars to origin states

Return OS_VB [NOS, Nsites, 3] mapping of origin state to a vector basis

rateexpansions (*jumpnetwork, jumptype, omega2=False*)

Construct the omega0 and omega1 matrix expansions in terms of the jumpnetwork; includes the escape terms separately. The escape terms are tricky because they have probability factors that differ from the transitions; the PS (pair stars) is useful for finding this. We just call it the 'probfactor' below. *Note:* this used to be separated into rate0expansion, and rate1expansion, and partly in bias1expansion. Note also that if jumpnetwork_omega2 is passed, it also works for that. However, in that case we have a different approach for the calculation of rate0expansion: if there are origin states, then we need to "jump" to those; if there is a non-empty VectorBasis we will want to account for them there.

Parameters

- **jumpnetwork** – jumpnetwork of symmetry unique omega1-type jumps, corresponding to our starset. List of lists of (IS, FS), dx tuples, where IS and FS are indices corresponding to states in our starset.
- **jumptype** – specific omega0 jump type that the jump corresponds to
- **omega2** – (optional) are we dealing with the omega2 list, so we need to remove origin states? (default=False)

Return rate0expansion array[Nsv, Nsv, Njump_omega0] the omega0 matrix[i, j] = sum(rate0expansion[i, j, k] * omega0[k]); IF NVB>0 we "hijack" this and use it for [NVB, Nsv, Njump_omega0], as we're doing an omega2 calc and rate0expansion won't be used *anyway*.

Return rate0escape array[Nsv, Njump_omega0] the escape contributions: omega0[i,i] += sum(rate0escape[i,k]*omega0[k]*probfactor0(PS[k]))

Return rate1expansion array[Nsv, Nsv, Njump_omega1] the omega1 matrix[i, j] = sum(rate1expansion[i, j, k] * omega1[k])

Return rate1escape array[Nsv, Njump_omega1] the escape contributions: omega1[i,i] += sum(rate1escape[i,k]*omega1[k]*probfactor(PS[k]))

crystalStars. **array2PSlist** (*ij, R, dx*)

Take in arrays of ij, R, dx (from HDF5), return a list of PairStates

Parameters

- **ij** – int_array[N][2] = (i,j)
- **R** – int[N][3]
- **dx** – float[N][3]

Return PSlist list of pair states

crystalStars. **doublelist2flatlistindex** (*listlist*)

Takes a list of lists, returns a flattened list and an index array

Parameters listlist – list of lists of objects

Return flatlist flat list of objects (preserving order)

Return indexarray array indexing which original list it came from

crystalStars. **flatlistindex2doublelist** (*flatlist, indexarray*)

Takes a flattened list and an index array, returns a list of lists

Parameters

- **flatlist** – flat list of objects (preserving order)
- **indexarray** – array indexing which original list it came from

Return listlist list of lists of objects

`crystalStars.zeroclean (x, threshold=1e-08)`

Modify *x* in place, return 0 if *x* is below a threshold; useful for “symmetrizing” our expansions

SUPERCCELL

Supercell:

The supercell module defines the `supercell` class for building supercells from `crystal.Crystal` classes.
Supercell class

Class to store supercells of crystals. A supercell is a lattice model of a crystal, with periodically repeating unit cells. In that framework we can

1. add/remove/substitute atoms
2. find the transformation map between two different representations of the same supercell
3. output POSCAR format (possibly other formats?)

class `supercell.Supercell` (*crys, super, interstitial=(), Nsolute=0, empty=False, NOSYM=False*)

A class that defines a Supercell of a crystal.

Takes in a crystal, a supercell (3x3 integer matrix). We can identify sites as interstitial sites, and specify if we'll have solutes.

KrogerVink ()

Attempt to make a "simple" string based on the defectindices, using Kroger-Vink notation. That is, we identify: vacancies, antisites, and interstitial sites, and return a string. NOTE: there is no relative charges, so this is a pseudo-KV notation.

Return KV string representation

POSCAR (*name=None, stoichiometry=True*)

Return a VASP-style POSCAR, returned as a string.

Parameters

- **name** – (optional) name to use for first list
- **stoichiometry** – (optional) if True, append stoichiometry to name

Return POSCAR string

__eq__ (*other*)

Return True if two supercells are equal; this means they should have the same occupancy. *and* the same ordering

Parameters other – supercell for comparison

Returns True if same crystal, supercell, occupancy, and ordering; False otherwise

__getitem__ (*key*)

Index into supercell

Parameters key – index (either an int, a slice, or a position)

Returns chemical occupation at that point

__imul__ (*other*)

Multiply by a GroupOp, in place.

Parameters **other** – must be a GroupOp (and *should* be a GroupOp of the supercell!)

Returns self

__init__ (*crys*, *super*, *interstitial*=(), *Nsolute*=0, *empty*=False, *NOSYM*=False)

Initialize our supercell to an empty supercell.

Parameters

- **crys** – crystal object
- **super** – 3x3 integer matrix
- **interstitial** – (optional) list/tuple of indices that correspond to interstitial sites
- **Nsolute** – (optional) number of substitutional solute elements to consider; default=0
- **empty** – (optional) designed to allow “copy” to work–skips all derived info
- **NOSYM** – (optional) does not do symmetry analysis (intended ONLY for testing purposes)

__mul__ (*other*)

Multiply by a GroupOp; returns a new supercell (constructed via copy).

Parameters **other** – must be a GroupOp (and *should* be a GroupOp of the supercell!)

Returns rotated supercell

__ne__ (*other*)

Inequality == not **__eq__**

__rmul__ (*other*)

Multiply by a GroupOp; returns a new supercell (constructed via copy).

Parameters **other** – must be a GroupOp (and *should* be a GroupOp of the supercell!)

Returns rotated supercell

__sane__ ()

Return True if supercell occupation and chemorder are consistent

__setitem__ (*key*, *value*)

Set specific composition for site; uses same indexing as **__getitem__**

Parameters

- **key** – index (either an int, a slice, or a position)
- **value** – chemical occupation at that point

__str__ ()

Human readable version of supercell

__weakref__

list of weak references to the object (if defined)

copy ()

Make a copy of the supercell; initializes, then copies over **__copyattr__** and **__eqattr__** .

Returns new supercell object, copy of the original

defectindices ()

Return a dictionary that corresponds to the “defect” content of the supercell.

Return defects dictionary, keyed by defect type, with a set of indices of corresponding defects

definesolute (c, chemistry)

Set the name of the chemistry of chemical index c. Only works for substitutional solutes.

Parameters

- **c** – index
- **chemistry** – string

equivalencemap (other)

Given the super *other* we want to find a group operation that transforms *self* into *other*. This is a *GroupOp* *along* with an index mapping of chemorder. The index mapping is to get the *occposlist* to match up: `(g*self).occposlist()[c][mapping[c][i]] == other.occposlist()[c][i]` (We can write a similar expression using *chemorder*, since *chemorder* indexes into *pos*). We’re going to return both *g* and *mapping*.

Remember: *g* does not change the presentation ordering; *mapping* is necessary for full equivalence. If no such equivalence, return *None, None*.

Parameters other – Supercell

Return g *GroupOp* to transform sites from *self* to *other*

Return mapping list of maps, such that `(g*self).chemorder[c][mapping[c][i]] == other.chemorder[c][i]`

fillperiodic (ci, Wyckoff=True)

Occupies all of the (Wyckoff) sites corresponding to chemical index with the appropriate chemistry.

Parameters

- **ci** – tuple of (chem, index) in crystal
- **Wyckoff** – (optional) if *False*, *only* occupy the specific tuple, but still periodically

Return self

gengroup ()

Generate the group operations internal to the supercell

Return G set of *GroupOps*

index (pos, threshold=1.0)

Return the index that corresponds to the position *closest* to *pos* in the supercell. Done in direct coordinates of the supercell, using periodic boundary conditions.

Parameters

- **pos** – 3-vector
- **threshold** – (optional) minimum squared “distance” in supercell for a match; default=1.

Return index index of closest position

makesites ()

Generate the array corresponding to the sites; the indexing is based on the translations and the atomindices in *crys*. These may not all be filled when the supercell is finished.

Return pos array [N*size, 3] of supercell positions in direct coordinates

static maketrans (*super*)

Takes in a supercell matrix, and returns a list of all translations of the unit cell that remain inside the supercell

Parameters **super** – 3x3 integer matrix

Return size integer, corresponding to number of unit cells

Return invsuper integer matrix inverse of supercell (needs to be divided by size)

Return translist list of integer vectors (to be divided by size) corresponding to unit cell positions

Return transdict dictionary of tuples and their corresponding index (inverse of trans)

occposlist ()

Returns a list of lists of occupied positions, in (chem)order.

Return occposlist list of lists of supercell coord. positions

reorder (*mapping*)

Reorder (in place) the occupied sites. Does not change the occupancies, only the ordering for “presentation”.

Parameters **mapping** – list of maps; will make `newchemorder[c][i] = chemorder[c][mapping[c][i]]`

Return self

If mapping is not a proper permutation, raises ValueError.

setocc (*ind*, *c*)

Set the occupancy of position indexed by ind, to chemistry c. Used by all the other algorithms.

Parameters

- **ind** – integer index
- **c** – chemistry index

stoichiometry ()

Return a string representing the current stoichiometry

POWEREXPANSION

PowerExpansion:

The PowerExpansion module defines the `Taylor3D` class, which is for 3-dimensional (xyz) Taylor expansions of functions. It's primary purpose is to be used in the calculation of the vacancy Green function, as it allows fairly straightforward block evaluation of the small k (large distance) transition matrix, and its inverse. This is key to removing the pole in the Green function evaluation. Power expansion class

Class to store and manipulate 3-dimensional Taylor (power) expansions of functions Particularly useful for inverting the FT of the evolution matrix, and subtracting off analytically calculated IFT for the Green function.

Really designed to get used by other code.

class PowerExpansion. **Taylor3D** (*coefflist=[]*, *Lmax=4*, *nodeeppcopy=False*)

Class that stores a Taylor expansion of a function in 3D, and defines some arithmetic

__add__ (*other*)

Add a set of Taylor expansions

__call__ (*u*, *fnu=None*)

Method for evaluating our 3D Taylor expansion. We have two approaches: if we are passed a dictionary in *fnu* that will map (n,l) tuple pairs to either (a) values or (b) functions of a single parameter *umagn*, then we will compute and return the function value. Otherwise, we return a dictionary mapping (n,l) tuple pairs into values, and leave it at that.

Parameters

- **u** – three vector to evaluate; may (or may not) be normalized
- **fnu** – dictionary of (n,l): value or function pairs.

Return value or dictionary depending on *fnu*; default is dictionary

__getitem__ (*key*)

Indexes (or even slices) into our Taylor expansion.

Parameters **key** – indices for our Taylor expansion

Return **Taylor3D** Taylor expansion after indexing

__iadd__ (*other*)

Add a set of Taylor expansions

classmethod **__initTaylor3Dindexing__** (*Lmax*)

This calls *all* the class methods defined above, and stores them *for the class*. This is intended to be done *once*

Parameters **Lmax** – maximum power / orbital angular momentum

__init__ (*coefflist*=[], *Lmax*=4, *nodeepcopy*=False)

Initializes a Taylor3D object, with coefflist (default = empty)

Parameters

- **coefflist** – list((n, lmax, powexpansion)). No type checking; default empty
- **Lmax** – maximum power / orbital angular momentum; can be set only once the first time a Taylor expansion is constructed, and is set for all objects
- **nodeepcopy** – true if we don't want to copy the matrices on creation of object (i.e., deep copy, which is the default) **Note:** deep copy is strongly preferred. The *only* real reason to use nodeepcopy is when returning slices / indexing in arrays, but even then we have to be careful about doing things like reductions, etc., that modify matrices *in place*. We always copy the list, but that doesn't make copies of the underlying matrices.

__isub__ (*other*)

Subtract a set of Taylor expansions

__mul__ (*other*)

Multiply our expansion

Parameters other –

Return Taylor3D expansion of product

__neg__ ()

Return -T3D

__pos__ ()

Return +T3D

__radd__ (*other*)

Add a set of Taylor expansions

__rmul__ (*other*)

Multiply our expansion

Parameters other –

Return Taylor3D expansion of product

__rsub__ (*other*)

Subtract a set of Taylor expansions

__setitem__ (*key*, *value*)

Indexes (or even slices) into our Taylor expansion and “sets”; really only intended to work with another Taylor expansion

Parameters

- **key** – indices for our Taylor expansion
- **value** – assignment value; really, should be

Returns Taylor expansion after indexing

__str__ ()

Human readable string representation

__sub__ (*other*)

Subtract a set of Taylor expansions

__weakref__

list of weak references to the object (if defined)

addhdf5 (*HDF5group*)

Adds an HDF5 representation of object into an HDF5group (needs to already exist). Example: if *f* is an open HDF5, then `T3D.addhdf5(f.create_group('T3D'))` will (1) create the group named 'T3D', and then (2) put the T3D representation in that group.

Parameters **HDF5group** – HDF5 group

addterms (*coefflist*)

Add additional coefficients into our object. No type checking. Only works if terms are completely non-overlapping (otherwise, need to use `sum`).

Parameters **coefflist** – list((*n*, *lmax*, *powexpansion*))

classmethod checkinternalsHDF5 (*HDF5group*)

Reads the power expansion internals into an HDF5group, and performs sanity check

Parameters **HDF5group** – HDF5 group

classmethod coeffproductcoeff (*a*, *b*)

Takes a direction expansion *a* and *b*, and returns the product expansion.

Parameters

- **a** – list((*n*, *lmax*, *powexpansion*))
- **b** – list((*n*, *lmax*, *powexpansion*)) written as a series of coefficients; *n* defines the magnitude function, which is additive; *lmax* is the largest cumulative power of coefficients, and *powexpansion* is a numpy array that can be multiplied. We assume that *a* and *b* have consistent shapes throughout—we *do not test this*; runtime will likely fail if not true. The entries in the list are *tuples* of *n*, *lmax*, *pow*

Return **c** list((*n*, *lmax*, *powexpansion*)), product of *a* and *b*

classmethod collectcoeff (*a*, *inplace=False*, *atol=1e-10*)

Collects coefficients: sums up all the common *n* values. Best to be done *after* `reduce` is called.

Parameters

- **a** – list((*n*, *lmax*, *powexpansion*)), expansion of function in powers
- **inplace** – modify *a* in place?

Return **coefflist** *a*

classmethod constructexpansion (*basis*, *N=-1*, *pre=None*)

Takes a "basis" for constructing an expansion – list of vectors and matrices – and constructs the expansions up to power *N* (default = *Lmax*) Takes a direction expansion *a* and *b*, and returns the sum of the expansions.

Parameters

- **= list((coeffmatrix, vect))** (*basis*) – expansions to create; `sum(coeffmatrix * (vect*q)^n)`, for powers *n* = 0..*N*
- **N** – maximum power to consider; for *N*=-1, use *Lmax*
- **pre** – list of prefactors, defining the Taylor expansion. Default = 1

Return list((*n*, *lmax*, *powexpansion*)),... our expansion, as input to create Taylor3D objects

copy ()

Returns a copy of the current expansion

dumpinternalsHDF5 (HDF5group)

Adds the initialized power expansion internals into an HDF5group—should be stored for a sanity check

Parameters **HDF5group** – HDF5 group

ildot (c)

Computes $c \cdot self$ in place

inv (Nmax=0)

Return the inverse of the expansion, up to order Nmax

Parameters **Nmax** – maximum order in the inverse expansion

Return **Taylor3D⁻¹** Taylor series of inverse

classmethod inversecoeff (a, Nmax=0)

Takes a direction expansion , and returns the inversion expansion (approximated based on the Taylor expansion of $1/(1-x) = \sum_{i=0}^{\infty} x^i$, or $(A+B)^{-1} = ((1+BA^{-1})A)^{-1} = A^{-1}(1-(-BA^1))^{-1} = A^{-1} \sum_{i=0}^{\infty} (-BA^{-1})^i$

NOTE: assumes SMALLEST n coefficient is the leading order; only works if that coefficient is also isotropic (l=0). Otherwise, raises an error. NOTE: there is no sanity check on whether Nmax is reasonable given the expansion and Lmax values; *caveat emptor*.

Parameters

- **a** – = list((n, lmax, powexpansion) written as a series of coefficients; n defines the magnitude function, which is additive; lmax is the largest cumulative power of coefficients, and powexpansion is a numpy array that can multiplied. We assume that a and b have consistent shapes throughout—we *do not test this*; runtime will likely fail if not true. The entries in the list are *tuples* of n, lmax, pow
- **Nmax** – maximum remaining n value in expansion. Default value of 0 means up to a discontinuity correction in an inversion, but higher (or lower) values are possible.

Return **c** list((n, lmax, powexpansion)), inverse of a

irdot (c)

Computes $self \cdot c$ in place

irotrate (powtrans)

Rotate in place.

Parameters **powtrans** – Npow x Npow matrix, of [oldpow,newpow] corresponding to the rotation

Returns self

ldot (c)

Returns $c \cdot self$

classmethod loadhdf5 (HDF5group)

Creates a new T3D from an HDF5 group.

Parameters **HDFgroup** – HDF5 group

Return **T3D** new T3D object

classmethod makeLprojections ()

Constructs a series of projection matrices for each l component in our power series

Returns `projL[l][p][p']` projection of powers containing *only* `l` component. -1 component = `sum(l=0..Lmax, projL[l])` = simplification projection

classmethod `makeYlmpow ()`

Construct the expansion of the Ylm's in powers of x,y,z. Done via brute force.

Return `Ylmpow[lm, p]` expansion of each Ylm in powers

classmethod `makedirectmult ()`

Return `directmult[p][p']` index that corresponds to the multiplication of power indices `p` and `p'`

static `makeindexPowerYlm (Lmax)`

Analyzes the spherical harmonics and powers for a given `Lmax`; returns a series of index functions.

Parameters `Lmax` – maximum `l` value to consider; equal to the sum of powers

Return `NYlm` number of Ylm coefficients

Return `Npower` number of power coefficients

Return `pow2ind[n1][n2][n3]` powers to index

Return `ind2pow[n]` powers for a given index

Return `Ylm2ind[l][m]` (l,m) to index

Return `ind2Ylm[lm]` (l,m) for a given index

Return `powlrange[l]` upper limit of power indices for a given `l` value; note: [-1] = 0

classmethod `makepowYlm ()`

Construct the expansion of the powers in Ylm's. Done using recursion relations instead of direct calculation. Note: an alternative approach would be Gaussian quadrature.

Return `powYlm[p][lm]` expansion of powers in Ylm; uses indexing scheme above

classmethod `makepowercoeff ()`

Make our power coefficients for our construct expansion method

Return `powercoeff[n][p]` vector we multiply by our power expansion to get the `n`'th coefficients

classmethod `negcoeff (a)`

Negates a coefficient expansion `a`

Parameters = `list((n, lmax, powexpansion) (a) – expansion of function in powers`

Return `coefflist` -a

nl ()

Returns a list of (n,l) pairs in the `coefflist`

Return `nl_list` all of the (n,l) pairs that are present in our `coefflist`

classmethod `powexp (u, normalize=True)`

Given a vector `u`, normalize it and return the power expansion of `uvec`

Parameters

- `u[3]` – vector to apply
- `normalize` – do we normalize `u` first?

Return `upow[Npower]` `ux uy uz` products of powers

Return umagn magnitude of u (if normalized)

rdot (c)

Returns $self \cdot c$

reduce ()

Reduce the coefficients: eliminate any n that has zero coefficients, collect all of the same values of n together. Done in place.

classmethod reducecoeff (a, inplace=False, atol=1e-10)

Projects coefficients through Ylm space, then eliminates any zero contributions (including possible reduction in l values, too).

Parameters

- **a** – list((n, lmax, powexpansion), expansion of function in powers
- **inplace** – modify a in place?

Return coefflist a

rotate (powtrans)

Return a rotated version of the expansion.

Parameters powtrans – Npow x Npow matrix, of [oldpow,newpow] corresponding to the rotation

Return rTaylor3D Taylor expansion, rotated

classmethod rotatecoeff (a, npowtrans, inplace=False)

Return a rotated version of the expansion. Needs to use pad to work with reduced representations.

Parameters

- **a** – coefficient list
- **npowtrans** – Lmax+1 x Npow x Npow matrix, of [n,oldpow,newpow] corresponding to the rotation

Return rcoeff coefficient list, rotated

classmethod rotatedirections (qptrans)

Takes a transformation matrix qptrans, where $q[i] = \sum_j qptrans[i][j] p[j]$, and returns the Npow x Npow transformation matrix for the new components in terms of the old. NOTE: This is more complex than one might first realize. If we only work with cases where all of the entries for a given power n have those same n (that is, not reduced), then this is straightforward. However, we run into problems with *reductions*: e.g., for n=2, the power $x^0 y^0 z^0$ is, in reality, $x^2 + y^2 + z^2$, and hence *it must be transformed* because we allow non-orthogonal transformation matrices.

Parameters qptrans – 3x3 matrix

Return npowtrans [Lmax +1][Npow][Npow] transformation matrix [n][original pow][new pow] for each n from 0 up to Lmax

classmethod scalarproductcoeff (c, a, inplace=False)

Multiplies an coefficient expansion a by a scalar c

Parameters

- **c** – scalar or dictionary mapping (n,l) to scalars
- **= list((n, lmax, powexpansion) (a)** – expansion of function in powers
- **inplace** – modify a in place?

Return coefflist c*a

separate ()

Separate out the coefficients into (n,l) terms where *only* l contributions appear in each.

classmethod separatecoeff (a, inplace=False, atol=1e-10)

Projects coefficients through Ylm space, one by one. Assumes they've already been reduced and collected first; if not, could lead to duplicated (n,l) entries in list, which is inefficient (should still *evaluate* the same, just with extra steps). After this, each (n,l) term *only* contains terms equal to l, rather than terms $\leq l$.

Parameters

- **a** – list((n, lmax, powexpansion), expansion of function in powers
- **inplace** – modify a in place?

Return coefflist a

classmethod sumcoeff (a, b, alpha=1, beta=1, inplace=False)

Takes Taylor3D expansion a and b, and returns the sum of the expansions.

Param a, b = list((n, lmax, powexpansion) written as a series of coefficients; n defines the magnitude function, which is additive; lmax is the largest cumulative power of coefficients, and powexpansion is a numpy array that can multiplied. We assume that a and b have consistent shapes throughout—we *do not test this*; runtime will likely fail if not true. The entries in the list are *tuples* of n, lmax, pow

Parameters

- **beta (alpha,)** – optional scalars: $c = \alpha*a + \beta*b$; allows for more efficient expansions
- **inplace** – True if the summation should modify a in place

Return c coeff of sum of a and b (! NOTE ! does not return the class!) sum of a and b

classmethod tensorproductcoeff (c, a, leftmultiply=True)

Multiplies an coefficient expansion a by a scalar c

Parameters

- **c** – array or dictionary mapping (n,l) to arrays
- **= list((n, lmax, powexpansion) (a)** – expansion of function in powers
- **leftmultiply** – tensordot(c,a) vs. tensordot(a,c)

Return coefflist c.a (or a.c)

truncate (Nmax, inplace=False)

Remove the coefficients above a given Nmax; normally returns a new object

Parameters

- **Nmax** – maximum coefficient to include
- **inplace** – do it in place?

classmethod truncatecoeff (a, Nmax, inplace=False)

Remove the coefficients above a given Nmax; normally returns a new object

Parameters

- **Nmax** – maximum coefficient to include
- **a** – list((n, lmax, powexpansion), expansion of function in powers
- **inplace** – do it in place?

classmethod zeros (*nmin*, *nmax*, *shape*, *dtype*=<class 'complex'>)

Constructs (and returns) a “zero” Taylor expansion with the prescribed shape. This will be useful for doing slicing assignments. Because of the manner in which slicing works for assignment, we create what looks like a *lot* of zeros, by explicitly making the full range of *l* values.

Parameters

- **nmin** – minimum value of *n*
- **nmax** – maximum value of *n* (inclusive)
- **shape** – shape of matrix, as zeros would expect.

Return Taylor3D Taylor3D, with a zero coefficient list

GFCALC

Gfcalc:

The Gfcalc module defines the `GFCrystalcalc` class for the evaluation of the vacancy Green function. Gfcalc module

Code to compute the lattice Green function for diffusion; this entails inverting the “diffusion” matrix, which is infinite, singular, and has translational invariance. The solution involves fourier transforming to reciprocal space, inverting, and inverse fourier transforming back to real (lattice) space. The complication is that the inversion produces a second order pole which must be treated analytically. Subtracting off the pole then produces a discontinuity at the gamma-point ($q=0$), which also should be treated analytically. Then, the remaining function can be numerically inverse fourier transformed.

class Gfcalc. GFCrystalcalc (*crys, chem, sitelist, jumpnetwork, Nmax=4*)

Class calculator for the Green function, designed to work with the Crystal class.

This computes the bare vacancy GF. It requires a crystal, chemical identity for the vacancy, list of symmetry unique sites (to define energies / entropies uniquely), and a corresponding jumpnetwork for that vacancy.

BlockInvertOmegaTaylor (*dd, dr, rd, rr, D*)

Returns block inverted omega as a Taylor expansion, up to $N_{max} = 0$ (discontinuity correction). Needs to be rotated such that leading order of D is isotropic.

Parameters

- **dd** – diffusive/diffusive block (upper left)
- **dr** – diffusive/relaxive block (lower left)
- **rd** – relaxive/diffusive block (upper right)
- **rr** – relaxive/relaxive block (lower right)
- **D** – $dd - dr(rr)^{-1}rd$ (diffusion)

Return gT Taylor expansion of g in block form, and reduced (collected terms)

BlockRotateOmegaTaylor (*omega_Taylor_rotate*)

Returns block partitioned Taylor expansion of a rotated omega Taylor expansion.

Parameters

- **omega_Taylor_rotate** – rotated into diffusive [0] / relaxive [1:] basis
- **dd** – diffusive/diffusive block (upper left)
- **dr** – diffusive/relaxive block (lower left)
- **rd** – relaxive/diffusive block (upper right)

- **rr** – relaxive/relaxive block (lower right)
- **D** – $dd - dr(rr)^{-1}rd$ (diffusion)

BreakdownGroups ()

Takes in a crystal, and a chemistry, and constructs the indexing breakdown for each (i,j) pair. :return grouparray: array[NG][3][3] of the NG group operations :return indexpair: array[N][N][NG][2] of the index pair for each group operation

DiagGamma (omega=None)

Diagonalize the gamma point (q=0) term

Parameters **omega** – optional; the Taylor expansion to use. If None, use self.omega_Taylor

Return **r** array of eigenvalues, sorted from 0 to decreasing values.

Return **vr** array of eigenvectors where vr[:,i] is the vector for eigenvalue r[i]

Diffusivity (omega_Taylor_D=None)

Return the diffusivity, or compute it if it's not already known. Uses omega_Taylor_D to compute with maximum efficiency.

Parameters **omega_Taylor_D** – Taylor expansion of the diffusivity component

Return **D** diffusivity [3,3] array

FourierTransformJumps (jumpnetwork, N, kpts)

Generate the Fourier transform coefficients for each jump

Parameters

- **jumpnetwork** – list of unique transitions, as lists of ((i,j), dx)
- **N** – number of sites
- **kpts** – array[Nkpt][3], in Cartesian (same coord. as dx)

Return **FTjumps** array[Njump][Nkpt][Nsite][Nsite] of FT of the jump network

Return **SEjumps** array[Nsite][Njump] multiplicity of jump on each site

SetRates (pre, betaene, preT, betaeneT, pmaxerror=1e-08)

(Re)sets the rates, given the prefactors and Arrhenius factors for the sites and transitions, using the ordering according to sitelist and jumpnetwork. Initiates all of the calculations so that GF calculation is (fairly) efficient for each input.

Parameters

- **pre** – list of prefactors for site probabilities
- **betaene** – list of beta*E (energy/kB T) for each site
- **preT** – list of prefactors for transition states
- **betaeneT** – list of beta*ET (energy/kB T) for each transition state
- **pmaxerror** – parameter controlling error from pmax value. Should be same order as integration error.

SymmRates (pre, betaene, preT, betaeneT)

Returns a list of lists of symmetrized rates, matched to jumpnetwork

TaylorExpandJumps (jumpnetwork, N)

Generate the Taylor expansion coefficients for each jump

Parameters

- **jumpnetwork** – list of unique transitions, as lists of $((i,j), dx)$
- **N** – number of sites

Return T3Djumps list of Taylor3D expansions of the jump network

__call__ (*i, j, dx*)

Evaluate the Green function from site *i* to site *j*, separated by vector *dx*

Parameters

- **i** – site index
- **j** – site index
- **dx** – vector pointing from *i* to *j* (can include lattice contributions)

Return G Green function value

__init__ (*crys, chem, sitelist, jumpnetwork, Nmax=4*)

Initializes our calculator with the appropriate topology / connectivity. Doesn't require, at this point, the site probabilities or transition rates to be known.

Parameters

- **crys** – Crystal object
- **chem** – index identifying the diffusing species
- **sitelist** – list, grouped into Wyckoff common positions, of unique sites
- **jumpnetwork** – list of unique transitions as lists of $((i,j), dx)$
- **Nmax** – maximum range as estimator for kpt mesh generation

__weakref__

list of weak references to the object (if defined)

addhdf5 (*HDF5group*)

Adds an HDF5 representation of object into an HDF5group (needs to already exist).

Example: if *f* is an open HDF5, then `GFcalc.addhdf5(f.create_group('GFcalc'))` will (1) create the group named 'GFcalc', and then (2) put the GFcalc representation in that group.

Parameters HDF5group – HDF5 group

biascorrection (*etav=None*)

Return the bias correction, or compute it if it's not already known. Uses *etav* to compute.

Parameters etav – Taylor expansion of the bias correction

Return eta [N,3] array

exp_dxq (*dx*)

Return the array of $\exp(-i \cdot q \cdot dx)$ evaluated over the *q*-points, and accounting for symmetry

Parameters dx – vector

Return exp(-i q.dx) array of $\exp(-i \cdot dx)$

classmethod loadhdf5 (*crys, HDF5group*)

Creates a new GFcalc from an HDF5 group.

Parameters

- **crys** – crystal object–MUST BE PASSED IN as it is not stored with the GFcalc
- **HDFgroup** – HDF5 group

Return GFcalc new GFcalc object

networkcount (*jumpnetwork*)

Return a count of how many separate connected networks there are

ONSAGERCALC

OnsagerCalc:

The OnsagerCalc module defines the `Interstitial` class (for computation of interstitial-mediated diffusion), and `VacancyMediated` class (for computation of vacancy-mediated diffusion). Onsager calculator module: Interstitialcy mechanism and Vacancy-mediated mechanism

Class to create an Onsager “calculator”, which brings two functionalities: 1. determines *what* input is needed to compute the Onsager (mobility, or L) tensors 2. constructs the function that calculates those tensors, given the input values.

This class is designed to be combined with code that can, e.g., automatically run some sort of atomistic-scale (DFT, classical potential) calculation of site energies, and energy barriers, and then in concert with scripts to convert such data into rates and probabilities, this will allow for efficient evaluation of transport coefficients.

This implementation will be for vacancy-mediated solute diffusion assumes the dilute limit. The mathematics is based on a Green function solution for the vacancy diffusion. The computation of the GF is included in the GFcalc module.

Now with HDF5 write / read capability for VacancyMediated module

class OnsagerCalc. **Interstitial** (*crys, chem, sitelist, jumpnetwork*)

A class to compute interstitial diffusivity; uses structure of crystal to do most of the heavy lifting in terms of symmetry.

Takes in a crystal that contains the interstitial as one of the chemical elements, to be specified by *chem*, the *sitelist* (list of symmetry equivalent sites), and *jumpnetwork*. Both of the latter can be computed automatically from *crys* methods, but as they are lists, can also be edited or constructed by hand.

__init__ (*crys, chem, sitelist, jumpnetwork*)

Initialization; takes an underlying crystal, a choice of atomic chemistry, a corresponding Wyckoff site list and jump network.

Parameters

- **crys** – Crystal object
- **chem** – integer, index into the basis of *crys*, corresponding to the chemical element that hops
- **sitelist** – list of lists of indices, site indices where the atom may hop; grouped by symmetry equivalency
- **jumpnetwork** – list of lists of tuples: (*i, j*), *dx*) symmetry unique transitions; each list is all of the possible transitions from site *i* to site *j* with jump vector *dx*; includes *i*->*j* and *j*->*i*

__str__ ()

Human readable version of diffuser

__weakref__

list of weak references to the object (if defined)

diffusivity (*pre, betaene, preT, betaeneT, CalcDeriv=False*)

Computes the diffusivity for our element given prefactors and energies/kB T. Also returns the negative derivative of diffusivity with respect to beta (used to compute the activation barrier tensor) if CalcDeriv = True The input list order corresponds to the sitelist and jumpnetwork

Parameters

- **pre** – list of prefactors for unique sites
- **betaene** – list of site energies divided by kB T
- **preT** – list of prefactors for transition states
- **betaeneT** – list of transition state energies divided by kB T

Return D[3,3] diffusivity as a 3x3 tensor

Return DE[3,3] diffusivity times activation barrier (if CalcDeriv == True)

elastodiffusion (*pre, betaene, dipole, preT, betaeneT, dipoleT*)

Computes the elastodiffusion tensor for our element given prefactors, energies/kB T, and elastic dipoles/kB T The input list order corresponds to the sitelist and jumpnetwork

Parameters

- **pre** – list of prefactors for unique sites
- **betaene** – list of site energies divided by kB T
- **dipole** – list of elastic dipoles divided by kB T
- **preT** – list of prefactors for transition states
- **betaeneT** – list of transition state energies divided by kB T
- **dipoleT** – list of elastic dipoles divided by kB T

Return D[3,3] diffusivity as 3x3 tensor

Return dD[3,3,3,3] elastodiffusion tensor as 3x3x3x3 tensor

generateJumpGroupOps ()

Generates a list of group operations that transform the first jump in the jump network into all of the other members; one group operation for each.

Return siteGroupOps list of list of group ops that mirrors the structure of jumpnetwork.

generateJumpSymmTensorBasis ()

Generates a list of symmetric tensor bases for the first representative transition in our jump network

Return TensorSet list of list of symmetric tensors

generateSiteGroupOps ()

Generates a list of group operations that transform the first site in each site list into all of the other members; one group operation for each.

Return siteGroupOps list of list of group ops that mirrors the structure of site list

generateSiteSymmTensorBasis ()

Generates a list of symmetric tensor bases for the first representative site in our site list.

Return TensorSet list of symmetric tensors

generatetags ()

Create tags for unique interstitial states, and transition states.

Return tags dictionary of tags; each is a list-of-lists

Return tagdict dictionary that maps tag into the index of the corresponding list.

Return tagdicttype dictionary that maps tag into the key for the corresponding list.

jumpDipoles (*dipoles*)

Returns a list of the elastic dipole for each transition, given the dipoles for the representatives. ("populating" the full set of dipoles)

Parameters **dipoles** – list of dipoles for the first representative transition

Return dipolelist list of lists of dipole for each jump[site][3][3]

static jumpnetworkYAML (*jumpnetwork*)

Dumps a "sample" YAML formatted version of the jumpnetwork with data to be entered

makesupercells (*super_n*)

Take in a supercell matrix, then generate all of the supercells needed to compute site energies and transitions (corresponding to the representatives).

Parameters **super_n** – 3x3 integer matrix to define our supercell

Return superdict dictionary of states , transitions , transmapping , and indices that correspond to dictionaries with tags.

- superdict['states'][i] = supercell of site;
- superdict['transitions'][n] = (supercell initial, supercell final);
- superdict['transmapping'][n] = ((site tag, groupop, mapping), (site tag, groupop, mapping))
- superdict['indices'][tag] = index of tag, where tag is either a state or transition tag.

ratelist (*pre, betaene, preT, betaeneT*)

Returns a list of lists of rates, matched to jumpnetwork

siteDipoles (*dipoles*)

Returns a list of the elastic dipole on each site, given the dipoles for the representatives. ("populating" the full set of dipoles)

Parameters **dipoles** – list of dipoles for the first representative site

Return dipolelist array of dipole for each site [site][3][3]

static sitelistYAML (*sitelist*)

Dumps a "sample" YAML formatted version of the sitelist with data to be entered

siteprob (*pre, betaene*)

Returns our site probabilities, normalized, as a vector

symmratelist (*pre, betaene, preT, betaeneT*)

Returns a list of lists of symmetrized rates, matched to jumpnetwork

class OnsagerCalc. VacancyMediated (*crys, chem, sitelist, jumpnetwork, Nthermo=0, NGFmax=4*)

A class to compute vacancy-mediated solute transport coefficients, specifically L_{vv} (vacancy diffusion), L_{ss} (solute), and L_{sv} (off-diagonal). As part of that, it determines *what* quantities are needed as inputs in order to perform this calculation.

Based on crystal class. Also now includes its own GF calculator and cacheing, and storage in HDF5 format.

Requires a crystal, chemical identity of vacancy, list of symmetry-equivalent sites for that chemistry, and a jumpnetwork for the vacancy. The thermodynamic range (number of “shells” – see `crystalStars.StarSet` for precise definition).

GFcalculator (*NGFmax=0*)

Return the GF calculator; create a new one if NGFmax is being changed

Lij (*bFV, bFS, bFSV, bFT0, bFT1, bFT2, large_om2=100000000.0*)

Calculates the transport coefficients: L_{0vv} , L_{ss} , L_{sv} , L_{1vv} from the scaled free energies. The Green function entries are calculated from the ω_0 info. As this is the most time-consuming part of the calculation, we cache these values with a dictionary and hash function.

Parameters

- **bFV[NWyckoff]** – $\beta \cdot \text{eneV} - \ln(\text{preV})$ (relative to minimum value)
- **bFS[NWyckoff]** – $\beta \cdot \text{eneS} - \ln(\text{preS})$ (relative to minimum value)
- **bFSV[Nthermo]** – $\beta \cdot \text{eneSV} - \ln(\text{preSV})$ (excess)
- **bFT0[Nomega0]** – $\beta \cdot \text{eneT0} - \ln(\text{preT0})$ (relative to minimum value of bFV)
- **bFT1[Nomega1]** – $\beta \cdot \text{eneT1} - \ln(\text{preT1})$ (relative to minimum value of bFV + bFS)
- **bFT2[Nomega2]** – $\beta \cdot \text{eneT2} - \ln(\text{preT2})$ (relative to minimum value of bFV + bFS)
- **large_om2** – threshold for changing treatment of omega2 contributions (default: 10^8)

Return $L_{vv}[3, 3]$ vacancy-vacancy; needs to be multiplied by cv/kBT

Return $L_{ss}[3, 3]$ solute-solute; needs to be multiplied by $cv \cdot cs/kBT$

Return $L_{sv}[3, 3]$ solute-vacancy; needs to be multiplied by $cv \cdot cs/kBT$

Return $L_{vv1}[3, 3]$ vacancy-vacancy correction due to solute; needs to be multiplied by $cv \cdot cs/kBT$

__init__ (*crys, chem, sitelist, jumpnetwork, Nthermo=0, NGFmax=4*)

Create our diffusion calculator for a given crystal structure, chemical identity, jumpnetwork (for the vacancy) and thermodynamic shell.

Parameters

- **crys** – Crystal object
- **chem** – index identifying the diffusing species
- **sitelist** – list, grouped into Wyckoff common positions, of unique sites
- **jumpnetwork** – list of unique transitions as lists of $((i,j), dx)$
- **Nthermo** – range of thermodynamic interaction (in successive jumpnetworks)
- **NGFmax** – parameter controlling k-point density of GF calculator; 4 seems reasonably accurate

__str__ ()

Human readable version of diffuser

__weakref__

list of weak references to the object (if defined)

addhdf5 (*HDF5group*)

Adds an HDF5 representation of object into an HDF5group (needs to already exist).

Example: if `f` is an open HDF5, then `VacancyMediated.addhdf5(f.create_group('Diffuser'))` will (1) create the group named 'Diffuser', and then (2) put the `VacancyMediated` representation in that group.

Parameters `HDF5group` – HDF5 group

clearcache ()

Clear out the GF cache values

generate (*Nthermo*)

Generate the necessary stars, vector-stars, and jump networks based on the thermodynamic range.

Parameters `Nthermo` – range of thermodynamic interactions, in terms of “shells”, which is multiple summations of jumpvect

generatematrices ()

Generates all the matrices and “helper” pieces, based on our jump networks. This has been separated out in case the user wants to, e.g., prune / modify the networks after they’ve been created with `generate()`, then `generatematrices()` can be rerun.

generatetags ()

Create tags for vacancy states, solute states, solute-vacancy complexes; `omega0`, `omega1`, and `omega2` transition states.

Return tags dictionary of tags; each is a list-of-lists

Return tagdict dictionary that maps tag into the index of the corresponding list.

Return tagdicttype dictionary that maps tag into the key for the corresponding list.

interactlist ()

Return a list of solute-vacancy configurations for interactions. The points correspond to a vector between a solute atom and a vacancy. Defined by Stars.

Return statelist list of `PairStates` for the solute-vacancy interactions

classmethod loadhdf5 (*HDF5group*)

Creates a new `VacancyMediated` diffuser from an HDF5 group.

Parameters `HDFgroup` – HDF5 group

Return `VacancyMediated` new `VacancyMediated` diffuser object from HDF5

makeLIMBpreene (*preS, eneS, preSV, eneSV, preT0, eneT0, **ignoredextraarguments*)

Generates corresponding energies / prefactors for corresponding to LIMB (Linearized interpolation of migration barrier approximation). Returns a dictionary. (we ignore extra arguments so that a dictionary including additional entries can be passed)

Parameters

- **preS[NWyckoff]** – prefactor for solute formation
- **eneS[NWyckoff]** – solute formation energy
- **preSV[Nthermo]** – prefactor for solute-vacancy interaction
- **eneSV[Nthermo]** – solute-vacancy binding energy
- **preT0[Nomega0]** – prefactor for vacancy jump transitions (follows jumpnetwork)
- **eneT0[Nomega0]** – transition energy for vacancy jumps

Return **preT1[Nomega1]** prefactor for omega1-style transitions (follows om1_jn)

Return **eneT1[Nomega1]** transition energy/kBT for omega1-style jumps

Return `preT2[Nomega2]` prefactor for omega2-style transitions (follows `om2_jn`)

Return `eneT2[Nomega2]` transition energy/kBT for omega2-style jumps

makesupercells (*super_n*)

Take in a supercell matrix, then generate all of the supercells needed to compute site energies and transitions (corresponding to the representatives).

Note: the states are lone vacancy, lone solute, solute-vacancy complexes in our thermodynamic range. Note that there will be escape states are endpoints of some omega1 jumps. They are not relaxed, and have no pre-existing tag. They will only be output as a single endpoint of an NEB run; there may be symmetry equivalent duplicates, as we construct these supercells on an as needed basis.

We've got a few classes of warnings (from most egregious to least) that can issued if the supercell is too small; the analysis will continue despite any warnings:

1. Thermodynamic shell states map to different states in supercell
2. Thermodynamic shell states are not unique in supercell (multiplicity)
3. Kinetic shell states map to different states in supercell
4. Kinetic shell states are not unique in supercell (multiplicity)

The lowest level can still be run reliably but runs the risk of errors in escape transition barriers. Extreme caution should be used if any of the other warnings are raised.

Parameters `super_n` – 3x3 integer matrix to define our supercell

Return `superdict` dictionary of `states`, `transitions`, `transmapping`, `indices` that correspond to dictionaries with tags; the final tag `reference` is the basesupercell for calculations without defects.

- `superdict['states'][i]` = supercell of state;
- `superdict['transitions'][n]` = (supercell initial, supercell final);
- `superdict['transmapping'][n]` = ((site tag, groupop, mapping), (site tag, groupop, mapping))
- `superdict['indices'][tag]` = (type, index) of tag, where tag is either a state or transition tag.
- `superdict['reference']` = supercell reference, no defects

maketracerpreene (*preT0*, *eneT0*, ***ignoredextraarguments*)

Generates corresponding energies / prefactors for an isotopic tracer. Returns a dictionary. (we ignore extra arguments so that a dictionary including additional entries can be passed)

Parameters

- `preT0[Nomega0]` – prefactor for vacancy jump transitions (follows `jumpnetwork`)
- `eneT0[Nomega0]` – transition energy state for vacancy jumps

Return `preS[NWyckoff]` prefactor for solute formation

Return `eneS[NWyckoff]` solute formation energy

Return `preSV[Nthermo]` prefactor for solute-vacancy interaction

Return `eneSV[Nthermo]` solute-vacancy binding energy

Return `preT1[Nomega1]` prefactor for omega1-style transitions (follows `om1_jn`)

Return `eneT1[Nomega1]` transition energy for omega1-style jumps

Return preT2[Nomega2] prefactor for omega2-style transitions (follows om2_jn)

Return eneT2[Nomega2] transition energy for omega2-style jumps

omegalist (*fivefreqindex=1*)

Return a list of pairs of endpoints for a vacancy jump, corresponding to omega1 or omega2 Solute at the origin, vacancy hopping between two sites. Defined by om1_jumpnetwork

Parameters fivefreqindex – 1 or 2, corresponding to omega1 or omega2

Return omegalist list of tuples of PairStates

Return omegajumptype index of corresponding omega0 jumptype

static preene2betafree (*kT, preV, eneV, preS, eneS, preSV, eneSV, preT0, eneT0, preT1, eneT1, preT2, eneT2, **ignoredextraarguments*)

Read in a series of prefactors ($\exp(S/k_B)$) and energies, and return βF for energies and transition state energies. Used to provide scaled values to Lij(). Can specify all of the entries using a dictionary; e.g., `preene2betafree(kT, **data_dict)` and then send that output as input to Lij: `Lij(*preene2betafree(kT, **data_dict))` (we ignore extra arguments so that a dictionary including additional entries can be passed)

Parameters

- **kT** – temperature times Boltzmann’s constant k_B
- **preV** – prefactor for vacancy formation (prod of inverse vibrational frequencies)
- **eneV** – vacancy formation energy
- **preS** – prefactor for solute formation (prod of inverse vibrational frequencies)
- **eneS** – solute formation energy
- **preSV** – excess prefactor for solute-vacancy binding
- **eneSV** – solute-vacancy binding energy
- **preT0** – prefactor for vacancy transition state
- **eneT0** – energy for vacancy transition state (relative to eneV)
- **preT1** – prefactor for vacancy swing transition state
- **eneT1** – energy for vacancy swing transition state (relative to eneV + eneS + eneSV)
- **preT2** – prefactor for vacancy exchange transition state
- **eneT2** – energy for vacancy exchange transition state (relative to eneV + eneS + eneSV)

Return bFV $\beta \text{eneV} - \ln(\text{preV})$ (relative to minimum value)

Return bFS $\beta \text{eneS} - \ln(\text{preS})$ (relative to minimum value)

Return bFSV $\beta \text{eneSV} - \ln(\text{preSV})$ (excess)

Return bFT0 $\beta \text{eneT0} - \ln(\text{preT0})$ (relative to minimum value of bFV)

Return bFT1 $\beta \text{eneT1} - \ln(\text{preT1})$ (relative to minimum value of bFV + bFS)

Return bFT2 $\beta \text{eneT2} - \ln(\text{preT2})$ (relative to minimum value of bFV + bFS)

tags2preene (*usertagdict, VERBOSE=False*)

Generates energies and prefactors based on a dictionary of tags.

Parameters

- **usertagdict** – dictionary where the keys are tags, and the values are tuples: (pre, ene)
- **VERBOSE** – (optional) if True, also return a dictionary of missing tags, duplicate tags, and bad tags

Return thermodict dictionary of ene’s and pre’s corresponding to usertagdict

Return missingdict dictionary with keys corresponding to tag types, and the values are lists of lists of symmetry equivalent tags that are missing

Return duplicatelist list of lists of tags in usertagdict that are (symmetry) duplicates

Return badtaglist list of all tags in usertagdict that aren’t found in our dictionary

OnsagerCalc. **arrays2vTKdict** (*vTKarray, valarray, vTKsplits*)

Takes two arrays of vTK keys and values, and the splits to separate vTKarray back into vTK and returns a dictionary indexed by the vTK.

Parameters

- **vTKarray** – array of vTK entries
- **valarray** – array of values
- **vTKsplits** – split placement for vTK entries

Return vTKdict dictionary, indexed by vTK objects, whose entries are arrays

OnsagerCalc. **vTKdict2arrays** (*vTKdict*)

Takes a dictionary indexed by vTK objects, returns two arrays of vTK keys and values, and the splits to separate vTKarray back into vTK

Parameters vTKdict – dictionary, indexed by vTK objects, whose entries are arrays

Return vTKarray array of vTK entries

Return valarray array of values

Return vTKsplits split placement for vTK entries

class OnsagerCalc. **vacancyThermoKinetics**

Class to store (in a hashable manner) the thermodynamics and kinetics for the vacancy

Parameters

- **pre** – prefactors for sites
- **betaene** – energy for sites / kBT
- **preT** – prefactors for transition states
- **betaeneT** – transition state energy for sites / kBT

static vacancyThermoKinetics_constructor (*loader, node*)

Construct a GroupOp from YAML

static vacancyThermoKinetics_representer (*dumper, data*)

Output a PairState

AUTOMATOR

Automator:

The automator module defines functions that create a tarball from a supercell dictionary. Automator code Functions to convert from a supercell dictionary (output from a Diffuser) into a tarball that contains all of the input files in an organized directory structure to run the atomic-scale transition state calculations. This includes:

1. All positions in POSCAR format (POSCAR files for states to relax, POS as reference for transition endpoints that need to be relaxed)
2. Transformation information from relaxed states to initial states.
3. INCAR files for relaxation and NEB runs; KPOINTS for each.
4. perl script to transform CONTCAR output from a state relaxation to NEB endpoints.
5. perl script to linearly interpolate between NEB endpoints.*
6. Makefile to run NEB construction.

Note: the NEB interpolator script (nebmake.pl) is part of the [VTST scripts](#).

`automator.map2string (tag, groupop, mapping)`

Takes in a map: tag, groupop, mapping and constructs a string representation to be dumped to a file. If we want to call using the tuple, `map2string(*(map))` will suffice.

Parameters

- **tag** – string of initial state to rotate
- **groupop** – see `crystal.GroupOp`; we use the rot and trans. This is in the supercell coord.
- **mapping** – in “chemorder” format; list by chemistry of lists of indices of position in initial cell to use.

Return string_rep string representation (to be used by an external script)

```

automator. supercelltar ( tar, superdict, filemode=436, directmode=509, timestamp=None, INCARrelax='SYSTEM = {system}\nPREC = High\nISIF = 2\nEDIFF = 1E-8\nEDIFFG = -10E-3\nIBRION = 2\nNSW = 50\nISMear = 1\nSIGMA = 0.1\n# ENCUT =\n# NGX =\n# NGY =\n# NGZ =\n# NGXF =\n# NGYF =\n# NGZF =\n# NPAR =\nLWAVE = .FALSE.\nLCHARG = .FALSE.\nLREAL = .FALSE.\nVOSKOWN = 1\n', INCARNEB='SYSTEM = {system}\nPREC = High\nISIF = 2\nEDIFF = 1E-8\nEDIFFG = -10E-3\nIBRION = 2\nNSW = 50\nISMear = 1\nSIGMA = 0.1\n# ENCUT =\n# NGX =\n# NGY =\n# NGZ =\n# NGXF =\n# NGYF =\n# NGZF =\n# NPAR =\nLWAVE = .FALSE.\nLCHARG = .FALSE.\nLREAL = .FALSE.\nVOSKOWN = 1\nIMAGES = 1\nSPRING = -5\nLCLIMB = .TRUE.\nNELMIN = 4\nNFREE = 10\n', KPOINTS='Gamma\n1\nReciprocal\n0. 0. 0. 1\n', basedir='', statename='relax.', transitionname='neb.', IDformat='{:02d}', JSONdict='tags.json', YAMLdef='supercell.yaml')

```

Takes in a tarfile (needs to be open for writing) and a supercelldict (from a diffuser) and creates the full directory structure inside the tarfile. Best used in a form like

```

with tarfile.open('supercells.tar.gz', mode='w:gz') as tar:
    automator.supercelltar(tar, supercelldict)

```

Parameters

- **tar** – tarfile open for writing; may contain other files in advance.
- **superdict** – dictionary of states, transitions, transmapping, indices that correspond to dictionaries with tags; the final tag reference is the basesupercell for calculations without defects.
 - superdict['states'][i] = supercell of state;
 - superdict['transitions'][n] = (supercell initial, supercell final);
 - superdict['transmapping'][n] = ((site tag, groupop, mapping), (site tag, groupop, mapping))
 - superdict['indices'][tag] = (type, index) of tag, where tag is either a state or transition tag; or...
 - superdict['indices'][tag] = index of tag, where tag is either a state or transition tag.
 - superdict['reference'] = (optional) supercell reference, no defects
- **filemode** – mode to use for files (default: 664)
- **directmode** – mode to use for directories (default: 775)
- **timestamp** – UNIX time for files; if None, use current time (default)
- **INCARrelax** – contents of INCAR file to use for relaxation; must contain {system} to be replaced by tag value (default: automator.INCARrelax)
- **INCARNEB** – contents of INCAR file to use for NEB; must contain {system} to be replaced by tag value (default: automator.INCARNEB)
- **KPOINTS** – contents of KPOINTS file (default: gamma-point only calculation); if None or empty, no KPOINTS file at all
- **basedir** – prepended to all files/directories (default: "")
- **statename** – prepended to all state names, before 2 digit number (default: relax.)

- **transitionname** – prepended to all transition names, before 2 digit number (default: neb.)
- **IDformat** – format for integer tags (default: {:02d})
- **JSONdict** – name of JSON file storing the tags corresponding to each directory (default: tags.json)
- **YAMLdef** – YAML file containing full definition of supercells, relationship, etc. (default: supercell.yaml); set to None to not output. **may want to change this to None for the future**

NOTEBOOKS

Example notebooks:

Below are several jupyter notebooks with example input and output from onsager .

12.1 Fe-C diffusion and elastodiffusivity

Taking data from R.G.A. Veiga, M. Perez, C. Becquart, E. Clouet and C. Domain, Acta mater. **59** (2011) p. 6963 doi:10.1016/j.actamat.2011.07.048

Fe in the body-centered cubic phase, $a_0 = 0.28553$ nm; C sit at octahedral sites, where the transition states between octahedral sites are represented by tetrahedral sites. The data is obtained from an EAM potential, where $C_{11} = 243$ GPa, $C_{12} = 145$ GPa, and $C_{44} = 116$ GPa. The tetrahedral transition state is 0.816 eV above the octahedral site, and the attempt frequency is taken as 10 THz (10^{13} Hz).

The dipole tensors can be separated into *parallel* and *perpendicular* components; the parallel direction points towards the closest Fe atoms for the C, while the perpendicular components lie in the interstitial plane. For the octahedral, the parallel component is 8.03 eV, and the perpendicular is 3.40 eV; for the tetrahedral transition state, the parallel component is 4.87 eV, and the perpendicular is 6.66 eV.

```
In [1]: import sys
        sys.path.extend(['../', '../../..'])
        import numpy as np
        import matplotlib.pyplot as plt
        plt.style.use('seaborn-whitegrid')
        %matplotlib inline
        import onsager.crystal as crystal
        import onsager.OnsagerCalc as onsager
        from scipy.constants import physical_constants
        kB = physical_constants['Boltzmann constant in eV/K'][0]
```

Create BCC lattice (lattice constant in nm).

```
In [2]: a0 = 0.28553
        Fe = crystal.Crystal.BCC(a0, "Fe")
        print(Fe)

#Lattice:
a1 = [-0.142765  0.142765  0.142765]
a2 = [ 0.142765 -0.142765  0.142765]
a3 = [ 0.142765  0.142765 -0.142765]

#Basis:
(Fe) 0.0 = [ 0.  0.  0.]
```

Elastic constants converted from GPa (10^9 J/m³) to eV/(atomic volume).

```
In [3]: stressconv = 1e9*1e-27*Fe.volume/physical_constants['electron volt'][0]
        c11, c12, c44 = 243*stressconv, 145*stressconv, 116*stressconv
        s11, s12, s44 = (c11+c12)/((c11-c12)*(c11+2*c12)), -c12/((c11-c12)*(c11+2*c12)), 1/c44
        print('S11 = {:.4f} S12 = {:.4f} S44 = {:.4f}'.format(s11, s12, s44))
        stensor = np.zeros((3,3,3,3))
        for a in range(3):
            for b in range(3):
                for c in range(3):
                    for d in range(3):
                        if a==b and b==c and c==d: stensor[a,b,c,d] = s11
                        elif a==b and c==d: stensor[a,b,c,d] = s12
                        elif (a==d and b==c) or (a==c and b==d): stensor[a,b,c,d] = s44/4

S11 = 0.1023 S12 = -0.0382 S44 = 0.1187
```

Add carbon interstitial sites at octahedral sites in the lattice. This code (1) gets the set of symmetric Wyckoff positions corresponding to the single site $[00\frac{1}{2}]$ (first translated into unit cell coordinates), and then adds that new basis to our Fe crystal to generate a *new* crystal structure, that we name “FeC”.

```
In [4]: uoct = np.dot(Fe.invlatt, np.array([0, 0, 0.5*a0]))
        FeC = Fe.addbasis(Fe.Wyckoffpos(uoct), ["C"])
        print(FeC)
```

```
#Lattice:
a1 = [-0.142765  0.142765  0.142765]
a2 = [ 0.142765 -0.142765  0.142765]
a3 = [ 0.142765  0.142765 -0.142765]
#Basis:
(Fe) 0.0 = [ 0.  0.  0.]
(C) 1.0 = [ 0.  0.5 0.5]
(C) 1.1 = [ 0.5 0.5 0. ]
(C) 1.2 = [ 0.5 0.  0.5]
```

Next, we construct a *diffuser* based on our interstitial. We need to create a *sitelist* (which will be the Wyckoff positions) and a *jumpnetwork* for the transitions between the sites. There are tags that correspond to the unique states and transitions in the diffuser.

```
In [5]: chem = 1 # 1 is the index corresponding to our C atom in the crystal
        sitelist = FeC.sitelist(chem)
        jumpnetwork = FeC.jumpnetwork(chem, 0.6*a0) # 0.6*a0 is the cutoff distance for finding jumps
        FeCdiffuser = onsager.Interstitial(FeC, chem, sitelist, jumpnetwork)
        print(FeCdiffuser)
```

Diffuser for atom 1 (C)

```
#Lattice:
a1 = [-0.142765  0.142765  0.142765]
a2 = [ 0.142765 -0.142765  0.142765]
a3 = [ 0.142765  0.142765 -0.142765]
#Basis:
(Fe) 0.0 = [ 0.  0.  0.]
(C) 1.0 = [ 0.  0.5 0.5]
(C) 1.1 = [ 0.5 0.5 0. ]
(C) 1.2 = [ 0.5 0.  0.5]
states:
i:+0.000,+0.500,+0.500
transitions:
```

```
i:+0.500,+0.000,+0.500*i:+0.000,-0.500,+0.500
```

Next, we assemble our data: the energies, prefactors, and dipoles for the C atom in Fe, matched to the *representative* states: these are the first states in the lists, which are also identified by the tags above.

A note about units: If ν_0 is in THz, and a_0 is in nm, then $a_0^2\nu_0 = 10^{-2} \text{ cm}^2/\text{s}$. Thus, we multiply by $D_{\text{conv}} = 10^{-2}$ so that our diffusivity is output in cm^2/s .

```
In [6]: Dconv = 1e-2
        vu0 = 10*Dconv
        Etrans = 0.816
        dipoledict = {'Poctpara': 8.03, 'Poctperp': 3.40,
                      'Ptetpara': 4.87, 'Ptetperp': 6.66}
        FeCthermodict = {'pre': np.ones(len(sitelist)), 'ene': np.zeros(len(sitelist)),
                         'preT': vu0*np.ones(len(jumpnetwork)),
                         'eneT': Etrans*np.ones(len(jumpnetwork))}
        # now to construct the site and transition dipole tensors; we use a "direction"--either
        # the site position or the jump direction--to determine the parallel and perpendicular
        # directions.
        for dipname, Pname, direction in zip(('dipole', 'dipoleT'), ('Poct', 'Ptet'),
                                             (np.dot(FeC.lattice, FeC.basis[chem][sitelist[0][0]]),
                                              jumpnetwork[0][0][1])):
            # identify the non-zero index in our direction:
            paraindex = [n for n in range(3) if not np.isclose(direction[n], 0)][0]
            Ppara, Pperp = dipoledict[Pname + 'para'], dipoledict[Pname + 'perp']
            FeCthermodict[dipname] = np.diag([Ppara if i==paraindex else Pperp
                                              for i in range(3)])

        for k,v in FeCthermodict.items():
            print('{}: {}'.format(k, v))

ene: [ 0.]
pre: [ 1.]
preT: [ 0.1]
dipoleT: [[ 6.66  0.  0. ]
          [ 0.  6.66  0. ]
          [ 0.  0.  4.87]]
dipole: [[ 8.03  0.  0. ]
          [ 0.  3.4  0. ]
          [ 0.  0.  3.4 ]]
eneT: [ 0.816]
```

We look at the diffusivity D , the elastodiffusivity d , and the activation volume tensor V over a range of temperatures from 300K to 1200K.

First, we calculate all of the pieces, including the diffusivity prefactor and activation barrier. As we only *have* one barrier, we compute the barrier at $k_B T = 1$.

```
In [7]: Trange = np.linspace(300, 1200, 91)
        Tlabels = Trange[0::30]
        Dlist, dDlist, Vlist = [], [], []
        for T in Trange:
            beta = 1./(kB*T)
            D, dD = FeCdifuser.elastodiffusion(FeCthermodict['pre'],
                                                beta*FeCthermodict['ene'],
                                                [beta*FeCthermodict['dipole']],
                                                FeCthermodict['preT'],
```

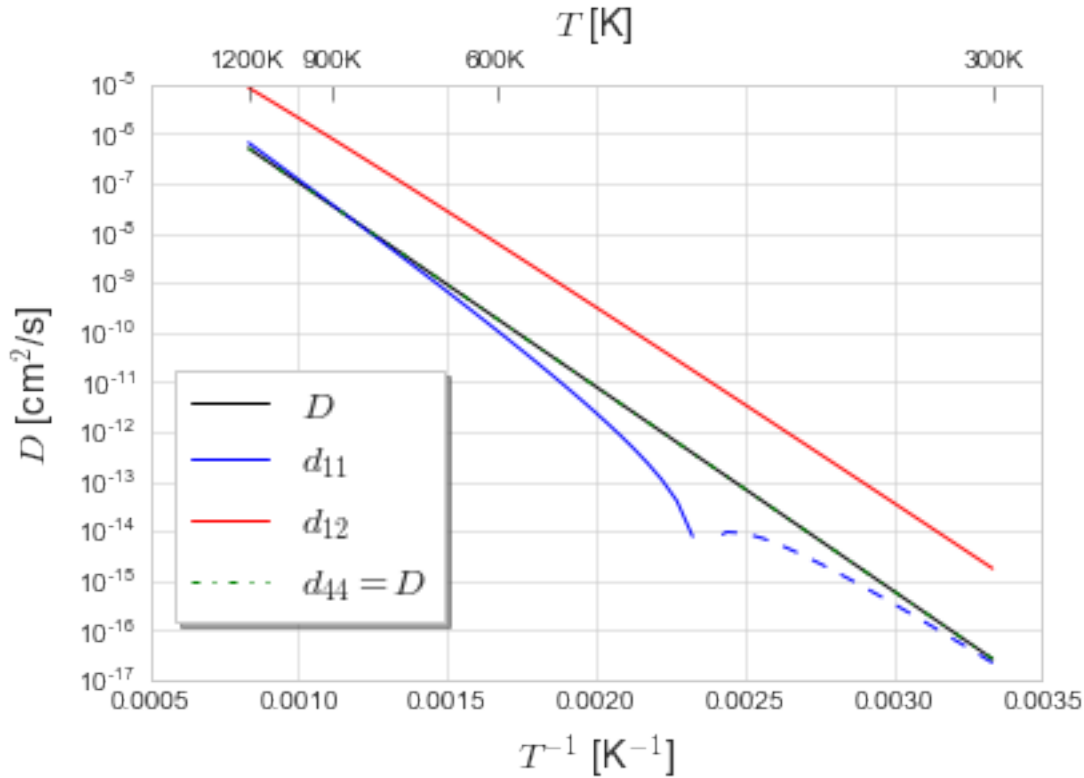
```

        beta*FeCthermodict['eneT'],
        [beta*FeCthermodict['dipoleT']]])

Dlist.append(D[0,0])
dDlist.append([dD[0,0,0,0], dD[0,0,1,1], dD[0,1,0,1]])
Vtensor = (kB*T/(D[0,0]))*np.tensordot(dD, stensor, axes=((2,3),(0,1)))
Vlist.append([np.trace(np.trace(Vtensor))/3,
              Vtensor[0,0,0,0], Vtensor[0,0,1,1], Vtensor[0,1,0,1]])
D0 = FeCdiffuser.diffusivity(FeCthermodict['pre'],
                             np.zeros_like(FeCthermodict['ene']),
                             FeCthermodict['preT'],
                             np.zeros_like(FeCthermodict['eneT']))
D, dbeta = FeCdiffuser.diffusivity(FeCthermodict['pre'],
                                   FeCthermodict['ene'],
                                   FeCthermodict['preT'],
                                   FeCthermodict['eneT'],
                                   CalcDeriv=True)

print('D0: {:.4e} cm^2/s\nEact: {:.3f} eV'.format(D0[0,0], dbeta[0,0]/D[0,0]))
D0: 1.3588e-03 cm^2/s
Eact: 0.816 eV

In [8]: D, dD = np.array(Dlist), np.array(dDlist)
        d11_T = np.vstack((Trange, dD[:,0])).T
        d11pos = np.array([(T,d) for T,d in d11_T if d>=0])
        d11neg = np.array([(T,d) for T,d in d11_T if d<0])
        fig, ax1 = plt.subplots()
        ax1.plot(1./Trange, D, 'k', label='$D$')
        # ax1.plot(1./Trange, dD[:,0], 'b', label='$d_{11}$')
        ax1.plot(1./d11pos[:,0], d11pos[:,1], 'b', label='$d_{11}$')
        ax1.plot(1./d11neg[:,0], -d11neg[:,1], 'b--')
        ax1.plot(1./Trange, dD[:,1], 'r', label='$d_{12}$')
        ax1.plot(1./Trange, dD[:,2], 'g-', label='$d_{44} = D$')
        ax1.set_yscale('log')
        ax1.set_ylabel('$D$ [cm$^2$/s]', fontsize='x-large')
        ax1.set_xlabel('$T^{-1}$ [K$^{-1}$]', fontsize='x-large')
        ax1.legend(bbox_to_anchor=(0.15,0.15,0.2,0.4), ncol=1,
                  shadow=True, frameon=True, fontsize='x-large')
        ax2 = ax1.twinx()
        ax2.set_xlim(ax1.get_xlim())
        ax2.set_xticks([1./t for t in Tlabels])
        ax2.set_xticklabels(["{:.0f}K".format(t) for t in Tlabels])
        ax2.set_xlabel('$T$ [K]', fontsize='x-large')
        ax2.grid(False)
        ax2.tick_params(axis='x', top='on', direction='in', length=6)
        plt.show()
        # plt.savefig('Fe-C-diffusivity.pdf', transparent=True, format='pdf')
    
```

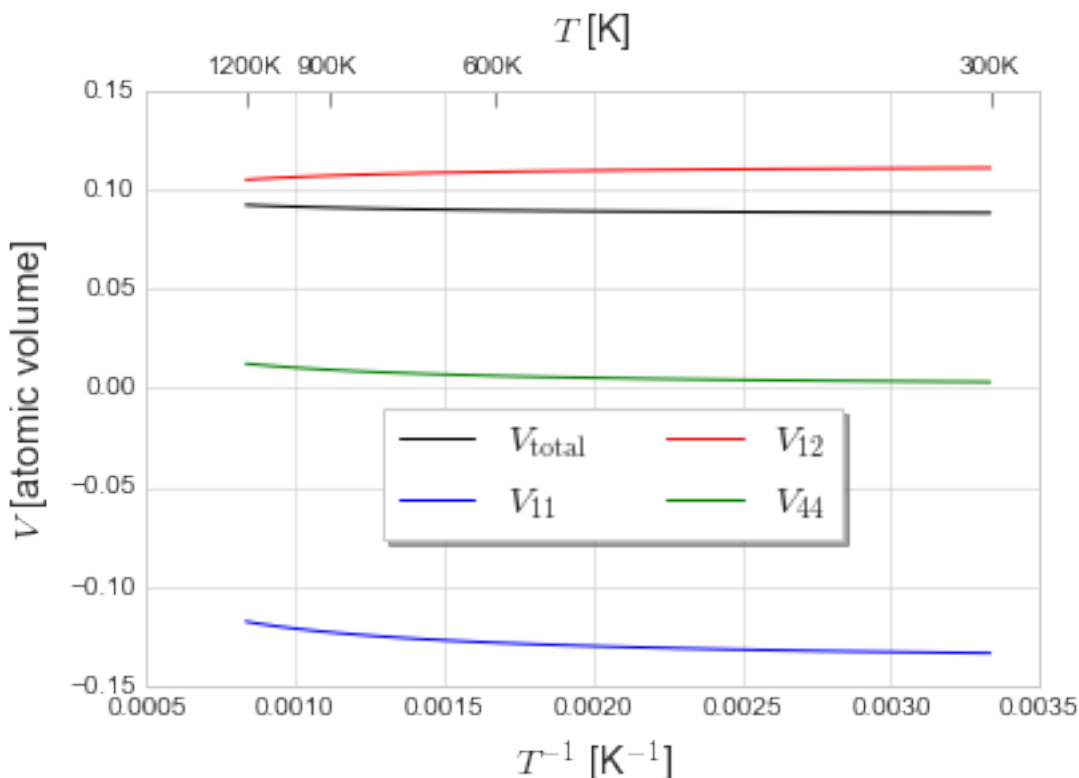



```
In [9]: d11pos[0,0], d11neg[-1,0]
```

```
Out[9]: (430.0, 420.0)
```

Activation volume. We plot the isotropic value (change in diffusivity with respect to pressure), but also the V_{xxxx} , V_{xyyy} , and V_{xyxy} terms. Interestingly, the V_{xxxx} term is negative—which indicates that diffusivity along the [100] direction *increases* with compressive stress in the [100] direction.

```
In [10]: V = np.array(Vlist)
fig, ax1 = plt.subplots()
ax1.plot(1./Trange, V[:,0], 'k', label='$V_{\rm total}$')
ax1.plot(1./Trange, V[:,1], 'b', label='$V_{11}$')
ax1.plot(1./Trange, V[:,2], 'r', label='$V_{12}$')
ax1.plot(1./Trange, 2*V[:,3], 'g', label='$V_{44}$')
ax1.set_yscale('linear')
ax1.set_ylabel('$V$ [atomic volume]', fontsize='x-large')
ax1.set_xlabel('$T^{-1}$ [K$^{-1}$]', fontsize='x-large')
ax1.legend(bbox_to_anchor=(0.3,0.3,0.5,0.2), ncol=2,
          shadow=True, frameon=True, fontsize='x-large')
ax2 = ax1.twinx()
ax2.set_xlim(ax1.get_xlim())
ax2.set_xticks([1./t for t in Tlabels])
ax2.set_xticklabels(["{:0f}K".format(t) for t in Tlabels])
ax2.set_xlabel('$T$ [K]', fontsize='x-large')
ax2.grid(False)
ax2.tick_params(axis='x', top='on', direction='in', length=6)
plt.show()
# plt.savefig('Fe-C-activation-volume.pdf', transparent=True, format='pdf')
```



```
In [11]: print('Total volume: {v[0]:.4f}, {V[0]:.4f}A^3\nV_xxxx: {v[1]:.4f}, {V[1]:.4f}A^3\nV_xxyy: {v[2]:.4f}, {V[2]:.4f}A^3\nV_xyxy: {v[3]:.4f}, {V[3]:.4f}A^3'.format(v=v, V=V))
Total volume: 0.0921, 1.0722A^3
V_xxxx: -0.1175, -1.3681A^3
V_xxyy: 0.1048, 1.2202A^3
V_xyxy: 0.0061, 0.0714A^3

In [12]: Vsph = 0.2*(3*V[-1,1] + 2*V[-1,2] + 4*V[-1,3]) # (3V11 + 2V12 + 2V44)/5
print('Spherical average uniaxial activation volume: {:.4f} {:.4f}A^3'.format(Vsph, Vsph*1e3*F))
Spherical average uniaxial activation volume: -0.0237 -0.2757A^3
```

12.2 Garnet correlation coefficients

Comparing to correlation coefficients from William D. Carlson and Clark R. Wilson, *Phys Chem Minerals* **43**, 363-369 (2016) [doi:10.1007/s00269-016-0800-2](https://doi.org/10.1007/s00269-016-0800-2)

Garnet structure includes pyrope, which we use as our example structure, with space group 230 (Ia3d) with stoichiometry $\text{Mg}_3\text{Al}_2\text{Si}_3\text{O}_{12}$. The occupied [Wyckoff positions](#) for this are (lattice constant $a_0=1.1459$ nm):

Wyckoff site	chemistry	position
24c	Mg	1/8 0 1/4
16a	Al	0 0 0
24d	Si	3/8 0 1/4
96h	O	.03284 .05014 .65330

Data from G. V. Gibbs and J. V. Smith, “Refinement of the crystal structure of synthetic pyrope.” *American Mineralogist* **50** 2023-2039 (1965), [PDF](#).

```
In [1]: import sys
        sys.path.extend(['../'])
        import numpy as np
        import onsager.crystal as crystal
        import onsager.OnsagerCalc as onsager
```

Create garnet crystal (lattice constant in nm). Wyckoff positions cut and pasted from Bilbao crystallographic server.

```
In [2]: # a0 = 1.1459
        # alatt = a0*np.eye(3)
        a0 = 1.
        alatt = a0*np.array([[ -0.5, 0.5, 0.5], [0.5, -0.5, 0.5], [0.5, 0.5, -0.5]])
        invlatt = np.array([[0, 1, 1], [1, 0, 1], [1, 1, 0]])
        x,y,z = (.03284, .05014, .65330)
        uMg = ((1/8, 0, 1/4), (3/8, 0, 3/4), (1/4, 1/8, 0), (3/4, 3/8, 0),
                (0, 1/4, 1/8), (0, 3/4, 3/8), (7/8, 0, 3/4), (5/8, 0, 1/4),
                (3/4, 7/8, 0), (1/4, 5/8, 0), (0, 3/4, 7/8), (0, 1/4, 5/8))
        uAl = ((0, 0, 0), (1/2, 0, 1/2), (0, 1/2, 1/2), (1/2, 1/2, 0),
                (3/4, 1/4, 1/4), (3/4, 3/4, 3/4), (1/4, 1/4, 3/4), (1/4, 3/4, 1/4))
        uSi = ((3/8, 0, 1/4), (1/8, 0, 3/4), (1/4, 3/8, 0), (3/4, 1/8, 0),
                (0, 1/4, 3/8), (0, 3/4, 1/8), (3/4, 5/8, 0), (3/4, 3/8, 1/2),
                (1/8, 1/2, 1/4), (7/8, 0, 1/4), (0, 1/4, 7/8), (1/2, 1/4, 1/8))
        uO = ((x,y,z), (-x+1/2, -y, z+1/2), (-x, y+1/2, -z+1/2), (x+1/2, -y+1/2, -z),
                (z,x,y), (z+1/2, -x+1/2, -y), (-z+1/2, -x, y+1/2), (-z, x+1/2, -y+1/2),
                (y,z,x), (-y, z+1/2, -x+1/2), (y+1/2, -z+1/2, -x), (-y+1/2, -z, x+1/2),
                (y+3/4, x+1/4, -z+1/4), (-y+3/4, -x+3/4, -z+3/4), (y+1/4, -x+1/4, z+3/4), (-y+1/4, x+3/4, z+1/4),
                (x+3/4, z+1/4, -y+1/4), (-x+1/4, z+3/4, y+1/4), (-x+3/4, -z+3/4, -y+3/4), (x+1/4, -z+1/4, y+3/4),
                (z+3/4, y+1/4, -x+1/4), (z+1/4, -y+1/4, x+3/4), (-z+1/4, y+3/4, x+1/4), (-z+3/4, -y+3/4, -x+3/4),
                (-x, -y, -z), (x+1/2, y, -z+1/2), (x, -y+1/2, z+1/2), (-x+1/2, y+1/2, z),
                (-z, -x, -y), (-z+1/2, x+1/2, y), (z+1/2, x, -y+1/2), (z, -x+1/2, y+1/2),
                (-y, -z, -x), (y, -z+1/2, x+1/2), (-y+1/2, z+1/2, x), (y+1/2, z, -x+1/2),
                (-y+1/4, -x+3/4, z+3/4), (y+1/4, x+1/4, z+1/4), (-y+3/4, x+3/4, -z+1/4), (y+3/4, -x+1/4, -z+3/4),
                (-x+1/4, -z+3/4, y+3/4), (x+3/4, -z+1/4, -y+3/4), (x+1/4, z+1/4, y+1/4), (-x+3/4, z+3/4, -y+1/4),
                (-z+1/4, -y+3/4, x+3/4), (-z+3/4, y+3/4, -x+1/4), (z+3/4, -y+1/4, -x+3/4), (z+1/4, y+1/4, x+1/4))
        # tovec = lambda x: np.array(x)
        # tovec2 = lambda x: np.array((x[0]+1/2, x[1]+1/2, x[2]+1/2))
        tovec = lambda x: np.dot(invlatt, x)
        pyrope = crystal.Crystal(alatt, [[vec(w) for w in ulist for vec in (tovec,)]
                                         for ulist in (uMg, uAl, uSi, uO)],
                                ['Mg', 'Al', 'Si', 'O'])
        # print(pyrope)
```

Next, we construct a *diffuser* based on vacancies for our Mg ion. We need to create a *sitelist* (which will be the Wyckoff positions) and a *jumpnetwork* for the transitions between the sites. There are tags that correspond to the unique states and transitions in the diffuser. The first cutoff is $\sim 0.31a_0$, but that connects half of the Mg cation sites to each other; increasing the cutoff to $\sim 0.51a_0$ introduces a second network that completes the connections.

```
In [3]: chem = 0 # 0 is the index corresponding to our Mg atom in the crystal
        cutoff = 0.31*a0 # had been 0.51*a0
        sitelist = pyrope.sitelist(chem)
        jumpnetwork = pyrope.jumpnetwork(chem, cutoff)
        Mgdiffuser = onsager.VacancyMediated(pyrope, chem, sitelist, jumpnetwork, 1)
        print(Mgdiffuser)
```

```

Diffuser for atom 0 (Mg), Nthermo=1
#Lattice:
a1 = [-0.5  0.5  0.5]
a2 = [ 0.5 -0.5  0.5]
a3 = [ 0.5  0.5 -0.5]
#Basis:
(Mg) 0.0 = [ 0.25  0.375  0.125]
(Mg) 0.1 = [ 0.75  0.125  0.375]
(Mg) 0.2 = [ 0.125  0.25  0.375]
(Mg) 0.3 = [ 0.375  0.75  0.125]
(Mg) 0.4 = [ 0.375  0.125  0.25 ]
(Mg) 0.5 = [ 0.125  0.375  0.75 ]
(Mg) 0.6 = [ 0.75  0.625  0.875]
(Mg) 0.7 = [ 0.25  0.875  0.625]
(Mg) 0.8 = [ 0.875  0.75  0.625]
(Mg) 0.9 = [ 0.625  0.25  0.875]
(Mg) 0.10 = [ 0.625  0.875  0.75 ]
(Mg) 0.11 = [ 0.875  0.625  0.25 ]
(Al) 1.0 = [ 0.  0.  0.]
(Al) 1.1 = [ 0.5  0.  0.5]
(Al) 1.2 = [ 0.  0.5  0.5]
(Al) 1.3 = [ 0.5  0.5  0. ]
(Al) 1.4 = [ 0.5  0.  0. ]
(Al) 1.5 = [ 0.5  0.5  0.5]
(Al) 1.6 = [ 0.  0.  0.5]
(Al) 1.7 = [ 0.  0.5  0. ]
(Si) 2.0 = [ 0.25  0.625  0.375]
(Si) 2.1 = [ 0.75  0.875  0.125]
(Si) 2.2 = [ 0.375  0.25  0.625]
(Si) 2.3 = [ 0.125  0.75  0.875]
(Si) 2.4 = [ 0.625  0.375  0.25 ]
(Si) 2.5 = [ 0.875  0.125  0.75 ]
(Si) 2.6 = [ 0.625  0.75  0.375]
(Si) 2.7 = [ 0.875  0.25  0.125]
(Si) 2.8 = [ 0.75  0.375  0.625]
(Si) 2.9 = [ 0.25  0.125  0.875]
(Si) 2.10 = [ 0.125  0.875  0.25 ]
(Si) 2.11 = [ 0.375  0.625  0.75 ]
(O) 3.0 = [ 0.70344  0.68614  0.08298]
(O) 3.1 = [ 0.10316  0.62046  0.41702]
(O) 3.2 = [ 0.39684  0.81386  0.5173 ]
(O) 3.3 = [ 0.79656  0.87954  0.9827 ]
(O) 3.4 = [ 0.08298  0.70344  0.68614]
(O) 3.5 = [ 0.41702  0.10316  0.62046]
(O) 3.6 = [ 0.5173  0.39684  0.81386]
(O) 3.7 = [ 0.9827  0.79656  0.87954]
(O) 3.8 = [ 0.68614  0.08298  0.70344]
(O) 3.9 = [ 0.62046  0.41702  0.10316]
(O) 3.10 = [ 0.81386  0.5173  0.39684]
(O) 3.11 = [ 0.87954  0.9827  0.79656]
(O) 3.12 = [ 0.87954  0.39684  0.08298]
(O) 3.13 = [ 0.81386  0.79656  0.41702]
(O) 3.14 = [ 0.62046  0.70344  0.5173 ]
(O) 3.15 = [ 0.68614  0.10316  0.9827 ]

```

```

(O) 3.16 = [ 0.10316  0.9827  0.68614]
(O) 3.17 = [ 0.70344  0.5173  0.62046]
(O) 3.18 = [ 0.79656  0.41702  0.81386]
(O) 3.19 = [ 0.39684  0.08298  0.87954]
(O) 3.20 = [ 0.5173   0.62046  0.70344]
(O) 3.21 = [ 0.9827   0.68614  0.10316]
(O) 3.22 = [ 0.08298  0.87954  0.39684]
(O) 3.23 = [ 0.41702  0.81386  0.79656]
(O) 3.24 = [ 0.29656  0.31386  0.91702]
(O) 3.25 = [ 0.89684  0.37954  0.58298]
(O) 3.26 = [ 0.60316  0.18614  0.4827 ]
(O) 3.27 = [ 0.20344  0.12046  0.0173 ]
(O) 3.28 = [ 0.91702  0.29656  0.31386]
(O) 3.29 = [ 0.58298  0.89684  0.37954]
(O) 3.30 = [ 0.4827   0.60316  0.18614]
(O) 3.31 = [ 0.0173   0.20344  0.12046]
(O) 3.32 = [ 0.31386  0.91702  0.29656]
(O) 3.33 = [ 0.37954  0.58298  0.89684]
(O) 3.34 = [ 0.18614  0.4827   0.60316]
(O) 3.35 = [ 0.12046  0.0173   0.20344]
(O) 3.36 = [ 0.12046  0.60316  0.91702]
(O) 3.37 = [ 0.18614  0.20344  0.58298]
(O) 3.38 = [ 0.37954  0.29656  0.4827 ]
(O) 3.39 = [ 0.31386  0.89684  0.0173 ]
(O) 3.40 = [ 0.89684  0.0173   0.31386]
(O) 3.41 = [ 0.29656  0.4827   0.37954]
(O) 3.42 = [ 0.20344  0.58298  0.18614]
(O) 3.43 = [ 0.60316  0.91702  0.12046]
(O) 3.44 = [ 0.4827   0.37954  0.29656]
(O) 3.45 = [ 0.0173   0.31386  0.89684]
(O) 3.46 = [ 0.91702  0.12046  0.60316]
(O) 3.47 = [ 0.58298  0.18614  0.20344]
vacancy configurations:
v:+0.250,+0.375,+0.125
solute configurations:
s:+0.250,+0.375,+0.125
solute-vacancy configurations:
s:+0.375,+0.125,+0.250-v:+0.750,+0.125,+0.375
omega0 jumps:
omega0:v:+0.625,+0.250,+0.875^v:+0.250,-0.125,+0.625
omega1 jumps:
omega1:s:+0.875,+0.625,+0.250-v:+0.625,+0.250,-0.125^v:+0.250,-0.125,-0.375
omega1:s:+0.750,+0.625,+0.875-v:+0.625,+0.250,+0.875^v:+0.250,-0.125,+0.625
omega1:s:+0.625,+0.875,+0.750-v:+0.625,+1.250,+0.875^v:+0.250,+0.875,+0.625
omega2 jumps:
omega2:s:+0.250,+0.875,+0.625-v:+0.625,+1.250,+0.875^s:+0.625,+0.250,+0.875-v:+0.250,-0.125,+0.625

```

Quick analysis on our jump network:

1. What is the connectivity, Z ?
2. What is the individual contribution to $\delta x \otimes \delta x$? And $1/3 \text{ Tr}$ (which will be the symmetrized contribution)?
3. What is the squared magnitude δx^2 ?

```
In [4]: for jlist in jumpnetwork:
        Z = 0
        dx2 = np.zeros((3,3))
        for (i,j), dx in jlist:
            if i==0:
                Z += 1
                dx2 += np.outer(dx,dx)
        print("coordination number:", Z)
        print(dx2)
        print("1/3 Tr dx dx:", dx2.trace()/3)
        print("dx^2:", np.dot(dx,dx))
```

```
coordination number: 4
[[ 0.0625  0.      0.      ]
 [ 0.      0.15625 -0.125  ]
 [ 0.     -0.125  0.15625]]
1/3 Tr dx dx: 0.125
dx^2: 0.09375
```

Next, we assemble our data: the energies and prefactors, for a VMg in pyrope for our *representative* states and transitions: these are the first states in the lists, which are also identified by the tags above. As we are computing a tracer, we make the choice to set $\nu_0 = 1/Z$ where $Z = 4$ is the coordination number.

```
In [5]: nu0 = 0.25
        Etrans = 0.
        # we don't need to use the tags, since there's only one site and jump type, and
        # we want to build a tracer.
        Mgthermodict = {'preV': np.ones(len(sitelist)),
                        'eneV': np.zeros(len(sitelist)),
                        'preT0': nu0*np.ones(len(jumpnetwork)),
                        'eneT0': Etrans*np.ones(len(jumpnetwork))}
        Mgthermodict.update(Mgdiffuser.maketracerpreene(**Mgthermodict))
        for k,v in Mgthermodict.items():
            print('{}: {}'.format(k, v))

eneSV: [ 0.]
eneT0: [ 0.]
preS: [ 1.]
preT1: [ 0.25  0.25  0.25]
preSV: [ 1.]
preT0: [ 0.25]
eneV: [ 0.]
eneT2: [ 0.]
eneS: [ 0.]
preV: [ 1.]
eneT1: [ 0.  0.  0.]
preT2: [ 0.25]
```

We compute the Onsager matrices, and look at $-L_{ss}/L_{sv}$ to get our correlation coefficient.

Note: we can define f (for our tracer) as the ratio of L_{ss} to $Z(\delta x)^2 w_2 c_v c_s / 6 = \frac{1}{16} \nu_0 a_0^2$ in this case, the same as what we get for L_{vv} and $-L_{sv}$.

```
In [6]: Lvv, Lss, Lsv, L1vv = Mgdiffuser.Lij(*Mgdiffuser.preene2betafree(1, **Mgthermodict))
        print(Lvv)
        print(Lss)
        print(Lsv)
```

```

print(L1vv)
print("Correlation coefficient:", -Lss[0,0]/Lsv[0,0])
[[ 0.015625 -0.         -0.         ]
 [-0.         0.015625 -0.         ]
 [-0.         -0.         0.015625]]
[[ 0.00585895  0.         0.         ]
 [ 0.         0.00585895  0.         ]
 [ 0.         0.         0.00585895]]
[[-0.015625  0.         0.         ]
 [ 0.        -0.015625  0.         ]
 [ 0.         0.        -0.015625]]
[[ -1.17108470e-34  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -1.17108470e-34  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00 -1.17108470e-34]]
Correlation coefficient: 0.374972670783

```

Compare with tabulated GF data from Carlson and Wilson paper. They use the notation (l, m, n) for a δx vector that is $a_0(l\hat{x} + m\hat{y} + n\hat{z})/8$. We will need to find a corresponding site that lands at that displacement from our origin site.

Unfortunately, it looks like in two cases ((800), (444)) there are two distinct sites that are mapped in that displacement vector, which have different GF values; the CW reported values appear to be the averaged values. In two other cases, ((640), (420)) the reported values are half of what the computed values are here.

As Carlson and Wilson used a stochastic approach to compute their GF values, all of their other data has errors $\sim 10^{-4}$.

In [7]: # tabulated data from paper

```

CarlsonWilsonGFdata = \
{(0,0,0): 2.30796022, (2,1,1): 1.30807261, (3,3,2): 0.80669536,
 (4,2,0): 0.40469085, (4,4,4): 0.50242046, (5,3,2): 0.56195744,
 (6,1,1): 0.56071092, (6,4,0): 0.22460654, (6,5,3): 0.42028488,
 (6,5,5): 0.40137897, (7,2,1): 0.44437878, (8,0,0): 0.41938675}

```

In [8]: print('CW index\t dx match\t GF (FT eval)\t GF(CW stoch.)\terror')

```

GF = Mgdifuser.GFcalc # get our GF calculator; should already have rates set
basis = pyrope.basis[chem]
x0 = np.dot(alatt, basis[0])

```

```

for vec,gCW in CarlsonWilsonGFdata.items():

```

```

    dx0 = np.array(vec,dtype=float)/8

```

```

    nmatch, Gave, Gmatch = 0, 0, {}

```

```

    for g in pyrope.G:

```

```

        dx = np.dot(g.cartrot, dx0)

```

```

        j = pyrope.cart2pos(x0+dx)[1]

```

```

        if j is not None and j[0]==chem and j[1]<6:

```

```

            G = GF(0, j[1], dx)

```

```

            Gmatch[tuple((8*dx).astype(int))] = G

```

```

            nmatch += 1

```

```

            Gave += G

```

```

    Gave /= nmatch

```

```

    for t,G in Gmatch.items():

```

```

        print('{}\t{}\t{:.12f}\t{:.8f}\t{:.4e}'.format(vec, t, -G, gCW, abs(G+gCW)))

```

```

    print('{}\taverage value\t{:.12f}\t{:.8f}\t{:.4e}'.format(vec, -Gave, gCW, abs(Gave+gCW)))

```

CW index	dx match	GF (FT eval)	GF(CW stoch.)	error
(8, 0, 0)	(0, 0, -8)	0.427361034009	0.41938675	7.9743e-03

(8, 0, 0)	(8, 0, 0)	0.403566247455	0.41938675	1.5821e-02
(8, 0, 0)	(0, 8, 0)	0.427361034009	0.41938675	7.9743e-03
(8, 0, 0)	(-8, 0, 0)	0.403566247455	0.41938675	1.5821e-02
(8, 0, 0)	(0, -8, 0)	0.427361034009	0.41938675	7.9743e-03
(8, 0, 0)	(0, 0, 8)	0.427361034009	0.41938675	7.9743e-03
(8, 0, 0)	average value	0.419429438491	0.41938675	4.2688e-05
(6, 1, 1)	(-1, 6, 1)	0.560766700022	0.56071092	5.5780e-05
(6, 1, 1)	(-1, -6, -1)	0.560766700022	0.56071092	5.5780e-05
(6, 1, 1)	(1, 1, 6)	0.560766700022	0.56071092	5.5780e-05
(6, 1, 1)	(1, -1, -6)	0.560766700022	0.56071092	5.5780e-05
(6, 1, 1)	average value	0.560766700022	0.56071092	5.5780e-05
(3, 3, 2)	(-3, 3, -2)	0.806767995595	0.80669536	7.2636e-05
(3, 3, 2)	(3, -2, 3)	0.806767995595	0.80669536	7.2636e-05
(3, 3, 2)	(3, 2, -3)	0.806767995595	0.80669536	7.2636e-05
(3, 3, 2)	(-3, -3, 2)	0.806767995595	0.80669536	7.2636e-05
(3, 3, 2)	average value	0.806767995595	0.80669536	7.2636e-05
(2, 1, 1)	(-1, -2, 1)	1.308081132926	1.30807261	8.5229e-06
(2, 1, 1)	(-1, 2, -1)	1.308081132926	1.30807261	8.5229e-06
(2, 1, 1)	(1, 1, -2)	1.308081132926	1.30807261	8.5229e-06
(2, 1, 1)	(1, -1, 2)	1.308081132926	1.30807261	8.5229e-06
(2, 1, 1)	average value	1.308081132926	1.30807261	8.5229e-06
(6, 5, 3)	(3, -6, 5)	0.420386782427	0.42028488	1.0190e-04
(6, 5, 3)	(-3, -5, 6)	0.420386782427	0.42028488	1.0190e-04
(6, 5, 3)	(5, -3, -6)	0.420386782427	0.42028488	1.0190e-04
(6, 5, 3)	(-3, 5, -6)	0.420386782427	0.42028488	1.0190e-04
(6, 5, 3)	(-5, -6, -3)	0.420386782427	0.42028488	1.0190e-04
(6, 5, 3)	(-5, 6, 3)	0.420386782427	0.42028488	1.0190e-04
(6, 5, 3)	(5, 3, 6)	0.420386782427	0.42028488	1.0190e-04
(6, 5, 3)	(3, 6, -5)	0.420386782427	0.42028488	1.0190e-04
(6, 5, 3)	average value	0.420386782427	0.42028488	1.0190e-04
(6, 4, 0)	(-6, 0, 4)	0.449091350780	0.22460654	2.2448e-01
(6, 4, 0)	(6, 4, 0)	0.449091350780	0.22460654	2.2448e-01
(6, 4, 0)	(6, -4, 0)	0.449091350780	0.22460654	2.2448e-01
(6, 4, 0)	(-6, 0, -4)	0.449091350780	0.22460654	2.2448e-01
(6, 4, 0)	average value	0.449091350780	0.22460654	2.2448e-01
(0, 0, 0)	(0, 0, 0)	2.308081141615	2.30796022	1.2092e-04
(0, 0, 0)	average value	2.308081141615	2.30796022	1.2092e-04
(4, 2, 0)	(2, 0, -4)	0.809394258097	0.40469085	4.0470e-01
(4, 2, 0)	(-2, -4, 0)	0.809394258097	0.40469085	4.0470e-01
(4, 2, 0)	(-2, 4, 0)	0.809394258097	0.40469085	4.0470e-01
(4, 2, 0)	(2, 0, 4)	0.809394258097	0.40469085	4.0470e-01
(4, 2, 0)	average value	0.809394258097	0.40469085	4.0470e-01
(5, 3, 2)	(-5, 2, -3)	0.561961239416	0.56195744	3.7994e-06
(5, 3, 2)	(3, 2, 5)	0.561961239416	0.56195744	3.7994e-06
(5, 3, 2)	(3, -2, -5)	0.561961239416	0.56195744	3.7994e-06
(5, 3, 2)	(-5, -2, 3)	0.561961239416	0.56195744	3.7994e-06
(5, 3, 2)	(5, 3, -2)	0.561961239416	0.56195744	3.7994e-06
(5, 3, 2)	(5, -3, 2)	0.561961239416	0.56195744	3.7994e-06
(5, 3, 2)	(-3, -5, -2)	0.561961239416	0.56195744	3.7994e-06
(5, 3, 2)	(-3, 5, 2)	0.561961239416	0.56195744	3.7994e-06
(5, 3, 2)	average value	0.561961239416	0.56195744	3.7994e-06
(7, 2, 1)	(-1, -2, -7)	0.444350262895	0.44437878	2.8517e-05
(7, 2, 1)	(1, 7, 2)	0.444350262895	0.44437878	2.8517e-05
(7, 2, 1)	(7, 2, -1)	0.444350262895	0.44437878	2.8517e-05

(7, 2, 1)	(-7, 1, -2)	0.444350262895	0.44437878	2.8517e-05
(7, 2, 1)	(-7, -1, 2)	0.444350262895	0.44437878	2.8517e-05
(7, 2, 1)	(7, -2, 1)	0.444350262895	0.44437878	2.8517e-05
(7, 2, 1)	(-1, 2, 7)	0.444350262895	0.44437878	2.8517e-05
(7, 2, 1)	(1, -7, -2)	0.444350262895	0.44437878	2.8517e-05
(7, 2, 1)	average value	0.444350262895	0.44437878	2.8517e-05
(4, 4, 4)	(-4, 4, 4)	0.457297218361	0.50242046	4.5123e-02
(4, 4, 4)	(-4, 4, -4)	0.547635344309	0.50242046	4.5215e-02
(4, 4, 4)	(-4, -4, -4)	0.457297218361	0.50242046	4.5123e-02
(4, 4, 4)	(4, 4, -4)	0.547635344309	0.50242046	4.5215e-02
(4, 4, 4)	(-4, -4, 4)	0.547635344309	0.50242046	4.5215e-02
(4, 4, 4)	(4, -4, -4)	0.457297218361	0.50242046	4.5123e-02
(4, 4, 4)	(4, -4, 4)	0.547635344309	0.50242046	4.5215e-02
(4, 4, 4)	(4, 4, 4)	0.457297218361	0.50242046	4.5123e-02
(4, 4, 4)	average value	0.502466281335	0.50242046	4.5821e-05
(6, 5, 5)	(-5, 6, -5)	0.401425331863	0.40137897	4.6362e-05
(6, 5, 5)	(-5, -6, 5)	0.401425331863	0.40137897	4.6362e-05
(6, 5, 5)	(5, -5, 6)	0.401425331863	0.40137897	4.6362e-05
(6, 5, 5)	(5, 5, -6)	0.401425331863	0.40137897	4.6362e-05
(6, 5, 5)	average value	0.401425331863	0.40137897	4.6362e-05

12.3 Convergence of Green function calculation

We check the convergence with N_{kpt} for the calculation of the vacancy Green function for FCC and HCP structures. In particular, we will look at:

1. The $\mathbf{R} = 0$ value,
2. The largest \mathbf{R} value in the calculation of a first neighbor thermodynamic interaction range,
3. The difference of the Green function value for (1) and (2),

with increasing k-point density.

```
In [1]: import sys
        sys.path.extend(['../'])
        import numpy as np
        import matplotlib.pyplot as plt
        plt.style.use('seaborn-whitegrid')
        %matplotlib inline
        import onsager.crystal as crystal
        import onsager.GFcalc as GFcalc
```

Create an FCC and HCP lattice.

```
In [2]: a0 = 1.
        FCC, HCP = crystal.Crystal.FCC(a0, "fcc"), crystal.Crystal.HCP(a0, chemistry="hcp")
        print(FCC)
        print(HCP)

#Lattice:
a1 = [ 0.  0.5  0.5]
a2 = [ 0.5  0.  0.5]
a3 = [ 0.5  0.5  0. ]

#Basis:
(fcc) 0.0 = [ 0.  0.  0.]
```

```
#Lattice:
a1 = [ 0.5      -0.8660254  0.      ]
a2 = [ 0.5      0.8660254  0.      ]
a3 = [ 0.        0.        1.63299316]
#Basis:
(hcp) 0.0 = [ 0.33333333  0.66666667  0.25      ]
(hcp) 0.1 = [ 0.66666667  0.33333333  0.75      ]
```

We will put together our vectors for consideration: * Maximum \mathbf{R} for FCC = (400) , or $\mathbf{x} = 2\hat{j} + 2\hat{k}$. * Maximum \mathbf{R} for HCP = (440) , or $\mathbf{x} = 4\hat{i}$, and (222) , or $\mathbf{x} = 2\hat{i} + 2\sqrt{8/3}\hat{k}$.

and our sitelists and jumpnetworks.

```
In [3]: FCCR = np.array([0,2.,2.])
        HCPR1, HCPR2 = np.array([4.,0.,0.]), np.array([2.,0.,2*np.sqrt(8/3)])
```

```
In [4]: FCCsite, FCCjn = FCC.sitelist(0), FCC.jumpnetwork(0, 0.75)
        HCPsite, HCPjn = HCP.sitelist(0), HCP.jumpnetwork(0, 1.01)
```

We use N_{\max} parameter, which controls the automated generation of k-points to iterate through successively denser k-point meshes.

```
In [5]: FCCdata = {pmaxerror:[] for pmaxerror in range(-16,0)}
        print('kpt\tNkpt\tG(0)\tG(R)\tG diff')
        for Nmax in range(1,9):
            GFFCC = GFcalc.GFCrystalcalc(FCC, 0, FCCsite, FCCjn, Nmax=Nmax)
            Nreduce, Nkpt, kpt = GFFCC.Nkpt, np.prod(GFFCC.kptgrid), GFFCC.kptgrid
            for pmax in sorted(FCCdata.keys(), reverse=True):
                GFFCC.SetRates(np.ones(1), np.zeros(1), np.ones(1)/12, np.zeros(1), 10**(pmax))
                g0,gR = GFFCC(0,0,np.zeros(3)), GFFCC(0,0,FCCR)
                FCCdata[pmax].append((Nkpt, g0, gR))
            Nkpt,g0,gR = FCCdata[-8][-1] # print the 10^-8 values
            print("{k[0]}x{k[1]}x{k[2]}\t".format(k=kpt) +
                  "{:5d} ({})\t{:12f}\t{:12f}\t{:12f}".format(Nkpt, Nreduce,
                                                                g0, gR,g0-gR))
```

kpt	Nkpt	G(0)	G(R)	G diff
6x6x6	216 (16)	-1.344901582401	-0.119888361621	-1.225013220779
10x10x10	1000 (48)	-1.344674624975	-0.084566077531	-1.260108547444
14x14x14	2744 (106)	-1.344663672542	-0.084541308263	-1.260122364278
18x18x18	5832 (200)	-1.344661890661	-0.084539383601	-1.260122507060
22x22x22	10648 (337)	-1.344661442418	-0.084538941204	-1.260122501213
26x26x26	17576 (528)	-1.344661295591	-0.084538798573	-1.260122497018
30x30x30	27000 (778)	-1.344661238153	-0.084538742761	-1.260122495392
34x34x34	39304 (1095)	-1.344661212587	-0.084538717850	-1.260122494737

```
In [6]: HCPdata = []
        print('kpt\tNkpt\tG(0)\tG(R1)\tG(R2)\tG(R1)-G(0)\tG(R2)-G(0)')
        for Nmax in range(1,13):
            GFHCP = GFcalc.GFCrystalcalc(HCP, 0, HCPsite, HCPjn, Nmax=Nmax)
            GFHCP.SetRates(np.ones(1), np.zeros(1), np.ones(2)/12, np.zeros(2), 1e-8)
            g0,gR1,gR2 = GFHCP(0,0,np.zeros(3)), GFHCP(0,0,HCPR1), GFHCP(0,0,HCPR2)
            Nreduce, Nkpt, kpt = GFHCP.Nkpt, np.prod(GFHCP.kptgrid), GFHCP.kptgrid
            HCPdata.append((Nkpt, g0, gR1, gR2))
            print("{k[0]}x{k[1]}x{k[2]}\t".format(k=kpt) +
                  "{:5d} ({})\t{:12f}\t{:12f}\t{:12f}\t{:12f}\t{:12f}".format(Nkpt, Nreduce,
                                                                g0, gR1, gR2,
                                                                g0-gR1, g0-gR2))
```

kpt	Nkpt	G(0)	G(R1)	G(R2)	G(R1)-G(0)	G(R2)-G0		
6x6x4	144	(21)	-1.367909503563	-0.192892722514	-0.131552967388	-1.175016781049	-1.236356536175	
10x10x6	600	(56)	-1.345034474341	-0.087913619020	-0.089866654871	-1.257120855321	-1.255167819470	
16x16x8	2048	(150)	-1.344668575390	-0.084546609595	-0.088212957806	-1.260121965795	-1.256455617584	
20x20x12		4800 (308)		-1.344662392185	-0.084539941251	-0.088166498574	-1.260122450934	-1.2564
26x26x14		9464 (560)		-1.344661615456	-0.084539088966	-0.088165768509	-1.260122526490	-1.2564
30x30x16		14400 (819)		-1.344661401027	-0.084538892419	-0.088165529659	-1.260122508608	-1.2564
36x36x20		25920 (1397)		-1.344661260564	-0.084538764009	-0.088165374312	-1.260122496555	-1.2564
40x40x22		35200 (1848)		-1.344661230214	-0.084538734661	-0.088165342770	-1.260122495553	-1.2564
46x46x24		50784 (2600)		-1.344661210808	-0.084538715598	-0.088165322977	-1.260122495211	-1.2564
50x50x28		70000 (3510)		-1.344661197817	-0.084538703416	-0.088165309065	-1.260122494400	-1.2564
56x56x30		94080 (4640)		-1.344661192649	-0.084538698279	-0.088165303871	-1.260122494370	-1.2564
60x60x32		115200 (5627)		-1.344661189980	-0.084538695678	-0.088165301128	-1.260122494302	-1.2564

First, look at the behavior of the error with $p_{\max}(\text{error})$ parameter. The k-point integration error scales as $N_{\text{kpt}}^{5/3}$, and we see the p_{\max} error is approximately 10^{-8} .

```
In [7]: print('pmax\tGinf\talpha (Nkpt^-5/3 prefactor)')
Ginflist=[]
for pmax in sorted(FCCdata.keys(), reverse=True):
    data = FCCdata[pmax]
    Nk53 = np.array([N**(5/3) for (N,g0,gR) in data])
    gval = np.array([g0 for (N,g0,gR) in data])
    N10,N5 = np.average(Nk53*Nk53),np.average(Nk53)
    g10,g5 = np.average(gval*Nk53*Nk53),np.average(gval*Nk53)
    denom = N10-N5**2
    Ginf,alpha = (g10-g5*N5)/denom, (g10*N5-g5*N10)/denom
    Ginflist.append(Ginf)
    print('{}\t{}\t{}'.format(pmax, Ginf, alpha))
```

pmax	Ginf	alpha (Nkpt^-5/3 prefactor)
-1	-1.3622431228203618	107.68032003382153
-2	-1.3452258994701751	12.629186631942448
-3	-1.3446884353187338	2.082017161988394
-4	-1.3446627918154508	1.0633204684019726
-5	-1.344661290378736	1.0462291512843422
-6	-1.3446611905262338	1.141759242474714
-7	-1.3446611825889647	1.2677586299509196
-8	-1.344661179858607	1.4411728790136409
-9	-1.3446611749706006	1.6992758893513724
-10	-1.3446611661040953	2.084685390450087
-11	-1.3446611519367564	2.6380532642193937
-12	-1.3446611313776884	3.3938198836995825
-13	-1.344661103589501	4.378495907854502
-14	-1.3446610679820363	5.610776776620662
-15	-1.3446610241750612	7.102718730176203
-16	-1.3446609719473421	8.861351881915382

Plot the error in the Green function for FCC (at 0, maximum R, and difference between those GF). We extract the infinite value by fitting the error to $N_{\text{kpt}}^{-5/3}$, which empirically matches the numerical error.

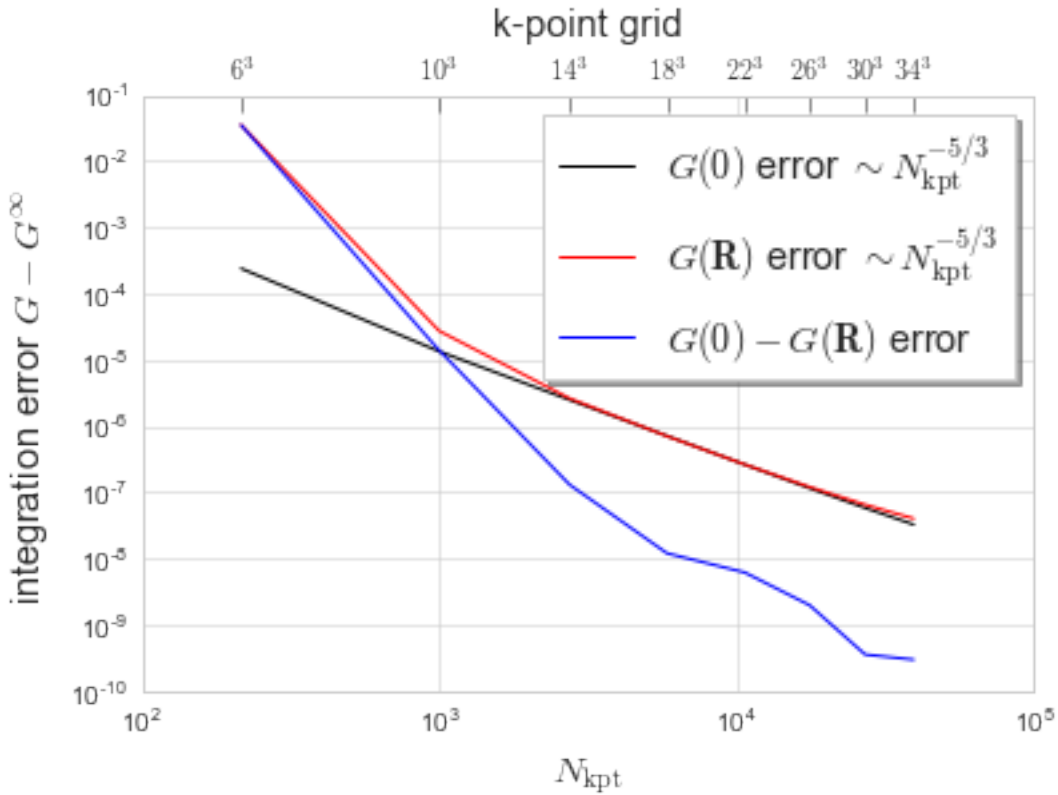
```
In [8]: # plot the errors from pmax = 10^-8
data = FCCdata[-8]
Nk = np.array([N for (N,g0,gR) in data])
g0val = np.array([g0 for (N,g0,gR) in data])
gRval = np.array([gR for (N,g0,gR) in data])
```

```

gplot = []
Nk53 = np.array([N**(5/3) for (N,g0,gR) in data])
for gdata, start in zip((g0val, gRval, g0val-gRval), (0,1,2)):
    N10,N5 = np.average(Nk53[start:]*Nk53[start:]),np.average(Nk53[start:])
    denom = N10-N5**2
    g10 = np.average(gdata[start:]*Nk53[start:]*Nk53[start:])
    g5 = np.average(gdata[start:]*Nk53[start:])
    Ginf,alpha = (g10-g5*N5)/denom, (g10*N5-g5*N10)/denom
    gplot.append(np.abs(gdata-Ginf))

fig, ax1 = plt.subplots()
ax1.plot(Nk, gplot[0], 'k', label='$G(\mathbf{0})$ error $\sim N_{\mathrm{kpt}}^{-5/3}$')
ax1.plot(Nk, gplot[1], 'r', label='$G(\mathbf{R})$ error $\sim N_{\mathrm{kpt}}^{-5/3}$')
ax1.plot(Nk, gplot[2], 'b', label='$G(\mathbf{0})-G(\mathbf{R})$ error')
ax1.set_xscale('log')
ax1.set_yscale('log')
ax1.set_xlabel('$N_{\mathrm{kpt}}$', fontsize='x-large')
ax1.set_ylabel('integration error $G-G^{\infty}$', fontsize='x-large')
ax1.legend(bbox_to_anchor=(0.6,0.6,0.4,0.4), ncol=1,
          shadow=True, frameon=True, fontsize='x-large')
ax2 = ax1.twinx()
ax2.set_xscale('log')
ax2.set_xlim(ax1.get_xlim())
ax2.set_xticks([n for n in Nk])
ax2.set_xticklabels(["${:.0f}^3$".format(n**(1/3)) for n in Nk])
ax2.set_xlabel('k-point grid', fontsize='x-large')
ax2.grid(False)
ax2.tick_params(axis='x', top='on', direction='in', length=6)
plt.show()
# plt.savefig('FCC-GError.pdf', transparent=True, format='pdf')

```



Plot the error in Green function for HCP.

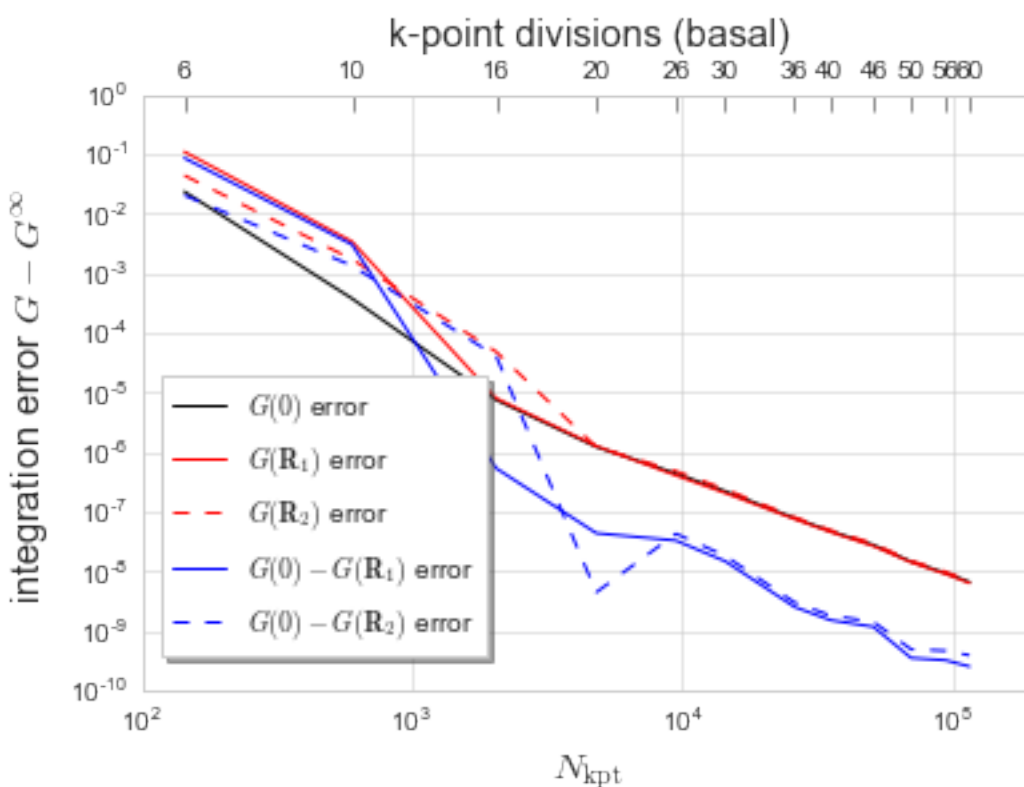
```
In [9]: # plot the errors from pmax = 10^-8
data = HCPdata
Nk = np.array([N for (N,g0,gR1,gR2) in data])
g0val = np.array([g0 for (N,g0,gR1,gR2) in data])
gR1val = np.array([gR1 for (N,g0,gR1,gR2) in data])
gR2val = np.array([gR2 for (N,g0,gR1,gR2) in data])

gplot = []
Nk53 = np.array([N**(5/3) for (N,g0,gR1,gR2) in data])
for gdata, start in zip((g0val, gR1val, gR2val, g0val-gR1val, g0val-gR2val), (3,3,3,3,3)):
    N10,N5 = np.average(Nk53[start:]*Nk53[start:]),np.average(Nk53[start:])
    denom = N10-N5**2
    g10 = np.average(gdata[start:]*Nk53[start:]*Nk53[start:])
    g5 = np.average(gdata[start:]*Nk53[start:])
    Ginf,alpha = (g10-g5*N5)/denom, (g10*N5-g5*N10)/denom
    gplot.append(np.abs(gdata-Ginf))

fig, ax1 = plt.subplots()
ax1.plot(Nk, gplot[0], 'k', label='$G(\mathbf{0})$ error')
ax1.plot(Nk, gplot[1], 'r', label='$G(\mathbf{R}_1)$ error')
ax1.plot(Nk, gplot[2], 'r--', label='$G(\mathbf{R}_2)$ error')
ax1.plot(Nk, gplot[3], 'b', label='$G(\mathbf{0})-G(\mathbf{R}_1)$ error')
ax1.plot(Nk, gplot[4], 'b--', label='$G(\mathbf{0})-G(\mathbf{R}_2)$ error')
ax1.set_xlim((1e2,2e5))
ax1.set_xscale('log')
ax1.set_yscale('log')
```

```

ax1.set_xlabel('$N_{\mathrm{kpt}}$', fontsize='x-large')
ax1.set_ylabel('integration error $G-G^{\infty}$', fontsize='x-large')
ax1.legend(bbox_to_anchor=(0.1,0.05,0.3,0.5), ncol=1,
           shadow=True, frameon=True, fontsize='medium')
ax2 = ax1.twinx()
ax2.set_xscale('log')
ax2.set_xlim(ax1.get_xlim())
ax2.set_xticks([n for n in Nk])
# ax2.set_xticklabels(["${:.0f}$".format((n*1.875)**(1/3)) for n in Nk])
ax2.set_xticklabels(['6', '10', '16', '20', '26', '30', '36', '40', '46', '50', '56', '60'])
ax2.set_xlabel('k-point divisions (basal)', fontsize='x-large')
ax2.grid(False)
ax2.tick_params(axis='x', top='on', direction='in', length=6)
plt.show()
# plt.savefig('HCP-GError.pdf', transparent=True, format='pdf')
    
```



12.4 Tracer correlation coefficients

We want (for testing purposes) to compute correlation coefficients for tracers for several different crystal structures:

- Simple cubic
- Body-centered cubic
- Face-centered cubic
- Diamond

- Wurtzite
- Hexagonal closed-packed
- NbO
- omega
- octahedral-tetrahedral network in HCP

Some are well-known (previously published) others are new.

```
In [1]: import sys
        sys.path.extend(['../'])
        import numpy as np
        from onsager import crystal, OnsagerCalc
```

Create all of our lattices, with lattice constant a0.

```
In [2]: a0 = 1.
        SC = crystal.Crystal(a0*np.eye(3), [np.array([0.,0.,0.])], ["SC"])
        BCC = crystal.Crystal.BCC(a0, ["BCC"])
        FCC = crystal.Crystal.FCC(a0, ["FCC"])
        diamond = crystal.Crystal(a0*np.array([[0,1/2,1/2],[1/2,0,1/2],[1/2,1/2,0]]),
                                   [np.array([1/8,1/8,1/8]), np.array([-1/8,-1/8,-1/8])],
                                   ["diamond"])
        wurtzite = crystal.Crystal(a0*np.array([[1/2,1/2,0],
                                                [-np.sqrt(3/4),np.sqrt(3/4),0.],
                                                [0.,0.,np.sqrt(8/3)]]),
                                   [np.array([1/3,2/3,1/4-3/16]), np.array([1/3,2/3,1/4+3/16]),
                                    np.array([2/3,1/3,3/4-3/16]), np.array([2/3,1/3,3/4+3/16])],
                                   ["wurtzite"])
        HCP = crystal.Crystal.HCP(a0, np.sqrt(8/3), ["HCP"])
        NbO = crystal.Crystal(a0*np.eye(3),
                              [[np.array([0,1/2,1/2]), np.array([1/2,0,1/2]),np.array([1/2,1/2,0])],
                               [np.array([1/2,0,0]), np.array([0,1/2,0]), np.array([0,0,1/2])]],
                              ['Nb', 'O'])
        omega = crystal.Crystal(a0*np.array([[1/2,1/2,0],
                                                [-np.sqrt(3/4),np.sqrt(3/4),0.],
                                                [0.,0.,np.sqrt(3/8)]]),
                                   [np.array([0.,0.,0.]),
                                    np.array([1/3,2/3,1/2]), np.array([2/3,1/3,1/2])],
                                   ["omega"])
        octtet = crystal.Crystal(a0*np.array([[1/2,1/2,0],
                                                [-np.sqrt(3/4),np.sqrt(3/4),0.],
                                                [0.,0.,np.sqrt(8/3)]]),
                                   [[np.array([0.,0.,0.]), np.array([0.,0.,0.5]),
                                    np.array([1/3,2/3,5/8]), np.array([1/3,2/3,7/8]),
                                    np.array([2/3,1/3,3/8]), np.array([2/3,1/3,1/8])],
                                    [np.array([1/3,2/3,1/4]), np.array([2/3,1/3,3/4])]],
                                   ["O", "Ti"])
        crystallist = [SC, BCC, FCC, diamond, wurtzite, HCP, NbO, omega, octtet]
        crystalnames = ["simple cubic", "body-centered cubic", "face-centered cubic", "diamond",
                        "wurtzite", "hexagonal closed-packed", "NbO", "hexagonal omega",
                        "HCP octahedral-tetrahedral"]

In [3]: for name, crys in zip(crystalnames, crystallist):
        print(name)
```

```

        print(crys)
        print()

simple cubic
#Lattice:
    a1 = [ 1.  0.  0.]
    a2 = [ 0.  1.  0.]
    a3 = [ 0.  0.  1.]
#Basis:
    (SC) 0.0 = [ 0.  0.  0.]

body-centered cubic
#Lattice:
    a1 = [-0.5  0.5  0.5]
    a2 = [ 0.5 -0.5  0.5]
    a3 = [ 0.5  0.5 -0.5]
#Basis:
    (BCC) 0.0 = [ 0.  0.  0.]

face-centered cubic
#Lattice:
    a1 = [ 0.  0.5  0.5]
    a2 = [ 0.5  0.  0.5]
    a3 = [ 0.5  0.5  0. ]
#Basis:
    (FCC) 0.0 = [ 0.  0.  0.]

diamond
#Lattice:
    a1 = [ 0.  0.5  0.5]
    a2 = [ 0.5  0.  0.5]
    a3 = [ 0.5  0.5  0. ]
#Basis:
    (diamond) 0.0 = [ 0.625  0.625  0.625]
    (diamond) 0.1 = [ 0.375  0.375  0.375]

wurtzite
#Lattice:
    a1 = [ 0.5      -0.8660254  0.      ]
    a2 = [ 0.5      0.8660254  0.      ]
    a3 = [ 0.        0.        1.63299316]
#Basis:
    (wurtzite) 0.0 = [ 0.33333333  0.66666667  0.0625  ]
    (wurtzite) 0.1 = [ 0.33333333  0.66666667  0.4375  ]
    (wurtzite) 0.2 = [ 0.66666667  0.33333333  0.5625  ]
    (wurtzite) 0.3 = [ 0.66666667  0.33333333  0.9375  ]

hexagonal closed-packed
#Lattice:
    a1 = [ 0.5      -0.8660254  0.      ]
    a2 = [ 0.5      0.8660254  0.      ]
    a3 = [ 0.        0.        1.63299316]
#Basis:
    (HCP) 0.0 = [ 0.33333333  0.66666667  0.25  ]

```



```

(HCP) 0.1 = [ 0.66666667  0.33333333  0.75      ]

NbO
#Lattice:
a1 = [ 1.  0.  0.]
a2 = [ 0.  1.  0.]
a3 = [ 0.  0.  1.]
#Basis:
(Nb) 0.0 = [ 0.  0.5  0.5]
(Nb) 0.1 = [ 0.5  0.  0.5]
(Nb) 0.2 = [ 0.5  0.5  0. ]
(O) 1.0 = [ 0.5  0.  0. ]
(O) 1.1 = [ 0.  0.5  0. ]
(O) 1.2 = [ 0.  0.  0.5]

hexagonal omega
#Lattice:
a1 = [ 0.  0.  0.61237244]
a2 = [ 0.5 -0.8660254  0. ]
a3 = [ 0.5  0.8660254  0. ]
#Basis:
(omega) 0.0 = [ 0.  0.  0.]
(omega) 0.1 = [ 0.5  0.33333333  0.66666667]
(omega) 0.2 = [ 0.5  0.66666667  0.33333333]

HCP octahedral-tetrahedral
#Lattice:
a1 = [ 0.5 -0.8660254  0. ]
a2 = [ 0.5  0.8660254  0. ]
a3 = [ 0.  0.  1.63299316]
#Basis:
(O) 0.0 = [ 0.  0.  0.]
(O) 0.1 = [ 0.  0.  0.5]
(O) 0.2 = [ 0.33333333  0.66666667  0.625 ]
(O) 0.3 = [ 0.33333333  0.66666667  0.875 ]
(O) 0.4 = [ 0.66666667  0.33333333  0.375 ]
(O) 0.5 = [ 0.66666667  0.33333333  0.125 ]
(Ti) 1.0 = [ 0.33333333  0.66666667  0.25 ]
(Ti) 1.1 = [ 0.66666667  0.33333333  0.75 ]
    
```

Now we generate diffusers for *every crystal*. This is fairly automated, where the main input is the cutoff distance.

```

In [4]: cutoffs = [1.01*a0, 0.9*a0, 0.75*a0, 0.45*a0, 0.62*a0, 1.01*a0, 0.8*a0, 0.66*a0, 0.71*a0]
diffusers = []
for name, crys, cut in zip(crystalnames, crystallist, cutoffs):
    jn = crys.jumpnetwork(0, cut, 0.01)
    print(name)
    print(' Unique jumps:', len(jn))
    for jlist in jn:
        print(' connectivity:', len([i for (i,j), dx in jlist if i==jlist[0][0][0]]))
    diffusers.append(OnsagerCalc.VacancyMediated(crys, 0, crys.sitelist(0), jn, 1, 6))

simple cubic
    
```

```

Unique jumps: 1
  connectivity: 6
body-centered cubic
Unique jumps: 1
  connectivity: 8
face-centered cubic
Unique jumps: 1
  connectivity: 12
diamond
Unique jumps: 1
  connectivity: 4
wurtzite
Unique jumps: 2
  connectivity: 1
  connectivity: 3
hexagonal closed-packed
Unique jumps: 2
  connectivity: 6
  connectivity: 6
NbO
Unique jumps: 1
  connectivity: 8
hexagonal omega
Unique jumps: 4
  connectivity: 2
  connectivity: 12
  connectivity: 2
  connectivity: 3
HCP octahedral-tetrahedral
Unique jumps: 3
  connectivity: 6
  connectivity: 1
  connectivity: 3

```

Now run through each, creating the “tracer” and compute the correlation coefficient. We do this by giving all of the vacancy positions the same energy (may not apply for true omega and octahedral-tetrahedral networks, for example), and then assigning the same energy for all transitions (again, may not apply for cases where there is more than one unique jump). We compute the full Onsager matrix, then look at the diagonal of $f = -L_{ss}/L_{sv}$.

```

In [5]: print('crystal\tf_xx f_yy f_zz')
        for name, diff in zip(crystalnames, diffusers):
            nsites, njumps = len(diff.sitelist), len(diff.om0_jn)
            tdict = {'preV': np.ones(nsites), 'eneV': np.zeros(nsites),
                    'preT0': np.ones(njumps), 'eneT0': np.zeros(njumps)}
            # make a tracer out of it:
            tdict.update(diff.maketracerpreene(**tdict))
            Lss, Lsv = diff.Lij(*diff.preene2betafree(1, **tdict))[1:3] # just pull out ss and sv
            f = np.diag(-np.dot(Lss, np.linalg.inv(Lsv)))
            print('{name}\t{f[0]:.8f} {f[1]:.8f} {f[2]:.8f}'.format(name=name, f=f))

crystal f_xx f_yy f_zz
simple cubic    0.65310884 0.65310884 0.65310884
body-centered cubic    0.72719414 0.72719414 0.72719414
face-centered cubic    0.78145142 0.78145142 0.78145142

```

```
diamond 0.50000000 0.50000000 0.50000000
wurtzite      0.50000000 0.50000000 0.50000000
hexagonal closed-packed 0.78120488 0.78120488 0.78145142
NbO      0.68891612 0.68891612 0.68891612
hexagonal omega 0.78122649 0.78122649 0.78157339
HCP octahedral-tetrahedral      0.63052307 0.63052307 0.65230273
```

Look at variation in correlation coefficient for wurtzite structure by varying the ratio of the two rates.

```
In [6]: print('w(c)/w(basal)\tf_xx f_yy f_zz')
        crysindex = crystalnames.index('wurtzite')
        diff = diffusers[crysindex]
        nsites, njumps = len(diff.sitelist), len(diff.om0_jn)
        for w0_w1 in np.linspace(-2,2,num=17):
            w0 = 10**(w0_w1)
            w1 = 1
            tdict = {'preV': np.ones(nsites), 'eneV': np.zeros(nsites),
                    'preT0': np.array([w0,w1]), 'eneT0': np.zeros(njumps)}
            # make a tracer out of it:
            tdict.update(diff.maketracerpreene(**tdict))
            Lss, Lsv = diff.Lij(*diff.preene2betafree(1, **tdict))[1:3] # just pull out ss and sv
            f = np.diag(-np.dot(Lss, np.linalg.inv(Lsv)))
            print('10^{w0_w1:.2f}\tf{f[0]:.8f} {f[1]:.8f} {f[2]:.8f}'.format(w0_w1=w0_w1, f=f))

w(c)/w(basal)   f_xx f_yy f_zz
10^-2.00       0.34028415 0.34028415 0.97703525
10^-1.75       0.34436809 0.34436809 0.96308802
10^-1.50       0.35071960 0.35071960 0.94102894
10^-1.25       0.36039193 0.36039193 0.90707895
10^-1.00       0.37474153 0.37474153 0.85697089
10^-0.75       0.39527477 0.39527477 0.78719268
10^-0.50       0.42323211 0.42323211 0.69772423
10^-0.25       0.45877839 0.45877839 0.59612281
10^+0.00       0.50000000 0.50000000 0.50000000
10^+0.25       0.54260548 0.54260548 0.43237601
10^+0.50       0.58129067 0.58129067 0.40813890
10^+0.75       0.61218108 0.61218108 0.42268531
10^+1.00       0.63424155 0.63424155 0.45691154
10^+1.25       0.64870094 0.64870094 0.49255065
10^+1.50       0.65763077 0.65763077 0.52060064
10^+1.75       0.66293871 0.66293871 0.53970328
10^+2.00       0.66602090 0.66602090 0.55182811
```


INDICES AND TABLES

- genindex
- modindex
- search

a

automator, [53](#)

c

crystal, [11](#)

crystalStars, [21](#)

g

GFcalc, [41](#)

o

OnsagerCalc, [45](#)

p

PowerExpansion, [33](#)

s

supercell, [29](#)

Symbols

- `__add__` () (PowerExpansion.Taylor3D method), 33
- `__add__` () (crystal.GroupOp method), 17
- `__add__` () (crystalStars.PairState method), 22
- `__add__` () (crystalStars.StarSet method), 23
- `__call__` () (GFcalc.GFCrystalcalc method), 43
- `__call__` () (PowerExpansion.Taylor3D method), 33
- `__contains__` () (crystalStars.StarSet method), 23
- `__eq__` () (crystal.GroupOp method), 18
- `__eq__` () (crystalStars.PairState method), 22
- `__eq__` () (supercell.SuperCell method), 29
- `__getitem__` () (PowerExpansion.Taylor3D method), 33
- `__getitem__` () (supercell.SuperCell method), 29
- `__hash__` () (crystal.GroupOp method), 18
- `__hash__` () (crystalStars.PairState method), 22
- `__iadd__` () (PowerExpansion.Taylor3D method), 33
- `__iadd__` () (crystalStars.StarSet method), 23
- `__imul__` () (supercell.SuperCell method), 30
- `__initTaylor3Dindexing__` () (PowerExpansion.Taylor3D class method), 33
- `__init__` () (GFcalc.GFCrystalcalc method), 43
- `__init__` () (OnsagerCalc.Interstitial method), 45
- `__init__` () (OnsagerCalc.VacancyMediated method), 48
- `__init__` () (PowerExpansion.Taylor3D method), 33
- `__init__` () (crystal.Crystal method), 12
- `__init__` () (crystalStars.StarSet method), 23
- `__init__` () (crystalStars.VectorStarSet method), 25
- `__init__` () (supercell.SuperCell method), 30
- `__isub__` () (PowerExpansion.Taylor3D method), 34
- `__mul__` () (PowerExpansion.Taylor3D method), 34
- `__mul__` () (crystal.GroupOp method), 18
- `__mul__` () (supercell.SuperCell method), 30
- `__ne__` () (crystal.GroupOp method), 18
- `__ne__` () (crystalStars.PairState method), 22
- `__ne__` () (supercell.SuperCell method), 30
- `__neg__` () (PowerExpansion.Taylor3D method), 34
- `__neg__` () (crystalStars.PairState method), 22
- `__pos__` () (PowerExpansion.Taylor3D method), 34
- `__radd__` () (PowerExpansion.Taylor3D method), 34
- `__repr__` () (crystal.Crystal method), 13
- `__rmul__` () (PowerExpansion.Taylor3D method), 34
- `__rmul__` () (supercell.SuperCell method), 30
- `__rsub__` () (PowerExpansion.Taylor3D method), 34
- `__sane__` () (crystal.GroupOp method), 18
- `__sane__` () (crystalStars.PairState method), 22
- `__sane__` () (supercell.SuperCell method), 30
- `__setitem__` () (PowerExpansion.Taylor3D method), 34
- `__setitem__` () (supercell.SuperCell method), 30
- `__str__` () (OnsagerCalc.Interstitial method), 45
- `__str__` () (OnsagerCalc.VacancyMediated method), 48
- `__str__` () (PowerExpansion.Taylor3D method), 34
- `__str__` () (crystal.Crystal method), 13
- `__str__` () (crystal.GroupOp method), 18
- `__str__` () (crystalStars.PairState method), 22
- `__str__` () (crystalStars.StarSet method), 23
- `__str__` () (supercell.SuperCell method), 30
- `__sub__` () (PowerExpansion.Taylor3D method), 34
- `__sub__` () (crystal.GroupOp method), 18
- `__sub__` () (crystalStars.PairState method), 22
- `__weakref__` (GFcalc.GFCrystalcalc attribute), 43
- `__weakref__` (OnsagerCalc.Interstitial attribute), 45
- `__weakref__` (OnsagerCalc.VacancyMediated attribute), 48
- `__weakref__` (PowerExpansion.Taylor3D attribute), 34
- `__weakref__` (crystal.Crystal attribute), 13
- `__weakref__` (crystalStars.StarSet attribute), 23
- `__weakref__` (crystalStars.VectorStarSet attribute), 25
- `__weakref__` (supercell.SuperCell attribute), 30
- `__xor__` () (crystalStars.PairState method), 22

A

- `addbasis` () (crystal.Crystal method), 13
- `addhdf5` () (crystalStars.StarSet method), 23
- `addhdf5` () (crystalStars.VectorStarSet method), 25
- `addhdf5` () (GFcalc.GFCrystalcalc method), 43
- `addhdf5` () (OnsagerCalc.VacancyMediated method), 48
- `addhdf5` () (PowerExpansion.Taylor3D method), 35
- `addterms` () (PowerExpansion.Taylor3D method), 35

array2PSlist() (in module crystalStars), 27
 arrays2vTKdict() (in module OnsagerCalc), 52
 automator (module), 53

B

bareexpansions() (crystalStars.VectorStarSet method), 25
 BCC() (crystal.Crystal class method), 11
 biascorrection() (GFcalc.GFCrystalcalc method), 43
 biasexpansions() (crystalStars.VectorStarSet method), 26
 BlockInvertOmegaTaylor() (GFcalc.GFCrystalcalc method), 41
 BlockRotateOmegaTaylor() (GFcalc.GFCrystalcalc method), 41
 BreakdownGroups() (GFcalc.GFCrystalcalc method), 42

C

calcmetric() (crystal.Crystal method), 13
 cart2pos() (crystal.Crystal method), 13
 cart2unit() (crystal.Crystal method), 13
 center() (crystal.Crystal method), 13
 checkinternalsHDF5() (PowerExpansion.Taylor3D class method), 35
 chemindex() (crystal.Crystal method), 13
 clearcache() (OnsagerCalc.VacancyMediated method), 49
 coeffproductcoeff() (PowerExpansion.Taylor3D class method), 35
 collectcoeff() (PowerExpansion.Taylor3D class method), 35
 CombineTensorBasis() (in module crystal), 11
 CombineVectorBasis() (in module crystal), 11
 constructexpansion() (PowerExpansion.Taylor3D class method), 35
 copy() (crystalStars.StarSet method), 23
 copy() (PowerExpansion.Taylor3D method), 35
 copy() (supercell.SuperCell method), 30
 Crystal (class in crystal), 11
 crystal (module), 11
 crystalStars (module), 21

D

defectindices() (supercell.SuperCell method), 30
 definesolute() (supercell.SuperCell method), 31
 DiagGamma() (GFcalc.GFCrystalcalc method), 42
 diffgenerate() (crystalStars.StarSet method), 23
 Diffusivity() (GFcalc.GFCrystalcalc method), 42
 diffusivity() (OnsagerCalc.Interstitial method), 46
 doublelist2flatlistindex() (in module crystalStars), 27
 dumpinternalsHDF5() (PowerExpansion.Taylor3D method), 36

E

eigen() (crystal.GroupOp method), 18
 elastodiffusion() (OnsagerCalc.Interstitial method), 46
 equivalencemap() (supercell.SuperCell method), 31
 exp_dxq() (GFcalc.GFCrystalcalc method), 43

F

FCC() (crystal.Crystal class method), 11
 fillperiodic() (supercell.SuperCell method), 31
 flatlistindex2doublelist() (in module crystalStars), 27
 FourierTransformJumps() (GFcalc.GFCrystalcalc method), 42
 fromcrys() (crystalStars.PairState class method), 22
 fromcrys_latt() (crystalStars.PairState class method), 22
 fromdict() (crystal.Crystal class method), 13
 fullkptmesh() (crystal.Crystal method), 14
 FullVectorBasis() (crystal.Crystal method), 12

G

g() (crystalStars.PairState method), 22
 g_cart() (crystal.Crystal method), 14
 g_dirac() (crystal.Crystal static method), 14
 g_dirac_equivalent() (crystal.Crystal method), 14
 g_pos() (crystal.Crystal method), 14
 g_tensor() (crystal.Crystal static method), 14
 g_vect() (crystal.Crystal static method), 15
 genBZG() (crystal.Crystal method), 15
 generate() (crystalStars.StarSet method), 24
 generate() (crystalStars.VectorStarSet method), 26
 generate() (OnsagerCalc.VacancyMediated method), 49
 generateJumpGroupOps() (OnsagerCalc.Interstitial method), 46
 generateJumpSymmTensorBasis() (OnsagerCalc.Interstitial method), 46
 generatematrices() (OnsagerCalc.VacancyMediated method), 49
 generateouter() (crystalStars.VectorStarSet method), 26
 generateSiteGroupOps() (OnsagerCalc.Interstitial method), 46
 generateSiteSymmTensorBasis() (OnsagerCalc.Interstitial method), 46
 generatetags() (OnsagerCalc.Interstitial method), 46
 generatetags() (OnsagerCalc.VacancyMediated method), 49
 gengroup() (crystal.Crystal method), 15
 gengroup() (supercell.SuperCell method), 31
 genpoint() (crystal.Crystal method), 15
 genWyckoffsets() (crystal.Crystal method), 15
 GFcalc (module), 41

Gfcalculator() (OnsagerCalc.VacancyMediated method), 48
 GfCrystalcalc (class in Gfcalc), 41
 GFexpansion() (crystalStars.VectorStarSet method), 25
 GroupOp (class in crystal), 17
 GroupOp_constructor() (crystal.GroupOp static method), 17
 GroupOp_representer() (crystal.GroupOp static method), 17

H

HCP() (crystal.Crystal class method), 12

I

ident() (crystal.GroupOp class method), 18
 ildot() (PowerExpansion.Taylor3D method), 36
 inBZ() (crystal.Crystal method), 15
 incell() (crystal.GroupOp method), 18
 incell() (in module crystal), 19
 index() (supercell.SuperCell method), 31
 inhalf() (crystal.GroupOp method), 18
 inhalf() (in module crystal), 19
 interactlist() (OnsagerCalc.VacancyMediated method), 49
 Interstitial (class in OnsagerCalc), 45
 inv() (crystal.GroupOp method), 18
 inv() (PowerExpansion.Taylor3D method), 36
 inversecoeff() (PowerExpansion.Taylor3D class method), 36
 irdot() (PowerExpansion.Taylor3D method), 36
 irotate() (PowerExpansion.Taylor3D method), 36
 iszero() (crystalStars.PairState method), 22

J

jumpDipoles() (OnsagerCalc.Interstitial method), 47
 jumpnetwork() (crystal.Crystal method), 15
 jumpnetwork2lattice() (crystal.Crystal method), 16
 jumpnetwork_omega1() (crystalStars.StarSet method), 24
 jumpnetwork_omega2() (crystalStars.StarSet method), 24
 jumpnetworkYAML() (OnsagerCalc.Interstitial static method), 47

K

KrogerVink() (supercell.SuperCell method), 29

L

ldot() (PowerExpansion.Taylor3D method), 36
 Lij() (OnsagerCalc.VacancyMediated method), 48
 loadhdf5() (crystalStars.StarSet class method), 24

loadhdf5() (crystalStars.VectorStarSet class method), 26
 loadhdf5() (Gfcalc.GfCrystalcalc class method), 43
 loadhdf5() (OnsagerCalc.VacancyMediated class method), 49
 loadhdf5() (PowerExpansion.Taylor3D class method), 36

M

makedirectmult() (PowerExpansion.Taylor3D class method), 37
 makeindexPowerYlm() (PowerExpansion.Taylor3D static method), 37
 makeLIMBpreene() (OnsagerCalc.VacancyMediated method), 49
 makeLprojections() (PowerExpansion.Taylor3D class method), 36
 makepowercoeff() (PowerExpansion.Taylor3D class method), 37
 makepowYlm() (PowerExpansion.Taylor3D class method), 37
 makesites() (supercell.SuperCell method), 31
 makesupercells() (OnsagerCalc.Interstitial method), 47
 makesupercells() (OnsagerCalc.VacancyMediated method), 50
 maketracerpreene() (OnsagerCalc.VacancyMediated method), 50
 maketrans() (supercell.SuperCell static method), 32
 makeYlmpow() (PowerExpansion.Taylor3D class method), 37
 map2string() (in module automator), 53
 maptranslation() (in module crystal), 19
 minlattice() (crystal.Crystal method), 16

N

ndarray_representer() (in module crystal), 20
 negcoeff() (PowerExpansion.Taylor3D class method), 37
 networkcount() (Gfcalc.GfCrystalcalc method), 44
 nl() (PowerExpansion.Taylor3D method), 37
 nnlist() (crystal.Crystal method), 16

O

occposlist() (supercell.SuperCell method), 32
 omegalist() (OnsagerCalc.VacancyMediated method), 51
 OnsagerCalc (module), 45
 optype() (crystal.GroupOp static method), 18
 originstateVectorBasisfolddown() (crystalStars.VectorStarSet method), 26

P

PairState (class in crystalStars), 21

PairState_constructor() (crystalStars.PairState static method), 22
 PairState_representer() (crystalStars.PairState static method), 22
 pos2cart() (crystal.Crystal method), 16
 POSCAR() (supercell.Supercell method), 29
 PowerExpansion (module), 33
 powexp() (PowerExpansion.Taylor3D class method), 37
 preene2betafree() (OnsagerCalc.VacancyMediated static method), 51
 ProjectTensorBasis() (in module crystal), 18
 PSlist2array() (in module crystalStars), 21

R

rateexpansions() (crystalStars.VectorStarSet method), 27
 ratelist() (OnsagerCalc.Interstitial method), 47
 rdot() (PowerExpansion.Taylor3D method), 38
 reduce() (crystal.Crystal method), 16
 reduce() (PowerExpansion.Taylor3D method), 38
 reducecoeff() (PowerExpansion.Taylor3D class method), 38
 reducekptmesh() (crystal.Crystal method), 16
 remapbasis() (crystal.Crystal method), 17
 reorder() (supercell.Supercell method), 32
 rotate() (PowerExpansion.Taylor3D method), 38
 rotatecoeff() (PowerExpansion.Taylor3D class method), 38
 rotatedirections() (PowerExpansion.Taylor3D class method), 38

S

scalarproductcoeff() (PowerExpansion.Taylor3D class method), 38
 separate() (PowerExpansion.Taylor3D method), 38
 separatecoeff() (PowerExpansion.Taylor3D class method), 39
 setocc() (supercell.Supercell method), 32
 SetRates() (GFcalc.GFCrystalcalc method), 42
 simpleYAML() (crystal.Crystal method), 17
 siteDipoles() (OnsagerCalc.Interstitial method), 47
 sitelist() (crystal.Crystal method), 17
 sitelistYAML() (OnsagerCalc.Interstitial static method), 47
 siteprob() (OnsagerCalc.Interstitial method), 47
 starindex() (crystalStars.StarSet method), 24
 StarSet (class in crystalStars), 23
 stateindex() (crystalStars.StarSet method), 24
 stoichiometry() (supercell.Supercell method), 32
 strain() (crystal.Crystal method), 17
 sumcoeff() (PowerExpansion.Taylor3D class method), 39
 Supercell (class in supercell), 29

supercell (module), 29
 supercelltar() (in module automator), 53
 symmatch() (crystalStars.StarSet method), 24
 symmequivjumplist() (crystalStars.StarSet method), 25
 symmratelists() (OnsagerCalc.Interstitial method), 47
 SymmRates() (GFcalc.GFCrystalcalc method), 42
 SymmTensorBasis() (crystal.Crystal method), 12
 SymmTensorBasis() (in module crystal), 19

T

tags2preene() (OnsagerCalc.VacancyMediated method), 51
 Taylor3D (class in PowerExpansion), 33
 TaylorExpandJumps() (GFcalc.GFCrystalcalc method), 42
 tensorproductcoeff() (PowerExpansion.Taylor3D class method), 39
 truncate() (PowerExpansion.Taylor3D method), 39
 truncatecoeff() (PowerExpansion.Taylor3D class method), 39

U

unit2cart() (crystal.Crystal method), 17

V

VacancyMediated (class in OnsagerCalc), 47
 vacancyThermoKinetics (class in OnsagerCalc), 52
 vacancyThermoKinetics_constructor() (OnsagerCalc.vacancyThermoKinetics static method), 52
 vacancyThermoKinetics_representer() (OnsagerCalc.vacancyThermoKinetics static method), 52
 vectlist() (crystal.Crystal static method), 17
 VectorBasis() (crystal.Crystal method), 12
 VectorBasis() (in module crystal), 19
 VectorStarSet (class in crystalStars), 25
 Voigtstrain() (in module crystal), 19
 vTKdict2arrays() (in module OnsagerCalc), 52

W

Wyckoffpos() (crystal.Crystal method), 12

Z

zero() (crystalStars.PairState class method), 22
 zeroclean() (in module crystalStars), 28
 zeros() (PowerExpansion.Taylor3D class method), 39