In[2]:=

```
(* start rchart function definition *)
rchart[rdata_, n_] := Module[{ns, rbar, dlist, d3, d4, lcl, ucl},
  ns = Length[rdata];
  rbar = N[Mean[rdata]];
  dlist =
   {{0, 0, 0, 0, 0, 0.076, 0.136}, {3.267, 2.575, 2.282, 2.114, 2.004, 1.924, 1.864}};
  d3 = dlist[[1, n - 1]];
  d4 = dlist[[2, n - 1]];
  lcl = d3 rbar;
  ucl = d4 rbar;
  Show[
   ListLinePlot[ConstantArray[lcl, ns],
    PlotRange → {{-0.5, ns + 20}, {lcl - 0.5, ucl + 0.5}},
    PlotLabel → Style["R Chart", 16], Frame → True,
    FrameLabel → {Style["Sample Number", 12], Style["Sample Mean", 12]},
    Axes → False],
   ListLinePlot[rdata, PlotMarkers → {Automatic, 10}],
   ListLinePlot[ConstantArray[rbar, ns]],
   ListLinePlot[ConstantArray[ucl, ns]],
   Graphics[{
     Text[StringJoin["LCL = ", ToString[lcl]], {ns + 10, lcl}],
     Text[StringJoin["R̄ = ", ToString[rbar]], {ns + 10, rbar}],
     Text[StringJoin["UCL = ", ToString[ucl]], {ns + 10, ucl}]
    }
   ], ImageSize → {600, 500}
  ]
 ]
(* end rchart function definition *)
```

In[3]:=
```
(* start xbar chart function definition *)
(* This chart only works with range data. It can be modified to use *)
(* sample standard deviation data, see p. 781,782 *)
xbarchart[xbardata_, rdata_, n_] := Module[{ns, xbarbar, rbar, a2list, a2, lcl, ucl},
  ns = Length[xbardata];
  xbarbar = Mean[xbardata];
  rbar = Mean[rdata];
  a2list = {1.88, 1.023, 0.729, 0.577, 0.483, 0.419, 0.373};
  a2 = a2list[[n - 1]];
  lcl = xbarbar - a2 rbar;
  ucl = xbarbar + a2 rbar;
  Show[
   ListLinePlot[ConstantArray[lcl, ns],
    PlotRange → {{-0.5, ns + 20}, {lcl - 0.15 (ucl - lcl), ucl + 0.15 (ucl - lcl)}},
    PlotLabel → Style["X̅ Chart", 16], Frame → True,
    FrameLabel → {Style["Sample Number", 12], Style["Sample Mean", 12]},
    Axes → False],
   ListLinePlot[xbardata, PlotMarkers → {Automatic, 10}],
   ListLinePlot[ConstantArray[xbarbar, ns]],
   ListLinePlot[ConstantArray[ucl, ns]],
   Graphics[{
     Text[StringJoin["LCL = ", ToString[lcl]], {ns + 10, lcl}],
     Text[StringJoin["X̅ = ", ToString[xbarbar]], {ns + 10, xbarbar}],
     Text[StringJoin["UCL = ", ToString[ucl]], {ns + 10, ucl}]
    }
   ], ImageSize → {600, 500}
  ]
 ]
(* end xbar chart function definition *)
```

In[24]:=
```
(* start werchart function definition *)
werchart[xbardata_, rdata_, n_] :=
 Module[{ns, xbarbar, rbar, a2list, a2, lcl1, ucl1, lcl2, ucl2, lcl3, ucl3},
  ns = Length[xbardata];
  xbarbar = Mean[xbardata];
  rbar = Mean[rdata];
  a2list = {1.88, 1.023, 0.729, 0.577, 0.483, 0.419, 0.373};
  a2 = a2list[[n - 1]];
  lcl3 = xbarbar - a2 rbar;
  lcl2 = xbarbar - a2 rbar (2 / 3);
  lcl1 = xbarbar - a2 rbar (1 / 3);
  ucl3 = xbarbar + a2 rbar;
  ucl2 = xbarbar + a2 rbar (2 / 3);
  ucl1 = xbarbar + a2 rbar (1 / 3);
```

```
Show[
  ListLinePlot[ConstantArray[lcl3, ns],
   PlotRange → {{-0.5, ns + 24}, {lcl3 - 0.15 (ucl3 - lcl3), ucl3 + 0.15 (ucl3 - lcl3)}},
   PlotLabel → Style["WER X̄ Chart", 16], Frame → True,
   FrameLabel → {Style["Sample Number", 12], Style["Sample Mean", 12]},
   Axes → False],
  ListLinePlot[ConstantArray[lcl2, ns], PlotStyle → Dashed],
  ListLinePlot[ConstantArray[lcl1, ns], PlotStyle → Dashed],
  ListLinePlot[xbardata, PlotMarkers → {Automatic, 10}],
  ListLinePlot[ConstantArray[xbarbar, ns]],
  ListLinePlot[ConstantArray[ucl1, ns], PlotStyle → Dashed],
  ListLinePlot[ConstantArray[ucl2, ns], PlotStyle → Dashed],
  ListLinePlot[ConstantArray[ucl3, ns]],
  Graphics[{
    Text[StringJoin["X̿-3σ = ", ToString[lcl3]], {ns + 12, lcl3}],
    Text[StringJoin["X̿-2σ = ", ToString[lcl2]], {ns + 12, lcl2}],
    Text[StringJoin["X̿-σ = ", ToString[lcl1]], {ns + 12, lcl1}],
    Text[StringJoin["X̿ = ", ToString[xbarbar]], {ns + 12, xbarbar}],
    Text[StringJoin["X̿+σ = ", ToString[ucl1]], {ns + 12, ucl1}],
    Text[StringJoin["X̿+2σ = ", ToString[ucl2]], {ns + 12, ucl2}],
    Text[StringJoin["X̿+3σ = ", ToString[ucl3]], {ns + 12, ucl3}]
   }
  ], ImageSize → {600, 500}
 ]
]
(* end werchart function definition *)
```

In[218]:=
```
(* start schart function definition *)
schart[sdata_, n_] := Module[{ns, sbar, blist, b3, b4, lcl, ucl},
  ns = Length[sdata];
  sbar = N[Mean[sdata]];
  blist =
   {{0, 0, 0, 0, 0.03, 0.118, 0.185}, {3.267, 2.568, 2.266, 2.089, 1.97, 1.882, 1.815}};
  b3 = blist[[1, n - 1]];
  b4 = blist[[2, n - 1]];
  lcl = b3 sbar;
  ucl = b4 sbar;
  Show[
   ListLinePlot[ConstantArray[lcl, ns],
    PlotRange → {{-0.5, ns + 8}, {lcl - 0.05 (ucl - lcl), ucl + 0.05 (ucl - lcl)}},
    PlotLabel → Style["s Chart", 16], Frame → True,
    FrameLabel → {Style["Sample Number", 12], Style["Sample SD", 12]},
    Axes → False],
   ListLinePlot[sdata, PlotMarkers → {Automatic, 10}],
   ListLinePlot[ConstantArray[sbar, ns]],
   ListLinePlot[ConstantArray[ucl, ns]],
   Graphics[{
     Text[StringJoin["LCL = ", ToString[lcl]], {ns + 3, lcl}],
     Text[StringJoin["s̅ = ", ToString[sbar]], {ns + 3, sbar}],
     Text[StringJoin["UCL = ", ToString[ucl]], {ns + 3, ucl}]
    }
   ], ImageSize → {600, 500}
  ]
 ]
(* end schart function definition *)
```

Dallin Cawley - MAT330-01 - 3/24/2020

10.1 - 10.6 Hours Worked: 2 Score:

10.1 (10.1.2) True or false:

(a) Control charts are used to determine whether special causes are operating.

True.

(b) If no special causes are operating, then most of the output produced will meet specifications.

True

(c) Variability due to common causes does not increase or decrease much over short periods of time.

False

(d) Variability within the items sampled in a rational subgroup is due to special causes.

True

(e) If a process is in a state of statistical control, there will be almost no variation in the output.

False

10.2 (10.1.4) Fill in the blank:

Once a process has been brought into a state of statistical control,

(i) It must still be monitored continually.
(ii) Monitoring can be stopped for a while, since it is unlikely that the process will go out of control again right away.
(iii) The process need not be monitored again, unless it is redesigned.

10.3 (10.1.6) Fill in the blank:

When sampling units for rational subgroups,

(i) it is more important to choose large samples than to sample frequently, since large samples provide more precise information about the process.
(ii) it is more important to sample frequently than to choose large samples, so that special causes can be detected more quickly.

10.4 (10.2.3) The thickness, in mm, of metal washers is measured using samples of size 5. The following table presents the means, ranges, and standard deviations for 20 consecutive

samples.

The means are X = 2.505, R = 0.1395, and s = 0.057

(a) Calculate the 3σ control limits for the R chart.

In[53]:=
```
data4 = {{1, 2.49, 0.12, 0.07}, {2, 2.45, 0.17, 0.06},
   {3, 2.51, 0.13, 0.06}, {4, 2.53, 0.25, 0.09}, {5, 2.49, 0.11, 0.06},
   {6, 2.44, 0.11, 0.06}, {7, 2.44, 0.12, 0.05}, {8, 2.42, 0.18, 0.06},
   {9, 2.42, 0.08, 0.05}, {10, 2.47, 0.06, 0.02}, {11, 2.54, 0.19, 0.07},
   {12, 2.45, 0.09, 0.04}, {13, 2.54, 0.21, 0.07}, {14, 2.55, 0.10, 0.05},
   {15, 2.50, 0.25, 0.08}, {16, 2.53, 0.11, 0.04}, {17, 2.58, 0.16, 0.07},
   {18, 2.59, 0.09, 0.03}, {19, 2.60, 0.12, 0.05}, {20, 2.56, 0.14, 0.06}};
d3 = 0;
d4 = 2.114;
rBar = N[Mean[Flatten[data4[[All, 3]]]]];

LCL = d3 * rBar;
UCL = d4 * rBar;

Print["LCL: " , LCL , "\n" , "UCL: " , UCL , "\n"]
```

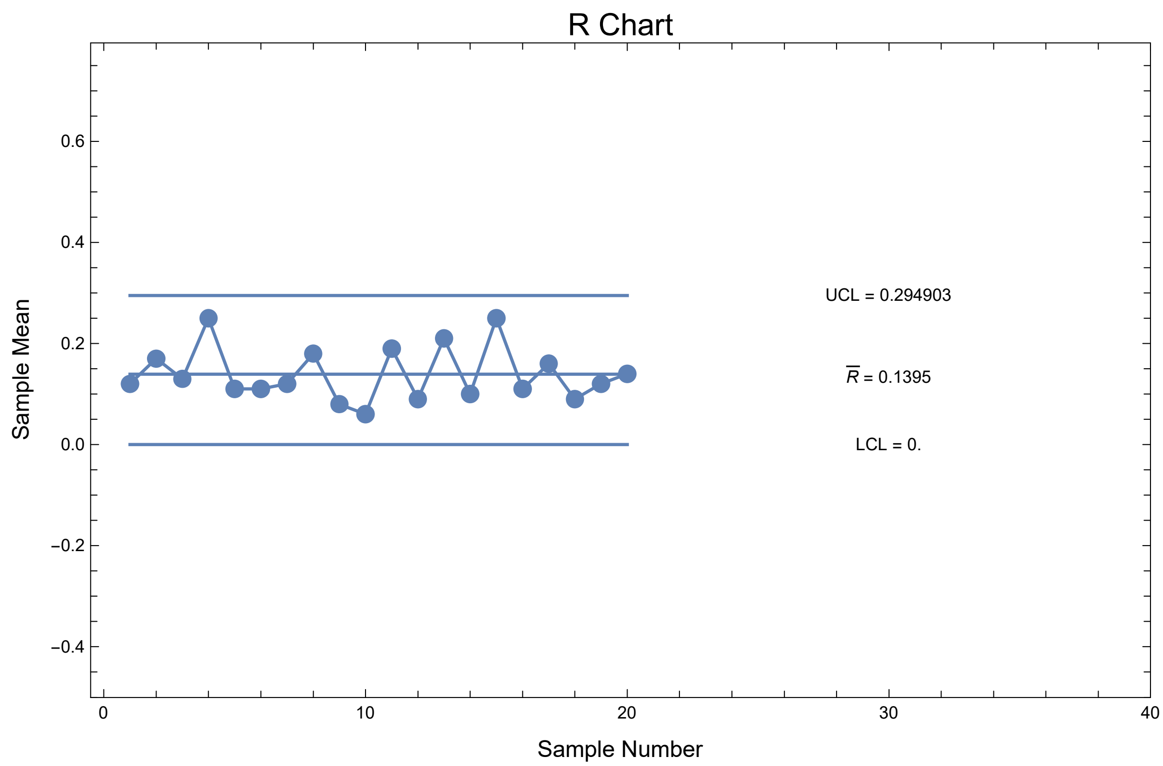
```
LCL: 0.
UCL: 0.294903
```

(b) Construct the R-chart. Is the variance under control? If not, delete the samples that are out of control and recompute X and R.

It is in control.

In[11]:=
```
n = 5;
rchart[Flatten[data4], n]
```

Out[12]=



(c) Based on the sample range R, calculate the 3σ control limits for the X chart.

In[67]:=
```
xBar4 = data4[[All, 2]];
xBarBar = Mean[Flatten[xBar4]];
A2 = 0.577;
n = 5;

LCL = xBarBar - (A2 * rBar);
UCL = xBarBar + (A2 * rBar);

Print["LCL: ", LCL, "\n", "UCL: ", UCL, "\n"]
```

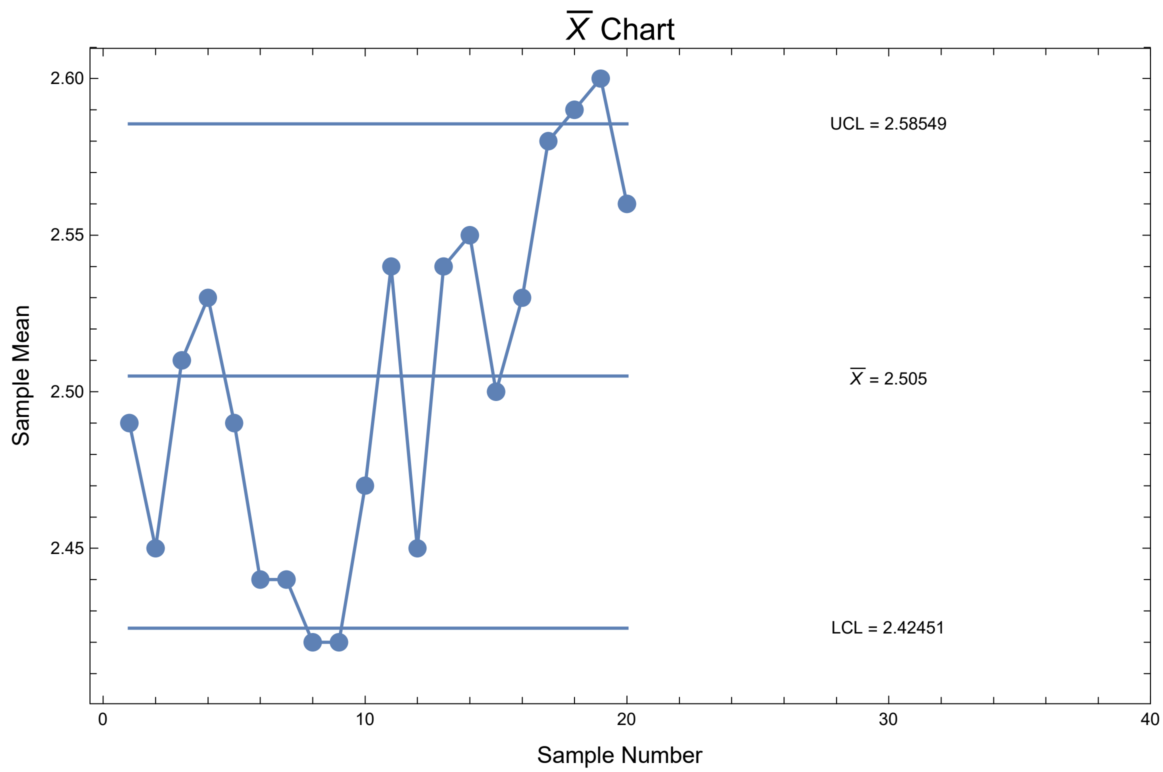
```
LCL: 2.42451
UCL: 2.58549
```

(d) Construct the X- chart. Is the process mean X in control? If not, when is it first detected to be out of control?

It is not in control. The first time the data exits the range is at sample 8.

In[74]:=
```
n = 5;
xbarchart[Flatten[xBar4], Flatten[data4[[All, 3]]], n]
```

Out[75]=

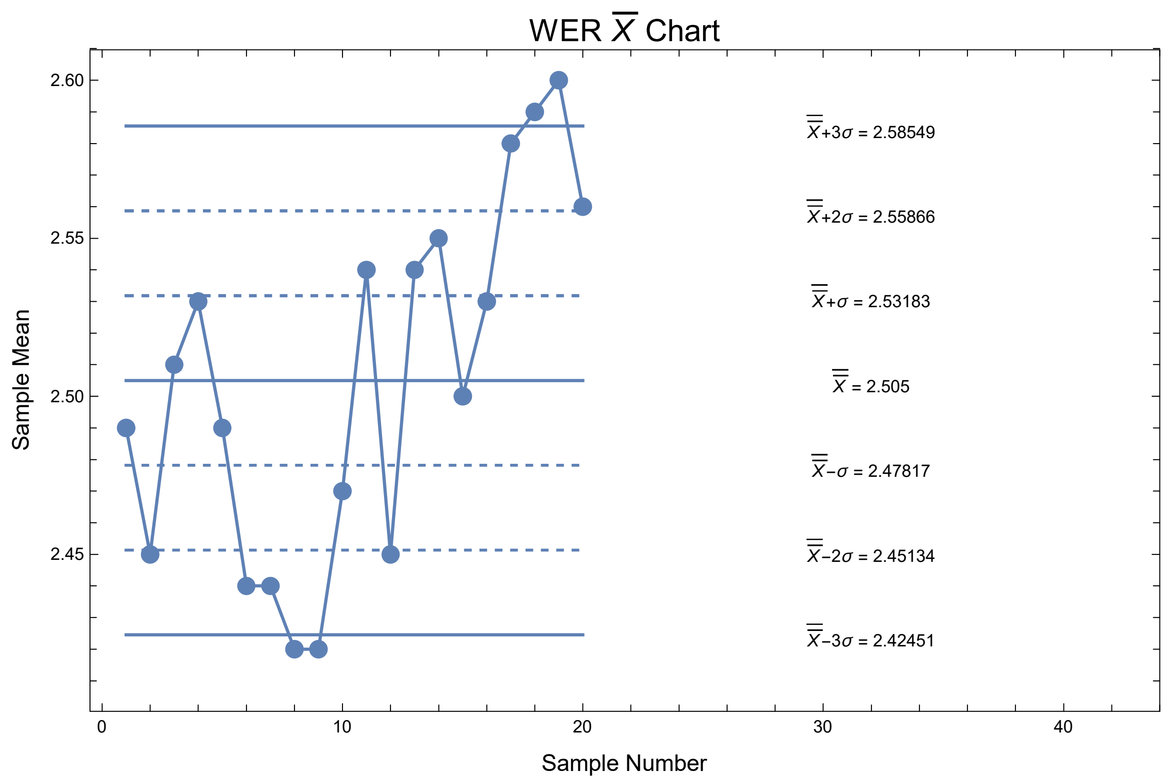


(e) Construct the Western Electric X- chart. Based on the Western Electric rules, is the process mean X in control? If not, when is it first detected to be out of control?

Out of Control. At point 5 is where the 2sigma rule is broken.

In[76]:=
```
werchart[Flatten[xBar4], Flatten[data4[[All, 3]]], 5]
```

Out[76]=



10.5 (10.2.6) A process has a mean μ = 8 and standard deviation σ = 2. The process is monitored by taking samples of size 4 at regular intervals. The process is declared to be out of control if a point plots outside the 3σ control limits of an X chart.

(a) If the process mean shifts to 9, what is the average number of samples that will be drawn before the shift is detected on an X chart?

In[101]:=
```
p = (1 - CDF[NormalDistribution[9.0, 2], 15]) + CDF[NormalDistribution[9, 2], 6];
arl = 1/p;
Print[Style["Probability of being out of control (p) = ", 18, Blue], Style[p, 18, Blue]]
Print[Style["ARL = 1/p = ", 18, Blue], Style[arl, 18, Blue]]
```

Probability of being out of control (p) = 0.0681571

$$\text{ARL} = \frac{1}{p} = 14.672$$

(b) An upward shift of what value will be detected with an average run length (ARL) of 8?

In[107]:=
```
nm = FindRoot[1.0 - CDF[NormalDistribution[m, 2], 15] +
     CDF[NormalDistribution[m, 6], -3.0] == (1 / 8), {m, 1}];

Print[Style["new mean = ", 18, Blue], Style[nm, 18, Blue]]
```

new mean = {m → 3.9021}

(c) If the sample size remains at 4, to what value must σ be reduced to produce an ARL of 8 when the process mean shifts to 9?

Since the ARL is already greater than 8, there would be no reduction.

In[208]:=
```
sigma = 2;
p =
  (1 - CDF[NormalDistribution[9.0, sigma], 15]) + CDF[NormalDistribution[9, sigma], 6];
arl = 1/p;
Print[Style["Probability of being out of control (p) = ", 18, Blue], Style[p, 18, Blue]]
Print[Style["ARL = 1/p = ", 18, Blue], Style[arl, 18, Blue]]
```

Probability of being out of control (p) = 0.0681571

$$\text{ARL} = \frac{1}{p} = 14.672$$

d) If the standard deviation remains 2, what sample size must be used to produce an ARL no greater than 8 when the process mean shifts to 9?

The ARL is already greater than 8.

In[213]:=
```
sigma = 2;
p =
  (1 - CDF[NormalDistribution[9.0, sigma], 15]) + CDF[NormalDistribution[9, sigma], 6];
arl = 1/p;
Print[Style["Probability of being out of control (p) = ", 18, Blue], Style[p, 18, Blue]]
Print[Style["ARL = 1/p = ", 18, Blue], Style[arl, 18, Blue]]
```

Probability of being out of control (p) = 0.0681571

$\text{ARL} = \frac{1}{p} = 14.672$

10.6 (10.2.9) Repeat Exercise 8, using the S chart in place of the R chart.

In[241]:=
```
data6 = {{1, 9.99, 0.28, 0.09}, {2, 10.02, 0.43, 0.13},
   {3, 10.10, 0.16, 0.05}, {4, 9.90, 0.26, 0.09}, {5, 9.92, 0.22, 0.07},
   {6, 10.05, 0.40, 0.15}, {7, 9.97, 0.08, 0.03}, {8, 9.93, 0.48, 0.15},
   {9, 10.01, 0.25, 0.09}, {10, 9.87, 0.29, 0.10}, {11, 9.90, 0.39, 0.14},
   {12, 9.97, 0.27, 0.08}, {13, 10.02, 0.20, 0.07}, {14, 9.99, 0.37, 0.13},
   {15, 9.99, 0.20, 0.06}, {16, 10.04, 0.26, 0.09}, {17, 10.07, 0.23, 0.07},
   {18, 10.04, 0.35, 0.12}, {19, 9.95, 0.25, 0.09}, {20, 9.98, 0.15, 0.06},
   {21, 9.98, 0.30, 0.10}, {22, 10.02, 0.14, 0.06}, {23, 9.94, 0.24, 0.07},
   {24, 10.04, 0.13, 0.04}, {25, 10.04, 0.24, 0.07}};
```

The means are X = 9.9892, R = 0.2628, and s = 0.088.

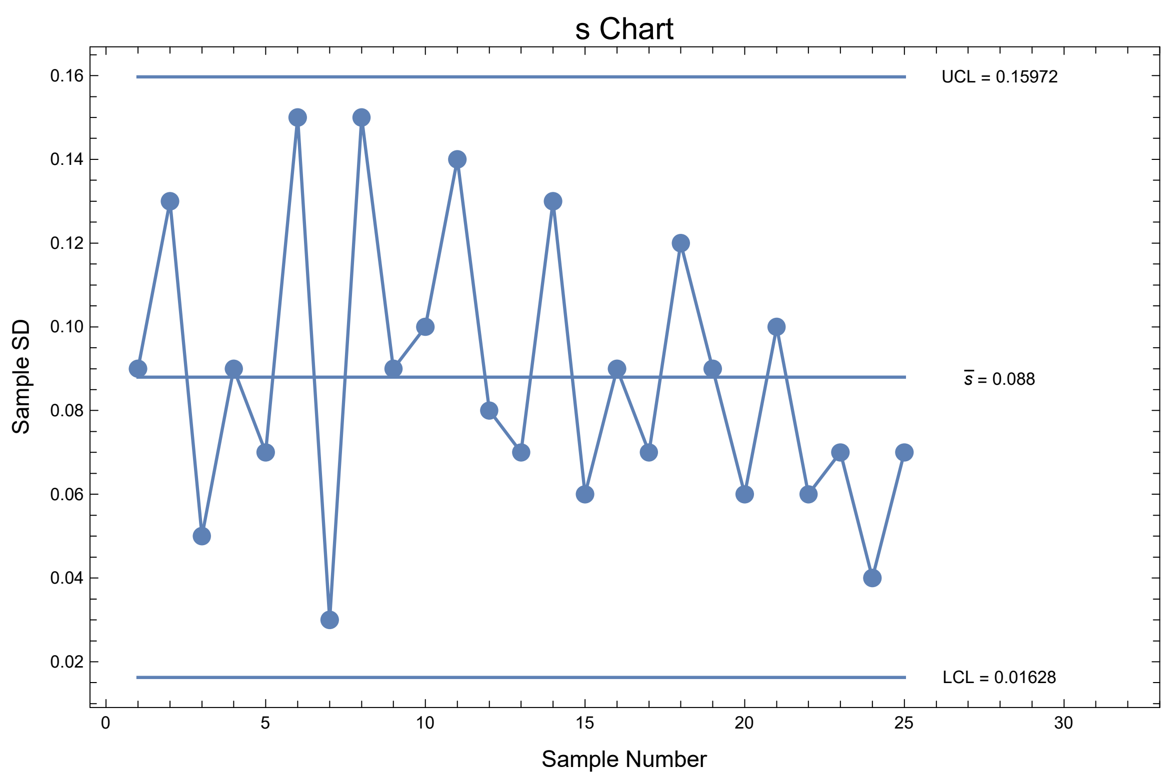
(a) Calculate the 3σ control limits for the s chart. Is the variance under control? If not, delete the samples that are out of control and recompute X and R.

It is in control.

In[250]:=
```
B4 = 1.815;
B3 = 0.185;
sBar = 0.088;

UCL = B4 * sBar;
LCL = B3 * sBar;

Print["\nUCL: ", UCL]
Print["LCL: ", LCL, "\n"]

sData = data6[[All, 4]];
schart[sData, 8]
```

```
UCL: 0.15972
LCL: 0.01628
```

Out[258]=

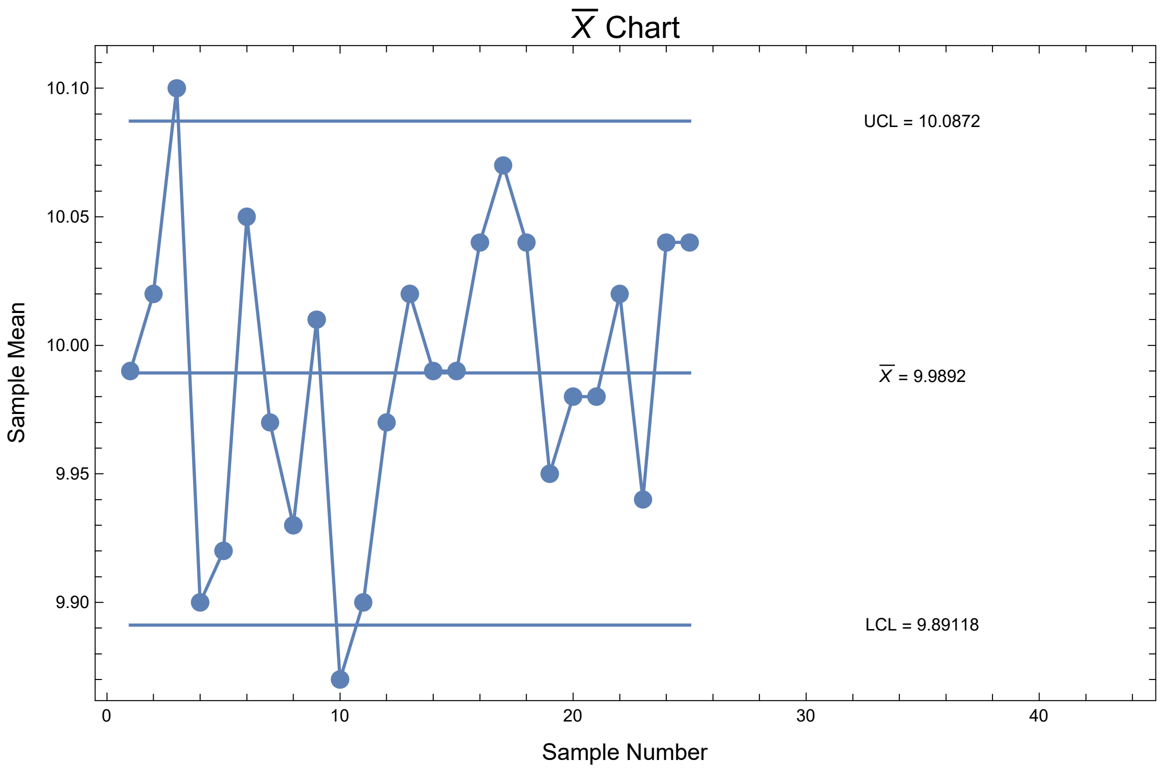


(b) Based on the sample range R, calculate the 3σ control limits for the X chart. Based on the 3σ limits, is the process mean X in control? If not, when is it first detected to be out of control?

It is out of control starting at point 3.

In[262]:=
```
xBar6 = data6[[All, 2]];
rangeData = data6[[All, 3]];
xbarchart[Flatten[xBar6], Flatten[rangeData], 8]
```

Out[264]=



(c) Construct the Western Electric X-chart. Based on the Western Electric rules, is the process mean X in control? If not, when is it first detected to be out of control?

Out of control with the first detected at point 3.

In[265]:=
```
werchart[Flatten[xBar6], Flatten[rangeData], 8]
```

Out[265]=