CS 5800 Distributed OS - Spring 2017
Homework 5

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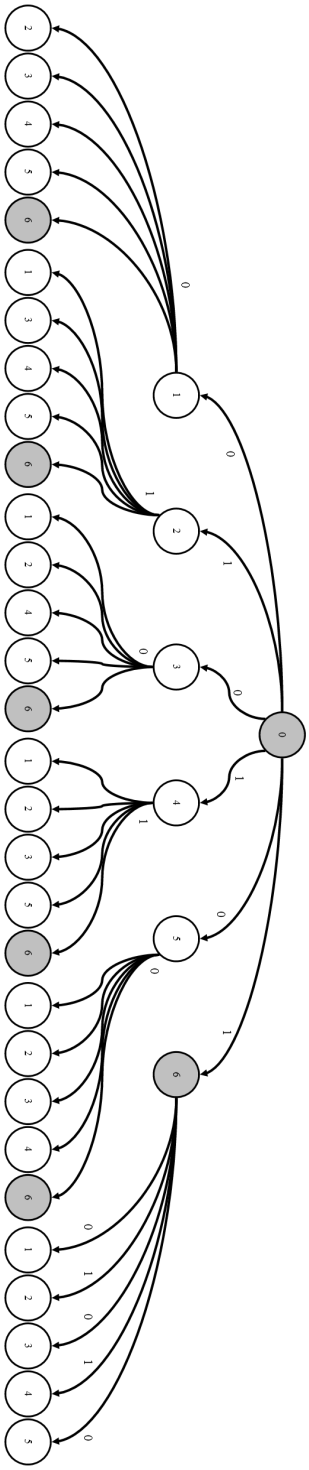
April 27, 2017

Problem 1 Assuming that the communication network is synchronous, does the proposed algorithm work? If not, provide an example where there are at least 2 traitors.

Let the default be 1 when a tie is reached and let the commander and lieutenant 6 be traitors. After tracing the tree, shown in Figure 1, it can be seen that lieutenant 1 would receive: (0,1,0,1,0,0) giving a majority value of 0, lieutenant 2 would receive: (1,0,0,1,0,1) giving a tie so going with the default of 1, lieutenant 3 would receive: (0,0,1,1,0,0) giving a majority value of 0, lieutenant 4 would receive: (1,0,1,0,0,1) giving a tie so going with the default of 1, and lieutenant 4 would receive: (0,0,1,0,1,0) giving a majority value of 0. Since the lieutenants do not all reach the same decision the proposed algorithm does not work.

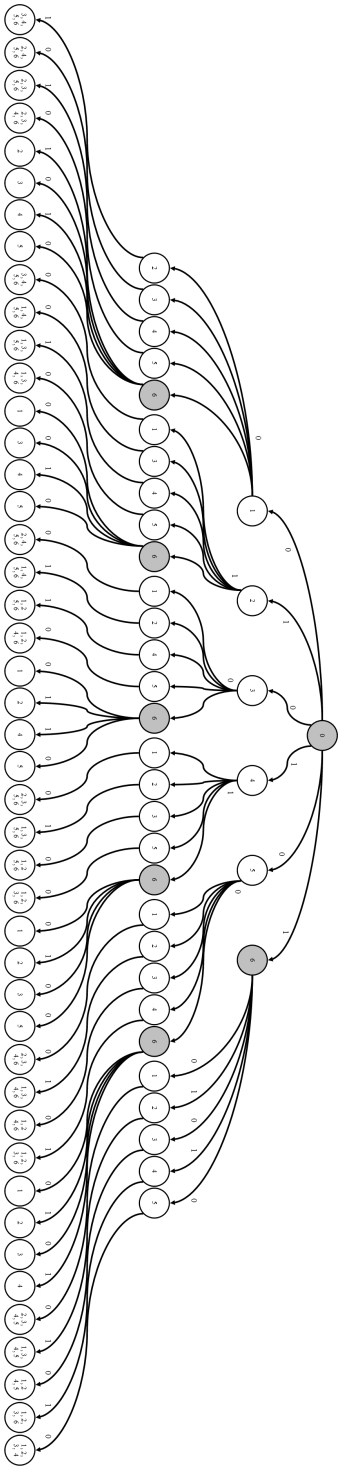
Figure 1: Problem 1



Problem 2 Using the same example, show, step by step, how Lamports algorithm allows the generals to reach an agreement.

Let the default be 1 when a tie is reached and let the commander and lieutenant 6 be traitors. After tracing the tree, shown in Figure 2, it can be seen that lieutenant 1 would receive: (1,0,1,0,0) giving a majority value of 0, lieutenant 2 would receive: (0,0,1,0,1) giving a majority value of 0, lieutenant 3 would receive: (0,1,1,0,0) giving a majority value of 0, lieutenant 4 would receive: (0,1,0,0,1) giving a majority value of 0, and lieutenant 4 would receive: (0,1,0,1,0) giving a majority value of 0. Following the algorithm all of the loyal generals were able to successfully reach an agreement of 0.

Figure 2: Problem 2



Problem 3 Suppose there are k traitors among n generals, calculate the number of messages sent by a lieutenant under the Lamports algorithm.

If there are k traitors and n generals, then the message complexity will be bounded by $O(n^{k+1})$ with an exact count of messages sent being:

$$\sum_{i=1}^{k+1} \frac{(n-1)!}{(n-1-i)!}$$

This is because, for the first row, or stage, $(n-1)$ messages are sent, as seen in question 2. In the second row, $(n-1)*(n-2)$ messages are sent. This pattern continues until the $k^{th}+1$ row, which is: $(n-1)*(n-2)...(n-(k+1))$. To get total message complexity, we must sum all of these stages, which results in the aforementioned equation.