COMP SCI 5401 FS2017 Assignment 1d

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MOEA

For this experiment, the second objective of the multi-objective EA is to minimize the width parameter of the stock. To determine fitness, a Pareto Front was created. Fitness was thus determined as the number of other boards in the population that a board dominates. To dominate, a board has to be greater than or equal to in one attribute and strictly greater than in the other attribute.

Mutation

Move and flip were compared as the mutation operations. For all three inputs, move proved better for minimizing stock length. This result can be seen in Figures 1, 9, and 17. As for stock width, neither operator proved more efficient, as can be seen in Figures 2, 10, and 18. This might be because in length, there is more material to work with, so, generally speaking, it's easier to minimize a bigger number than a smaller number when crunching the shapes close together.

K-Tournament W/ Replacement (k-tour) vs Fitness Proportional Selection (FPS)

When k-tour with FPS, neither parent selection operator proved better for minimizing stock length. This can be seen in Figures 3, 11, and 19. As for stock width, Inputs 1 and 2 yielded no significant difference between the two, as can be seen in Figures 4 and 12. For Input 3, k-tour was significantly better as can be seen in Figure 20. This may have been the case because there was less restrictive evolutionary pressure on the tournament than on the FPS, thus yielding more diverse solutions for the stock width, which should be noted, has a smaller space of possible values it can take.

K-Tournament W/ Replacement vs Uniform Random

For stock length, k-tour proved to be significantly better than uniform random for all inputs. This can be seen in Figures 5, 13, and 21. This result was also true for Input 3 of stock width, shown in Figure 22. For Inputs 1 and 2, there was no significant difference. For these two cases, there may not have been enough selective pressure given the tournament size of 5. K-tour may prove to be significantly better with a k-tour size of 2 for example. To decrease randomness of k-tour, a lower tournament size should be explored. Figures 6 and 14 show these cases.

Fitness Proportional Selection vs Uniform Random

FPS proved to be significantly better for all instances of stock length as shown in Figures 7, 15, and 23. For stock width, all inputs showed that neither parent selection method was superior, as shown in Figures 8, 16, and 24. A lower tournament size should be explored to see if stock width is impacted in a positive light, while also impacting stock length positively.

Conclusions

The above analysis along with the graphs shown in the Figures section shows that Inputs 1 and 2 are largely not affected by having a multi-objective EA. Stock length is largely minimized, while stock width is barely changed from previous assignments. As for Input 3, having a MOEA affected both objectives. The stock length is larger, but the stock width is smaller than in previous assignments. This may show that with larger data sets, such as the one provided for Input 3, MOEAs are more effective than with smaller inputs. It is possible that having a larger width aided in this as well. More tests are needed to come to a conclusion however.

Note

All graphs, pictures, and configurations are shown in the Figures section of this report.

Figure 1: Move vs Flip Tests for Input 1 Length

Input 1 - Length - Move vs Flip Mutation

F-Test Two-Sample for Variances Move Flip

	Variable 1	Variable 2
Mean	-26.0015	-33.7084
Variance	0.768971	13.13069
Observations	199	199
df	198	198
F	0.058563	
P(F<=f) one-tail	0	
F Critical one-tail	0.791084	

M(v1) > M(v2) and F < F-Critical => Equal Variance

t-Test: Two-Sample Assuming Equal Variances

	Variable 1	Variable 2
Mean	-26.0015	-33.7084
Variance	0.768971	13.13069
Observations	199	199
Pooled Variance	6.949832	
Hypothesized Mean Difference	0	
df	396	
t Stat	29.16099	
P(T<=t) one-tail	6.2E-101	
t Critical one-tail	1.648711	
P(T<=t) two-tail	1.2E-100	
t Critical two-tail	1.965973	

t > t Critical => Move is better

Figure 2: Move vs Flip Tests for Input 1 Width

Input 1 - Width - Move vs Flip Mutation
F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	-18.1534	-13.2685
Variance	0.043634	0.417668
Observations	199	199
df	198	198
F	0.104471	
P(F<=f) one-tail	0	
F Critical one-tail	0.791084	

Mean(v1) < Mean(v2) and F < F-Critical => Unequal Variance

t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	*Variable 2*
Mean	-18.1534	-13.2685
Variance	0.043634	0.417668
Observations	199	199
Hypothesized Mean Difference	0	
df	239	
t Stat	-101.459	
P(T<=t) one-tail	8.7E-199	
t Critical one-tail	1.651254	
P(T<=t) two-tail	1.7E-198	
t Critical two-tail	1.969939	

t < t Critical => Same

Figure 3: K-Tournament w/ Replacement vs Fitness Proportional Selection Tests for Input 1 Length

Input 1 - Length - k-T w/ Replace vs FPS

F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	-29.8549	-30.3086
Variance	21.81868	32.46742
Observations	398	398
df	397	397
F	0.672018	
P(F<=f) one-tail	4E-05	
F Critical one-tail	0.847636	

Mean(v1) > Mean(v2) and F < F-Critical => Equal Variance

t-Test: Two-Sample Assuming Equal Variances

	Variable 1	*Variable 2*
Mean	-29.8549	-30.3086
Variance	21.81868	32.46742
Observations	398	398
Pooled Variance	27.14305	
Hypothesized Mean Difference	0	
df	794	
t Stat	1.228434	
P(T<=t) one-tail	0.109824	
t Critical one-tail	1.646775	
P(T<=t) two-tail	0.219648	
t Critical two-tail	1.962956	

t < t Critical => Same

Figure 4: K-Tournament w/ Replacement vs Fitness Proportional Selection Tests for Input 1 Width

Input 1 - Width - k-T w/ Replace vs FPS

F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	-15.711	-15.7245
Variance	6.210719	7.403888
Observations	398	398
df	397	397
F	0.838846	
P(F<=f) one-tail	0.040192	
F Critical one-tail	0.847636	

Mean(v1) < Mean(v2) and F < F-Critical => Unequal Variance

t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	*Variable 2*
Mean	-15.711	-15.7245
Variance	6.210719	7.403888
Observations	398	398
Hypothesized Mean Difference	0	
df	788	
t Stat	0.073358	
P(T<=t) one-tail	0.47077	
t Critical one-tail	1.64679	
P(T<=t) two-tail	0.94154	
t Critical two-tail	1.962979	

t < t Critical => Same

Figure 5: K-Tournament w/ Replacement vs Uniform Random Tests for Input 1 Length

Input 1 - Length - k-T w/ Replace vs Random
F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	-29.8549	-31.6902
Variance	21.81868	61.77434
Observations	398	398
df	397	397
F	0.3532	
P(F<=f) one-tail	0	
F Critical one-tail	0.847636	

Mean(v1) < Mean(v2) and F < F-Critical => Unequal Variance

t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	*Variable 2*
Mean	-29.8549	-31.6902
Variance	21.81868	61.77434
Observations	398	398
Hypothesized Mean Difference	0	
df	646	
t Stat	4.004553	
P(T<=t) one-tail	3.47E-05	
t Critical one-tail	1.647216	
P(T<=t) two-tail	6.94E-05	
t Critical two-tail	1.963643	

t > t Critical => K-Tournament w/ replacement is Better

Figure 6: K-Tournament w/ Replacement vs Uniform Random Tests for Input 1 Width

Input 1 - Width - k-T w/ Replace vs Random
F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	-15.711	-15.8566
Variance	6.210719	8.626666
Observations	398	398
df	397	397
F	0.719944	
P(F<=f) one-tail	0.000549	
F Critical one-tail	0.847636	

Mean(v1) < Mean(v2) and F < F-Critical => Unequal Variance

t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	*Variable 2*
Mean	-15.711	-15.8566
Variance	6.210719	8.626666
Observations	398	398
Hypothesized Mean Difference	0	
df	773	
t Stat	0.754324	
P(T<=t) one-tail	0.225442	
t Critical one-tail	1.646827	
P(T<=t) two-tail	0.450884	
t Critical two-tail	1.963038	

t < t Critical => Same

Figure 7: Fitness Proportional Selection vs Uniform Random Tests for Input 1 Length

Input 1 - Length - FPS vs Random
F-Test Two-Sample for Variances

	Variable 1	Variable 2
Mean	-30.3086	-31.6902
Variance	32.46742	61.77434
Observations	398	398
df	397	397
F	0.525581	
P(F<=f) one-tail	1.07E-10	
F Critical one-tail	0.847636	

Mean(v1) > Mean(v2) and F < F-Critical => Equal Variance

t-Test: Two-Sample Assuming Equal Variances

	Variable 1	Variable 2
Mean	-30.3086	-31.6902
Variance	32.46742	61.77434
Observations	398	398
Pooled Variance	47.12088	
Hypothesized Mean Difference	0	
df	794	
t Stat	2.839188	
P(T<=t) one-tail	0.002319	
t Critical one-tail	1.646775	
P(T<=t) two-tail	0.004638	
t Critical two-tail	1.962956	

t > t Critical => FPS Better

Figure 8: Fitness Proportional Selection vs Uniform Random Tests for Input 1 Width

Input 1 - Width - FPS vs Random

F-Test Two-Sample for Variances

	Variable 1	Variable 2
Mean	-15.7245	-15.8566
Variance	7.403888	8.626666
Observations	398	398
df	397	397
F	0.858256	
P(F<=f) one-tail	0.064119	
F Critical one-tail	0.847636	

Mean(v1) > Mean(v2) and F > F-Critical => Unequal Variance

t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	Variable 2
Mean	-15.7245	-15.8566
Variance	7.403888	8.626666
Observations	398	398
Hypothesized Mean Difference	0	
df	789	
t Stat	0.658104	
P(T<=t) one-tail	0.255332	
t Critical one-tail	1.646787	
P(T<=t) two-tail	0.510663	
t Critical two-tail	1.962975	

t < t Critical => Same

Figure 9: Move vs Flip Tests for Input 2 Length

Input 2 - Length - Move vs Flip
F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	-80.4586	-134.965
Variance	6.268902	289.6804
Observations	199	199
df	198	198
F	0.021641	
P(F<=f) one-tail	0	
F Critical one-tail	0.791084	

Mean(v1) > Mean(v2) and F < F-Critical => Equal Variance

t-Test: Two-Sample Assuming Equal Variances

	Variable 1	*Variable 2*
Mean	-80.4586	-134.965
Variance	6.268902	289.6804
Observations	199	199
Pooled Variance	147.9746	
Hypothesized Mean Difference	0	
df	396	
t Stat	44.69564	
P(T<=t) one-tail	4.3E-157	
t Critical one-tail	1.648711	
P(T<=t) two-tail	8.5E-157	
t Critical two-tail	1.965973	

t > t Critical => Move is better

Figure 10: Move vs Flip Tests for Input 2 Width

Input 2 - Width - Move vs Flip
F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	-18.8695	-14.9707
Variance	0.029874	0.566549
Observations	199	199
df	198	198
F	0.05273	
P(F<=f) one-tail	0	
F Critical one-tail	0.791084	

Mean(v1) < Mean(v2) and F < F-Critical => Unequal Variance

t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	*Variable 2*
Mean	-18.8695	-14.9707
Variance	0.029874	0.566549
Observations	199	199
Hypothesized Mean Difference	0	
df	219	
t Stat	-71.2169	
P(T<=t) one-tail	9.8E-154	
t Critical one-tail	1.651841	
P(T<=t) two-tail	2E-153	
t Critical two-tail	1.970855	

t < t Critical => Same

Figure 11: K-Tournament w/ Replacement vs Fitness Proportional Selection Tests for Input 2 Length

Input 2 - Length - k-T w/ Replace vs FPS
F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	-107.712	-111.152
Variance	892.2087	1305.909
Observations	398	398
df	397	397
F	0.683209	
P(F<=f) one-tail	7.8E-05	
F Critical one-tail	0.847636	

Mean(v1) > Mean(v2) and F < F-Critical => Equal Variance

t-Test: Two-Sample Assuming Equal Variances

	Variable 1	*Variable 2*
Mean	-107.712	-111.152
Variance	892.2087	1305.909
Observations	398	398
Pooled Variance	1099.059	
Hypothesized Mean Difference	0	
df	794	
t Stat	1.463754	
P(T<=t) one-tail	0.071828	
t Critical one-tail	1.646775	
P(T<=t) two-tail	0.143657	
t Critical two-tail	1.962956	

t < t Critical => Same

Figure 12: K-Tournament w/ Replacement vs Fitness Proportional Selection Tests for Input 2 Width

Input 2 - Width - k-T w/ Replace vs FPS
F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	-16.9201	-16.8913
Variance	4.107247	4.434298
Observations	398	398
df	397	397
F	0.926245	
P(F<=f) one-tail	0.222817	
F Critical one-tail	0.847636	

Mean(v1) > Mean(v2) and F > F-Critical => Unequal Variance

t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	*Variable 2*
Mean	-16.9201	-16.8913
Variance	4.107247	4.434298
Observations	398	398
Hypothesized Mean Difference	0	
df	793	
t Stat	-0.19667	
P(T<=t) one-tail	0.42207	
t Critical one-tail	1.646777	
P(T<=t) two-tail	0.84414	
t Critical two-tail	1.96296	

t < t Critical => Same

Figure 13: K-Tournament w/ Replacement vs Uniform Random Tests for Input 2 Length

Input 2 - Length - k-T w/ Replace vs Random
F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	-107.712	-117.236
Variance	892.2087	1880.034
Observations	398	398
df	397	397
F	0.474571	
P(F<=f) one-tail	1.09E-13	
F Critical one-tail	0.847636	

Mean(v1) > Mean(v2) and F < F-Critical => Equal Variance

t-Test: Two-Sample Assuming Equal Variances

	Variable 1	*Variable 2*
Mean	-107.712	-117.236
Variance	892.2087	1880.034
Observations	398	398
Pooled Variance	1386.121	
Hypothesized Mean Difference	0	
df	794	
t Stat	3.6087	
P(T<=t) one-tail	0.000164	
t Critical one-tail	1.646775	
P(T<=t) two-tail	0.000327	
t Critical two-tail	1.962956	

t > t Critical => K-Tourn w/ Replacement is Better

Figure 14: K-Tournament w/ Replacement vs Uniform Random Tests for Input 2 Width

Input 2 - Width - k-T w/ Replace vs Random
F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	-16.9201	-17.0456
Variance	4.107247	3.790559
Observations	398	398
df	397	397
F	1.083546	
P(F<=f) one-tail	0.212212	
F Critical one-tail	1.179751	

Mean(v1) > Mean(v2) and F < F-Critical => Equal Variance

t-Test: Two-Sample Assuming Equal Variances

	Variable 1	*Variable 2*
Mean	-16.9201	-17.0456
Variance	4.107247	3.790559
Observations	398	398
Pooled Variance	3.948903	
Hypothesized Mean Difference	0	
df	794	
t Stat	0.891221	
P(T<=t) one-tail	0.18654	
t Critical one-tail	1.646775	
P(T<=t) two-tail	0.37308	
t Critical two-tail	1.962956	

t < t Critical => Same

Figure 15: Fitness Proportional Selection vs Uniform Random Tests for Input 2 Length

Input 2 - Length - FPS vs Random
F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	-111.152	-117.236
Variance	1305.909	1880.034
Observations	398	398
df	397	397
F	0.69462	
P(F<=f) one-tail	0.000148	
F Critical one-tail	0.847636	

Mean(v1) > Mean(v2) and F < F-Critical => Equal Variance

t-Test: Two-Sample Assuming Equal Variances

	Variable 1	*Variable 2*
Mean	-111.152	-117.236
Variance	1305.909	1880.034
Observations	398	398
Pooled Variance	1592.971	
Hypothesized Mean Difference	0	
df	794	
t Stat	2.150423	
P(T<=t) one-tail	0.015912	
t Critical one-tail	1.646775	
P(T<=t) two-tail	0.031823	
t Critical two-tail	1.962956	

t > t Critical => FPS is Better

Figure 16: Fitness Proportional Selection vs Uniform Random Tests for Input 2 Width

Input 2 - Width - FPS vs Random
F-Test Two-Sample for Variances

	Variable 1	Variable 2
Mean	-16.8913	-17.0456
Variance	4.434298	3.790559
Observations	398	398
df	397	397
F	1.169827	
P(F<=f) one-tail	0.059276	
F Critical one-tail	1.179751	

Mean(v1) > Mean(v2) and F < F-Critical => Equal Variance

t-Test: Two-Sample Assuming Equal Variances

	Variable 1	Variable 2
Mean	-16.8913	-17.0456
Variance	4.434298	3.790559
Observations	398	398
Pooled Variance	4.112429	
Hypothesized Mean Difference	0	
df	794	
t Stat	1.073738	
P(T<=t) one-tail	0.141633	
t Critical one-tail	1.646775	
P(T<=t) two-tail	0.283266	
t Critical two-tail	1.962956	

t < t Critical => Same

Figure 17: Move vs Flip Tests for Input 3 Length

Input 3 - Length - Move vs Flip
F-Test Two-Sample for Variances

	Variable 1	Variable 2
Mean	-281.112	-398.121
Variance	50.17884	1550.39
Observations	199	199
df	198	198
F	0.032365	
P(F<=f) one-tail	0	
F Critical one-tail	0.791084	

Mean(v1) > Mean(v2) and F < F-Critical => Equal Variance

t-Test: Two-Sample Assuming Equal Variances

	Variable 1	Variable 2
Mean	-281.112	-398.121
Variance	50.17884	1550.39
Observations	199	199
Pooled Variance	800.2847	
Hypothesized Mean Difference	0	
df	396	
t Stat	41.25817	
P(T<=t) one-tail	9.2E-146	
t Critical one-tail	1.648711	
P(T<=t) two-tail	1.8E-145	
t Critical two-tail	1.965973	

t > t Critical => Move is Better

Figure 18: Move vs Flip Tests for Input 3 Width

Input 3 - Width - Move vs Flip
F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	-28.1548	-25.3931
Variance	0.120335	0.992012
Observations	199	199
df	198	198
F	0.121304	
P(F<=f) one-tail	0	
F Critical one-tail	0.791084	

Mean(v1) < Mean(v2) and F < F-Critical => Unequal Variance

t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	*Variable 2*
Mean	-28.1548	-25.3931
Variance	0.120335	0.992012
Observations	199	199
Hypothesized Mean Difference	0	
df	245	
t Stat	-36.938	
P(T<=t) one-tail	2E-102	
t Critical one-tail	1.651097	
P(T<=t) two-tail	4E-102	
t Critical two-tail	1.969694	

t < t Critical => Same

Figure 19: K-Tournament w/ Replacement vs Fitness Proportional Selection Tests for Input 3 Length

Input 3 - Length - k-T w/ Replace vs FPS
F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	-339.617	-343.381
Variance	4229.689	6948.328
Observations	398	398
df	397	397
F	0.608735	
P(F<=f) one-tail	4.39E-07	
F Critical one-tail	0.847636	

Mean(v1) > Mean(v2) and F < F-Critical => Equal Variance

t-Test: Two-Sample Assuming Equal Variances

	Variable 1	*Variable 2*
Mean	-339.617	-343.381
Variance	4229.689	6948.328
Observations	398	398
Pooled Variance	5589.009	
Hypothesized Mean Difference	0	
df	794	
t Stat	0.710259	
P(T<=t) one-tail	0.238876	
t Critical one-tail	1.646775	
P(T<=t) two-tail	0.477752	
t Critical two-tail	1.962956	

t < t Critical => Same

Figure 20: K-Tournament w/ Replacement vs Fitness Proportional Selection Tests for Input 3 Width

Input 3 - Width - k-T w/ Replace vs FPS
F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	-26.774	-27.0102
Variance	2.466241	2.473184
Observations	398	398
df	397	397
F	0.997193	
P(F<=f) one-tail	0.488836	
F Critical one-tail	0.847636	

Mean(v1) > Mean(v2) and F > F-Critical => Unequal Variance

t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	*Variable 2*
Mean	-26.774	-27.0102
Variance	2.466241	2.473184
Observations	398	398
Hypothesized Mean Difference	0	
df	794	
t Stat	2.120812	
P(T<=t) one-tail	0.017124	
t Critical one-tail	1.646775	
P(T<=t) two-tail	0.034247	
t Critical two-tail	1.962956	

t > t Critical => K-Tourn w/ Replacement is Better

Figure 21: K-Tournament w/ Replacement vs Uniform Random Tests for Input 3 Length

Input 3 - Length - k-T w/ Replace vs Random
F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	-339.617	-365.739
Variance	4229.689	12282.68
Observations	398	398
df	397	397
F	0.344362	
P(F<=f) one-tail	0	
F Critical one-tail	0.847636	

Mean(v1) > Mean(v2) and F < F-Critical => Equal Variance

t-Test: Two-Sample Assuming Equal Variances

	Variable 1	*Variable 2*
Mean	-339.617	-365.739
Variance	4229.689	12282.68
Observations	398	398
Pooled Variance	8256.183	
Hypothesized Mean Difference	0	
df	794	
t Stat	4.055507	
P(T<=t) one-tail	2.75E-05	
t Critical one-tail	1.646775	
P(T<=t) two-tail	5.5E-05	
t Critical two-tail	1.962956	

t > t Critical => K-Tourn w/ Replacement is Better

Figure 22: K-Tournament w/ Replacement vs Uniform Random Tests for Input 3 Width

Input 3 - Width - k-T w/ Replace vs Random
F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	-26.774	-27.0182
Variance	2.466241	2.76247
Observations	398	398
df	397	397
F	0.892767	
P(F<=f) one-tail	0.129443	
F Critical one-tail	0.847636	

Mean(v1) > Mean(v2) and F > F-Critical => Unequal Variance

t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	*Variable 2*
Mean	-26.774	-27.0182
Variance	2.466241	2.76247
Observations	398	398
Hypothesized Mean Difference	0	
df	791	
t Stat	2.130725	
P(T<=t) one-tail	0.01671	
t Critical one-tail	1.646782	
P(T<=t) two-tail	0.03342	
t Critical two-tail	1.962968	

t > t Critical => K-Tourn w/ Replacement is Better

Figure 23: Fitness Proportional Selection vs Uniform Random Tests for Input 3 Length

Input 3 - Length -FPS vs Random
F-Test Two-Sample for Variances

	Variable 1	Variable 2
Mean	-343.381	-365.739
Variance	6948.328	12282.68
Observations	398	398
df	397	397
F	0.565701	
P(F<=f) one-tail	8.79E-09	
F Critical one-tail	0.847636	

Mean(v1) > Mean(v2) and F < F-Critical => Equal Variance

t-Test: Two-Sample Assuming Equal Variances

	Variable 1	Variable 2
Mean	-343.381	-365.739
Variance	6948.328	12282.68
Observations	398	398
Pooled Variance	9615.502	
Hypothesized Mean Difference	0	
df	794	
t Stat	3.216431	
P(T<=t) one-tail	0.000675	
t Critical one-tail	1.646775	
P(T<=t) two-tail	0.001351	
t Critical two-tail	1.962956	

t > t Critical => FPS is Better

Figure 24: Fitness Proportional Selection vs Uniform Random Tests for Input 3 Width

Input 3 - Width -FPS vs Random
F-Test Two-Sample for Variances

	Variable 1	Variable 2
Mean	-27.0102	-27.0182
Variance	2.473184	2.76247
Observations	398	398
df	397	397
F	0.89528	
P(F<=f) one-tail	0.135435	
F Critical one-tail	0.847636	

Mean(v1) > Mean(v2) and F > F-Critical => Unequal Variance

t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	Variable 2
Mean	-27.0102	-27.0182
Variance	2.473184	2.76247
Observations	398	398
Hypothesized Mean Difference	0	
df	792	
t Stat	0.069371	
P(T<=t) one-tail	0.472356	
t Critical one-tail	1.64678	
P(T<=t) two-tail	0.944712	
t Critical two-tail	1.962964	

t < t Critical => Same

Figures

Table 1: Figures 25 and 26 Configuration File

Search Algorithm	EA
Mutation Algorithm	Move
Penalty Coefficient	1
Population Size	100
Random Seed	1001
Parent Selection Algorithm	k-Tournament Selection with replacement
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Figure 25: Input 1 Length Fitness

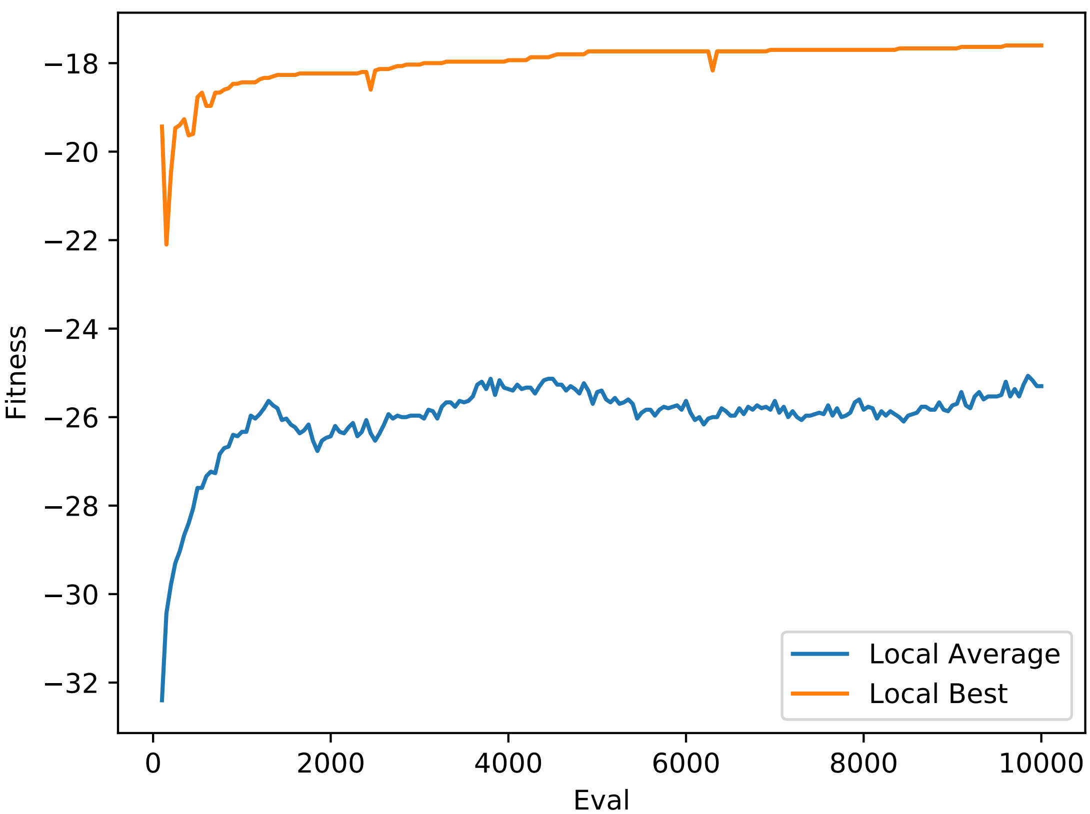

Figure 26: Input 1 Width Fitness

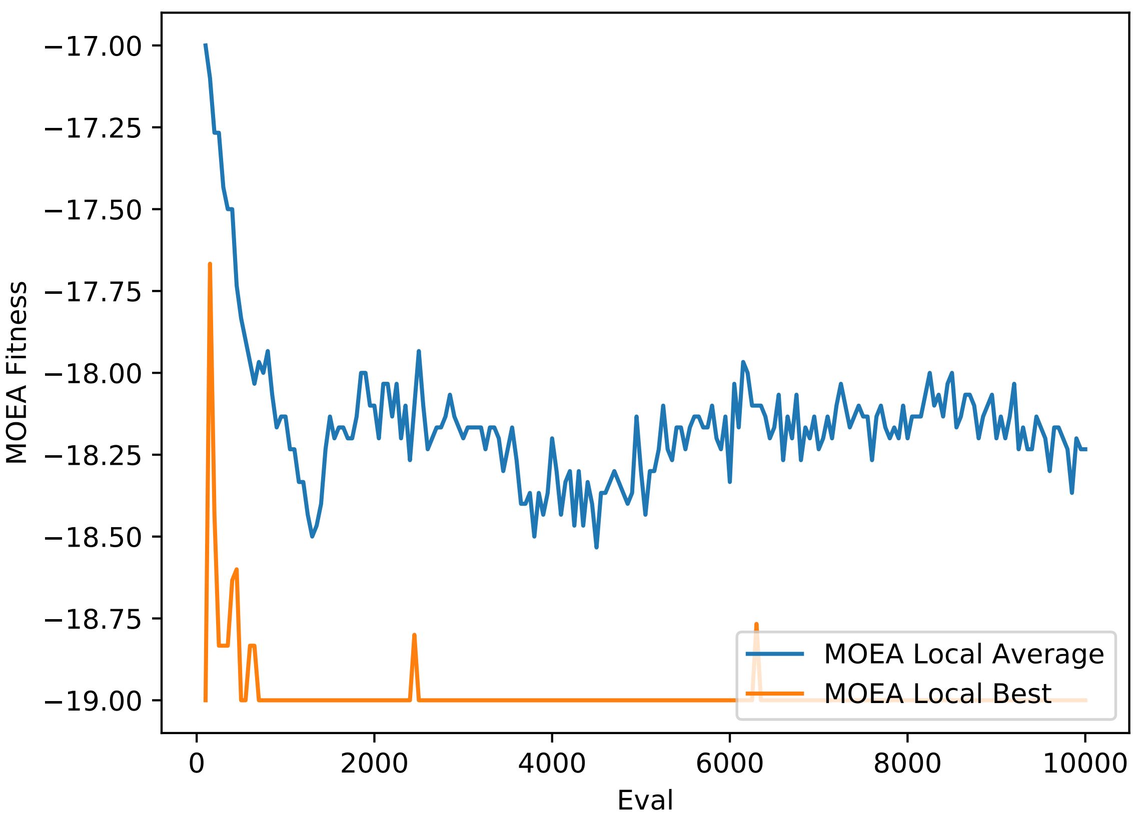

Figure 27: Figure 25 Representation

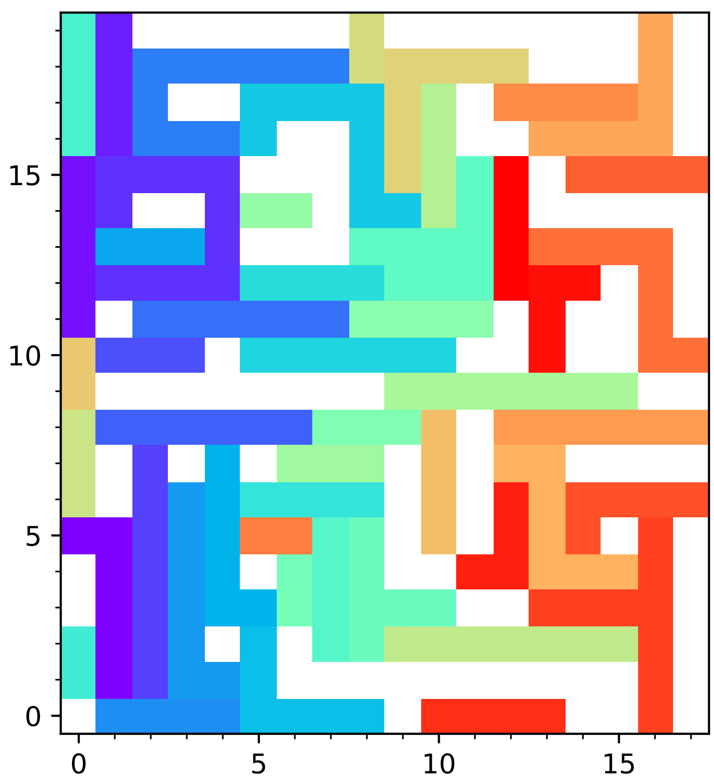

Table 2: Figures 28 and 29 Configuration File

Search Algorithm	EA
Mutation Algorithm	Flip
Penalty Coefficient	1
Population Size	100
Random Seed	1002
Parent Selection Algorithm	k-Tournament Selection with replacement
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Table 3: Figures 31 and 32 Configuration File

Search Algorithm	EA
Mutation Algorithm	Move
Penalty Coefficient	1
Population Size	100
Random Seed	1003
Parent Selection Algorithm	Fitness Proportional Selection
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Figure 28: Input 1 Length Fitness

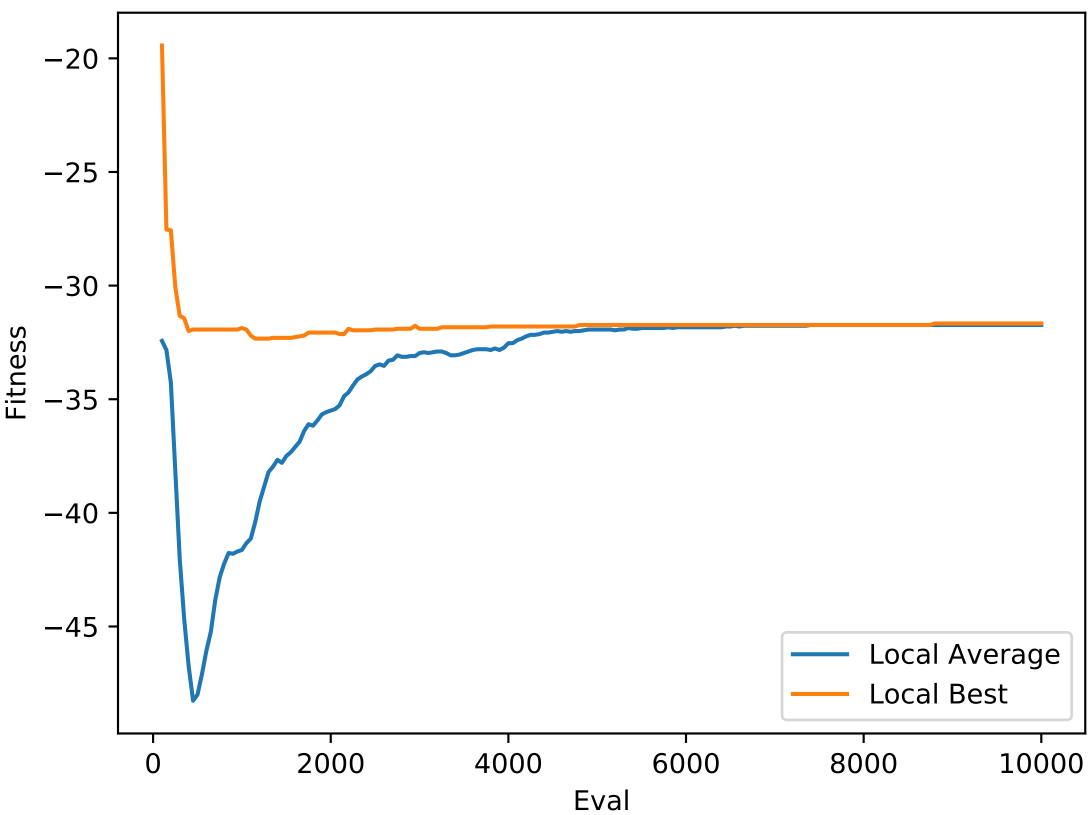

Figure 29: Input 1 Width Fitness

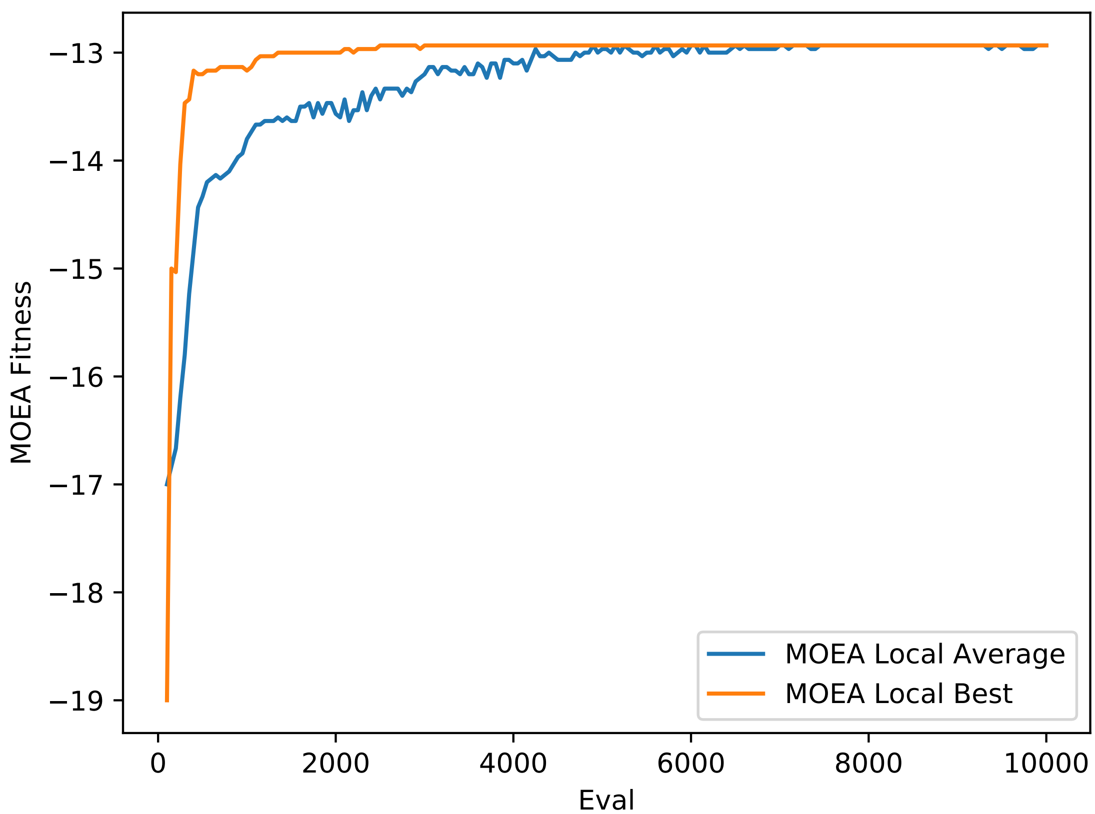

Figure 30: Figure 28 Representation

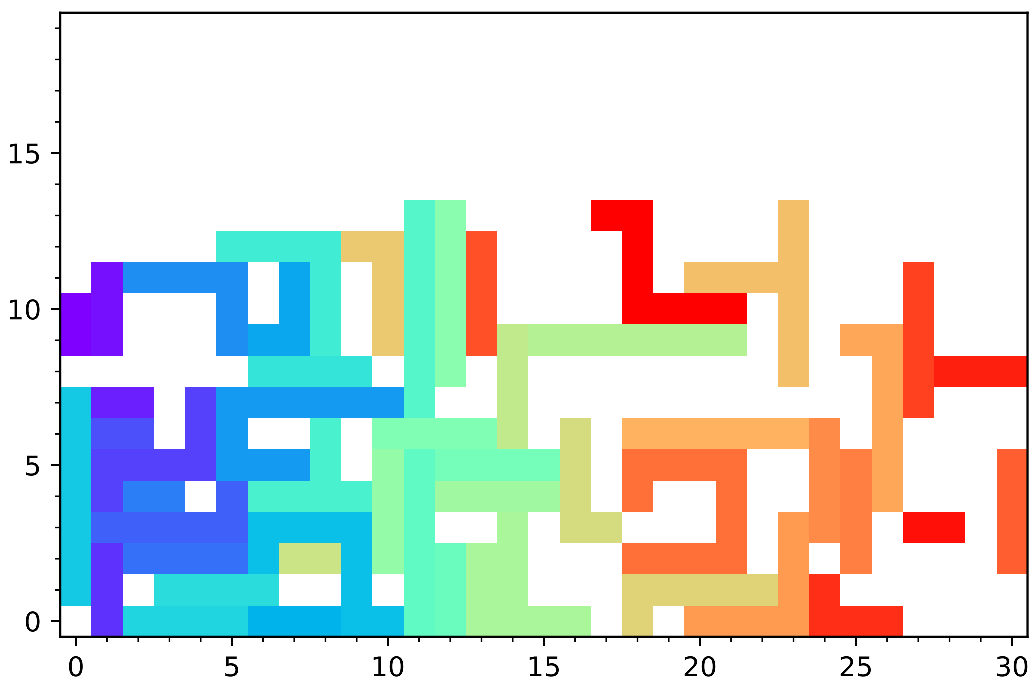

Figure 31: Input 1 Length Fitness

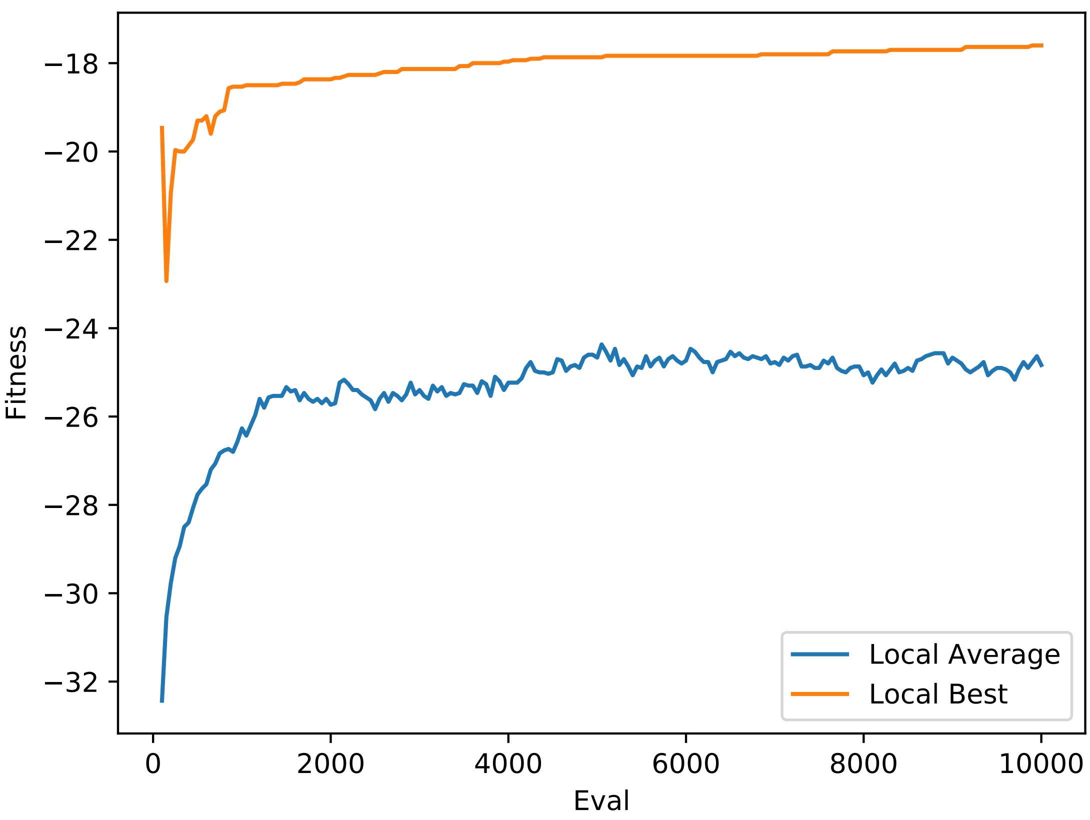

Figure 32: Input 1 Width Fitness

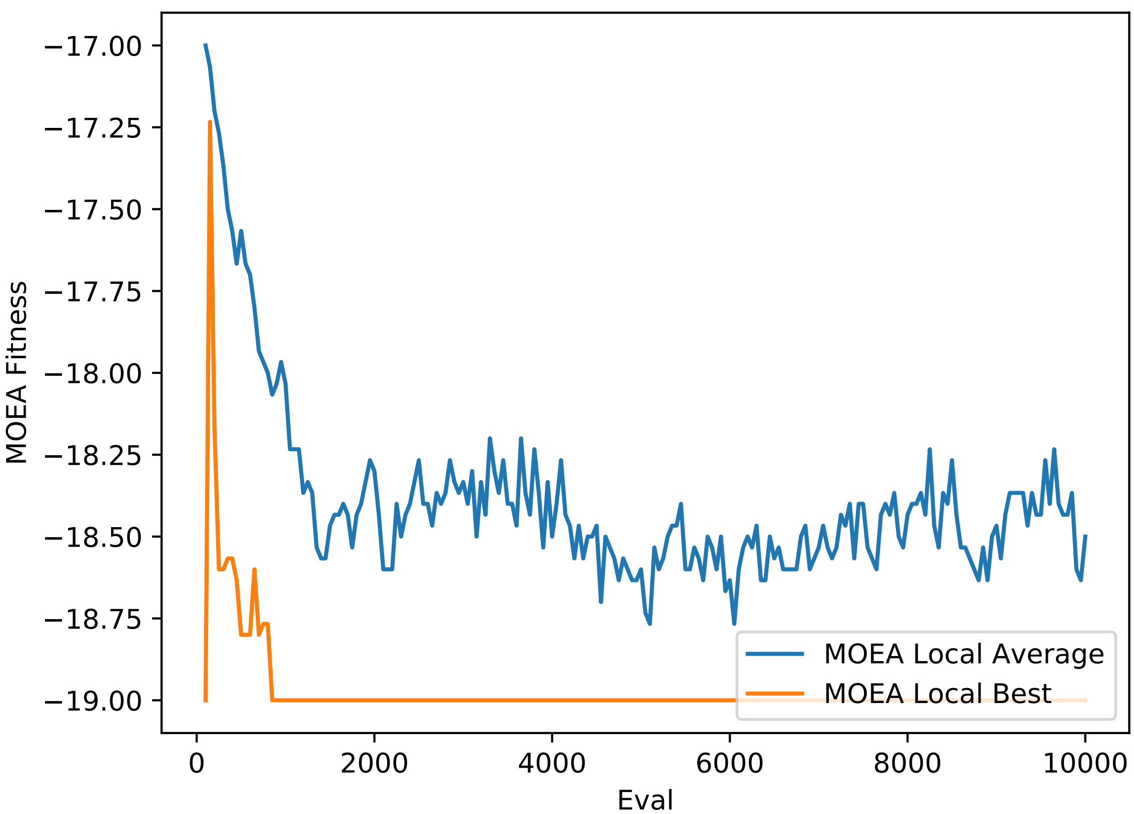

Figure 33: Figure 31 Representation

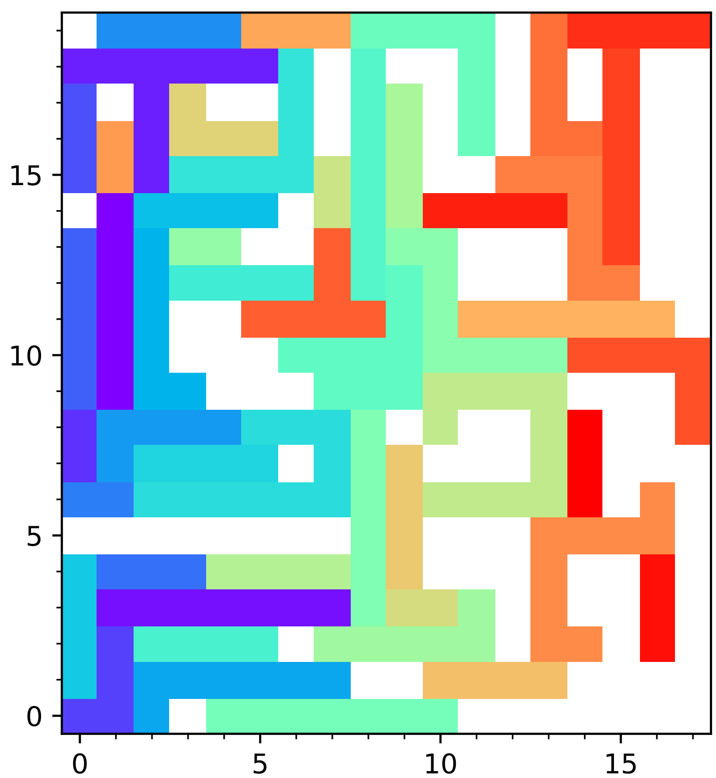

Table 4: Figures 34 and 35 Configuration File

Search Algorithm	EA
Mutation Algorithm	Flip
Penalty Coefficient	1
Population Size	100
Random Seed	1004
Parent Selection Algorithm	Fitness Proportional Selection
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Table 5: Figures 37 and 38 Configuration File

Search Algorithm	EA
Mutation Algorithm	Move
Penalty Coefficient	1
Population Size	100
Random Seed	1005
Parent Selection Algorithm	Uniform Random
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Figure 34: Input 1 Length Fitness

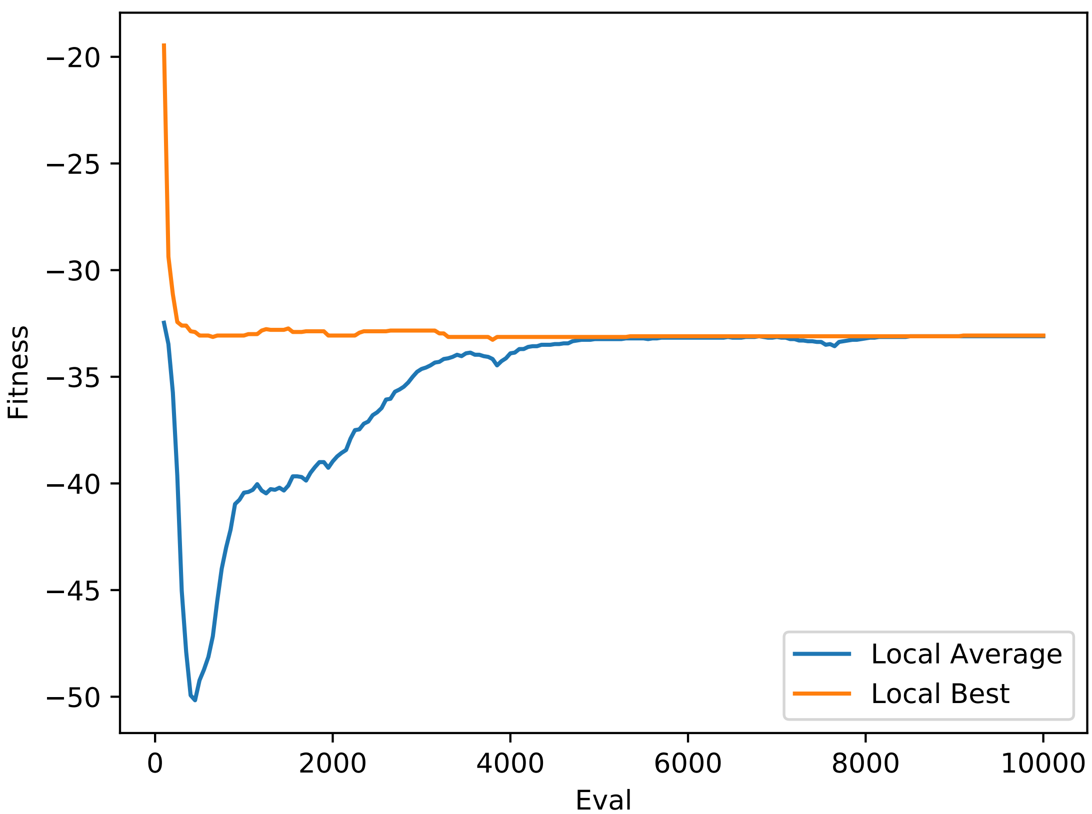

Figure 35: Input 1 Width Fitness

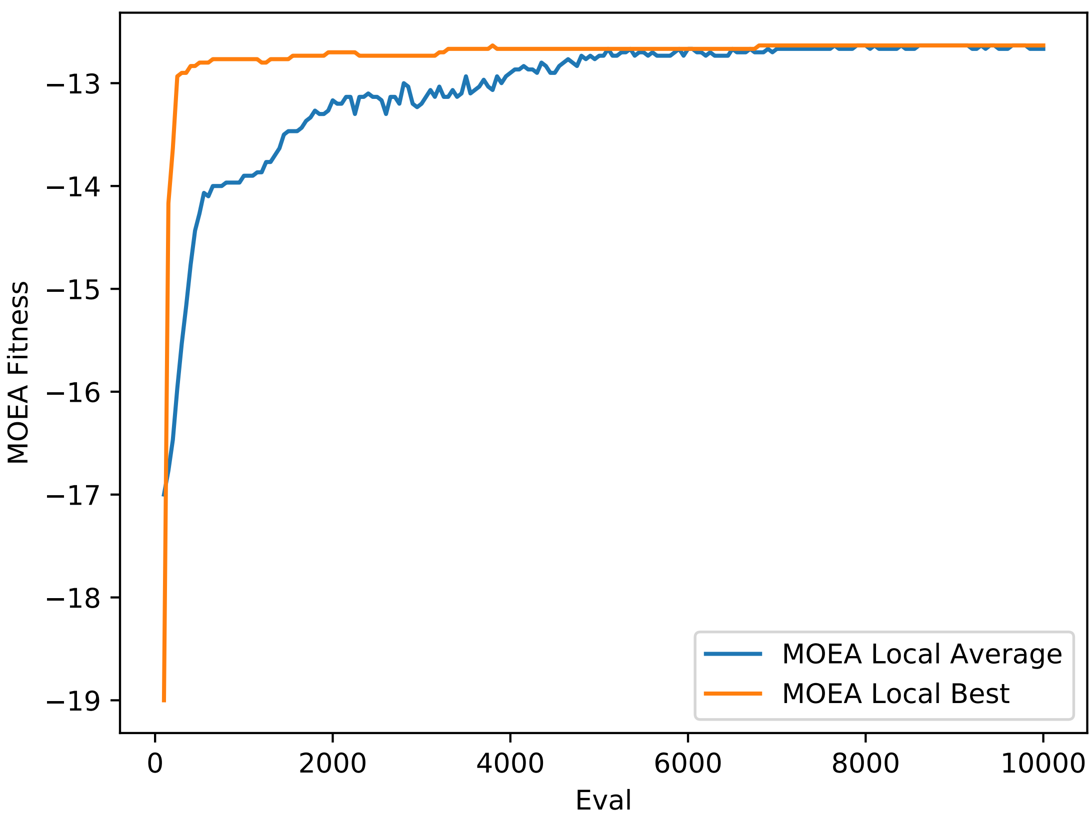

Figure 36: Figure 34 Representation

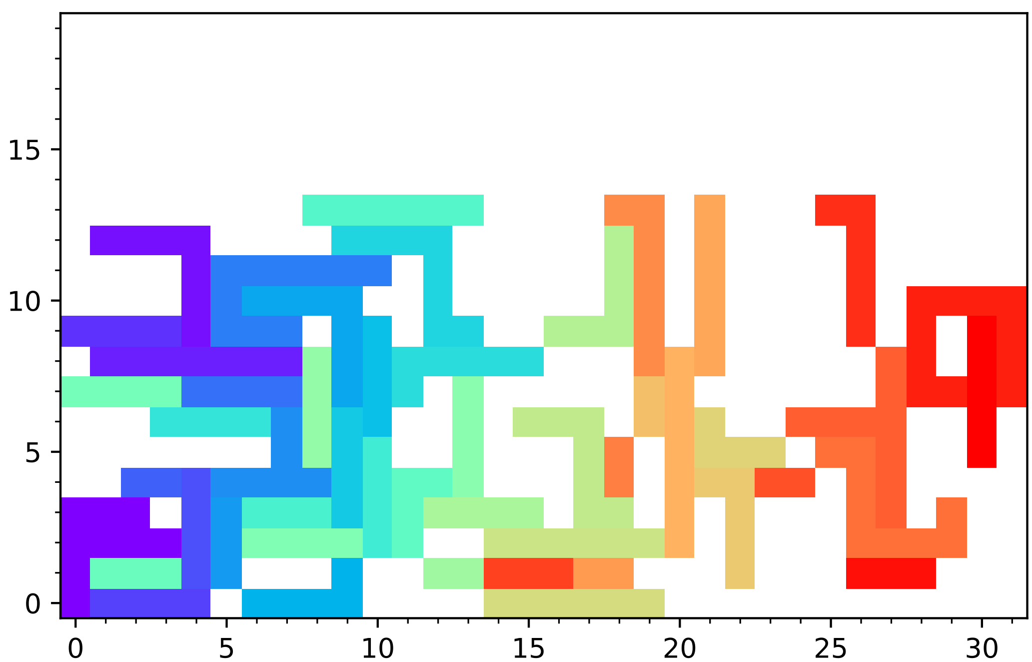

Figure 37: Input 1 Length Fitness

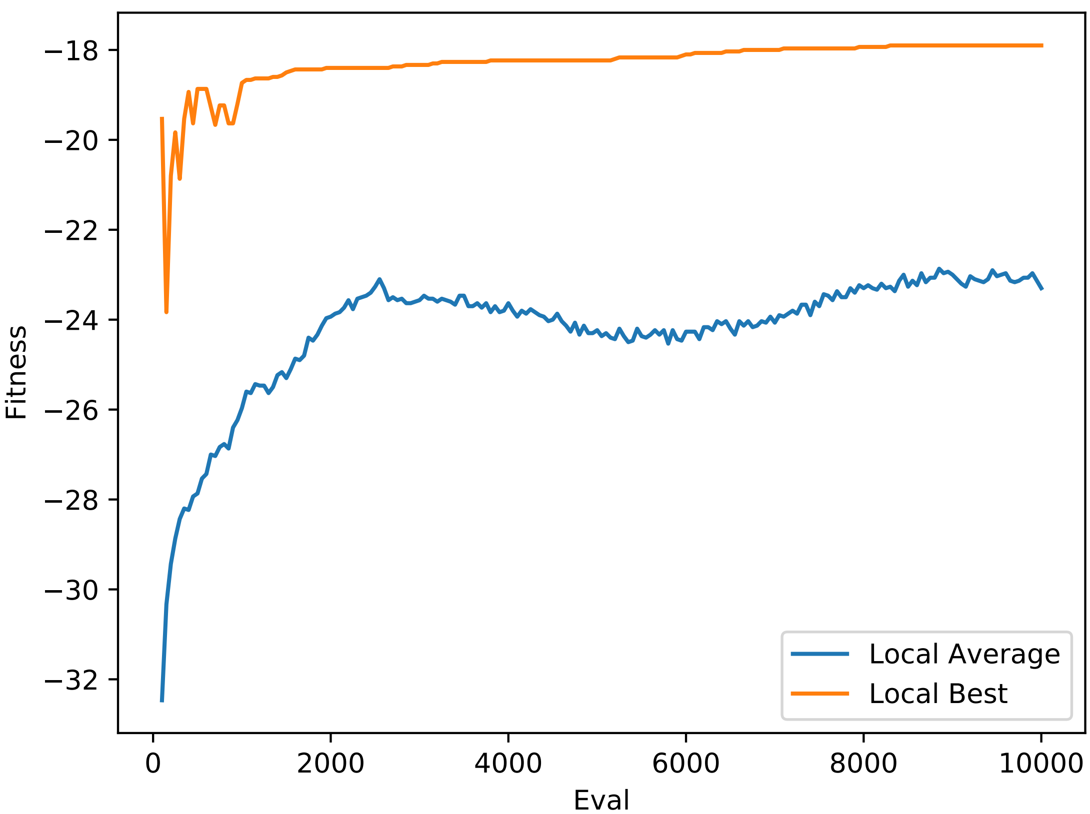

Figure 38: Input 1 Width Fitness

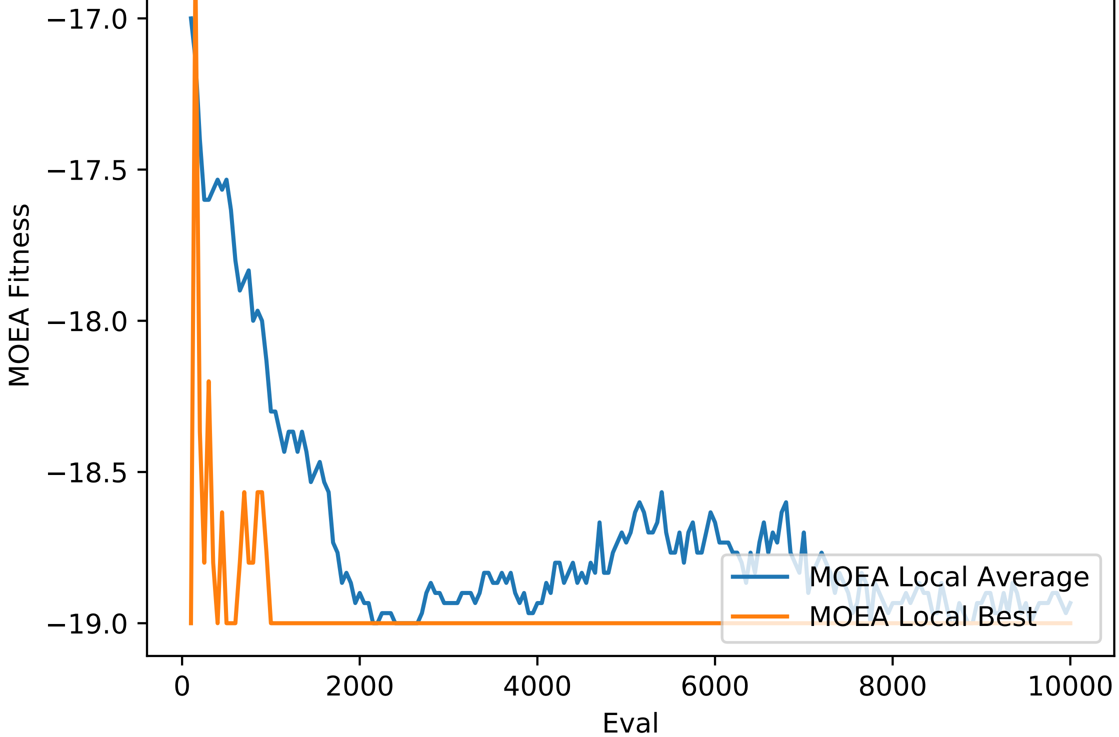

Figure 39: Figure 37 Representation

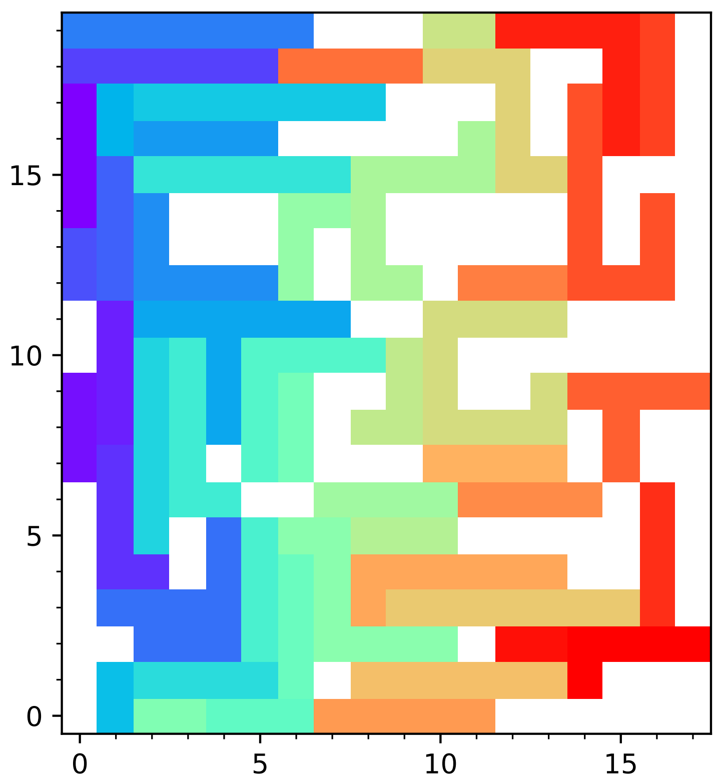

Table 6: Figures 40 and 41 Configuration File

Search Algorithm	EA
Mutation Algorithm	Flip
Penalty Coefficient	1
Population Size	100
Random Seed	1006
Parent Selection Algorithm	Uniform Random
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Table 7: Figures 43 and 44 Configuration File

Search Algorithm	EA
Mutation Algorithm	Move
Penalty Coefficient	1
Population Size	100
Random Seed	2001
Parent Selection Algorithm	k-Tournament Selection with replacement
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Figure 40: Input 1 Length Fitness

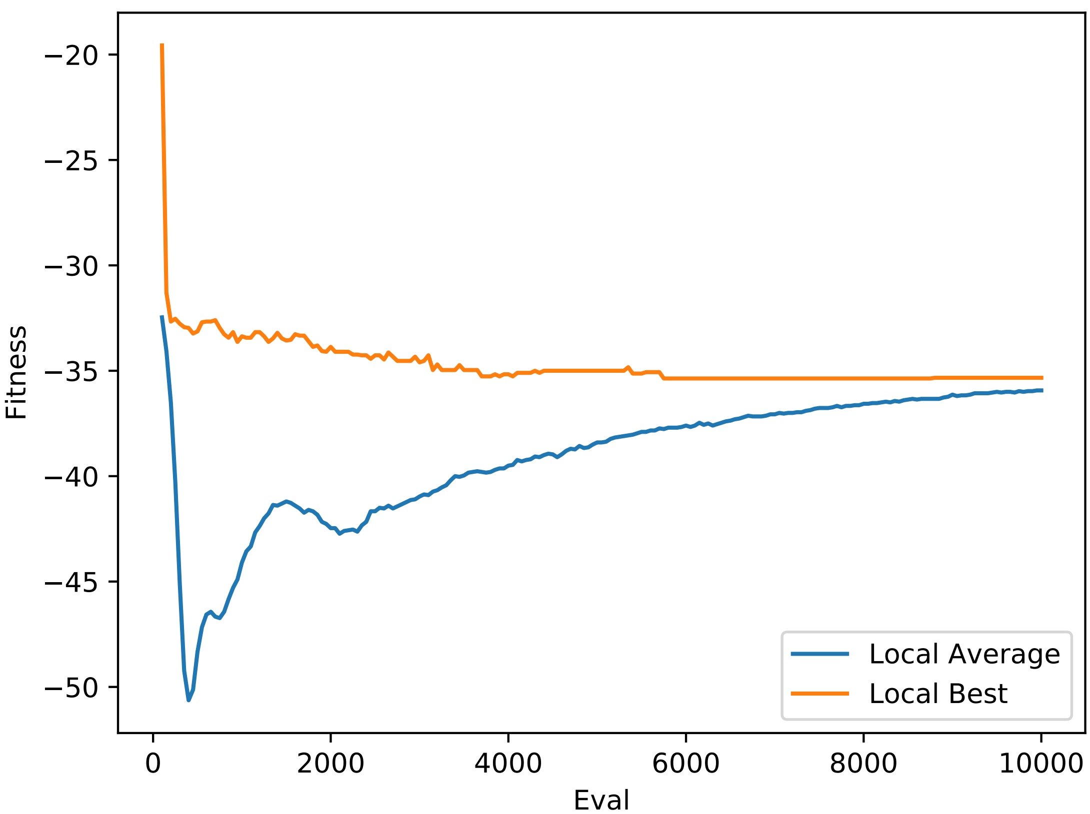

Figure 41: Input 1 Width Fitness

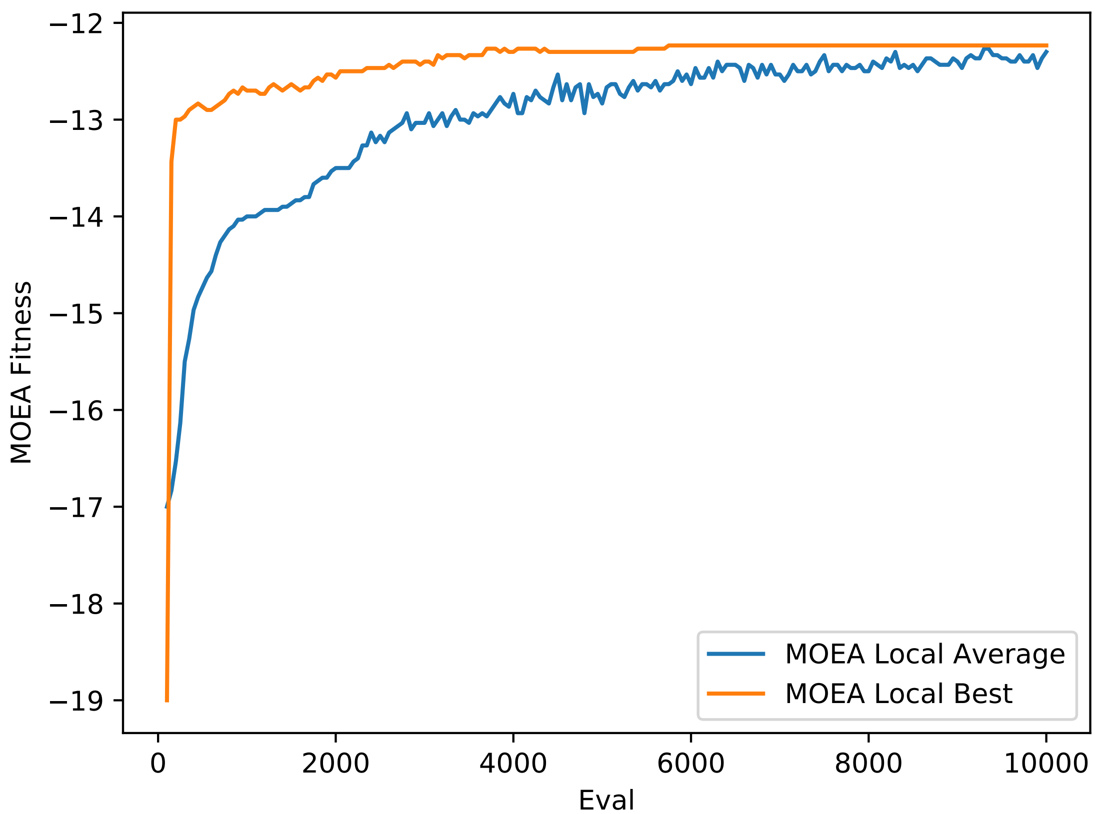

Figure 42: Figure 40 Representation

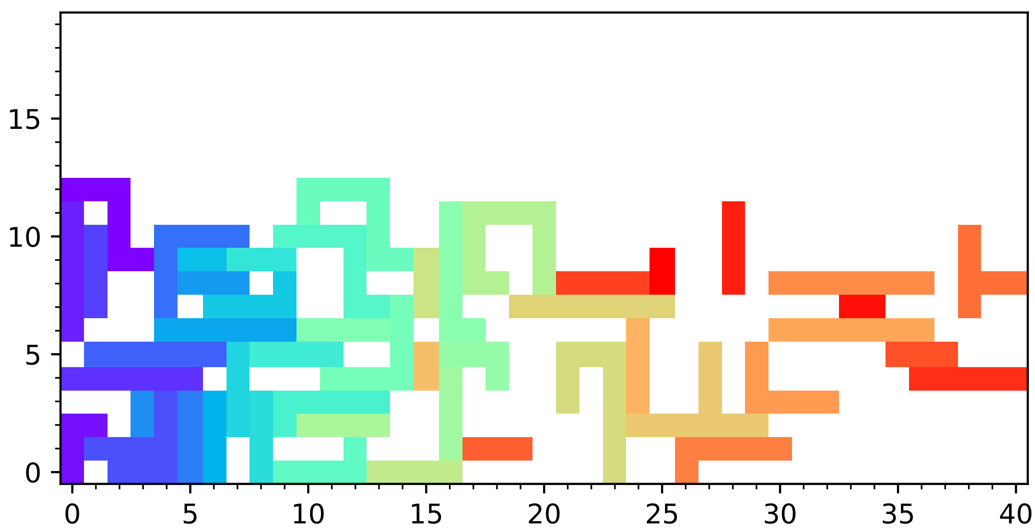

Figure 43: Input 2 Length Fitness

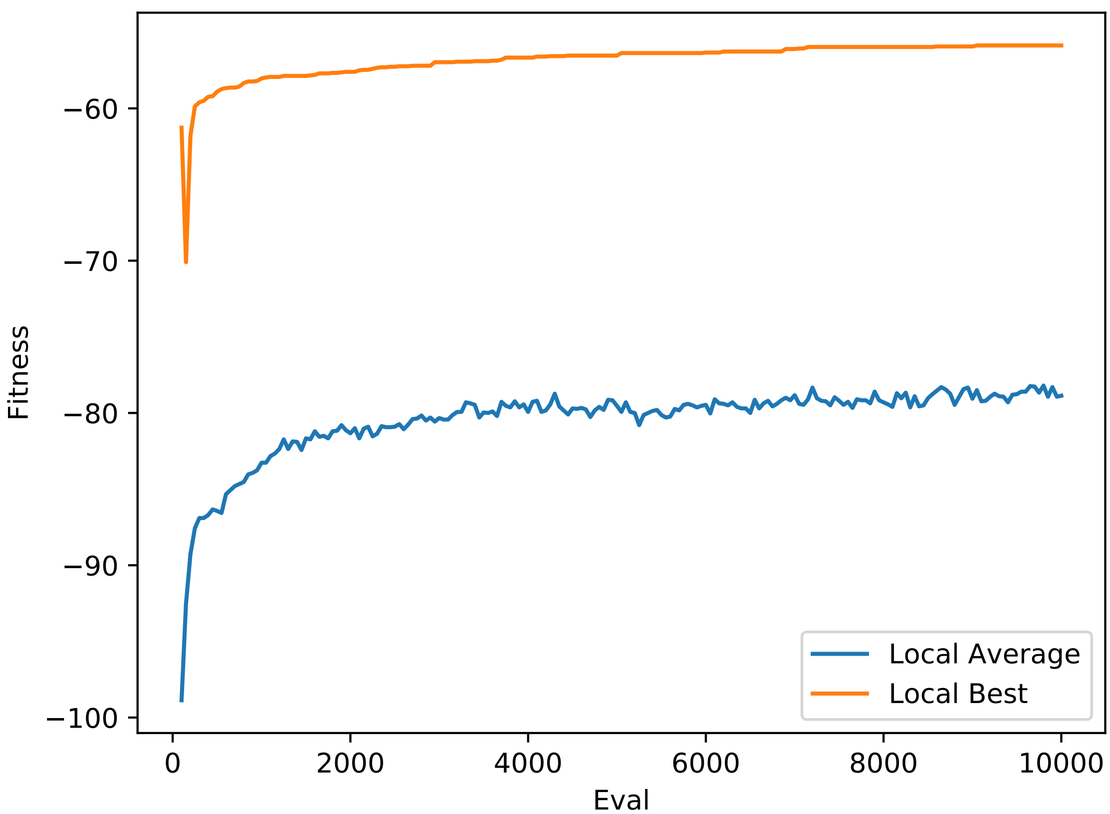

Figure 44: Input 2 Width Fitness

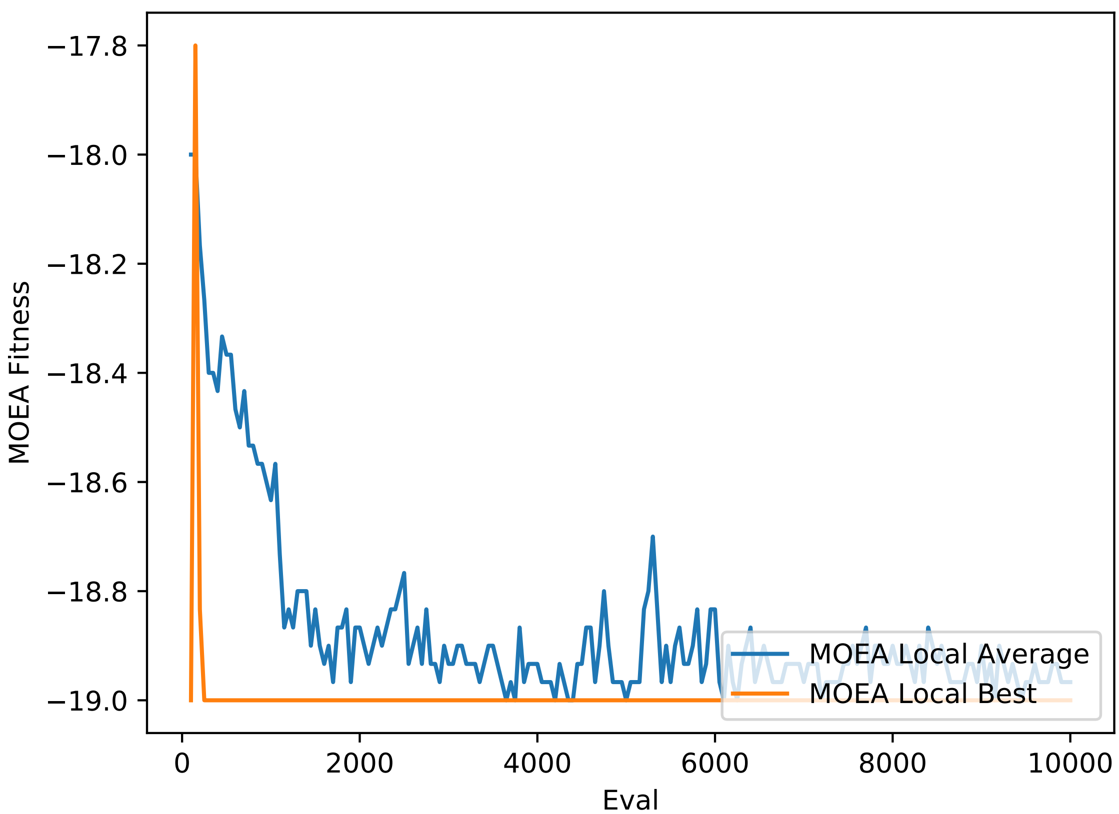

Figure 45: Figure 43 Representation

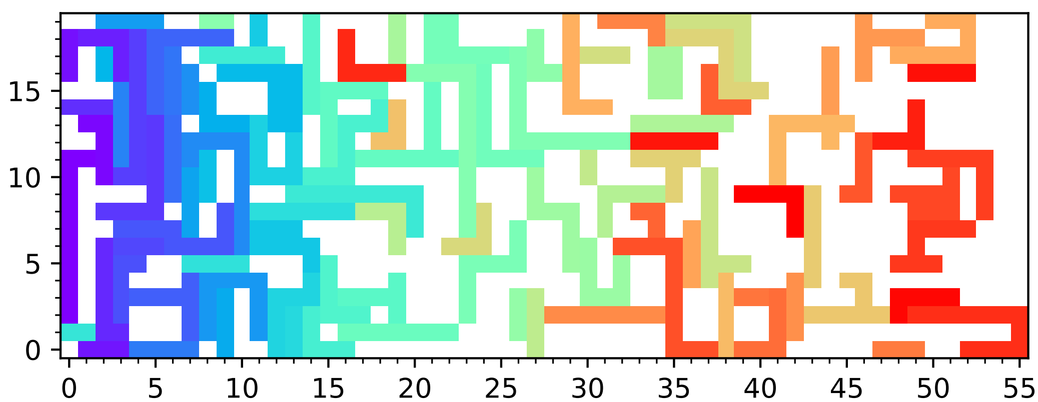

Table 8: Figures 46 and 47 Configuration File

Search Algorithm	EA
Mutation Algorithm	Flip
Penalty Coefficient	1
Population Size	100
Random Seed	2002
Parent Selection Algorithm	k-Tournament Selection with replacement
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Table 9: Figures 49 and 50 Configuration File

Search Algorithm	EA
Mutation Algorithm	Move
Penalty Coefficient	1
Population Size	100
Random Seed	2003
Parent Selection Algorithm	Fitness Proportional Selection
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Figure 46: Input 2 Length Fitness

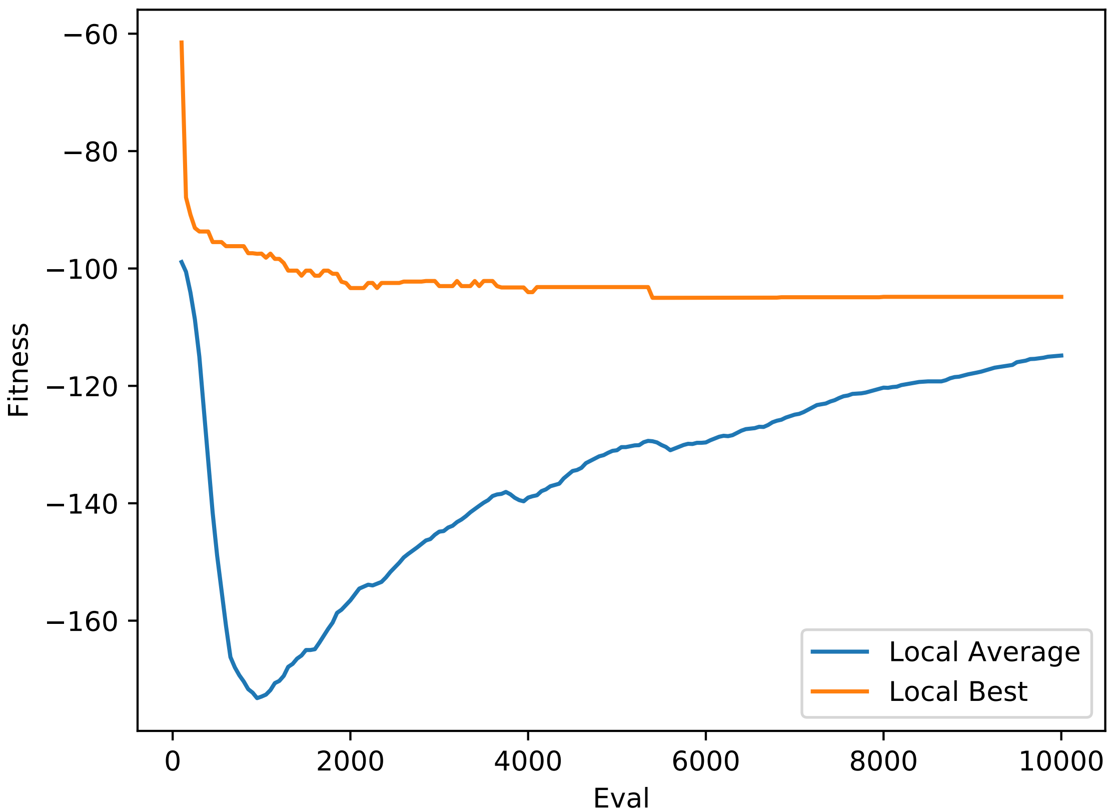

Figure 47: Input 2 Width Fitness

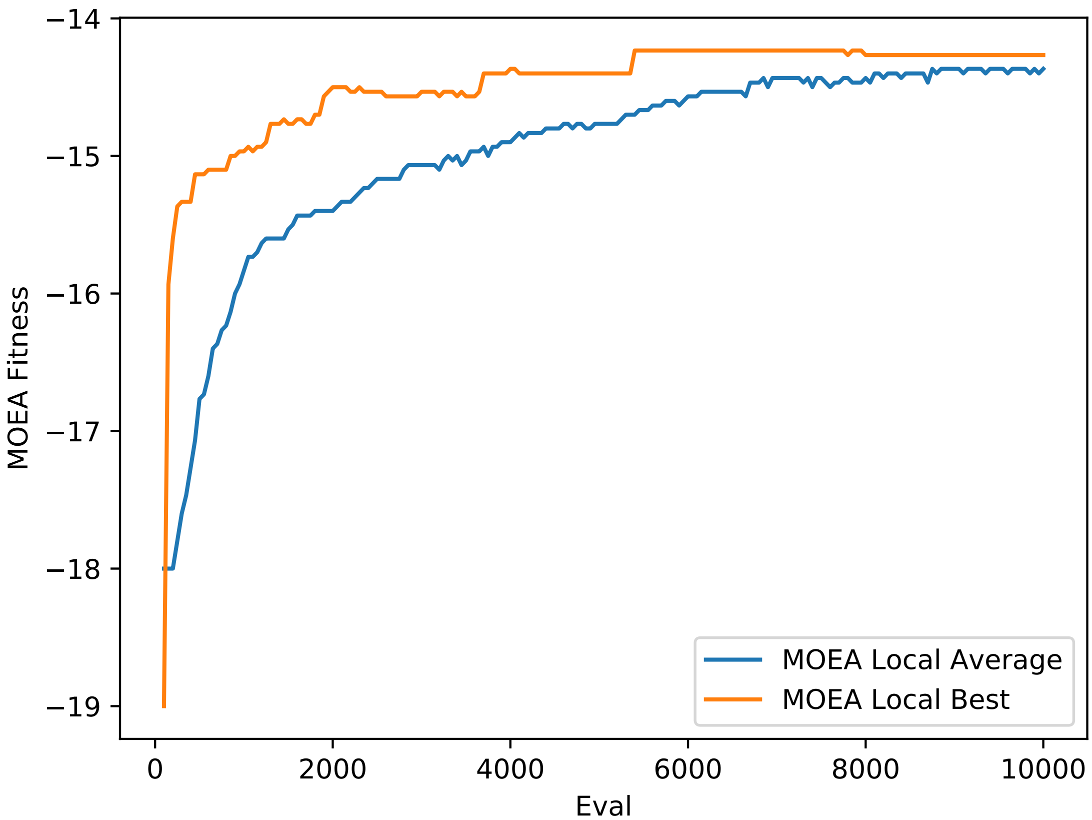

Figure 48: Figure 46 Representation

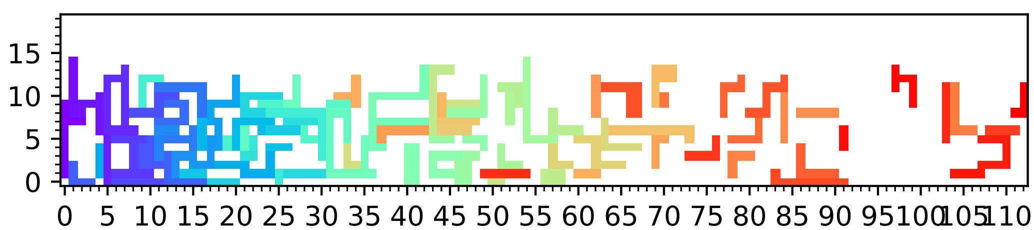

Figure 49: Input 2 Length Fitness

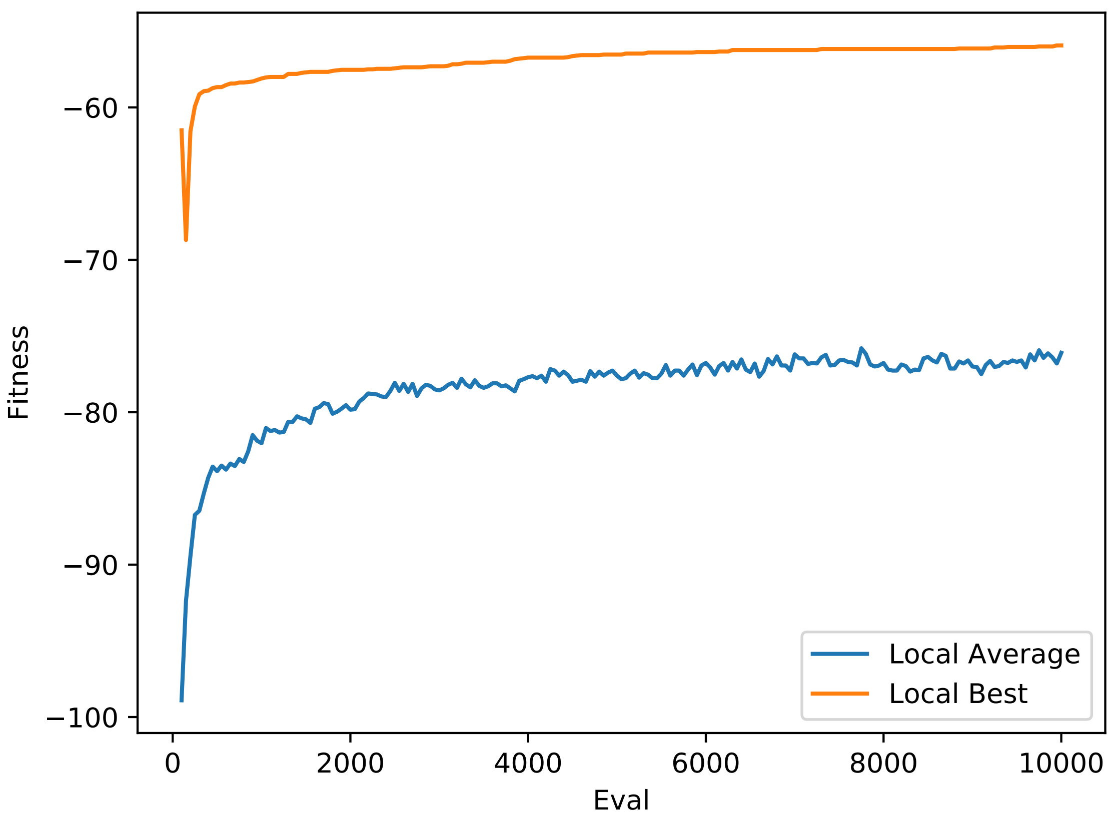

Figure 50: Input 2 Width Fitness

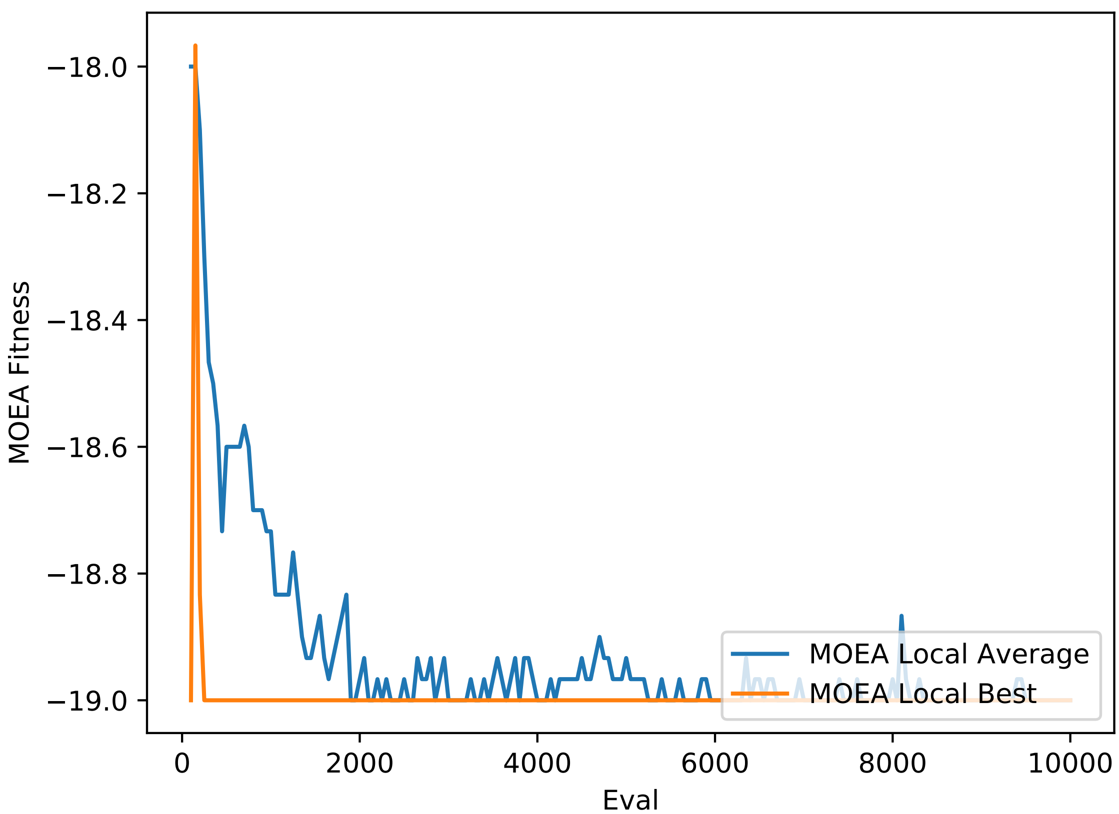

Figure 51: Figure 49 Representation

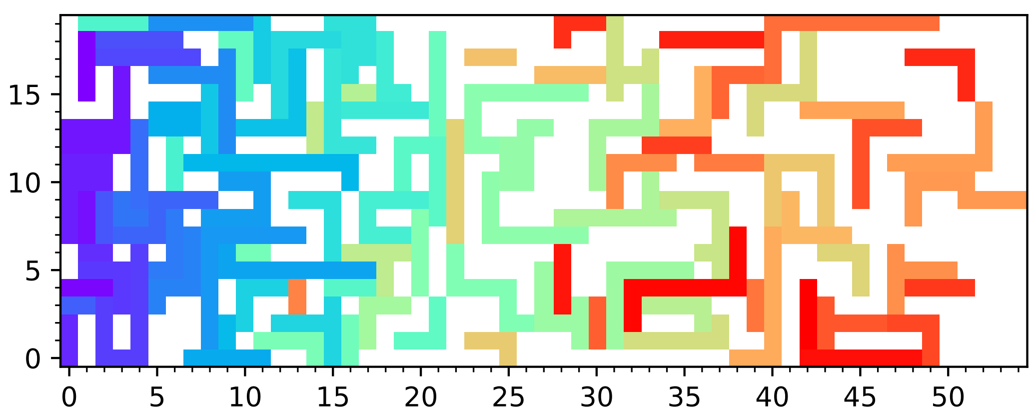

Table 10: Figures 52 and 53 Configuration File

Search Algorithm	EA
Mutation Algorithm	Flip
Penalty Coefficient	1
Population Size	100
Random Seed	2004
Parent Selection Algorithm	Fitness Proportional Selection
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Table 11: Figures 55 and 56 Configuration File

Search Algorithm	EA
Mutation Algorithm	Move
Penalty Coefficient	1
Population Size	100
Random Seed	2005
Parent Selection Algorithm	Uniform Random
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Figure 52: Input 2 Length Fitness

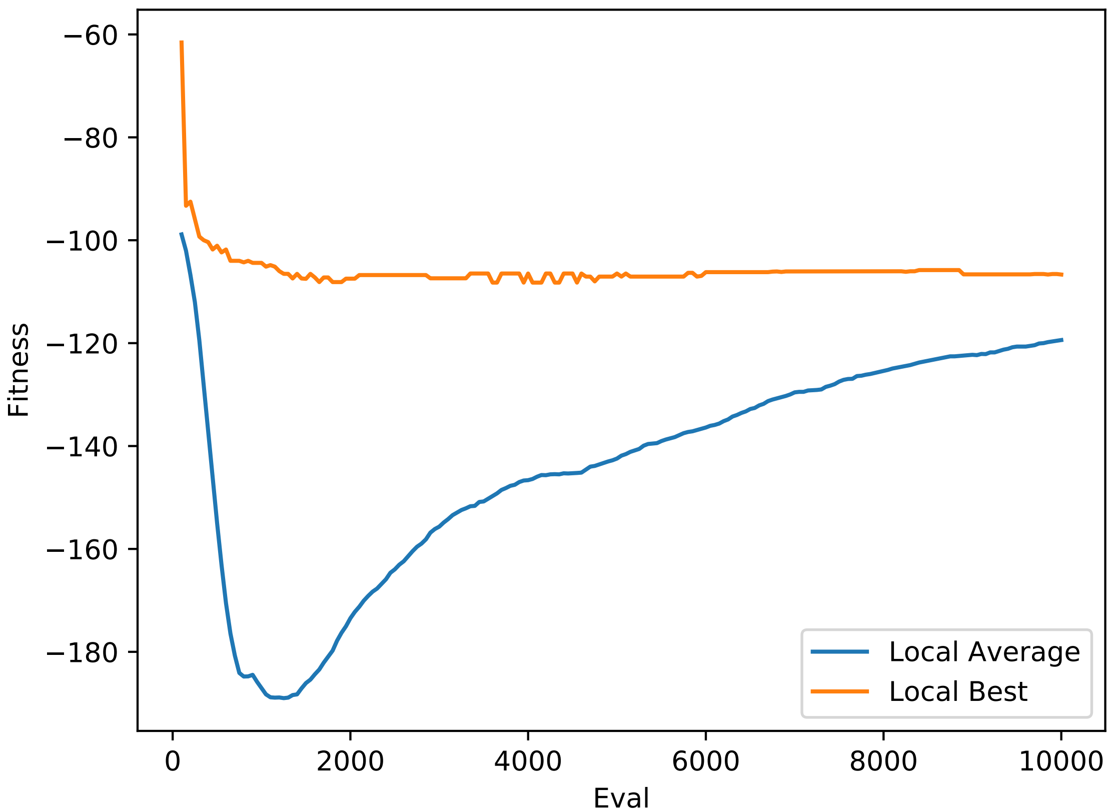

Figure 53: Input 2 Width Fitness

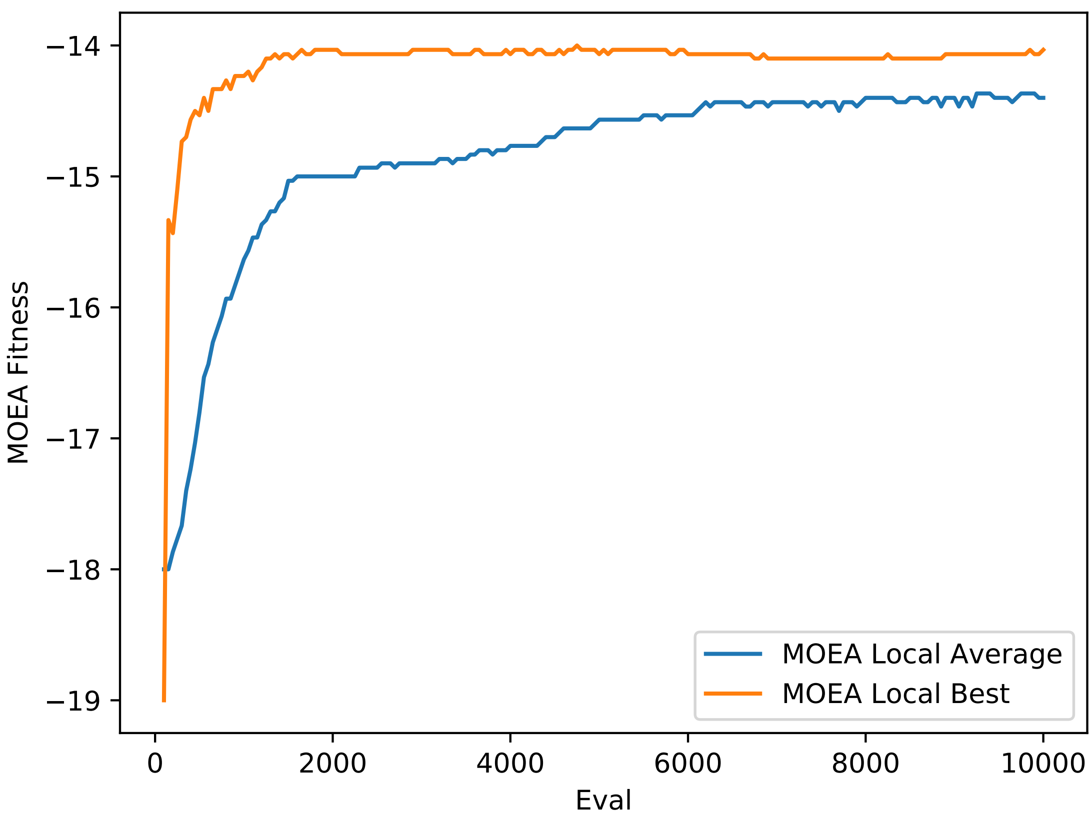

Figure 54: Figure 52 Representation

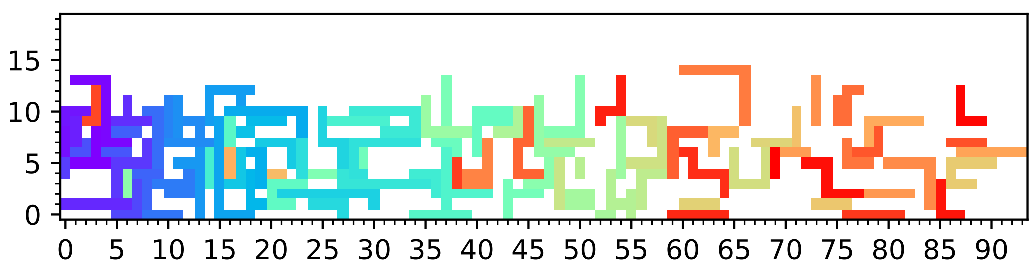

Figure 55: Input 2 Length Fitness

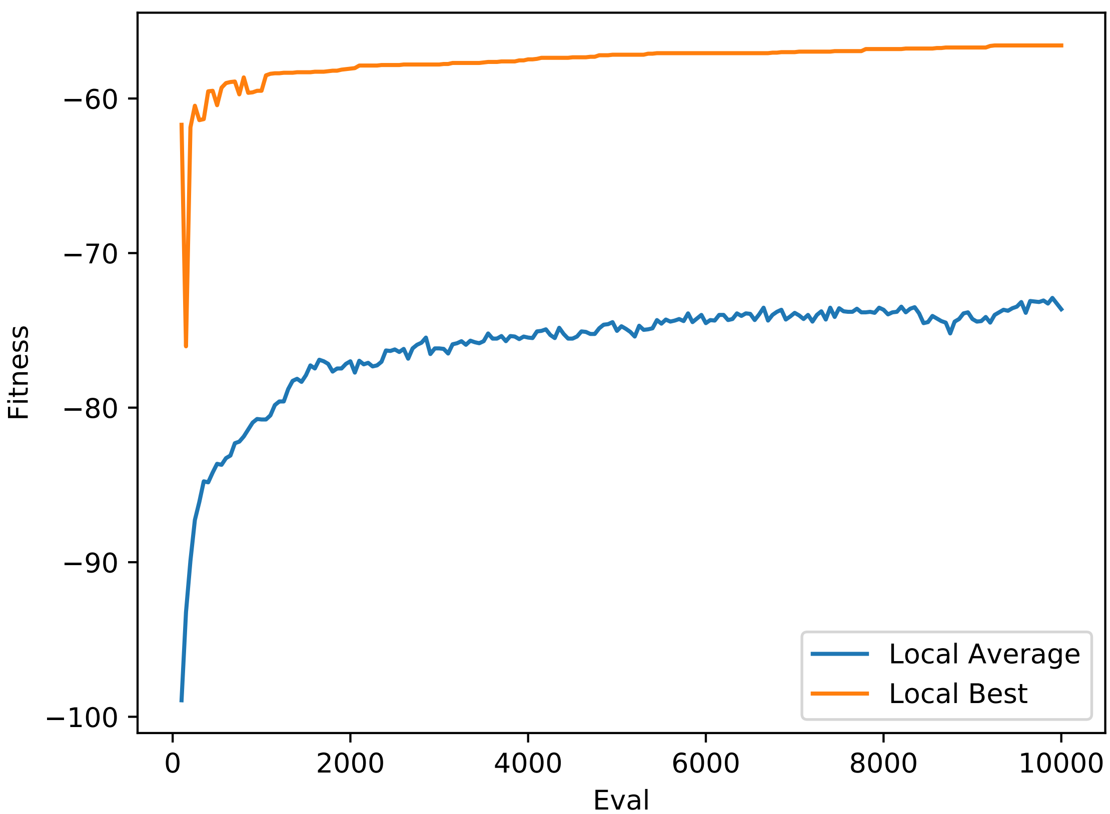

Figure 56: Input 2 Width Fitness

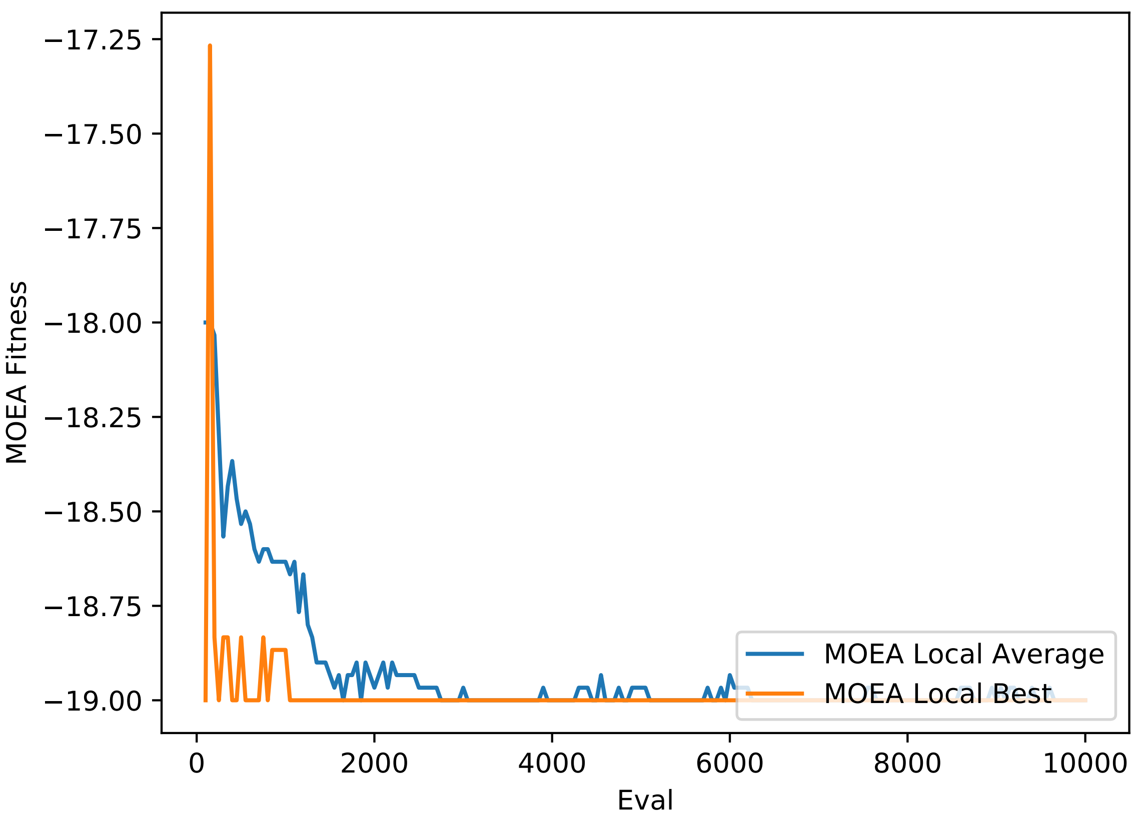

Figure 57: Figure 55 Representation

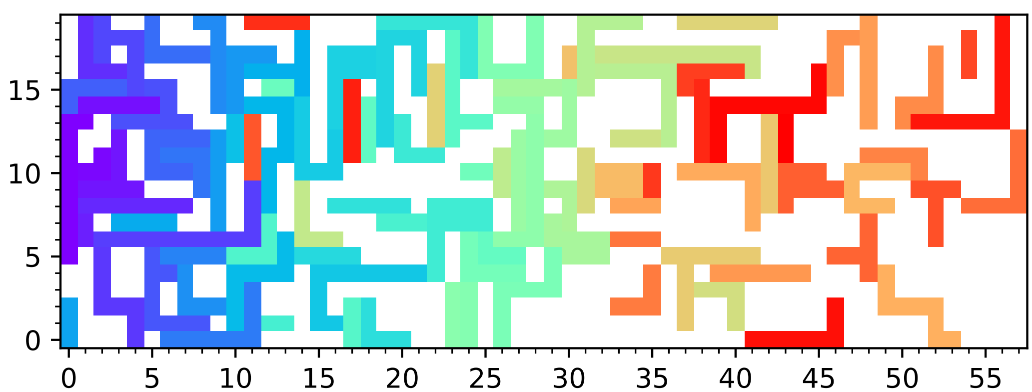

Table 12: Figures 58 and 59 Configuration File

Search Algorithm	EA
Mutation Algorithm	Flip
Penalty Coefficient	1
Population Size	100
Random Seed	2006
Parent Selection Algorithm	Uniform Random
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Table 13: Figures 61 and 62 Configuration File

Search Algorithm	EA
Mutation Algorithm	Move
Penalty Coefficient	1
Population Size	100
Random Seed	3001
Parent Selection Algorithm	k-Tournament Selection with replacement
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Figure 58: Input 2 Length Fitness

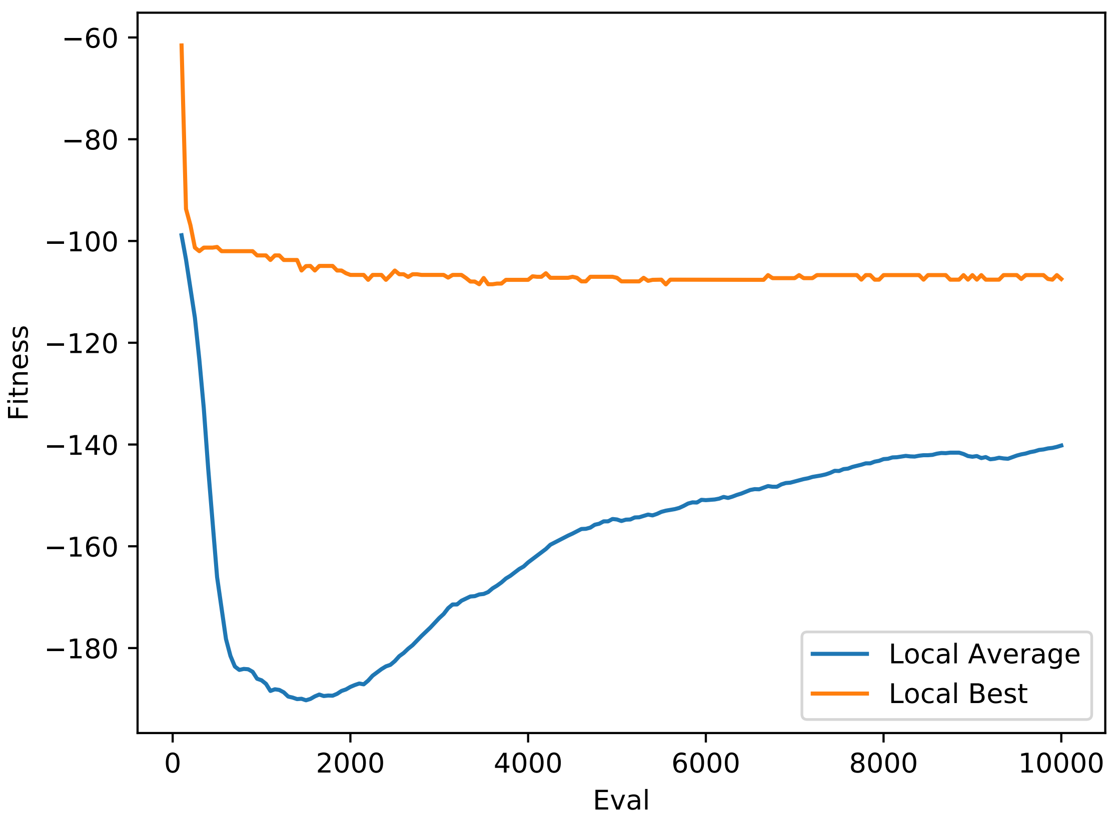

Figure 59: Input 2 Width Fitness

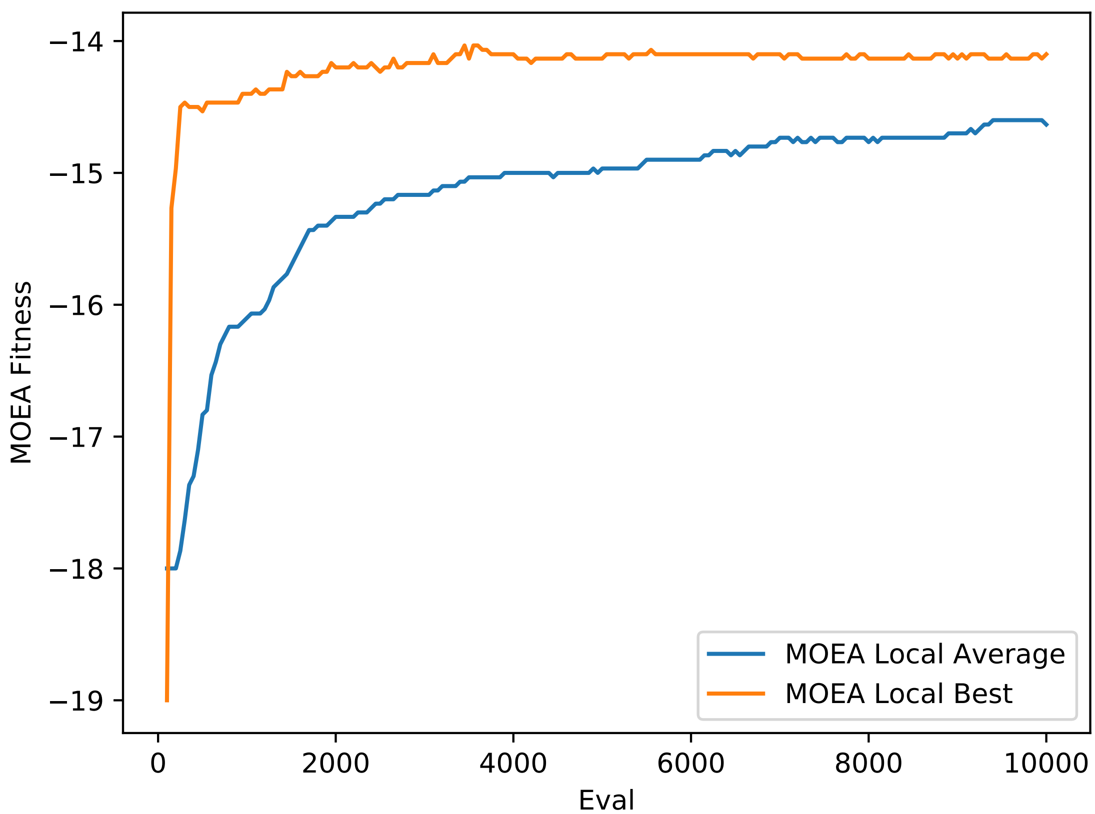

Figure 60: Figure 58 Representation

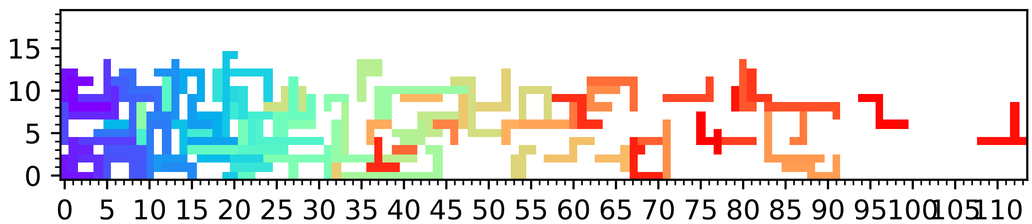

Figure 61: Input 3 Length Fitness

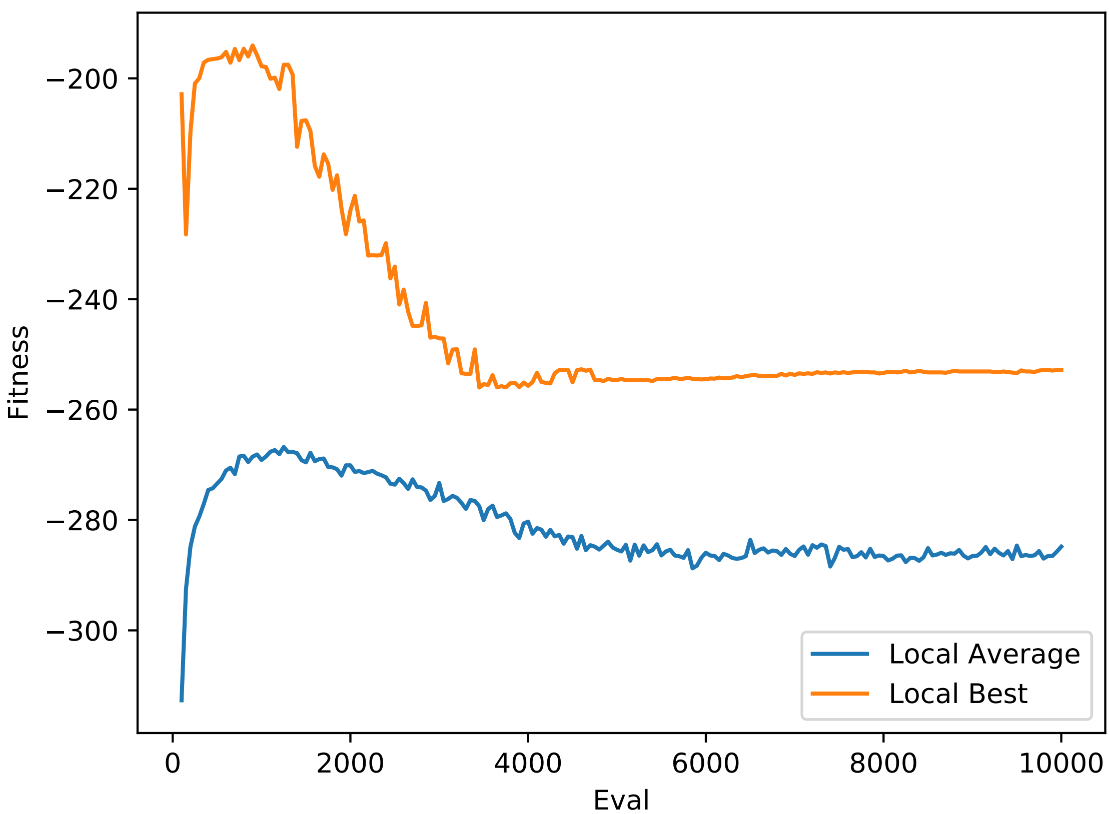

Figure 62: Input 3 Width Fitness

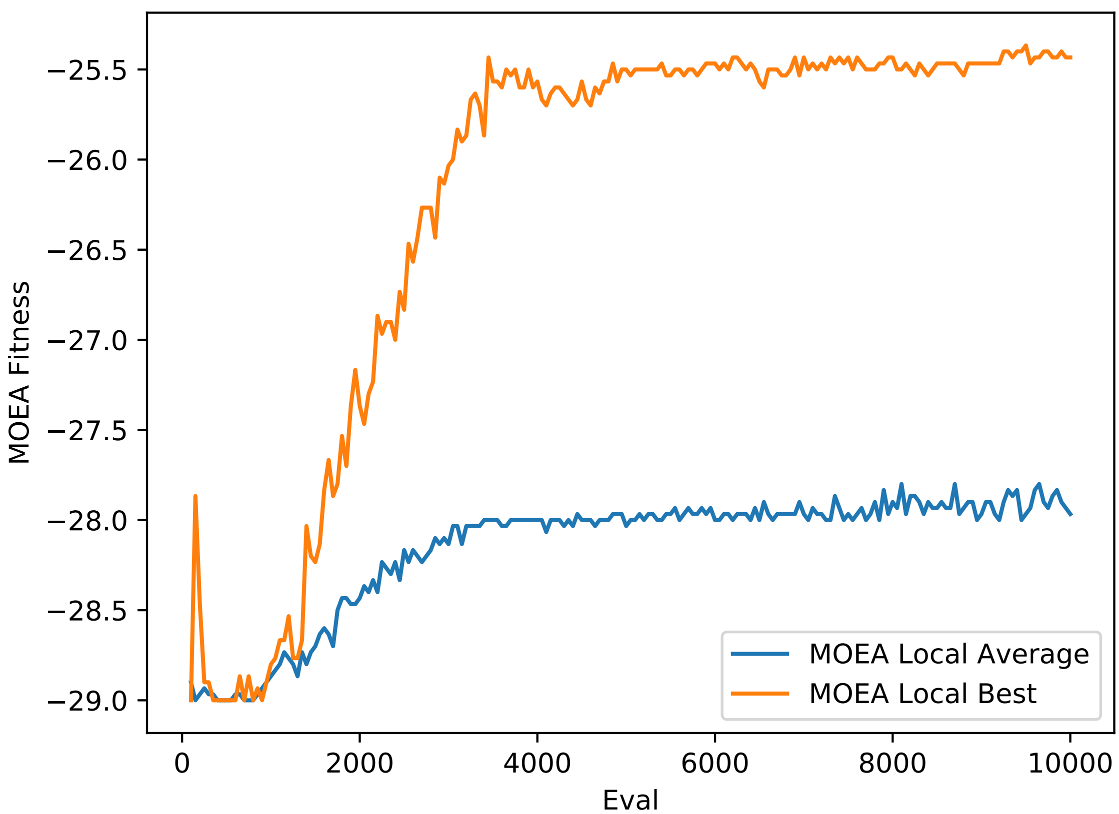

Figure 63: Figure 61 Representation

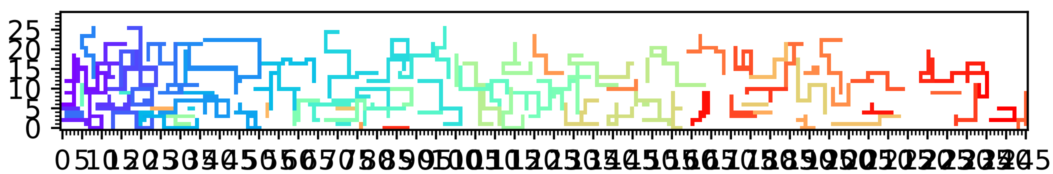

Table 14: Figures 64 and 65 Configuration File

Search Algorithm	EA
Mutation Algorithm	Flip
Penalty Coefficient	1
Population Size	100
Random Seed	3002
Parent Selection Algorithm	k-Tournament Selection with replacement
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Table 15: Figures 67 and 68 Configuration File

Search Algorithm	EA
Mutation Algorithm	Move
Penalty Coefficient	1
Population Size	100
Random Seed	3003
Parent Selection Algorithm	Fitness Proportional Selection
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Figure 64: Input 3 Length Fitness

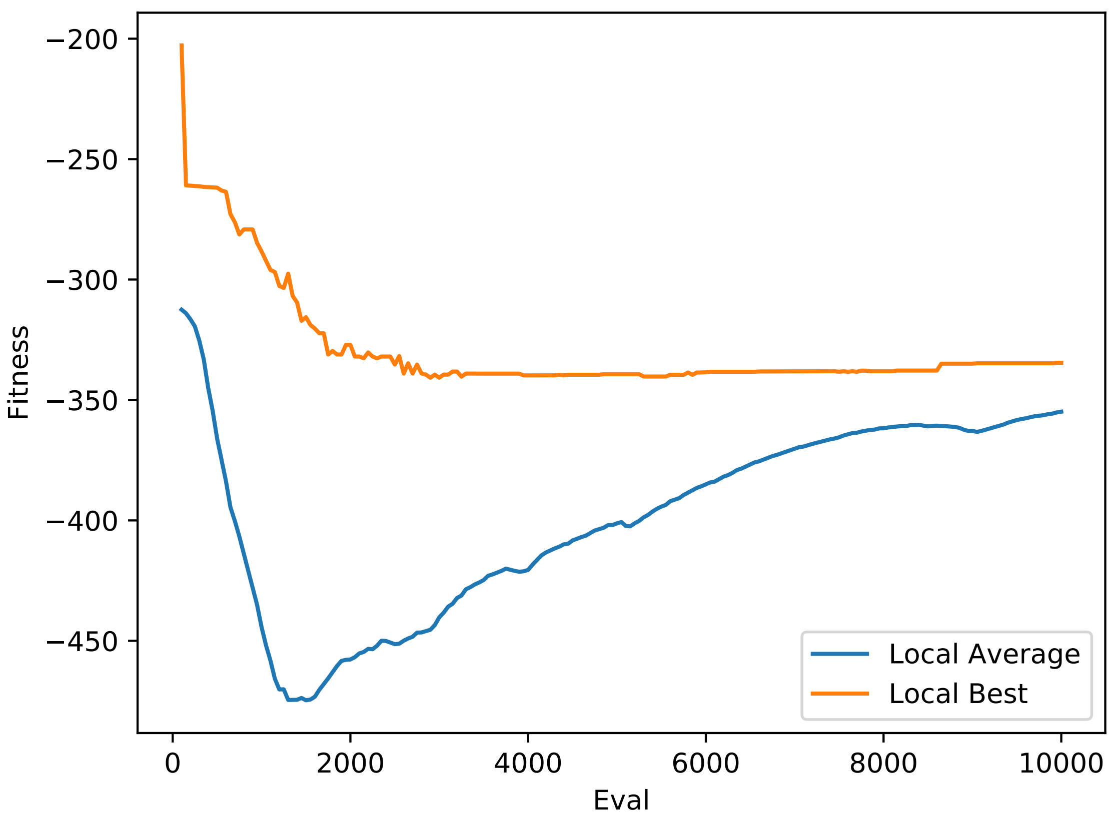

Figure 65: Input 3 Width Fitness

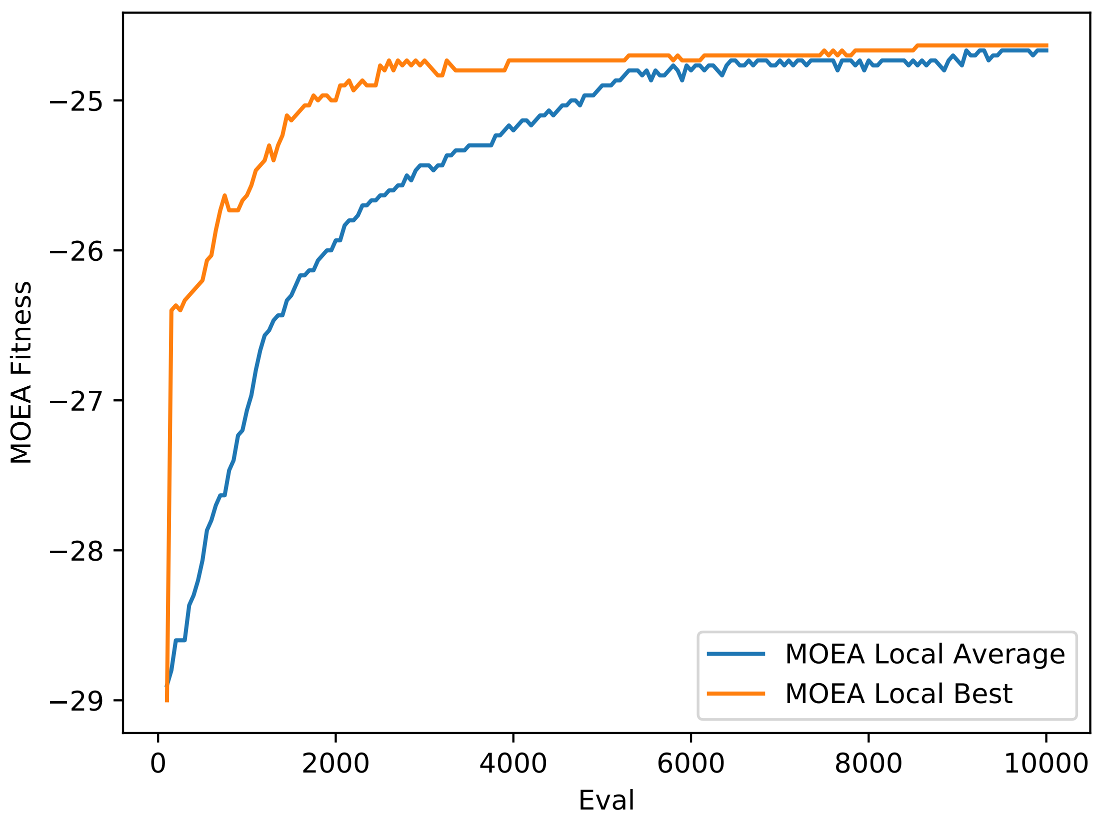

Figure 66: Figure 64 Representation

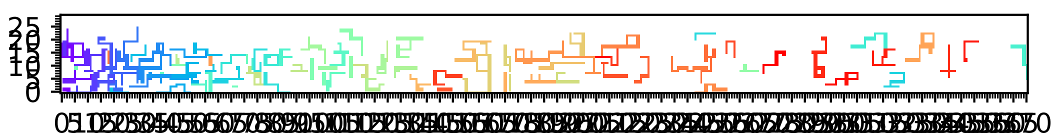

Figure 67: Input 3 Length Fitness

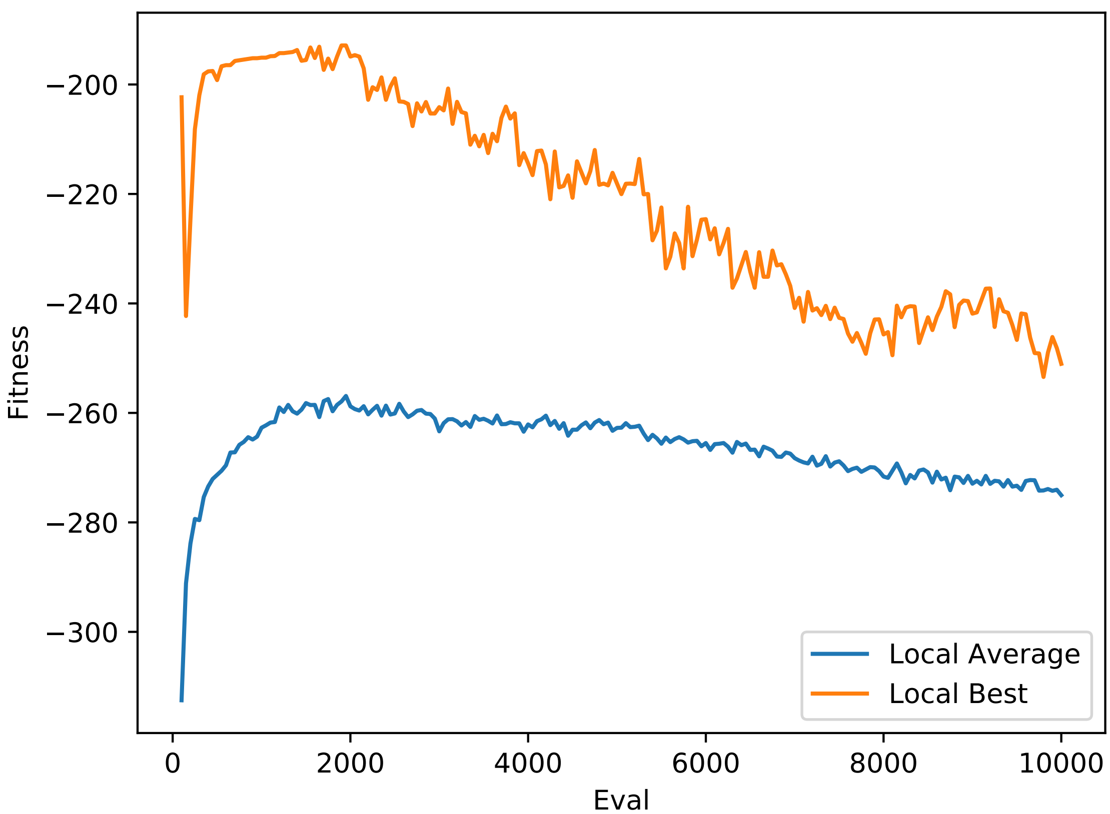

Figure 68: Input 3 Width Fitness

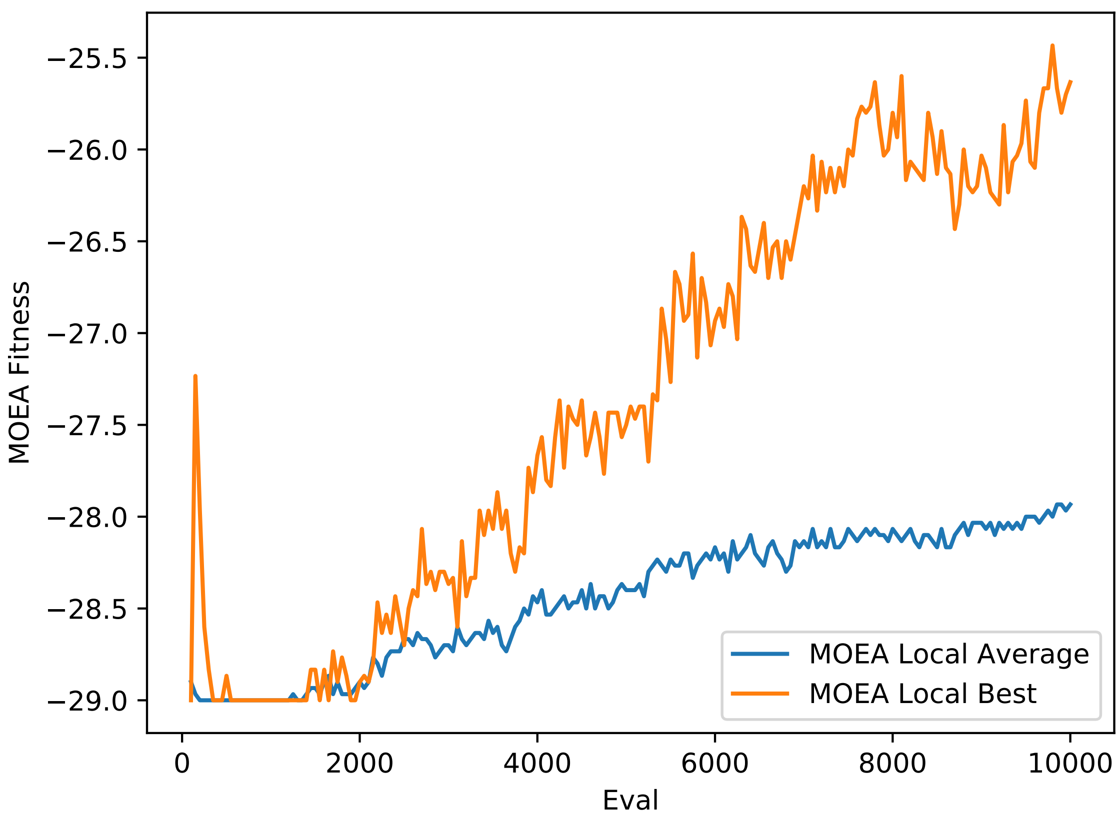

Figure 69: Figure 67 Representation

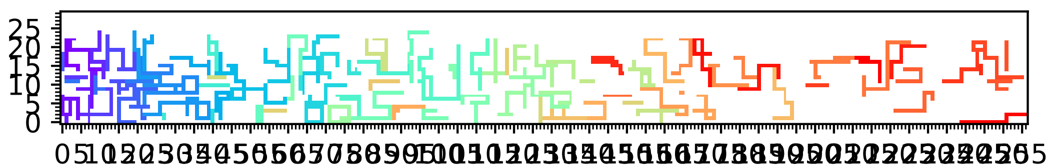

Table 16: Figures 70 and 71 Configuration File

Search Algorithm	EA
Mutation Algorithm	Flip
Penalty Coefficient	1
Population Size	100
Random Seed	3004
Parent Selection Algorithm	Fitness Proportional Selection
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Table 17: Figures 73 and 74 Configuration File

Search Algorithm	EA
Mutation Algorithm	Move
Penalty Coefficient	1
Population Size	100
Random Seed	3005
Parent Selection Algorithm	Uniform Random
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Figure 70: Input 3 Length Fitness

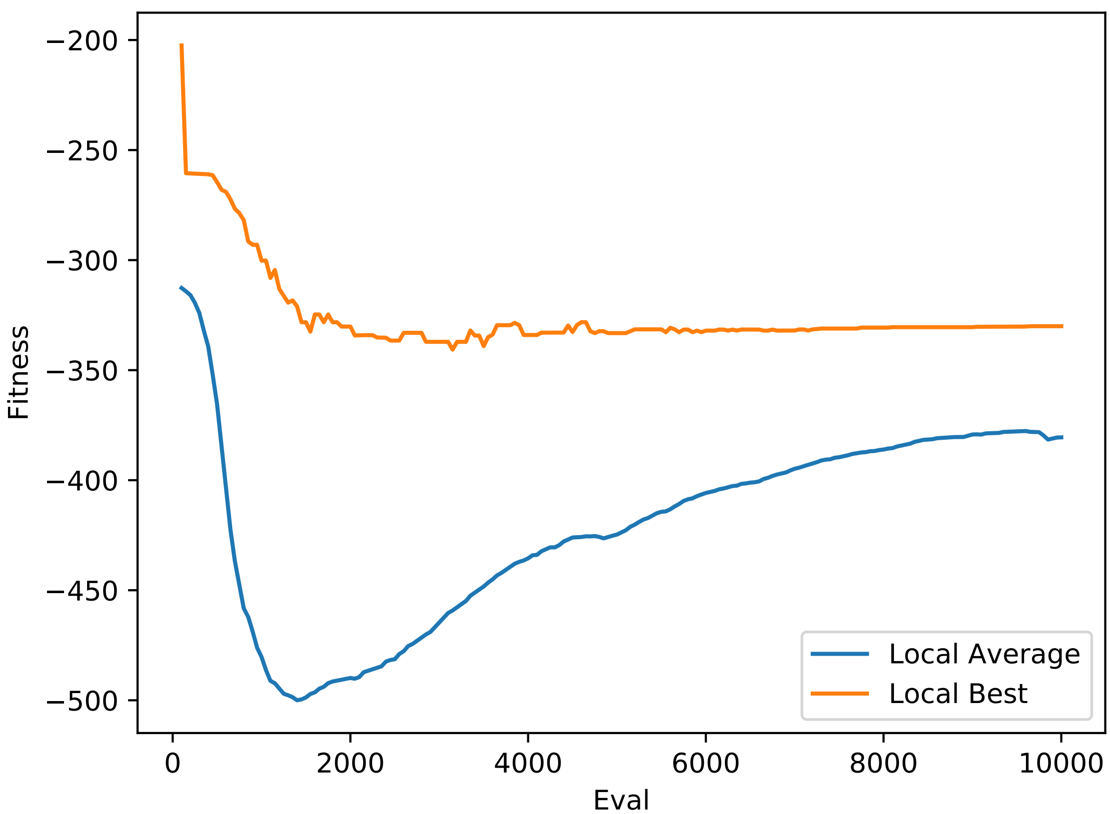

Figure 71: Input 3 Width Fitness

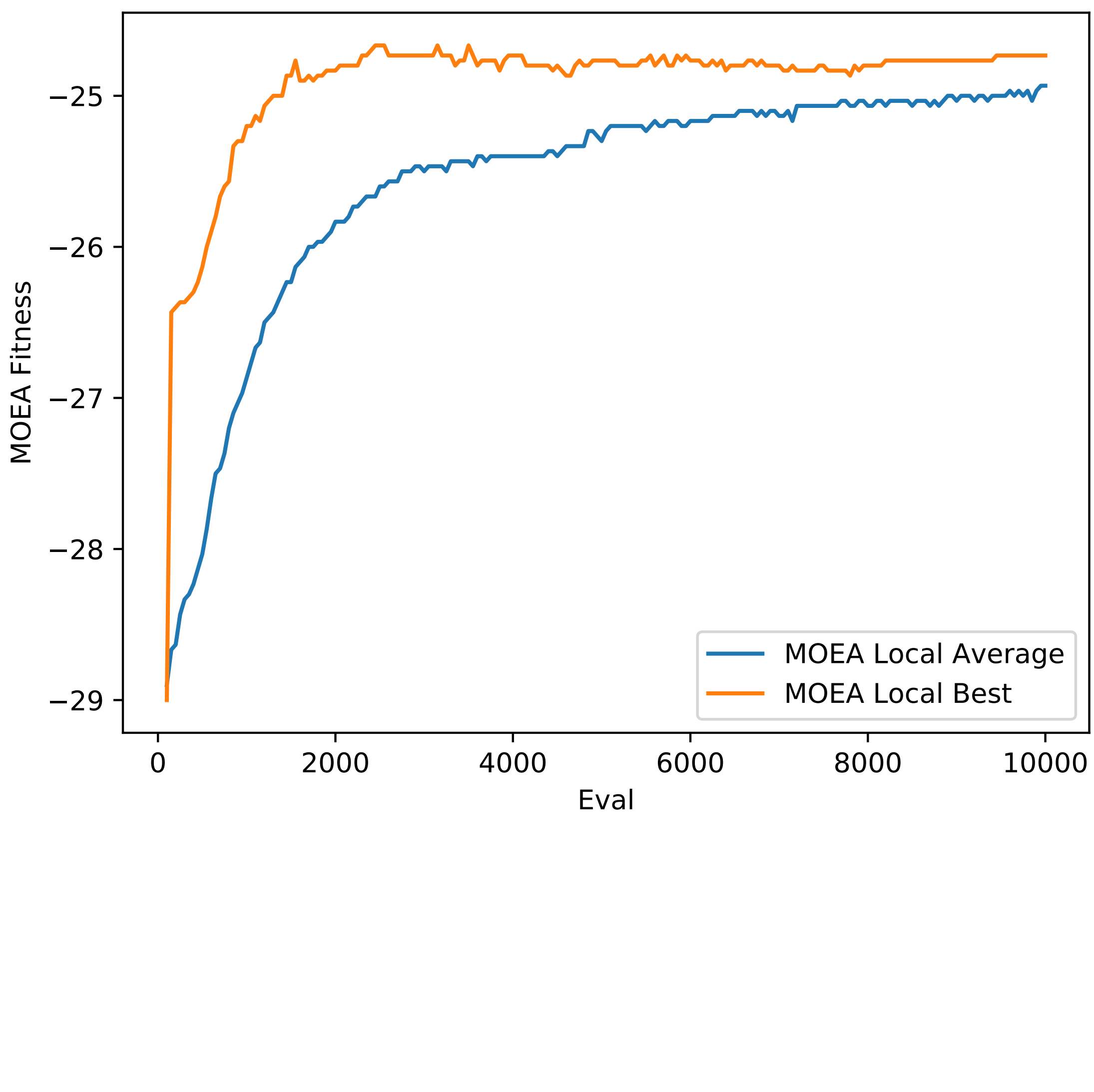

Figure 72: Figure 70 Representation

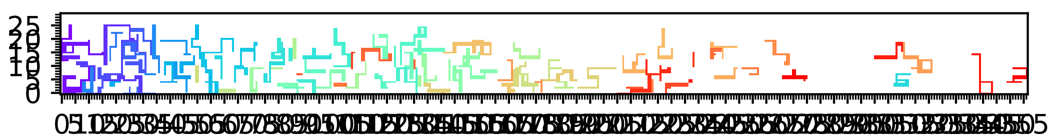

Figure 73: Input 3 Length Fitness

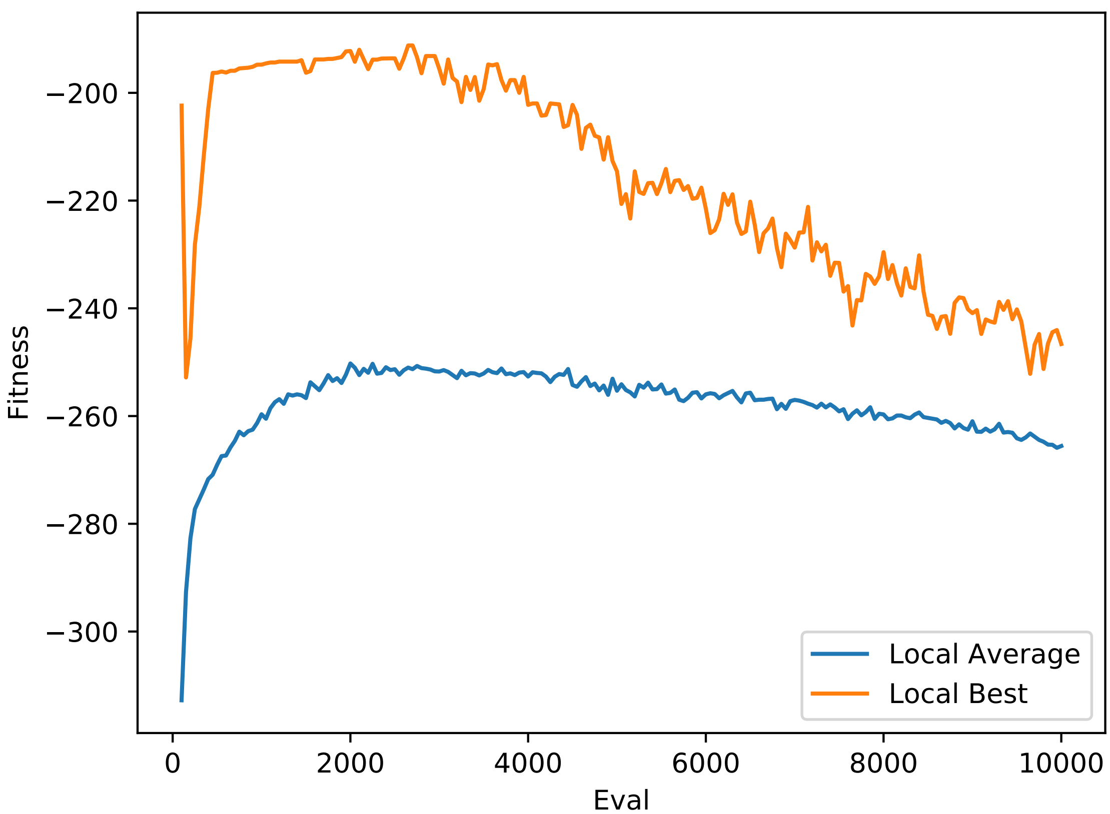

Figure 74: Input 3 Width Fitness

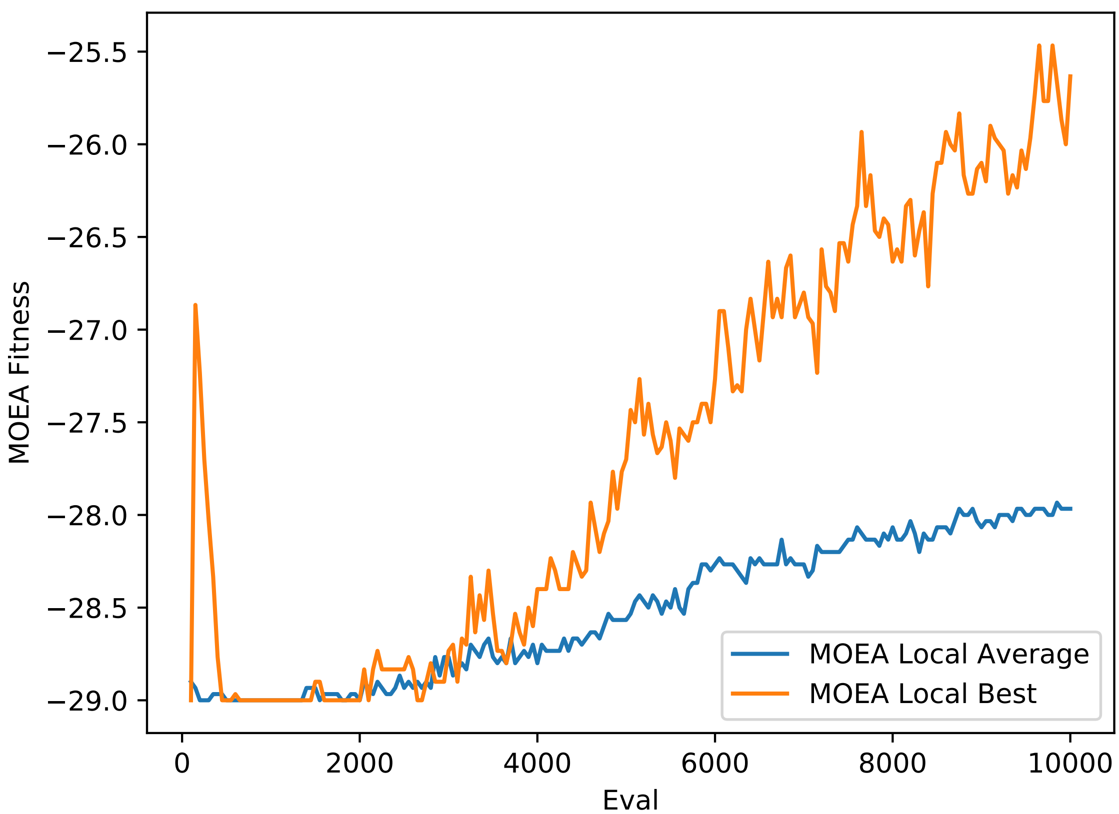

Figure 75: Figure 73 Representation

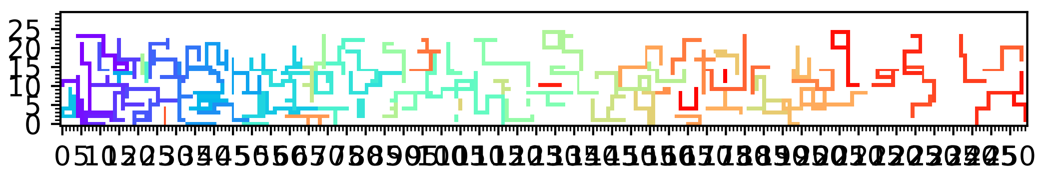

Table 18: Figures 76 and 77 Configuration File

Search Algorithm	EA
Mutation Algorithm	Flip
Penalty Coefficient	1
Population Size	100
Random Seed	3006
Parent Selection Algorithm	Uniform Random
Placement Algorithm	Random
Runs	30
Self Adaptive Offspring Count	False
Tournament Size For Survival Selection	5
Mutation Rate	0.1
Solution File Path	None
Survival Strategy	Plus
Termination Convergence Criterion	10000
Self Adaptive Penalty Coefficient	False
MOEA	True
Log File Path	None
Offspring Count	50
Recombination Algorithm	Partially Mapped Crossover
Self Adaptive Mutation Rate	False
Fitness Evaluations	10000
Survivor Algorithm	Truncation
Tournament Size For Parent Selection	5

Figure 76: Input 3 Length Fitness

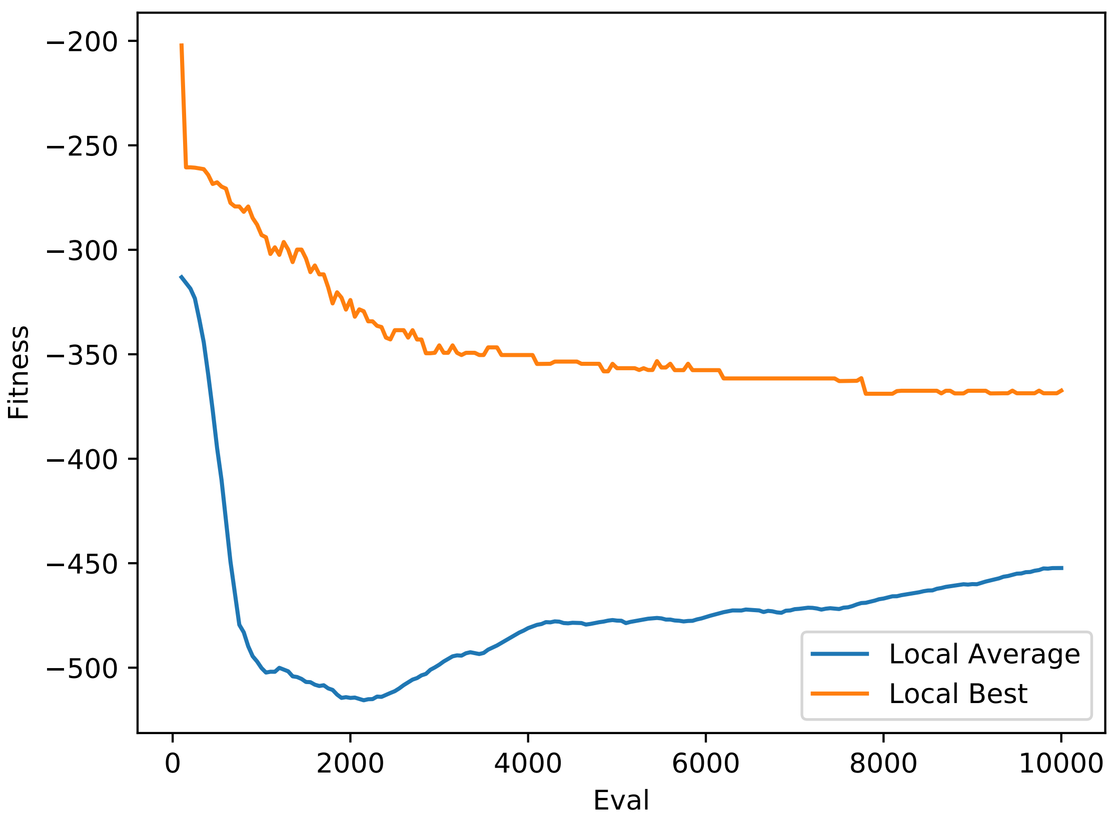

Figure 77: Input 3 Width Fitness

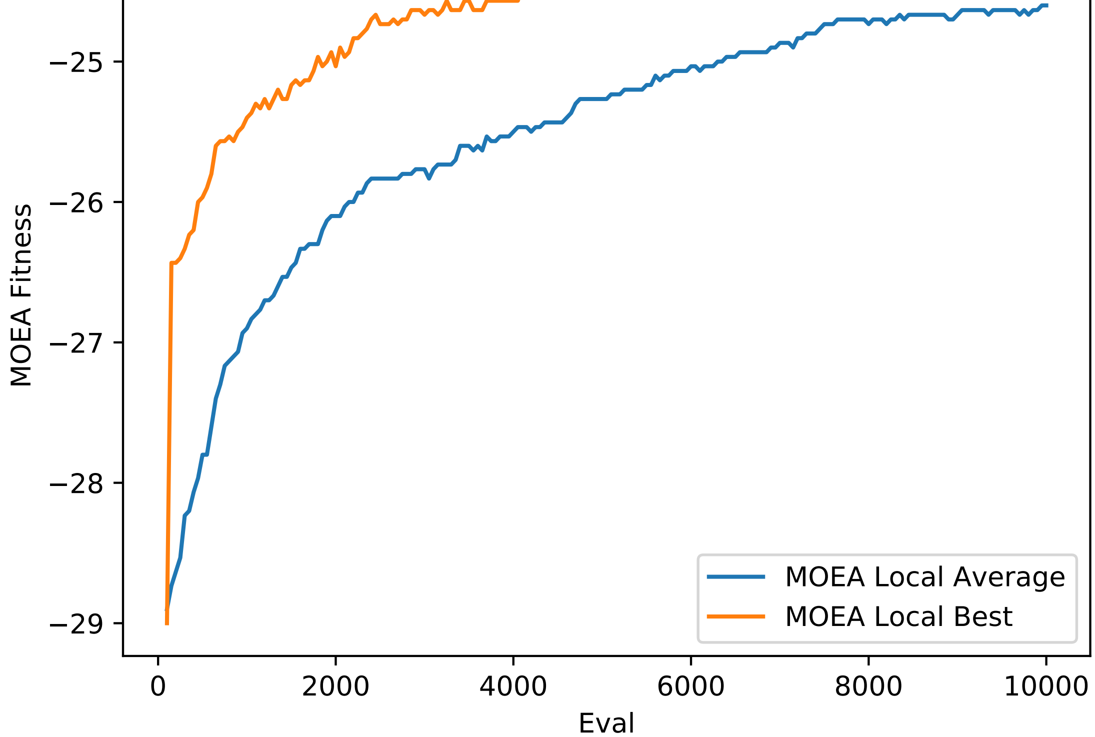

Figure 78: Figure 76 Representation