13. $\vec{T}(t) = \langle -\sin t, \cos t\rangle$; $\vec{N}(t) = \langle -\cos t, -\sin t\rangle$

15. $\vec{T}(t) = \left\langle -\frac{\sin t}{\sqrt{4\cos^2 t+\sin^2 t}}, \frac{2\cos t}{\sqrt{4\cos^2 t+\sin^2 t}} \right\rangle$;
$\vec{N}(t) = \left\langle -\frac{2\cos t}{\sqrt{4\cos^2 t+\sin^2 t}}, -\frac{\sin t}{\sqrt{4\cos^2 t+\sin^2 t}} \right\rangle$

17. (a) Be sure to show work
(b) $\vec{N}(\pi/4) = \langle -5/\sqrt{34}, -3/\sqrt{34}\rangle$

19. (a) Be sure to show work
(b) $\vec{N}(0) = \left\langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\rangle$

21. $\vec{T}(t) = \frac{1}{\sqrt{5}}\langle 2, \cos t, -\sin t\rangle$; $\vec{N}(t) = \langle 0, -\sin t, -\cos t\rangle$

23. $\vec{T}(t) = \frac{1}{\sqrt{a^2+b^2}}\langle -a\sin t, a\cos t, b\rangle$; $\vec{N}(t) = \langle -\cos t, -\sin t, 0\rangle$

25. $a_\text{T} = \frac{4t}{\sqrt{1+4t^2}}$ and $a_\text{N} = \sqrt{4 - \frac{16t^2}{1+4t^2}}$
At $t=0$, $a_\text{T} = 0$ and $a_\text{N} = 2$;
At $t=1$, $a_\text{T} = 4/\sqrt{5}$ and $a_\text{N} = 2/\sqrt{5}$.
At $t=0$, all acceleration comes in the form of changing the direction of velocity and not the speed; at $t=1$, more acceleration comes in changing the speed than in changing direction.

27. $a_\text{T} = 0$ and $a_\text{N} = 2$
At $t=0$, $a_\text{T} = 0$ and $a_\text{N} = 2$;
At $t=\pi/2$, $a_\text{T} = 0$ and $a_\text{N} = 2$.
The object moves at constant speed, so all acceleration comes from changing direction, hence $a_\text{T} = 0$. $\vec{a}(t)$ is always parallel to $\vec{N}(t)$, but twice as long, hence $a_\text{N} = 2$.

29. $a_\text{T} = 0$ and $a_\text{N} = a$
At $t=0$, $a_\text{T} = 0$ and $a_\text{N} = a$;
At $t=\pi/2$, $a_\text{T} = 0$ and $a_\text{N} = a$.
The object moves at constant speed, meaning that a_T is always 0. The object "rises" along the z-axis at a constant rate, so all acceleration comes in the form of changing direction circling the z-axis. The greater the radius of this circle the greater the acceleration, hence $a_\text{N} = a$.

Section 11.5

1. time and/or distance

3. Answers may include lines, circles, helixes

5. κ

7. $s = 3t$, so $\vec{r}(s) = \langle 2s/3, s/3, -2s/3\rangle$

9. $s = \sqrt{13}t$, so $\vec{r}(s) = \langle 3\cos(s/\sqrt{13}), 3\sin(s/\sqrt{13}), 2s/\sqrt{13}\rangle$

11. $\kappa = \frac{|6x|}{\left(1+(3x^2-1)^2\right)^{3/2}}$;
$\kappa(0) = 0$, $\kappa(1/2) = \frac{192}{17\sqrt{17}} \approx 2.74$.

13. $\kappa = \frac{|\cos x|}{\left(1+\sin^2 x\right)^{3/2}}$;
$\kappa(0) = 1$, $\kappa(\pi/2) = 0$

15. $\kappa = \frac{|2\cos t\cos(2t)+4\sin t\sin(2t)|}{\left(4\cos^2(2t)+\sin^2 t\right)^{3/2}}$;
$\kappa(0) = 1/4$, $\kappa(\pi/4) = 8$

17. $\kappa = \frac{|6t^2+2|}{\left(4t^2+(3t^2-1)^2\right)^{3/2}}$;
$\kappa(0) = 2$, $\kappa(5) = \frac{19}{1394\sqrt{1394}} \approx 0.0004$

19. $\kappa = 0$;
$\kappa(0) = 0$, $\kappa(1) = 0$

21. $\kappa = \frac{3}{13}$;
$\kappa(0) = 3/13$, $\kappa(\pi/2) = 3/13$

23. maximized at $x = \pm\frac{\sqrt{2}}{\sqrt[4]{5}}$

25. maximized at $t = 1/4$

27. radius of curvature is $5\sqrt{5}/4$.

29. radius of curvature is 9.

31. $x^2 + (y-1/2)^2 = 1/4$, or $\vec{c}(t) = \langle 1/2\cos t, 1/2\sin t + 1/2\rangle$

33. $x^2 + (y+8)^2 = 81$, or $\vec{c}(t) = \langle 9\cos t, 9\sin t - 8\rangle$

Chapter 12

Section 12.1

1. Answers will vary.

3. topographical

5. surface

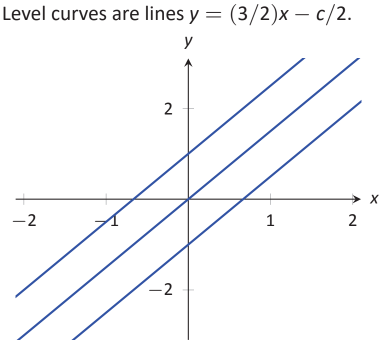
7. domain: $\mathbb{R}^2$
range: $z \geq 2$

9. domain: $\mathbb{R}^2$
range: $\mathbb{R}$

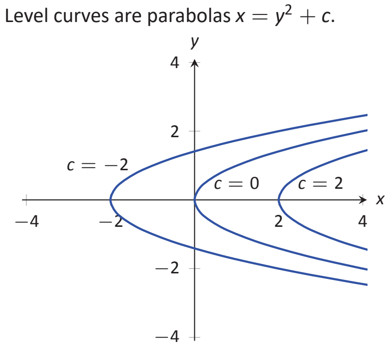
11. domain: $\mathbb{R}^2$
range: $0 < z \leq 1$

13. domain: $\{(x,y) \mid x^2+y^2 \leq 9\}$, i.e., the domain is the circle and interior of a circle centered at the origin with radius 3.
range: $0 \leq z \leq 3$

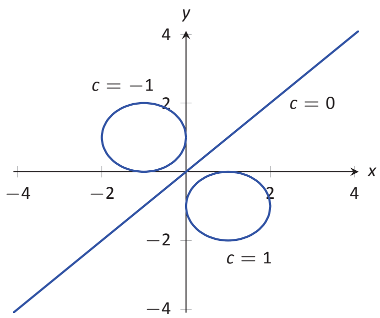
15. Level curves are lines $y = (3/2)x - c/2$.



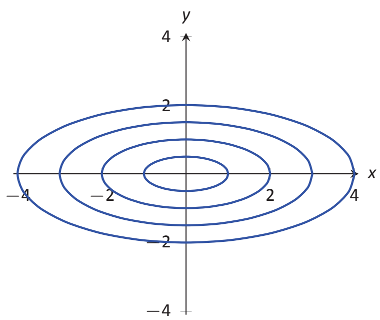
17. Level curves are parabolas $x = y^2 + c$.



19. When $c \neq 0$, the level curves are circles, centered at $(1/c, -1/c)$ with radius $\sqrt{2/c^2 - 1}$. When $c = 0$, the level curve is the line $y = x$.



21. Level curves are ellipses of the form $\frac{x^2}{c^2}+\frac{y^2}{c^2/4}=1$, i.e., $a=c$ and $b=c/2$.



23. domain: $x+2y-4z\neq 0$; the set of points in $\mathbb{R}^3$ NOT in the domain form a plane through the origin.
range: $\mathbb{R}$

25. domain: $z\geq x^2-y^2$; the set of points in $\mathbb{R}^3$ above (and including) the hyperbolic paraboloid $z=x^2-y^2$.
range: $[0,\infty)$

27. The level surfaces are spheres, centered at the origin, with radius $\sqrt{c}$.

29. The level surfaces are paraboloids of the form $z=\frac{x^2}{c}+\frac{y^2}{c}$; the larger c, the "wider" the paraboloid.

31. The level curves for each surface are similar; for $z=\sqrt{x^2+4y^2}$ the level curves are ellipses of the form $\frac{x^2}{c^2}+\frac{y^2}{c^2/4}=1$, i.e., $a=c$ and $b=c/2$; whereas for $z=x^2+4y^2$ the level curves are ellipses of the form $\frac{x^2}{c}+\frac{y^2}{c/4}=1$, i.e., $a=\sqrt{c}$ and $b=\sqrt{c}/2$. The first set of ellipses are spaced evenly apart, meaning the function grows at a constant rate; the second set of ellipses are more closely spaced together as c grows, meaning the function grows faster and faster as c increases.
The function $z=\sqrt{x^2+4y^2}$ can be rewritten as $z^2=x^2+4y^2$, an elliptic cone; the function $z=x^2+4y^2$ is a paraboloid, each matching the description above.

Section 12.2

1. Answers will vary.

3. Answers will vary.
One possible answer: $\{(x,y)|x^2+y^2\leq 1\}$

5. Answers will vary.
One possible answer: $\{(x,y)|x^2+y^2<1\}$

7. (a) Answers will vary.
interior point: $(1,3)$
boundary point: $(3,3)$
(b) S is a closed set
(c) S is bounded

9. (a) Answers will vary.
interior point: none
boundary point: $(0,-1)$
(b) S is a closed set, consisting only of boundary points
(c) S is bounded

11. (a) $D=\{(x,y)\mid 9-x^2-y^2\geq 0\}$.
(b) D is a closed set.
(c) D is bounded.

13. (a) $D=\{(x,y)\mid y>x^2\}$.
(b) D is an open set.
(c) D is unbounded.

15. (a) Along $y=0$, the limit is 1.
(b) Along $x=0$, the limit is -1.
Since the above limits are not equal, the limit does not exist.

17. (a) Along $y=mx$, the limit is $\dfrac{mx(1-m)}{m^2x+1}$.
(b) Along $x=0$, the limit is -1.
Since the above limits are not equal, the limit does not exist.

19. (a) Along $y=2$, the limit is:
$$\begin{aligned}\lim_{(x,y)\to(1,2)}\frac{x+y-3}{x^2-1}&=\lim_{x\to 1}\frac{x-1}{x^2-1}\\&=\lim_{x\to 1}\frac{1}{x+1}\\&=1/2.\end{aligned}$$
(b) Along $y=x+1$, the limit is:
$$\begin{aligned}\lim_{(x,y)\to(1,2)}\frac{x+y-3}{x^2-1}&=\lim_{x\to 1}\frac{2(x-1)}{x^2-1}\\&=\lim_{x\to 1}\frac{2}{x+1}\\&=1.\end{aligned}$$
Since the limits along the lines $y=2$ and $y=x+1$ differ, the overall limit does not exist.

21. -2

23. The limit does not exist.

Section 12.3

1. A constant is a number that is added or subtracted in an expression; a coefficient is a number that is being multiplied by a nonconstant function.

3. f_x

5. $f_x=2xy-1, f_y=x^2+2$
$f_x(1,2)=3, f_y(1,2)=3$

7. $f_x=-\sin x\sin y, f_y=\cos x\cos y$
$f_x(\pi/3,\pi/3)=-3/4, f_y(\pi/3,\pi/3)=1/4$

9. $f_x=2xy+6x, f_y=x^2+4$
$f_{xx}=2y+6, f_{yy}=0$
$f_{xy}=2x, f_{yx}=2x$

11. $f_x=1/y, f_y=-x/y^2$
$f_{xx}=0, f_{yy}=2x/y^3$
$f_{xy}=-1/y^2, f_{yx}=-1/y^2$

13. $f_x=2xe^{x^2+y^2}, f_y=2ye^{x^2+y^2}$
$f_{xx}=2e^{x^2+y^2}+4x^2e^{x^2+y^2}, f_{yy}=2e^{x^2+y^2}+4y^2e^{x^2+y^2}$
$f_{xy}=4xye^{x^2+y^2}, f_{yx}=4xye^{x^2+y^2}$

15. $f_x=\cos x\cos y, f_y=-\sin x\sin y$
$f_{xx}=-\sin x\cos y, f_{yy}=-\sin x\cos y$
$f_{xy}=-\sin y\cos x, f_{yx}=-\sin y\cos x$

17. $f_x = -5y^3 \sin(5xy^3)$, $f_y = -15xy^2 \sin(5xy^3)$
$f_{xx} = -25y^6 \cos(5xy^3)$,
$f_{yy} = -225x^2y^4 \cos(5xy^3) - 30xy \sin(5xy^3)$
$f_{xy} = -75xy^5 \cos(5xy^3) - 15y^2 \sin(5xy^3)$,
$f_{yx} = -75xy^5 \cos(5xy^3) - 15y^2 \sin(5xy^3)$

19. $f_x = \frac{2y^2}{\sqrt{4xy^2+1}}$, $f_y = \frac{4xy}{\sqrt{4xy^2+1}}$
$f_{xx} = -\frac{4y^4}{\sqrt{4xy^2+1}^3}$, $f_{yy} = -\frac{16x^2y^2}{\sqrt{4xy^2+1}^3} + \frac{4x}{\sqrt{4xy^2+1}}$
$f_{xy} = -\frac{8xy^3}{\sqrt{4xy^2+1}^3} + \frac{4y}{\sqrt{4xy^2+1}}$, $f_{yx} = -\frac{8xy^3}{\sqrt{4xy^2+1}^3} + \frac{4y}{\sqrt{4xy^2+1}}$

21. $f_x = -\frac{2x}{(x^2+y^2+1)^2}$, $f_y = -\frac{2y}{(x^2+y^2+1)^2}$
$f_{xx} = \frac{8x^2}{(x^2+y^2+1)^3} - \frac{2}{(x^2+y^2+1)^2}$, $f_{yy} = \frac{8y^2}{(x^2+y^2+1)^3} - \frac{2}{(x^2+y^2+1)^2}$
$f_{xy} = \frac{8xy}{(x^2+y^2+1)^3}$, $f_{yx} = \frac{8xy}{(x^2+y^2+1)^3}$

23. $f_x = yx^{y-1}$, $f_y = x^y \ln x$
$f_{xx} = y(y-1)x^{y-2}$, $f_{yy} = x^y(\ln x)^2$
$f_{xy} = x^{y-1}(1 + y\ln x)$, $f_{yx} = x^{y-1}(1 + y \ln x)$

25. $f_x = \frac{2x}{(x^2+y)}$, $f_y = \frac{1}{(x^2+y)}$
$f_{xx} = -\frac{4x^2}{(x^2+y)^2} + \frac{2}{(x^2+y)}$, $f_{yy} = -\frac{1}{(x^2+y)^2}$
$f_{xy} = -\frac{2x}{(x^2+y)^2}$, $f_{yx} = -\frac{2x}{(x^2+y)^2}$

27. $f_x = 5 + (2+\cos y)x^{1+\cos y}$, $f_y = -x^{2+\cos y} \ln x \sin y$
$f_{xx} = (2+\cos y)(1+\cos y)x^{\cos y}$,
$f_{yy} = x^{2+\cos y} \ln x \left(\ln x \sin^2 y - \cos y\right)$
$f_{xy} = -\sin y x^{1+\cos y}\left(1 + (2+\cos y)\ln x\right)$,
$f_{yx} = -\sin y x^{1+\cos y}\left(1 + (2+\cos y)\ln x\right)$

29. $f(x, y) = x \sin y + x + C$, where C is any constant.

31. $f(x, y) = \frac{1}{2}x^2 + xy + \frac{1}{2}y^2 + C$, where C is any constant.

33. No possible function $f(x, y)$ exists.

35. $f(x, y) = \ln(x^2 + y^2) + C$, where C is any constant.

37. $f_x = 3x^2y^2 + 3x^2z$, $f_y = 2x^3y + 2yz$, $f_z = x^3 + y^2$
$f_{yz} = 2y$, $f_{zy} = 2y$

39. $f_x = \frac{1}{x}$, $f_y = \frac{1}{y}$, $f_z = \frac{1}{z}$
$f_{yz} = 0$, $f_{zy} = 0$

Section 12.4

1. T

3. T

5. $dz = (\sin y + 2x)dx + (x\cos y)dy$

7. $dz = 5dx - 7dy$

9. $dz = \frac{x}{\sqrt{x^2+y}}dx + \frac{1}{2\sqrt{x^2+y}}dy$, with $dx = -0.05$ and $dy = .1$. At $(3, 7)$, $dz = 3/4(-0.05) + 1/8(.1) = -0.025$, so $f(2.95, 7.1) \approx -0.025 + 4 = 3.975$.

11. $dz = (2xy - y^2)dx + (x^2 - 2xy)dy$, with $dx = 0.04$ and $dy = 0.06$. At $(2, 3)$, $dz = 3(0.04) + (-8)(0.06) = -0.36$, so $f(2.04, 3.06) \approx -0.36 - 6 = -6.36$.

13. The total differential of volume is $dV = 4\pi dr + \pi dh$. The coefficient of dr is greater than the coefficient of dh, so the volume is more sensitive to changes in the radius.

15. Using trigonometry, $\ell = x \tan\theta$, so $d\ell = \tan\theta dx + x\sec^2\theta d\theta$. With $\theta = 85^\circ$ and $x = 30$, we have $d\ell = 11.43dx + 3949.38d\theta$. The measured length of the wall is much more sensitive to errors in θ than in x. While it can be difficult to compare sensitivities between measuring feet and measuring degrees (it is somewhat like "comparing apples to oranges"), here the coefficients are so different that the result is clear: a small error in degree has a much greater impact than a small error in distance.

17. $dw = 2xyz^3\,dx + x^2z^3\,dy + 3x^2yz^2\,dz$

19. $dx = 0.05$, $dy = -0.1$. $dz = 9(.05) + (-2)(-0.1) = 0.65$. So $f(3.05, 0.9) \approx 7 + 0.65 = 7.65$.

21. $dx = 0.5$, $dy = 0.1$, $dz = -0.2$.
$dw = 2(0.5) + (-3)(0.1) + 3.7(-0.2) = -0.04$, so
$f(2.5, 4.1, 4.8) \approx -1 - 0.04 = -1.04$.

Section 12.5

1. Because the parametric equations describe a level curve, z is constant for all t. Therefore $\frac{dz}{dt} = 0$.

3. $\frac{dx}{dt}$, and $\frac{\partial f}{\partial y}$

5. F

7. (a) $\frac{dz}{dt} = 3(2t) + 4(2) = 6t + 8$.
(b) At $t = 1$, $\frac{dz}{dt} = 14$.

9. (a) $\frac{dz}{dt} = 5(-2\sin t) + 2(\cos t) = -10\sin t + 2\cos t$
(b) At $t = \pi/4$, $\frac{dz}{dt} = -4\sqrt{2}$.

11. (a) $\frac{dz}{dt} = 2x(\cos t) + 4y(3\cos t)$.
(b) At $t = \pi/4$, $x = \sqrt{2}/2$, $y = 3\sqrt{2}/2$, and $\frac{dz}{dt} = 19$.

13. $t = -4/3$; this corresponds to a minimum

15. $t = \tan^{-1}(1/5) + n\pi$, where n is an integer

17. We find that
$$\frac{dz}{dt} = 38\cos t \sin t.$$
Thus $\frac{dz}{dt} = 0$ when $t = \pi n$ or $\pi n + \pi/2$, where n is any integer.

19. (a) $\frac{\partial z}{\partial s} = 2xy(1) + x^2(2) = 2xy + 2x^2$;
$\frac{\partial z}{\partial t} = 2xy(-1) + x^2(4) = -2xy + 4x^2$
(b) With $s = 1$, $t = 0$, $x = 1$ and $y = 2$. Thus $\frac{\partial z}{\partial s} = 6$ and $\frac{\partial z}{\partial t} = 0$

21. (a) $\frac{\partial z}{\partial s} = 2x(\cos t) + 2y(\sin t) = 2x\cos t + 2y \sin t$;
$\frac{\partial z}{\partial t} = 2x(-s\sin t) + 2y(s\cos t) = -2xs\sin t + 2ys\cos t$
(b) With $s = 2$, $t = \pi/4$, $x = \sqrt{2}$ and $y = \sqrt{2}$. Thus $\frac{\partial z}{\partial s} = 4$ and $\frac{\partial z}{\partial t} = 0$

23. $f_x = 2x\tan y$, $f_y = x^2\sec^2 y$;
$$\frac{dy}{dx} = -\frac{2\tan y}{x\sec^2 y}$$

25. $$f_x = \frac{(x+y^2)(2x) - (x^2+y)(1)}{(x+y^2)^2},$$
$$f_y = \frac{(x+y^2)(1) - (x^2+y)(2y)}{(x+y^2)^2};$$
$$\frac{dy}{dx} = -\frac{2x(x+y^2) - (x^2+y)}{x + y^2 - 2y(x^2+y)}$$

27. $\frac{dz}{dt} = 2(4) + 1(-5) = 3$.

29. $\frac{\partial z}{\partial s} = -4(5) + 9(-2) = -38$,
$\frac{\partial z}{\partial t} = -4(7) + 9(6) = 26$.

Section 12.6

1. A partial derivative is essentially a special case of a directional derivative; it is the directional derivative in the direction of x or y, i.e., $\langle 1, 0\rangle$ or $\langle 0, 1\rangle$.

3. $\vec{u} = \langle 0, 1\rangle$

5. maximal, or greatest

7. $\nabla f = \left\langle -2xy + y^2 + y, -x^2 + 2xy + x\right\rangle$

9. $\nabla f = \left\langle \frac{-2x}{(x^2+y^2+1)^2}, \frac{-2y}{(x^2+y^2+1)^2}\right\rangle$

11. $\nabla f = \langle 2x - y - 7, 4y - x \rangle$

13. $\nabla f = \left\langle -2xy + y^2 + y, -x^2 + 2xy + x \right\rangle$; $\nabla f(2,1) = \langle -2, 2 \rangle$. Be sure to change all directions to unit vectors.

    (a) $2/5$ ($\vec{u} = \langle 3/5, 4/5 \rangle$)

    (b) $-2\sqrt{5}$ ($\vec{u} = \left\langle -1/\sqrt{5}, -2\sqrt{5} \right\rangle$)

15. $\nabla f = \left\langle \frac{-2x}{(x^2+y^2+1)^2}, \frac{-2y}{(x^2+y^2+1)^2} \right\rangle$; $\nabla f(1,1) = \langle -2/9, -2/9 \rangle$. Be sure to change all directions to unit vectors.

    (a) 0 ($\vec{u} = \left\langle 1/\sqrt{2}, -1/\sqrt{2} \right\rangle$)

    (b) $2\sqrt{2}/9$ ($\vec{u} = \left\langle -1/\sqrt{2}, -1/\sqrt{2} \right\rangle$)

17. $\nabla f = \langle 2x - y - 7, 4y - x \rangle$; $\nabla f(4,1) = \langle 0, 0 \rangle$.

    (a) 0

    (b) 0

19. $\nabla f = \left\langle -2xy + y^2 + y, -x^2 + 2xy + x \right\rangle$

    (a) $\nabla f(2,1) = \langle -2, 2 \rangle$

    (b) $||\, \nabla f(2,1) \,|| = ||\, \langle -2, 2 \rangle \,|| = \sqrt{8}$

    (c) $\langle 2, -2 \rangle$

    (d) $\left\langle 1/\sqrt{2}, 1/\sqrt{2} \right\rangle$

21. $\nabla f = \left\langle \frac{-2x}{(x^2+y^2+1)^2}, \frac{-2y}{(x^2+y^2+1)^2} \right\rangle$

    (a) $\nabla f(1,1) = \langle -2/9, -2/9 \rangle$.

    (b) $||\, \nabla f(1,1) \,|| = ||\, \langle -2/9, -2/9 \rangle \,|| = 2\sqrt{2}/9$

    (c) $\langle 2/9, 2/9 \rangle$

    (d) $\left\langle 1/\sqrt{2}, -1/\sqrt{2} \right\rangle$

23. $\nabla f = \langle 2x - y - 7, 4y - x \rangle$

    (a) $\nabla f(4,1) = \langle 0, 0 \rangle$

    (b) 0

    (c) $\langle 0, 0 \rangle$

    (d) All directions give a directional derivative of 0.

25. (a) $\nabla F(x,y,z) = \left\langle 6xz^3 + 4y, 4x, 9x^2z^2 - 6z \right\rangle$

    (b) $113/\sqrt{3}$

27. (a) $\nabla F(x,y,z) = \left\langle 2xy^2, 2y(x^2 - z^2), -2y^2z \right\rangle$

    (b) 0

Section 12.7

1. Answers will vary. The displacement of the vector is one unit in the x-direction and 3 units in the z-direction, with no change in y. Thus along a line parallel to $\vec{v}$, the change in z is 3 times the change in x – i.e., a "slope" of 3. Specifically, the line in the x-z plane parallel to z has a slope of 3.

3. T

5. (a) $\ell_x(t) = \begin{cases} x = 2 + t \\ y = 3 \\ z = -48 - 12t \end{cases}$

    (b) $\ell_y(t) = \begin{cases} x = 2 \\ y = 3 + t \\ z = -48 - 40t \end{cases}$

    (c) $\ell_{\vec{u}}(t) = \begin{cases} x = 2 + t/\sqrt{10} \\ y = 3 + 3t/\sqrt{10} \\ z = -48 - 66\sqrt{2/5}t \end{cases}$

7. (a) $\ell_x(t) = \begin{cases} x = 4 + t \\ y = 2 \\ z = 2 + 3t \end{cases}$

    (b) $\ell_y(t) = \begin{cases} x = 4 \\ y = 2 + t \\ z = 2 - 5t \end{cases}$

    (c) $\ell_{\vec{u}}(t) = \begin{cases} x = 4 + t/\sqrt{2} \\ y = 2 + t/\sqrt{2} \\ z = 2 - \sqrt{2}t \end{cases}$

9. $\ell_{\vec{n}}(t) = \begin{cases} x = 2 - 12t \\ y = 3 - 40t \\ z = -48 - t \end{cases}$

11. $\ell_{\vec{n}}(t) = \begin{cases} x = 4 + 3t \\ y = 2 - 5t \\ z = 2 - t \end{cases}$

13. $(1.425, 1.085, -48.078)$, $(2.575, 4.915, -47.952)$

15. $(5.014, 0.31, 1.662)$ and $(2.986, 3.690, 2.338)$

17. $-12(x-2) - 40(y-3) - (z+48) = 0$

19. $3(x-4) - 5(y-2) - (z-2) = 0$ (Note that this tangent plane is the same as the original function, a plane.)

21. $\nabla F = \langle x/4, y/2, z/8 \rangle$; at P, $\nabla F = \left\langle 1/4, \sqrt{2}/2, \sqrt{6}/8 \right\rangle$

    (a) $\ell_{\vec{n}}(t) = \begin{cases} x = 1 + t/4 \\ y = \sqrt{2} + \sqrt{2}t/2 \\ z = \sqrt{6} + \sqrt{6}t/8 \end{cases}$

    (b) $\frac{1}{4}(x-1) + \frac{\sqrt{2}}{2}(y - \sqrt{2}) + \frac{\sqrt{6}}{8}(z - \sqrt{6}) = 0.$

23. $\nabla F = \left\langle y^2 - z^2, 2xy, -2xz \right\rangle$; at P, $\nabla F = \langle 0, 4, 4 \rangle$

    (a) $\ell_{\vec{n}}(t) = \begin{cases} x = 2 \\ y = 1 + 4t \\ z = -1 + 4t \end{cases}$

    (b) $4(y-1) + 4(z+1) = 0.$

Section 12.8

1. F; it is the "other way around."

3. T

5. One critical point at $(-4, 2)$; $f_{xx} = 1$ and $D = 4$, so this point corresponds to a relative minimum.

7. One critical point at $(6, -3)$; $D = -4$, so this point corresponds to a saddle point.

9. Two critical points: at $(0, -1)$; $f_{xx} = 2$ and $D = -12$, so this point corresponds to a saddle point;
   at $(0, 1)$, $f_{xx} = 2$ and $D = 12$, so this corresponds to a relative minimum.

11. One critical point at $(0, 0)$. $D = -12x^2y^2$, so at $(0, 0)$, $D = 0$ and the test is inconclusive. (Some elementary thought shows that it is the absolute minimum.)

13. Six critical points: $f_x = 0$ when $x = -1, 0$ and 1; $f_y = 0$ when $y = -3, 3$. Together, we get the points:
    $(-1, -3)$ saddle point; $(-1, 3)$ rel. min
    $(0, -3)$ rel. max; $(0, 3)$ saddle point
    $(1, -3)$ saddle point; $(1, 3)$ relative min
    where $f_{xx} = 12x^2 - 4$ and $D = 24y(3x^2 - 1)$.

15. One critical point: $f_x = 0$ when $x = 0$; $f_y = 0$ when $y = 0$, so one critical point at $(0, 0)$ (although it should be noted that at $(0, 0)$, both f_x and f_y are undefined.) The Second Derivative Test fails at $(0, 0)$, with $D = 0$. A graph, or simple calculation, shows that $(0, 0)$ is the absolute minimum of f.

17. The region has two "corners" at $(1, 1)$ and $(-1, 1)$.
    Along $y = 1$, there is no critical point.
    Along $y = x^2$, there is a critical point at
    $(5/14, 25/196) \approx (0.357, 0.128)$.

The function f itself has no critical points. Checking the value of f at the corners $(1,1)$, $(-1,1)$ and the critical point $(5/14, 25/196)$, we find the absolute maximum is at $(5/14, 25/196, 25/28) \approx (0.357, 0.128, 0.893)$ and the absolute minimum is at $(-1, 1, -12)$.

19. The region has two "corners" at $(-1,-1)$ and $(1,1)$.
Along the line $y = x$, $f(x,y)$ becomes $f(x) = 3x - 2x^2$. Along this line, we have a critical point at $(3/4, 3/4)$.
Along the curve $y = x^2 + x - 1$, $f(x,y)$ becomes $f(x) = x^2 + 3x - 3$. There is a critical point along this curve at $(-3/2, -1/4)$. Since $x = -3/2$ lies outside our bounded region, we ignore this critical point.
The function f itself has no critical points.
Checking the value of f at $(-1,-1)$, $(1,1)$, $(3/4,3/4)$, we find the absolute maximum is at $(3/4, 3/4, 9/8)$ and the absolute minimum is at $(-1,-1,-5)$.

21. $10\text{m} \times 16\text{m} \times 48\text{m}$

Section 12.9

1. perpendicular or orthogonal

3. T

5. $f_{\max} = \dfrac{2}{3\sqrt{3}}$ at $\left(\pm\sqrt{\dfrac{2}{3}}, \dfrac{1}{\sqrt{3}}\right)$, and $f_{\min} = -\dfrac{2}{3\sqrt{3}}$ at $\left(\pm\sqrt{\dfrac{2}{3}}, -\dfrac{1}{\sqrt{3}}\right)$

7. $f_{\max} = \dfrac{\sqrt{82}}{3}$ at $\left(\dfrac{9}{\sqrt{82}}, \dfrac{1}{3\sqrt{82}}\right)$, and $f_{\min} = -\dfrac{\sqrt{82}}{3}$ at $\left(-\dfrac{9}{\sqrt{82}}, -\dfrac{1}{3\sqrt{82}}\right)$

9. $f_{\max} = 1$ at $(1,1)$

11. $f_{\min} = \dfrac{1}{2}$ at the point $\left(\dfrac{1}{2}, 0, -\dfrac{1}{2}\right)$

Chapter 13

Section 13.1

1. $C(y)$, meaning that instead of being just a constant, like the number 5, it is a function of y, which acts like a constant when taking derivatives with respect to x.

3. curve to curve, then from point to point

5. (a) $18x^2 + 42x - 117$
   (b) -108

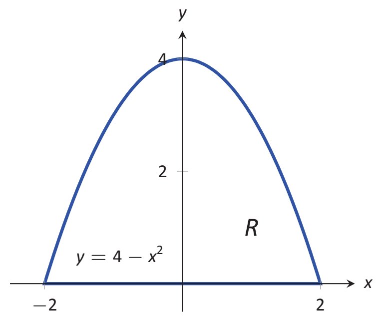
7. (a) $x^4/2 - x^2 + 2x - 3/2$
   (b) $23/15$

9. (a) $\sin^2 y$
   (b) $\pi/2$

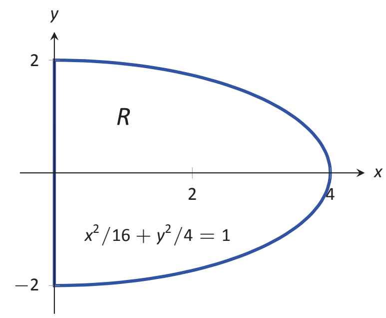
11. $\displaystyle\int_1^4 \int_{-2}^1 dy\,dx$ and $\displaystyle\int_{-2}^1 \int_1^4 dx\,dy.$
area of $R = 9u^2$

13. $\displaystyle\int_2^4 \int_{x-1}^{7-x} dy\,dx$. The order $dx\,dy$ needs two iterated integrals as x is bounded above by two different functions. This gives:
$$\int_1^3 \int_2^{y+1} dx\,dy + \int_3^5 \int_2^{7-y} dx\,dy.$$
area of $R = 4u^2$

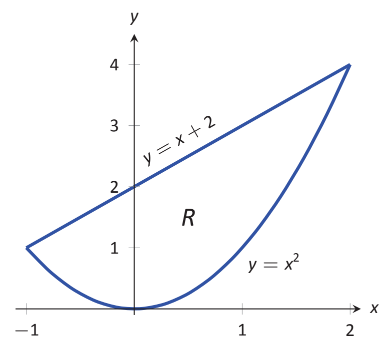
15. $\displaystyle\int_0^1 \int_{x^4}^{\sqrt{x}} dy\,dx$ and $\displaystyle\int_0^1 \int_{y^2}^{\sqrt[4]{y}} dx\,dy$
area of $R = 7/15u^2$

17. 

area of $R = \displaystyle\int_0^4 \int_{-\sqrt{4-y}}^{\sqrt{4-y}} dx\,dy$

19. 

area of $R = \displaystyle\int_0^4 \int_{-\sqrt{4-x^2/4}}^{\sqrt{4-x^2/4}} dy\,dx$

21. 

area of $R = \displaystyle\int_{-1}^2 \int_{x^2}^{x+2} dy\,dx$

23. 20

25. $2\sqrt{3} - \sqrt{6}$

27. 756

29. The integrand $\sqrt{x+y}$ cannot be written as $f(x)g(y)$ where f is a function of only x and g is a function of only y.

Section 13.2

1. volume

3. The double integral gives the signed volume under the surface. Since the surface is always positive, it is always above the x-y plane and hence produces only "positive" volume.

5. 6; $\displaystyle\int_{-1}^1 \int_1^2 \left(\frac{x}{y} + 3\right) dy\,dx$

7. 112/3; $\displaystyle\int_0^2 \int_0^{4-2y} \left(3x^2 - y + 2\right) dx\,dy$

9. 16/5; $\displaystyle\int_{-1}^1 \int_0^{1-x^2} (x + y + 2)\; dy\,dx$