A: Solutions To Selected Problems

Chapter 1

Section 1.1

1. Answers will vary.
3. F
5. Answers will vary.
7. -5
9. 2
11. Limit does not exist.
13. 7
15. Limit does not exist.

17.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	9
-0.01	9
0.01	9
0.1	9

The limit seems to be exactly 9.

19.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	-0.114943
-0.01	-0.111483
0.01	-0.110742
0.1	-0.107527

The limit is approx. -0.11.

21.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	0.202027
-0.01	0.2002
0.01	0.1998
0.1	0.198026

The limit is approx. 0.2.

23.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	-0.0499583
-0.01	-0.00499996
0.01	0.00499996
0.1	0.0499583

The limit is approx. 0.

Section 1.2

1. ε should be given first, and the restriction $|x-a|<\delta$ implies $|f(x)-K|<\varepsilon$, not the other way around.

3. T

5. Let $\varepsilon>0$ be given. We wish to find $\delta>0$ such that when $|x-5|<\delta$, $|f(x)-(-2)|<\varepsilon$.
Consider $|f(x)-(-2)|<\varepsilon$:
$$\begin{gathered}|f(x)+2|<\varepsilon\\ |(3-x)+2|<\varepsilon\\ |5-x|<\varepsilon\\ -\varepsilon<5-x<\varepsilon\\ -\varepsilon<x-5<\varepsilon.\end{gathered}$$
This implies we can let $\delta=\varepsilon$. Then:
$$\begin{gathered}|x-5|<\delta\\ -\delta<x-5<\delta\\ -\varepsilon<x-5<\varepsilon\\ -\varepsilon<(x-3)-2<\varepsilon\\ -\varepsilon<(-x+3)-(-2)<\varepsilon\\ |3-x-(-2)|<\varepsilon,\end{gathered}$$
which is what we wanted to prove.

7. Let $\varepsilon>0$ be given. We wish to find $\delta>0$ such that when $|x-4|<\delta$, $|f(x)-15|<\varepsilon$.
Consider $|f(x)-15|<\varepsilon$, keeping in mind we want to make a statement about $|x-4|$:
$$\begin{gathered}|f(x)-15|<\varepsilon\\ |x^2+x-5-15|<\varepsilon\\ |x^2+x-20|<\varepsilon\\ |x-4|\cdot|x+5|<\varepsilon\\ |x-4|<\varepsilon/|x+5|\end{gathered}$$
Since x is near 4, we can safely assume that, for instance, $3<x<5$. Thus
$$\begin{gathered}3+5<x+5<5+5\\ 8<x+5<10\\ \frac{1}{10}<\frac{1}{x+5}<\frac{1}{8}\\ \frac{\varepsilon}{10}<\frac{\varepsilon}{x+5}<\frac{\varepsilon}{8}\end{gathered}$$
Let $\delta=\frac{\varepsilon}{10}$. Then:
$$\begin{gathered}|x-4|<\delta\\ |x-4|<\frac{\varepsilon}{10}\\ |x-4|<\frac{\varepsilon}{x+5}\\ |x-4|\cdot|x+5|<\frac{\varepsilon}{x+5}\cdot|x+5|\end{gathered}$$
Assuming x is near 4, $x+5$ is positive and we can drop the absolute value signs on the right.
$$\begin{gathered}|x-4|\cdot|x+5|<\frac{\varepsilon}{x+5}\cdot(x+5)\\ |x^2+x-20|<\varepsilon\\ |(x^2+x-5)-15|<\varepsilon,\end{gathered}$$
which is what we wanted to prove.

9. Let $\varepsilon>0$ be given. We wish to find $\delta>0$ such that when $|x-2|<\delta$, $|f(x)-5|<\varepsilon$. However, since $f(x)=5$, a constant function, the latter inequality is simply $|5-5|<\varepsilon$, which is always true. Thus we can choose any δ we like; we arbitrarily choose $\delta=\varepsilon$.

11. Let $\varepsilon>0$ be given. We wish to find $\delta>0$ such that when $|x-0|<\delta$, $|f(x)-0|<\varepsilon$. In simpler terms, we want to show that when $|x|<\delta$, $|\sin x|<\varepsilon$.
Set $\delta=\varepsilon$. We start with assuming that $|x|<\delta$. Using the hint, we have that $|\sin x|<|x|<\delta=\varepsilon$. Hence if $|x|<\delta$, we know immediately that $|\sin x|<\varepsilon$.

Section 1.3

1. Answers will vary.
3. Answers will vary.
5. As x is near 1, both f and g are near 0, but f is approximately twice the size of g. (I.e., $f(x)\approx 2g(x)$.)
7. 6

9. Limit does not exist.
11. Not possible to know; as x approaches 6, $g(x)$ approaches 3, but we know nothing of the behavior of $f(x)$ when x is near 3.
13. -45
15. -1
17. π
19. -1
21. Limit does not exist
23. 2
25. $1/3$
27. -8
29. 10
31. $-3/2$
33. $1/4$
35. $1/2$
37. $\frac{3}{\sqrt{3}}$
39. 0
41. 1
43. 3
45. 1
47. (a) Apply Part 1 of Theorem 1.
    (b) Apply Theorem 6; $g(x) = \frac{x}{x}$ is the same as $g(x) = 1$ everywhere except at $x = 0$. Thus $\lim_{x\to 0} g(x) = \lim_{x \to 0} 1 = 1$.
    (c) The function $f(x)$ is always 0, so $g\big(f(x)\big)$ is never defined as $g(x)$ is not defined at $x = 0$. Therefore the limit does not exist.
    (d) The Composition Rule requires that $\lim_{x\to 0} g(x)$ be equal to $g(0)$. They are not equal, so the conditions of the Composition Rule are not satisfied, and hence the rule is not violated.

Section 1.4

1. The function approaches different values from the left and right; the function grows without bound; the function oscillates.
3. F
5. (a) 2
   (b) 2
   (c) 2
   (d) 1
   (e) As f is not defined for $x < 0$, this limit is not defined.
   (f) 1
7. (a) Does not exist.
   (b) Does not exist.
   (c) Does not exist.
   (d) Not defined.
   (e) 0
   (f) 0
9. (a) 2
   (b) 2
   (c) 2
   (d) 2
11. (a) 2
    (b) 2
    (c) 2
    (d) 0
    (e) 2
    (f) 2
    (g) 2
    (h) Not defined
13. (a) 2
    (b) -4
    (c) Does not exist.
    (d) 2
15. (a) 0
    (b) 0
    (c) 0
    (d) 0
    (e) 2
    (f) 2
    (g) 2
    (h) 2
17. (a) $1 - \cos^2 a = \sin^2 a$
    (b) $\sin^2 a$
    (c) $\sin^2 a$
    (d) $\sin^2 a$
19. (a) 4
    (b) 4
    (c) 4
    (d) 3
21. (a) -1
    (b) 1
    (c) Does not exist
    (d) 0
23. $2/3$
25. -9

Section 1.5

1. Answers will vary.
3. A root of a function f is a value c such that $f(c) = 0$.
5. F
7. T
9. F
11. No; $\lim_{x\to 1} f(x) = 2$, while $f(1) = 1$.
13. No; $f(1)$ does not exist.
15. Yes
17. (a) No; $\lim_{x\to -2} f(x) \neq f(-2)$
    (b) Yes
    (c) No; $f(2)$ is not defined.
19. (a) Yes
    (b) No; the left and right hand limits at 1 are not equal.
21. (a) Yes

(b) No. $\lim_{x\to 8} f(x) = 16/5 \neq f(8) = 5$.

23. (a) Yes

    (b) Yes

25. (a) Yes

    (b) No. The left and right hand limits are not equal.

27. $(-\infty, \infty)$

29. $[-1, 1]$

31. $(-1, 1)$

33. $(-\infty, \infty)$

35. $(-\infty, \infty)$

37. $(-\infty, \infty)$

39. Yes, by the Intermediate Value Theorem. In fact, we can be more specific and state such a value c exists in $(0, 2)$, not just in $(-3, 7)$.

41. We cannot say; the Intermediate Value Theorem only applies to continuous functions. As we do know know if h is continuous, we cannot say.

43. Approximate root is $x = 0.52$. The intervals used are:
    $[0.5, 0.55]$ $[0.5, 0.525]$ $[0.5125, 0.525]$
    $[0.51875, 0.525]$ $[0.521875, 0.525]$

45. Approximate root is $x = 0.78$. The intervals used are:
    $[0.7, 0.8]$ $[0.75, 0.8]$ $[0.775, 0.8]$
    $[0.775, 0.7875]$ $[0.78125, 0.7875]$
    (A few more steps show that 0.79 is better as the root is $\pi/4 \approx 0.78539$.)

47.

x	$f(x)$
-0.81	-2.34129
-0.801	-2.33413
-0.79	-2.32542
-0.799	-2.33254

The top two lines give an approximation of the limit from the left: -2.33. The bottom two lines give an approximation from the right: -2.33 as well.

Section 1.6

1. F

3. T

5. Answers will vary.

7. (a) ∞

   (b) ∞

9. $-\infty$

11. Limit does not exist

13. $-\infty$

15. ∞

17. Limit does not exist

19. $-\infty$

21. $-\infty$

23. Limit does not exist

25. -∞

27. ∞

29. Limit does not exist

31. ∞

33. ∞

35. ∞

37. $-\infty$

39. Vertical asymptote at $x = 4$.

41. Vertical asymptotes at every integer value.

43. No vertical asymptotes.

45. Solution omitted.

Section 1.7

1. T

3. (a) 1

   (b) 0

   (c) $1/2$

   (d) $1/2$

5. (a) Limit does not exist

   (b) Limit does not exist

7. $-\infty$, 0

9. $-\infty$

11. ∞

13. ∞

15. ∞

17. $-\infty$

19. $1/2$

21. 0

23. ∞

25. $7/2$

27. $-\infty$

29. $-5/7$

31. 0

33. $-\infty$

35. 0

37. ∞

39. $-3/2$

41. 0

43. $\sqrt{5}/2$

45. $-\infty$

47. $3/2$

49. 0

51. $-3/\sqrt{5}$

53. ∞

55. $-3/\sqrt{7}$

57. 0

59. $-\frac{\pi}{2}$

61. $-\infty$

63. 0

65. ∞

67. $\frac{\pi}{2}$

69. $-\infty$

71. $1/2$

73. Limit does not exist.

75. ∞

77. Horizontal asymptote at $y = -3/5$; vertical asymptote at $x = 3$.

79. No horizontal asymptote; vertical asymptote at $x = 1$.

81. Horizontal asymptote at $y = -1$; no vertical asymptotes

83. 1

Chapter 2

Section 2.1

1. T

3. Answers will vary.

5. Answers will vary.

7. $f'(x) = 2$

9. $g'(x) = 2x$

11. $f'(x) = 3x^2$

13. $f'(x) = \frac{1}{2\sqrt{x}}$

15. (a) $y = 6$
 (b) $x = -2$

17. (a) $y = -3x + 4$
 (b) $y = \dfrac{1}{3}(x - 7) - 17$

19. (a) $y = -7(x + 1) + 8$
 (b) $y = \dfrac{1}{7}(x + 1) + 8$

21. (a) $y = -\dfrac{1}{4}(x + 2) - \dfrac{1}{2}$
 (b) $y = 4(x + 2) - \dfrac{1}{2}$

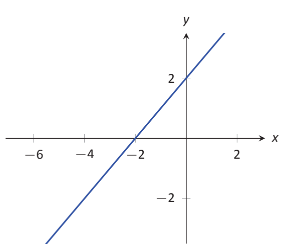
23. (a) $y = -1(x - 3) + 1$
 (b) $y = 1(x - 3) + 1$

25. $y = -0.099(x - 9) + 1$

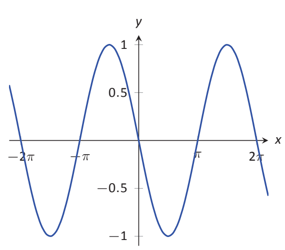
27. $y = -0.05x + 1$

29. (a) Approximations will vary; they should match (c) closely.
 (b) $f'(x) = -1/(x + 1)^2$
 (c) At $(0, 1)$, slope is -1. At $(1, 0.5)$, slope is $-1/4$.

31.



33.



35. Approximately 24.

37. (a) $(-\infty, \infty)$
 (b) $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$
 (c) $(-\infty, 5]$
 (d) $[-5, 5]$

Section 2.2

1. Velocity

3. Linear functions.

5. -17

7. $f(10.1)$ is likely most accurate, as accuracy is lost the farther from $x = 10$ we go.

9. 6

11. ft/s^2

13. (a) thousands of dollars per car
 (b) It is likely that $P(0) < 0$. That is, negative profit for not producing any cars.

15. $f(x) = g'(x)$

17. Either $g(x) = f'(x)$ or $f(x) = g'(x)$ is acceptable. The actual answer is $g(x) = f'(x)$, but is very hard to show that $f(x) \neq g'(x)$ given the level of detail given in the graph.

19. $f'(x) = 10x$

21. $f'(\pi) \approx 0$.

Section 2.3

1. Power Rule.

3. One answer is $f(x) = 10e^x$.

5. Answers may vary.

7. Answers will vary.

9. No such function exists.

11. $f'(x)$ is a velocity function, and $f''(x)$ is acceleration.

13. $f'(x) = 14x - 5$

15. $m'(t) = 45t^4 - \frac{3}{8}t^2 + 3$

17. $f'(\theta) = 9\cos\theta - 10\sin\theta$

19. $f'(r) = 6e^r$

21. $g'(t) = 40t^3 + \sin t + 7\cos t$

23. $f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} = \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x^3}}$

25. $f'(x) = -\dfrac{5}{4x^{5/4}}$

27. $g'(t) = 0$

29. $f'(x) = -5/x^2$

31. $h'(t) = e^t - \cos t + \sin t$

33. $g'(x) = 1 + 3/x^2$

35. $f'(t) = 0$

37. $f'(v) = \dfrac{9\ln(1/2)}{2^v} = -\dfrac{9\ln 2}{2^v}$

39. $g'(t) = 18t + 6$

41. $f'(x) = -3x^2 + 6x - 3$

43. $f'(x) = 18x - 12$

45. $f'(x) = 6x^5$ $f''(x) = 30x^4$ $f'''(x) = 120x^3$ $f^{(4)}(x) = 360x^2$

47. $h'(t) = 2t - e^t$ $h''(t) = 2 - e^t$ $h'''(t) = -e^t$ $h^{(4)}(t) = -e^t$

49. $p'(\theta) = 4\theta^3 - 3\theta^2$ $p''(\theta) = 12\theta^2 - 6\theta$ $p'''(\theta) = 24\theta - 6$ $p^{(4)}(\theta) = 24$

51. $f'(x) = f''(x) = f'''(x) = f^{(4)}(x) = 0$