Chapter 7

Section 7.1

1. T
3. Answers will vary.
5. $16/3$
7. π
9. $2\sqrt{2}$
11. 4.5
13. $2-\pi/2$
15. $1/6$
17. On regions such as $[\pi/6, 5\pi/6]$, the area is $3\sqrt{3}/2$. On regions such as $[-\pi/2, \pi/6]$, the area is $3\sqrt{3}/4$.
19. $5/3$
21. $9/4$
23. 1
25. 4
27. 219,000 ft^2

Section 7.2

1. T
3. Recall that "dx" does not just "sit there;" it is multiplied by $A(x)$ and represents the thickness of a small slice of the solid. Therefore dx has units of in, giving $A(x)\,dx$ the units of in^3.
5. $175\pi/3$ units^3
7. $\pi/6$ units^3
9. $35\pi/3$ units^3
11. $2\pi/15$ units^3
13. (a) $512\pi/15$
    (b) $256\pi/5$
    (c) $832\pi/15$
    (d) $128\pi/3$
15. (a) $104\pi/15$
    (b) $64\pi/15$
    (c) $32\pi/5$
17. (a) 8π
    (b) 8π
    (c) $16\pi/3$
    (d) $8\pi/3$
19. The cross–sections of this cone are the same as the cone in Exercise 18. Thus they have the same volume of $250\pi/3$ units^3.
21. Orient the solid so that the x-axis is parallel to long side of the base. All cross–sections are trapezoids (at the far left, the trapezoid is a square; at the far right, the trapezoid has a top length of 0, making it a triangle). The area of the trapezoid at x is $A(x) = 1/2(-1/2x+5+5)(5) = -5/4x+25$. The volume is 187.5 units^3.
23. (a) Answers may vary.
    (b) $V = \frac{1}{3}\pi hr^2$

Section 7.3

1. T
3. F
5. $9\pi/2$ units^3
7. $\pi^2 - 2\pi$ units^3
9. $48\pi\sqrt{3}/5$ units^3
11. $\pi^2/4$ units^3
13. (a) $4\pi/5$
    (b) $8\pi/15$
    (c) $\pi/2$
    (d) $5\pi/6$
15. (a) $4\pi/3$
    (b) $\pi/3$
    (c) $4\pi/3$
    (d) $2\pi/3$
17. (a) $2\pi\left(\sqrt{2}-1\right)$
    (b) $2\pi\left(1-\sqrt{2}+\ln\left(1+\sqrt{2}\right)\right)$

Section 7.4

1. T
3. $\sqrt{2}$
5. $4/3$
7. $109/2$
9. $12/5$
11. $-\ln(2-\sqrt{3}) \approx 1.31696$
13. $\int_0^1 \sqrt{1+4x^2}\,dx$
15. $\int_0^1 \sqrt{1+\frac{1}{4x}}\,dx$
17. $\int_{-1}^1 \sqrt{1+\frac{x^2}{1-x^2}}\,dx$
19. $\int_1^2 \sqrt{1+\frac{1}{x^4}}\,dx$
21. 1.4790
23. Simpson's Rule fails, as it requires one to divide by 0. However, recognize the answer should be the same as for $y=x^2$; why?
25. Simpson's Rule fails.
27. 1.4058
29. $2\pi\int_0^1 2x\sqrt{5}\,dx = 2\pi\sqrt{5}$
31. $2\pi\int_0^1 x^3\sqrt{1+9x^4}\,dx = \pi/27(10\sqrt{10}-1)$
33. $2\pi\int_0^1 \sqrt{1-x^2}\sqrt{1+x/(1-x^2)}\,dx = 4\pi$

Section 7.5

1. In SI units, it is one joule, i.e., one Newton–meter, or kg·m/s^2·m. In Imperial Units, it is ft–lb.
3. Smaller.
5. (a) 2450 J
   (b) 1568 J
7. 735 J
9. 11,100 ft–lb
11. 125 ft–lb
13. 12.5 ft–lb
15. $10/3$ ft $=$ 40 in
17. $f\cdot d/2$ Joules

19. 5 ft–lb
21. (a) 52,929.6 ft–lb
    (b) 18,525.3 ft–lb
    (c) When 3.83 ft of water have been pumped from the tank, leaving about 2.17 ft in the tank.
23. 212,135 ft–lb
25. 187,214 ft–lb
27. 4,917,150 J

Section 7.6

1. Answers will vary.
3. 499.2 lb
5. 6739.2 lb
7. 3920.7 lb
9. 2496 lb
11. 602.59 lb
13. (a) 2340 lb
    (b) 5625 lb
15. (a) 1597.44 lb
    (b) 3840 lb
17. (a) 56.42 lb
    (b) 135.62 lb
19. 5.1 ft

Section 7.7

1. $\left(\frac{6}{5}, \frac{15}{7}\right)$
3. $\left(0, -\frac{14}{3\pi+18}\right)$
5. $\left(\frac{\pi}{2}-1, \frac{\pi}{8}\right)$
7. $\left(0, \frac{4\pi-3\sqrt{3}}{4\pi-6\ln\left(2+\sqrt{3}\right)}\right)$
9. $\left(\frac{b-a}{3}, \frac{c}{3}\right)$

Chapter 8

Section 8.1

1. Answers will vary.
3. Answers will vary.
5. $2, \frac{8}{3}, \frac{8}{3}, \frac{32}{15}, \frac{64}{45}$
7. $\frac{1}{3}, 2, \frac{81}{5}, \frac{512}{3}, \frac{15625}{7}$
9. $a_n = 3n+1$
11. $a_n = 10 \cdot 2^{n-1}$
13. $1/7$
15. 0
17. diverges
19. converges to 0
21. diverges
23. converges to e
25. converges to 0
27. converges to 2
29. bounded
31. bounded
33. neither bounded above or below
35. monotonically increasing
37. never monotonic
39. Let $\{a_n\}$ be given such that $\lim\limits_{n\to\infty} |a_n| = 0$. By the definition of the limit of a sequence, given any $\varepsilon > 0$, there is a m such that for all $n > m$, $|\,|a_n| - 0| < \varepsilon$. Since $|\,|a_n| - 0| = |a_n - 0|$, this directly implies that for all $n > m$, $|a_n - 0| < \varepsilon$, meaning that $\lim\limits_{n\to\infty} a_n = 0$.
41. Left to reader

Section 8.2

1. Answers will vary.
3. One sequence is the sequence of terms $\{a_n\}$. The other is the sequence of n^{th} partial sums, $\{S_n\} = \{\sum_{i=1}^{n} a_i\}$.
5. F
7. (a) $1, \frac{5}{4}, \frac{49}{36}, \frac{205}{144}, \frac{5269}{3600}$
   (b) Plot omitted
9. (a) $1, 3, 6, 10, 15$
   (b) Plot omitted
11. (a) $\frac{1}{3}, \frac{4}{9}, \frac{13}{27}, \frac{40}{81}, \frac{121}{243}$
    (b) Plot omitted
13. (a) $0.1, 0.11, 0.111, 0.1111, 0.11111$
    (b) Plot omitted
15. $\lim\limits_{n\to\infty} a_n = \infty$; by Theorem 64 the series diverges.
17. $\lim\limits_{n\to\infty} a_n = 1$; by Theorem 64 the series diverges.
19. $\lim\limits_{n\to\infty} a_n = e$; by Theorem 64 the series diverges.
21. Diverges
23. Converges
25. (a) $S_n = \frac{1-(1/4)^n}{3/4}$
    (b) Converges to $4/3$.
27. (a) $S_n = \begin{cases} \frac{n+1}{2} & \text{n is odd} \\ -\frac{n}{2} & \text{n is even} \end{cases}$
    (b) Diverges
29. (a) $S_n = \frac{1-(1/e)^{n+1}}{1-1/e}$.
    (b) Converges to $1/(1-1/e) = e/(e-1)$.
31. (a) With partial fractions, $a_n = \frac{1}{n} - \frac{1}{n+1}$. Thus $S_n = 1 - \frac{1}{n+1}$.
    (b) Converges to 1.
33. (a) Use partial fraction decomposition to recognize the telescoping series: $a_n = \frac{1}{2}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$. Then $S_n = \frac{n}{2n+1}$.
    (b) Converges to $1/2$.
35. (a) $S_n = 1 - \frac{1}{(n+1)^2}$
    (b) Converges to 1.
37. (a) $a_n = 1/2^n + 1/3^n$ for $n \geq 0$. Thus $S_n = \frac{1-1/2^2}{1/2} + \frac{1-1/3^n}{2/3}$.
    (b) Converges to $2 + 3/2 = 7/2$.
39. (a) $S_n = \frac{1-(\sin 1)^{n+1}}{1-\sin 1}$
    (b) Converges to $\frac{1}{1-\sin 1}$..