3. All derivatives of e^x are e^x which evaluate to 1 at $x=0$.
The Taylor series starts $1+x+\frac{1}{2}x^2+\frac{1}{3!}x^3+\frac{1}{4!}x^4+\cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}\frac{x^n}{n!}$

5. The n^{th} derivative of $1/(1-x)$ is $f^{(n)}(x)=(n)!/(1-x)^{n+1}$, which evaluates to $n!$ at $x=0$.
The Taylor series starts $1+x+x^2+x^3+\cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}x^n$

7. The Taylor series starts
$0-(x-\pi/2)+0x^2+\frac{1}{6}(x-\pi/2)^3+0x^4-\frac{1}{120}(x-\pi/2)^5$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1}\frac{(x-\pi/2)^{2n+1}}{(2n+1)!}$

9. $f^{(n)}(x)=(-1)^n e^{-x}$; at $x=0$, $f^{(n)}(0)=-1$ when n is odd and $f^{(n)}(0)=1$ when n is even.
The Taylor series starts $1-x+\frac{1}{2}x^2-\frac{1}{3!}x^3+\cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^n}{n!}$.

11. $f^{(n)}(x)=(-1)^{n+1}\frac{n!}{(x+1)^{n+1}}$; at $x=1$, $f^{(n)}(1)=(-1)^{n+1}\frac{n!}{2^{n+1}}$
The Taylor series starts
$\frac{1}{2}+\frac{1}{4}(x-1)-\frac{1}{8}(x-1)^2+\frac{1}{16}(x-1)^3\cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{2^{n+1}}$.

13. Given a value x, the magnitude of the error term $R_n(x)$ is bounded by
$$\left|R_n(x)\right|\leq\frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}\left|x^{(n+1)}\right|,$$
where z is between 0 and x.
If $x>0$, then $z<x$ and $f^{(n+1)}(z)=e^z<e^x$. If $x<0$, then $x<z<0$ and $f^{(n+1)}(z)=e^z<1$. So given a fixed x value, let $M=\max\{e^x,1\}$; $f^{(n)}(z)<M$. This allows us to state
$$\left|R_n(x)\right|\leq\frac{M}{(n+1)!}\left|x^{(n+1)}\right|.$$
For any x, $\displaystyle\lim_{n\to\infty}\frac{M}{(n+1)!}\left|x^{(n+1)}\right|=0$. Thus by the Squeeze Theorem, we conclude that $\displaystyle\lim_{n\to\infty}R_n(x)=0$ for all x, and hence
$$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}\quad\text{for all } x.$$

15. Given a value x, the magnitude of the error term $R_n(x)$ is bounded by
$$\left|R_n(x)\right|\leq\frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}\left|(x-1)^{(n+1)}\right|,$$
where z is between 1 and x.
Note that $\left|f^{(n+1)}(x)\right|=\frac{n!}{x^{n+1}}$.
We consider the cases when $x>1$ and when $x<1$ separately.
If $x>1$, then $1<z<x$ and $f^{(n+1)}(z)=\frac{n!}{z^{n+1}}<n!$. Thus
$$\left|R_n(x)\right|\leq\frac{n!}{(n+1)!}\left|(x-1)^{(n+1)}\right|=\frac{(x-1)^{n+1}}{n+1}.$$
For a fixed x,
$$\lim_{n\to\infty}\frac{(x-1)^{n+1}}{n+1}=0.$$
If $0<x<1$, then $x<z<1$ and $f^{(n+1)}(z)=\frac{n!}{z^{n+1}}<\frac{n!}{x^{n+1}}$. Thus
$$\left|R_n(x)\right|\leq\frac{n!/x^{n+1}}{(n+1)!}\left|(x-1)^{(n+1)}\right|=\frac{x^{n+1}}{n+1}(1-x)^{n+1}.$$
Since $0<x<1$, $x^{n+1}<1$ and $(1-x)^{n+1}<1$. We can then extend the inequality from above to state
$$\left|R_n(x)\right|\leq\frac{x^{n+1}}{n+1}(1-x)^{n+1}<\frac{1}{n+1}.$$
As $n\to\infty$, $1/(n+1)\to 0$. Thus by the Squeeze Theorem, we conclude that $\displaystyle\lim_{n\to\infty}R_n(x)=0$ for all x, and hence
$$\ln x=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n}\quad\text{for all } 0<x\leq 2.$$

17. Given $\displaystyle\cos x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$,
$\displaystyle\cos(-x)=\sum_{n=0}^{\infty}(-1)^n\frac{(-x)^{2n}}{(2n)!}=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}=\cos x$, as all powers in the series are even.

19. Given $\displaystyle\sin x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$,
$\displaystyle\frac{d}{dx}\big(\sin x\big)=\frac{d}{dx}\left(\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}\right)=$
$\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(2n+1)x^{2n}}{(2n+1)!}=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}=\cos x$. (The summation still starts at $n=0$ as there was no constant term in the expansion of $\sin x$).

21. $1+\dfrac{x}{2}-\dfrac{x^2}{8}+\dfrac{x^3}{16}-\dfrac{5x^4}{128}$

23. $1+\dfrac{x}{3}-\dfrac{x^2}{9}+\dfrac{5x^3}{81}-\dfrac{10x^4}{243}$

25. $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(x^2)^{2n}}{(2n)!}=\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n}}{(2n)!}$

27. $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(2x+3)^{2n+1}}{(2n+1)!}$

29. $\displaystyle C+\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{n!(2n+1)}$

31. $x+x^2+\dfrac{x^3}{3}-\dfrac{x^5}{30}$

33. $\displaystyle\sum_{n=1}^{\infty}nx^n$

35. $\displaystyle\int_0^{\sqrt{\pi}}\sin\left(x^2\right)\,dx\approx\int_0^{\sqrt{\pi}}\left(x^2-\frac{x^6}{6}+\frac{x^{10}}{120}-\frac{x^{14}}{5040}\right)dx=0.8877$

37. $\displaystyle\sum_{n=1}^{\infty}\frac{x^n}{n^2}$

39. $\pi^2/6$

41. At $x=0$, both sides are 0. The derivative of the left side is $-\frac{\ln(1-x)}{x}-\frac{\ln(1+x)}{x}$. By algebra and rules of logarithms, this can be shown equivalent to the derivative of the right side, which is $-\frac{\ln\left(1-x^2\right)}{x}$.

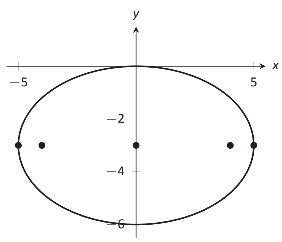
43. $\frac{\pi^2}{12}-\frac{(\ln 2)^2}{2}$

45. -1

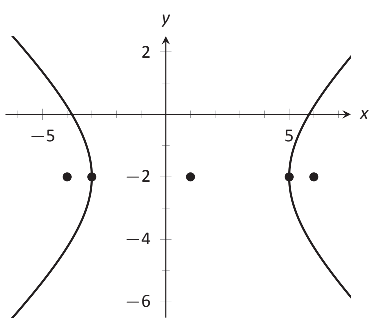
Chapter 9

Section 9.1

1. When defining the conics as the intersections of a plane and a double napped cone, degenerate conics are created when the plane intersects the tips of the cones (usually taken as the origin). Nondegenerate conics are formed when this plane does not contain the origin.
3. Hyperbola
5. With a horizontal transverse axis, the x^2 term has a positive coefficient; with a vertical transverse axis, the y^2 term has a positive coefficient.
7. $y = \frac{-1}{12}(x+1)^2 - 1$
9. $x = y^2$
11. $x = -\frac{1}{12}y^2$
13. $x = -\frac{1}{8}(y-3)^2 + 2$
15. focus: $(5, 2)$; directrix: $x = 1$. The point P is 10 units from each.



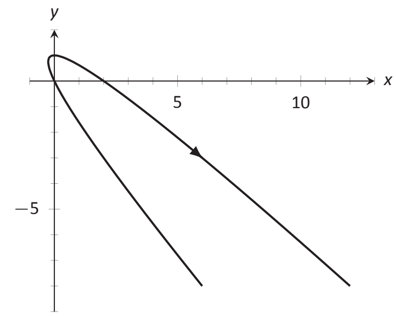
17.
19. $\frac{(x-1)^2}{1/4} + \frac{y^2}{9} = 1$; foci at $(1, \pm\sqrt{8.75})$; $e = \sqrt{8.75}/3 \approx 0.99$
21. $\frac{(x-2)^2}{25} + \frac{(y-3)^2}{16} = 1$
23. $\frac{(x+1)^2}{9} + \frac{(y-1)^2}{25} = 1$
25. $\frac{x^2}{3} + \frac{y^2}{5} = 1$
27. $\frac{(x-2)^2}{4} + \frac{(y-2)^2}{4} = 1$
29. $x^2 - \frac{y^2}{3} = 1$
31. $\frac{(y-3)^2}{4} - \frac{(x-1)^2}{9} = 1$



33.
35. $\frac{x^2}{4} - \frac{y^2}{5} = 1$
37. $\frac{(x-3)^2}{16} - \frac{(y-3)^2}{9} = 1$
39. $\frac{x^2}{4} - \frac{y^2}{3} = 1$
41. $(y-2)^2 - \frac{x^2}{10} = 1$
43. (a) Solve for c in $e = c/a$: $c = ae$. Thus $a^2e^2 = a^2 - b^2$, and $b^2 = a^2 - a^2e^2$. The result follows.
    (b) Mercury: $x^2/(0.387)^2 + y^2/(0.3787)^2 = 1$
    Earth: $x^2 + y^2/(0.99986)^2 = 1$
    Mars: $x^2/(1.524)^2 + y^2/(1.517)^2 = 1$
    (c) Mercury: $(x-0.08)^2/(0.387)^2 + y^2/(0.3787)^2 = 1$
    Earth: $(x-0.0167)^2 + y^2/(0.99986)^2 = 1$
    Mars: $(x-0.1423)^2/(1.524)^2 + y^2/(1.517)^2 = 1$

Section 9.2

1. T
3. rectangular



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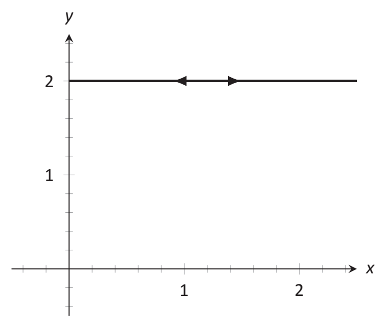

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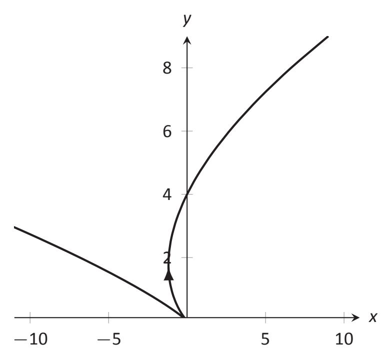

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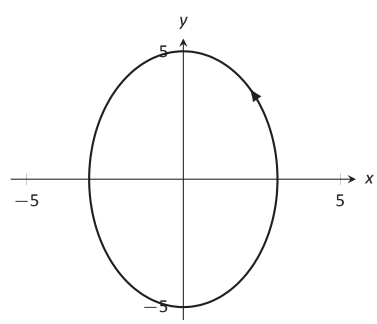

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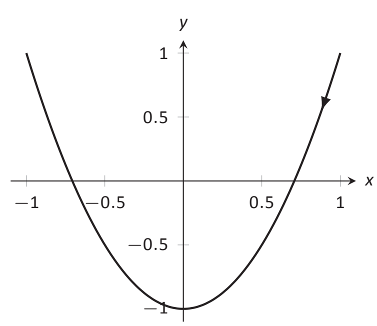

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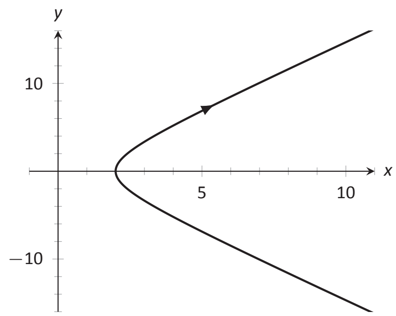
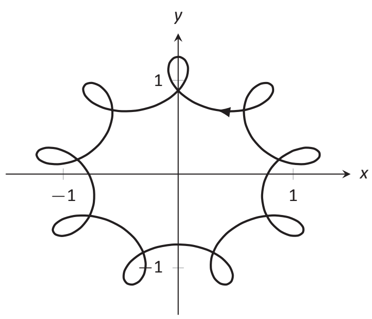
15. 

17. 

19. (a) Traces a circle of radius 1 counterclockwise once.
 (b) Traces a circle of radius 1 counterclockwise over 6 times.
 (c) Traces a circle of radius 1 clockwise infinite times.
 (d) Traces an arc of a circle of radius 1, from an angle of -1 radians to 1 radian, twice.

21. The point is on the curve, at $t = -2$.

23. The point is on the curve, at $t = \pi/3 + 2k\pi$ for any integer k.

25. $x^2 - y^2 = 1$

27. $y = x^{3/2}$

29. $y = x^3 - 3$

31. $y = e^{2x} - 1$

33. $x^2 - y^2 = 1$

35. $y = 1 - x^2$

37. $x^2 + y^2 = r^2$; circle centered at $(0, 0)$ with radius r.

39. $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$; hyperbola centered at (h, k) with horizontal transverse axis and asymptotes with slope b/a. The parametric equations only give half of the hyperbola. When $a > 0$, the right half; when $a < 0$, the left half.

41. $x = \ln t$, $y = t$. At $t = 1$, $x = 0$, $y = 1$.
 $y' = e^x$; when $x = 0$, $y' = 1$.

43. $x = 1/(4t^2)$, $y = 1/(2t)$. At $t = 1$, $x = 1/4$, $y = 1/2$.
 $y' = 1/(2\sqrt{x})$; when $x = 1/4$, $y' = 1$.

45. $t = -1,\ 2$

47. $t = \pi/6, \pi/2, 5\pi/6$

49. $t = 2$

51. $t = \ldots 0,\ 2\pi,\ 4\pi,\ \ldots$

53. $x = 50t$, $y = -16t^2 + 64t$

55. $x = 2\cos t$, $y = -2\sin t$; other answers possible

57. $x = \cos t + 1$, $y = 3\sin t + 3$; other answers possible

59. $x = \pm\sec t + 2$, $y = \sqrt{8}\tan t - 3$; other answers possible

Section 9.3

1. F

3. F

5. (a) $\frac{dy}{dx} = 2t$
 (b) Tangent line: $y = 2(x - 1) + 1$; normal line: $y = -1/2(x - 1) + 1$

7. (a) $\frac{dy}{dx} = \frac{2t+1}{2t-1}$
 (b) Tangent line: $y = 3x + 2$; normal line: $y = -1/3x + 2$

9. (a) $\frac{dy}{dx} = \csc t$
 (b) $t = \pi/4$: Tangent line: $y = \sqrt{2}(x - \sqrt{2}) + 1$; normal line: $y = -1/\sqrt{2}(x - \sqrt{2}) + 1$

11. (a) $\frac{dy}{dx} = \frac{\cos t\sin(2t) + \sin t\cos(2t)}{-\sin t\sin(2t) + 2\cos t\cos(2t)}$
 (b) Tangent line: $y = x - \sqrt{2}$; normal line: $y = -x - \sqrt{2}$

13. $t = 0$

15. $t = -1/2$

17. The graph does not have a horizontal tangent line.

19. The solution is non-trivial; use identities $\sin(2t) = 2\sin t\cos t$ and $\cos(2t) = \cos^2 t - \sin^2 t$ to rewrite $g'(t) = 2\sin t(2\cos^2 t - \sin^2 t)$. On $[0, 2\pi]$, $\sin t = 0$ when $t = 0, \pi, 2\pi$, and $2\cos^2 t - \sin^2 t = 0$ when $t = \tan^{-1}(\sqrt{2}),\ \pi \pm \tan^{-1}(\sqrt{2}),\ 2\pi - \tan^{-1}(\sqrt{2})$.

21. $t_0 = 0$; $\lim_{t\to 0} \frac{dy}{dx} = 0$.

23. $t_0 = 1$; $\lim_{t\to 1} \frac{dy}{dx} = \infty$.

25. $\frac{d^2y}{dx^2} = 2$; always concave up

27. $\frac{d^2y}{dx^2} = -\frac{4}{(2t-1)^3}$; concave up on $(-\infty, 1/2)$; concave down on $(1/2, \infty)$.

29. $\frac{d^2y}{dx^2} = -\cot^3 t$; concave up on $(-\infty, 0)$; concave down on $(0, \infty)$.

31. $\frac{d^2y}{dx^2} = \frac{4(13+3\cos(4t))}{(\cos t + 3\cos(3t))^3}$, obtained with a computer algebra system; concave up on $\big(-\tan^{-1}(\sqrt{2}/2), \tan^{-1}(\sqrt{2}/2)\big)$, concave down on $\big(-\pi/2, -\tan^{-1}(\sqrt{2}/2)\big) \cup \big(\tan^{-1}(\sqrt{2}/2), \pi/2\big)$

33. $L = 6\pi$

35. $L = 2\sqrt{34}$

37. $L = \displaystyle\int_0^6 \sqrt{(3t^2 - 1)^2 + (3t^2 + 2)^2}\, dt$

39. $8\sqrt{257} + \frac{1}{2}\ln\left(16 + \sqrt{257}\right)$

41. $L \approx 2.4416$ (actual value: $L = 2.42211$)

43. $L \approx 4.19216$ (actual value: $L = 4.18308$)

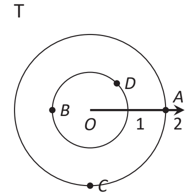
45. The answer is 16π for both (of course), but the integrals are different.

47. $SA \approx 8.50101$ (actual value $SA = 8.02851$

Section 9.4

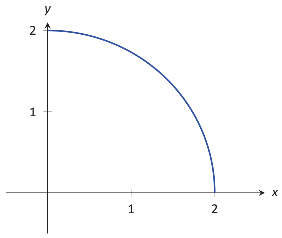
1. Answers will vary.

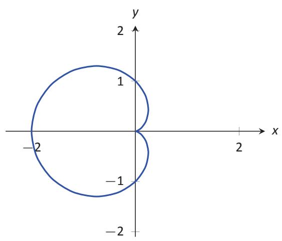
3. T

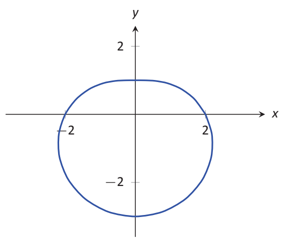
5. 

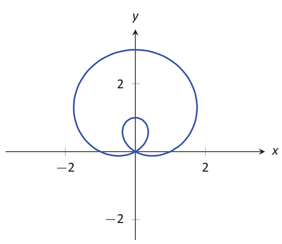
7. $A = P(2.5, \pi/4)$ and $P(-2.5, 5\pi/4)$;
$B = P(-1, 5\pi/6)$ and $P(1, 11\pi/6)$;
$C = P(3, 4\pi/3)$ and $P(-3, \pi/3)$;
$D = P(1.5, 2\pi/3)$ and $P(-1.5, 5\pi/3)$;

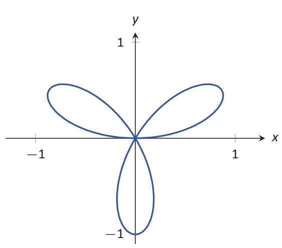
9. $A = (\sqrt{2}, \sqrt{2})$
$B = (\sqrt{2}, -\sqrt{2})$
$C = P(\sqrt{5}, -0.46)$
$D = P(\sqrt{5}, 2.68)$

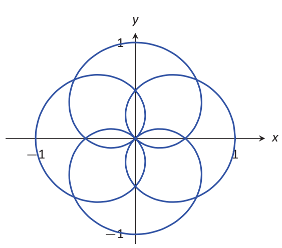
11. 

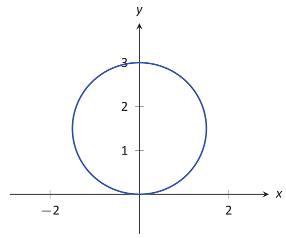
13. 

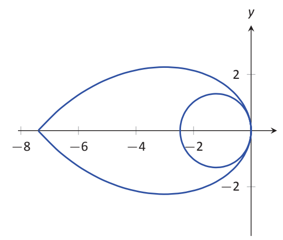
15. 

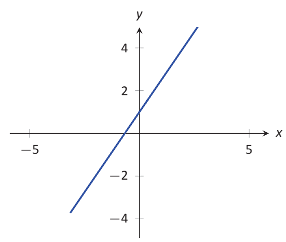
17. 

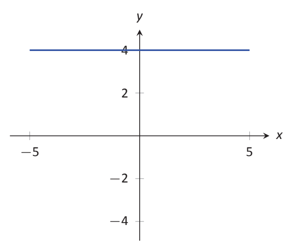
19. 

21. 

23. 

25. 

27. 

29. 

31. $x^2 + (y + 2)^2 = 4$

33. $y = 2/5x + 7/5$

35. $y = 4$

37. $x^2 + y^2 = 4$

39. $\theta = \pi/4$

41. $r = 5 \sec \theta$

43. $r = \cos \theta / \sin^2 \theta$

45. $r = \sqrt{7}$

47. $P(\sqrt{3}/2, \pi/6)$, $P(0, \pi/2)$, $P(-\sqrt{3}/2, 5\pi/6)$

49. $P(0, 0) = P(0, \pi/2)$, $P(\sqrt{2}, \pi/4)$

51. $P(\sqrt{2}/2, \pi/12)$, $P(-\sqrt{2}/2, 5\pi/12)$, $P(\sqrt{2}/2, 3\pi/4)$

53. For all points, $r = 1$; $\theta =$
$\pi/12,\ 5\pi/12,\ 7\pi/12,\ 11\pi/12,\ 13\pi/12,\ 17\pi/12,\ 19\pi/12,\ 23\pi/12.$

55. Answers will vary. If m and n do not have any common factors, then an interval of $2n\pi$ is needed to sketch the entire graph.

Section 9.5

1. Using $x = r\cos\theta$ and $y = r\sin\theta$, we can write $x = f(\theta)\cos\theta$, $y = f(\theta)\sin\theta$.

3. (a) $\frac{dy}{dx} = -\cot\theta$

   (b) tangent line: $y = -(x - \sqrt{2}/2) + \sqrt{2}/2$; normal line: $y = x$

5. (a) $\frac{dy}{dx} = \frac{\cos\theta(1+2\sin\theta)}{\cos^2\theta - \sin\theta(1+\sin\theta)}$

   (b) tangent line: $x = 3\sqrt{3}/4$; normal line: $y = 3/4$

7. (a) $\frac{dy}{dx} = \frac{\theta\cos\theta + \sin\theta}{\cos\theta - \theta\sin\theta}$

   (b) tangent line: $y = -2/\pi x + \pi/2$; normal line: $y = \pi/2 x + \pi/2$

9. (a) $\frac{dy}{dx} = \frac{4\sin(t)\cos(4t) + \sin(4t)\cos(t)}{4\cos(t)\cos(4t) - \sin(t)\sin(4t)}$

   (b) tangent line: $y = 5\sqrt{3}(x + \sqrt{3}/4) - 3/4$; normal line: $y = -1/5\sqrt{3}(x + \sqrt{3}/4) - 3/4$

11. horizontal: $\theta = \pi/2, 3\pi/2$;
    vertical: $\theta = 0, \pi, 2\pi$

13. horizontal: $\theta = \tan^{-1}(1/\sqrt{5}),\ \pi/2,\ \pi - \tan^{-1}(1/\sqrt{5}),\ \pi + \tan^{-1}(1/\sqrt{5}),\ 3\pi/2,\ 2\pi - \tan^{-1}(1/\sqrt{5})$;
    vertical: $\theta = 0,\ \tan^{-1}(\sqrt{5}),\ \pi - \tan^{-1}(\sqrt{5}),\ \pi,\ \pi + \tan^{-1}(\sqrt{5}),\ 2\pi - \tan^{-1}(\sqrt{5})$

15. In polar: $\theta = 0 \cong \theta = \pi$
    In rectangular: $y = 0$

17. area = 4π

19. area = $\pi/12$

21. area = $\pi - 3\sqrt{3}/2$

23. area = $\pi + 3\sqrt{3}$

25. area = $\int_{\pi/12}^{\pi/3} \frac{1}{2}\sin^2(3\theta)\,d\theta - \int_{\pi/12}^{\pi/6} \frac{1}{2}\cos^2(3\theta)\,d\theta = \frac{1}{12} + \frac{\pi}{24}$

27. area = $\int_0^{\pi/3} \frac{1}{2}(1 - \cos\theta)^2\,d\theta + \int_{\pi/3}^{\pi/2} \frac{1}{2}(\cos\theta)^2\,d\theta = \frac{7\pi}{24} - \frac{\sqrt{3}}{2} \approx 0.0503$

29. 4π

31. $L \approx 2.2592$; (actual value $L = 2.22748$)

33. $SA = 16\pi$

35. $SA = 32\pi/5$

37. $SA = 36\pi$

Chapter 10

Section 10.1

1. right hand

3. curve (a parabola); surface (a cylinder)

5. a hyperboloid of two sheets

7. $||\overline{AB}|| = \sqrt{6}$; $||\overline{BC}|| = \sqrt{17}$; $||\overline{AC}|| = \sqrt{11}$. Yes, it is a right triangle as $||\overline{AB}||^2 + ||\overline{AC}||^2 = ||\overline{BC}||^2$.

9. Center at $(4, 0, -7)$; radius = $\sqrt{118}$

11. $(x-6)^2 + y^2 + z^2 = 25$

13. Center at $(-2, 1, 2)$; radius = $\sqrt{5}$

15. $\frac{(x-3)^2}{16} + \frac{(y+2)^2}{25} + z^2 = 1$

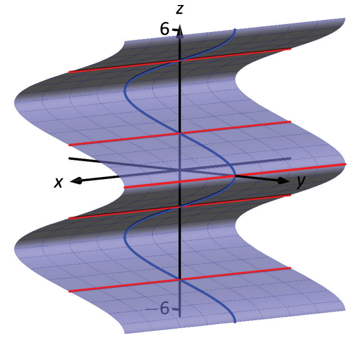
17. Circles

19. A single point.

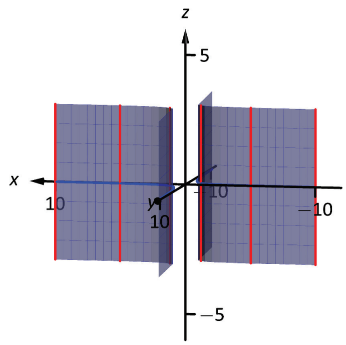
21. Region bounded between the planes $x = 0$ (the $y - z$ coordinate plane) and $x = 3$.

23. All points in space where the y value is greater than 3; viewing space as often depicted in this text, this is the region "to the right" of the plane $y = 3$ (which is parallel to the $x - z$ coordinate plane.)

25.



27.



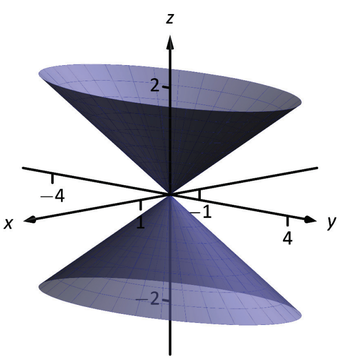
29. $y^2 + z^2 = x^4$

31. $z = \frac{1}{\sqrt{x^2+y^2}}$

33. (b) $x^2 - y^2 + z^2 = 0$

35. (a) $y^2 - x^2 - z^2 = 1$

37.



39.