33. The solid is the full cylinder of radius 1 with a height of 4 units

35. The solid is a cone opening down with spherical end, part of the solid unit sphere

37. -3

39. The limit does not exist.

Chapter 14

Section 14.1

1. When C is a curve in the plane and f is a surface defined over C, then $\int_C f(s)\, ds$ describes the area under the spatial curve that lies on f, over C.

3. The variable s denotes the arc-length parameter, which is generally difficult to use. The Key Idea allows one to parameterize a curve using another, ideally easier-to-use, parameter.

5. $12\sqrt{2}$

7. 1

9. $\frac{\sqrt{2}}{3} + \frac{1}{2}$

11. $10\pi^2$

13. Over the first subcurve of C, the line integral has a value of $2\sqrt{2}/3$; over the second subcurve, the line integral has a value of $\pi - 2$. The total value of the line integral is thus $\pi + 2\sqrt{2}/3 - 2$.

15. $\int_0^\pi t\sqrt{1+\cos^2 t}\, dt \approx 6.001$

17. $\int_{-1}^1 \left(3t^3+2t+5\right)\sqrt{9t^4+1}\, dt \approx 15.479$

19. 2π

21. $5/2$

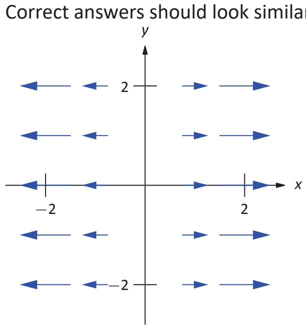
23. $M \approx 0.237$; center of mass is approximately $(0.173, 0.099, 0.065)$.

Section 14.2

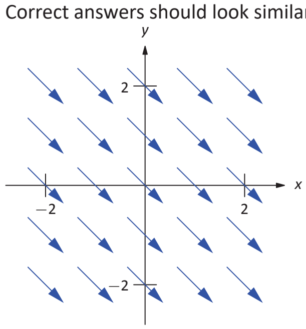
1. Answers will vary. Appropriate answers include velocities of moving particles (air, water, etc.); gravitational or electromagnetic forces.

3. Specific answers will vary, though should relate to the idea that the vector field is spinning clockwise at that point.

5. Correct answers should look similar to



7. Correct answers should look similar to



9. $\text{div}\,\vec{F} = 1 + 2y$
   $\text{curl}\,\vec{F} = 0$

11. $\text{div}\,\vec{F} = x\cos(xy) - y\sin(xy)$
   $\text{curl}\,\vec{F} = y\cos(xy) + x\sin(xy)$

13. $\text{div}\,\vec{F} = 3$
   $\text{curl}\,\vec{F} = \langle -1, -1, -1\rangle$

15. $\text{div}\,\vec{F} = 1 + 2y$
   $\text{curl}\,\vec{F} = 0$

17. $\text{div}\,\vec{F} = 2y - \sin z$
   $\text{curl}\,\vec{F} = \vec{0}$

Section 14.3

1. False. It is true for line integrals over scalar fields, though.

3. True.

5. We can conclude that $\vec{F}$ is conservative.

7. $\nabla f = \langle 3, 1\rangle$

9. $\nabla f = \langle 1, 2y\rangle$

11. $\nabla f = \langle 2x, -2y\rangle$

13. $\nabla f = \left\langle e^{x-y} - e^{x-y}\right\rangle$

15. $\nabla f = \left\langle y^x \ln y, xy^{x-1}\right\rangle$

17. Not conservative.

19. Not conservative.

21. Conservative. $f(x,y) = -4x^2 + 3xy + \frac{5y^2}{2} + C$

23. $5/3$. (One parameterization for C is $\vec{r}(t) = \langle t, t^2\rangle$ on $0 \le t \le 1$.)

25. $2/5$. (One parameterization for C is $\vec{r}(t) = \langle t, t^3\rangle$ on $-1 \le t \le 1$.)

27. 1.

29. $13/15$ joules. (One parameterization for C is $\vec{r}(t) = \langle t, \sqrt{t}\rangle$ on $0 \le t \le 1$.)

31. 24 ft-lbs.

33. (a) $f(x,y) = x^2 + xy + y^2$
   (b) $\text{curl}\,\vec{F} = 0$.
   (c) 0.
   (d) 0 (with $A = (0,0)$ and $B = (0,0)$, $f(B) - f(A) = 0$.)

35. (a) $f(x,y) = x^2 + y^2 + z^2$
   (b) $\text{curl}\,\vec{F} = \vec{0}$.
   (c) 0.
   (d) 0 (with $A = (1,0,0)$ and $B = (1,0,0)$, $f(B) - f(A) = 250$.)

Section 14.4

1. along, across

3. the curl of $\vec{F}$, or $\text{curl}\,\vec{F}$

5. $\text{curl}\,\vec{F}$

7. 12

9. $-2/3$

11. $1/2$

13. 4π

15. 0

17. -4π

19. The line integral $\oint_C \vec{F} \cdot d\vec{r}$, over the parabola, is $38/3$; over the line, it is -10. The total line integral is thus $38/3 - 10 = 8/3$. The double integral of curl $\vec{F} = 2$ over R also has value $8/3$.

21. Three line integrals need to be computed to compute $\oint_C \vec{F} \cdot d\vec{r}$. It does not matter which corner one starts from first, but be sure to proceed around the triangle in a counterclockwise fashion.
From $(0,0)$ to $(2,0)$, the line integral has a value of 0. From $(2,0)$ to $(1,1)$ the integral has a value of $7/3$. From $(1,1)$ to $(0,0)$ the line integral has a value of $-1/3$. Total value is 2.
The double integral of curl $\vec{F}$ over R also has value 2.

23. 3π

25. $256/15$

27. 410π

29. Any choice of $\vec{F}$ is appropriate as long as curl $\vec{F} = 1$. The choices of $\vec{F} = \langle -y, 0\rangle$ and $\langle 0, x\rangle$ each lead to reasonable integrands. The area of R is $4/3$.

31. Any choice of $\vec{F}$ is appropriate as long as curl $\vec{F} = 1$. The choice of $\vec{F} = \langle -y/2, x/2\rangle$ leads to a reasonable integrand after simplification. The area of R is $41\pi/10$.

33. Both the line integral and double integral have value of 0.

35. Two line integrals need to be computed to compute $\oint_C \vec{F} \cdot \vec{n}\, ds$. Along the parabola, the line integral has value $159/20$. Along the line, the line integral has value 6. Together, the total value is $279/20$.
The double integral of div $\vec{F}$ over R also has value $279/20$.

Section 14.5

1. Answers will vary, though generally should meaningfully include terms like "two sided".

3. (a) $\vec{r}(u,v) = \langle u, v, 3u^2v\rangle$ on $-1 \le u \le 1$, $0 \le v \le 2$.
(b) $\vec{r}(u,v) = \langle 3v\cos u + 1, 3v\sin u + 2, 3(3v\cos u + 1)^2(3v\sin u + 2)\rangle$, on $0 \le u \le 2\pi$, $0 \le v \le 1$.
(c) $\vec{r}(u,v) = \langle u, v(2-2u), 3u^2v(2-2u)\rangle$ on $0 \le u, v \le 1$.
(d) $\vec{r}(u,v) = \langle u, v(1-u^2), 3u^2v(1-u^2)\rangle$ on $-1 \le u \le 1$, $0 \le v \le 1$.

5. $\vec{r}(u,v) = \langle 0, u, v\rangle$ with $0 \le u \le 2$, $0 \le v \le 1$.

7. $\vec{r}(u,v) = \langle 3\sin u\cos v, 2\sin u\sin v, 4\cos u\rangle$ with $0 \le u \le \pi$, $0 \le v \le 2\pi$.

9. Answers may vary.
For $z = \frac{1}{2}(3-x)$: $\vec{r}(u,v) = \langle u, v, \frac{1}{2}(3-u)\rangle$, with $1 \le u \le 3$ and $0 \le v \le 2$.
For $x = 1$: $\vec{r}(u,v) = \langle 0, u, v\rangle$, with $0 \le u \le 2$, $0 \le v \le 1$
For $y = 0$: $\vec{r}(u,v) = \langle u, 0, v/2(3-u)\rangle$, with $1 \le u \le 3$, $0 \le v \le 1$
For $y = 2$: $\vec{r}(u,v) = \langle u, 2, v/2(3-u)\rangle$, with $1 \le u \le 3$, $0 \le v \le 1$
For $z = 0$: $\vec{r}(u,v) = \langle u, v, 0\rangle$, with $1 \le u \le 3$, $0 \le v \le 2$

11. Answers may vary.
For $z = 2y$: $\vec{r}(u,v) = \langle u, v(4-u^2), 2v(4-u^2)\rangle$ with $-2 \le u \le 2$ and $0 \le v \le 1$.
For $y = 4 - x^2$: $\vec{r}(u,v) = \langle u, 4-u^2, 2v(4-u^2)\rangle$ with $-2 \le u \le 2$ and $0 \le v \le 1$.
For $z = 0$: $\vec{r}(u,v) = \langle u, v(4-u^2), 0\rangle$ with $-2 \le u \le 2$ and $0 \le v \le 1$.

13. Answers may vary.
For $x + y^2/9 = 1$: $\vec{r}(u,v) = \langle \cos u, 3\sin u, v\rangle$ with $0 \le u \le 2\pi$ and $1 \le v \le 3$.
For $z = 1$: $\vec{r}(u,v) = \langle v\cos u, 3v\sin u, 1\rangle$ with $0 \le u \le 2\pi$ and $0 \le v \le 1$.
For $z = 3$: $\vec{r}(u,v) = \langle v\cos u, 3v\sin u, 3\rangle$ with $0 \le u \le 2\pi$ and $0 \le v \le 1$.

15. Answers may vary.
For $z = 1 - x^2$: $\vec{r}(u,v) = \langle u, v, 1-u^2\rangle$ with $-1 \le u \le 1$ and $-1 \le v \le 2$.
For $y = -1$: $\vec{r}(u,v) = \langle u, -1, v(1-u^2)\rangle$ with $-1 \le u \le 1$ and $0 \le v \le 1$.
For $y = 2$: $\vec{r}(u,v) = \langle u, 2, v(1-u^2)\rangle$ with $-1 \le u \le 1$ and $0 \le v \le 1$.
For $z = 0$: $\vec{r}(u,v) = \langle u, v, 0\rangle$ with $-1 \le u \le 1$ and $-1 \le v \le 2$.

17. $S = 2\sqrt{14}$.

19. $S = 4\sqrt{3}\pi$.

21. $\frac{\pi}{6}\left(17\sqrt{17} - 1\right)$

23. $\pi\sqrt{3}$

25. $\pi\left(2 - \sqrt{2}\right)$

27. $40\pi\sqrt{2}$

29. $\frac{3}{8}$

31. $S = \int_0^3 \int_0^{2\pi} \sqrt{v^2 + 4v^4}\, du\, dv = (37\sqrt{37} - 1)\pi/6 \approx 117.319$.

33. $S = \int_0^1 \int_{-1}^1 \sqrt{(5u^2 - 2uv - 5)^2 + u^4 + (1-u^2)^2}\, du\, dv \approx 7.084$.

Section 14.6

1. curve; surface

3. outside

5. $\frac{\sqrt{3}}{24}$

7. 0

9. $240\sqrt{3}$

Section 14.7

1. Answers will vary; in Section 14.4, the Divergence Theorem connects outward flux over a closed curve in the plane to the divergence of the vector field, whereas in this section the Divergence Theorem connects outward flux over a closed surface in space to the divergence of the vector field.

3. Answers will vary. Often the closed surface $\mathcal{S}$ is composed of several smooth surfaces. To measure total outward flux, this may require evaluating multiple double integrals. Each double integral requires the parameterization of a surface and the computation of the cross product of partial derivatives. One triple integral may require less work, especially as the divergence of a vector field is generally easy to compute.

5. Outward flux across the cylinder $x^2 + y^2 = 1$ is 0; across the plane $z = 3$ the outward flux is 3π; across the plane $z = -3$ the outward flux is 3π.
Total outward flux: 6π.
$\iint_D \text{div}\,\vec{F}\, dV = \int_0^{2\pi}\int_0^1\int_{-3}^3 r\, dz\, dr\, d\theta = 6\pi$.

7. Outward flux across the paraboloid is $112\pi/3$; across the disk the outward flux is 0.
Total outward flux: $112\pi/3$.
$\iint_D \text{div}\,\vec{F}\, dV = \int_0^{2\pi}\int_0^2\int_0^{4-r^2} (2z+2)r\, dz\, dr\, d\theta = 112\pi/3$.

9. 8

11. $64/3$

Section 14.8

1. Curl.

3. Circulation on C: $\oint_C \vec{F} \cdot d\vec{r} = \pi$
   $\iint_{\mathcal{S}} \left(\text{curl}\,\vec{F} \right) \cdot \vec{n}\, dS = \pi.$

5. Circulation on C: The flow along the line from $(0, 0, 2)$ to $(4, 0, 0)$ is 0; from $(4, 0, 0)$ to $(0, 3, 0)$ it is -6, and from $(0, 3, 0)$ to $(0, 0, 2)$ it is 6. The total circulation is $0 + (-6) + 6 = 0$.
   $\iint_{\mathcal{S}} \left(\text{curl}\,\vec{F} \right) \cdot \vec{n}\, dS = \iint_{\mathcal{S}} 0\, dS = 0.$

7. $5/3$

9. 23π

11. (a) $\text{curl}\,\vec{F} = 1$.

    (b) $\text{curl}\,\vec{F} \cdot \vec{n} = 1$, where $\vec{n}$ is a unit vector normal to $\mathcal{S}$.