41. With $\vec{u} = \langle u_1, u_2, u_3\rangle$ and $\vec{v} = \langle v_1, v_2, v_3\rangle$, we have

$$\begin{aligned}\vec{u}\cdot(\vec{u}\times\vec{v}) &= \langle u_1, u_2, u_3\rangle \cdot (\langle u_2v_3 - u_3v_2, -(u_1v_3 - u_3v_1), u_1v_2 - u_2v_1\rangle)\\ &= u_1(u_2v_3 - u_3v_2) - u2(u_1v_3 - u_3v_1) + u3(u_1v_2 - u_2v_1)\\ &= 0.\end{aligned}$$

43. 54

Section 10.5

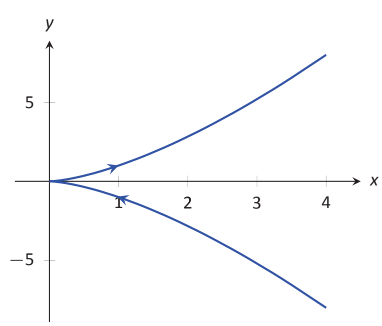
1. A point on the line and the direction of the line.
3. parallel, skew
5. vector: $\ell(t) = \langle 2, -4, 1\rangle + t\,\langle 9, 2, 5\rangle$
   parametric: $x = 2 + 9t$, $y = -4 + 2t$, $z = 1 + 5t$
   symmetric: $(x-2)/9 = (y+4)/2 = (z-1)/5$
7. vector: $\ell(t) = \langle -2, 5, 4\rangle + t\,\langle 0, 1, 3\rangle$
   parametric: $x = -2$, $y = 5 + t$, $z = 4 + 3t$
   symmetric: $x = -2$, $y - 5 = (z-4)/3$
9. Answers can vary: vector: $\ell(t) = \langle 2, 1, 5\rangle + t\,\langle 5, -3, -1\rangle$
   parametric: $x = 2 + 5t$, $y = 1 - 3t$, $z = 5 - t$
   symmetric: $(x-2)/5 = -(y-1)/3 = -(z-5)$
11. vector: $\ell(t) = \langle 1, 5, 5\rangle + t\,\langle 1, -3, 0\rangle$
   parametric: $x = 1 + t$, $y = 5 - 3t$, $z = 5$
   symmetric: $x - 1 = (y-5)/(-3)$, $z = 5$
13. Answers can vary; here the direction is given by $\vec{d}_1 \times \vec{d}_2$: vector: $\ell(t) = \langle 5, 1, 9\rangle + t\,\langle 0, -1, 0\rangle$
   parametric: $x = 5$, $y = 1 - t$, $z = 9$
   symmetric: $x = 5$, $z = 9$
15. Answers can vary; here the direction is given by $\vec{d}_1 \times \vec{d}_2$: vector: $\ell(t) = \langle 2, 2, 3\rangle + t\,\langle 5, -1, -3\rangle$
   parametric: $x = 2 + 5t$, $y = 2 - t$, $z = 3 - 3t$
   symmetric: $(x-2)/5 = -(y-2) = -(z-3)/3$
17. intersecting; $\ell_1(2) = \ell_2(-2) = \langle 12, 3, 7\rangle$
19. same
21. parallel
23. skew
25. $3\sqrt{2}$
27. 5
29. 2
31. (Note: this solution is easier once one has studied Section 10.6.) Since the two lines intersect, we can state $P_2 = P_1 + a\vec{d}_1 + b\vec{d}_2$ for some scalars a and b. (Here we abuse notation slightly and add points to vectors.) Thus $\overrightarrow{P_1P_2} = a\vec{d}_1 + b\vec{d}_2$. Vector $\vec{c}$ is the cross product of $\vec{d}_1$ and $\vec{d}_2$, hence is orthogonal to both, and hence is orthogonal to $\overrightarrow{P_1P_2}$. Thus $\overrightarrow{P_1P_2}\cdot\vec{c} = 0$, and the distance between lines is 0.

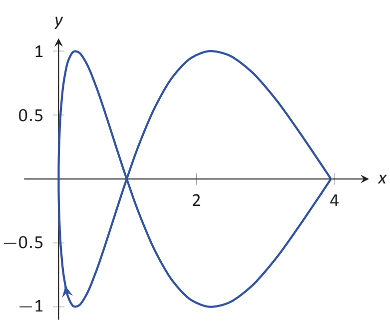
Section 10.6

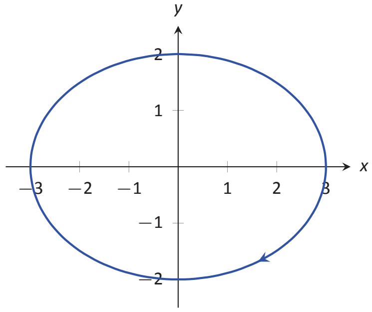
1. A point in the plane and a normal vector (i.e., a direction orthogonal to the plane).
3. Answers will vary.
5. Answers will vary.
7. Standard form: $3(x-2) - (y-3) + 7(z-4) = 0$
   general form: $3x - y + 7z = 31$
9. Answers may vary;
   Standard form: $8(x-1) + 4(y-2) - 4(z-3) = 0$
   general form: $8x + 4y - 4z = 4$
11. Answers may vary;
   Standard form: $-7(x-2) + 2(y-1) + (z-2) = 0$
   general form: $-7x + 2y + z = -10$
13. Answers may vary;
   Standard form: $2(x-1) - (y-1) = 0$
   general form: $2x - y = 1$
15. Answers may vary;
   Standard form: $2(x-2) - (y+6) - 4(z-1) = 0$
   general form: $2x - y - 4z = 6$
17. Answers may vary;
   Standard form: $(x-5) + (y-7) + (z-3) = 0$
   general form: $x + y + z = 15$
19. Answers may vary;
   Standard form: $3(x+4) + 8(y-7) - 10(z-2) = 0$
   general form: $3x + 8y - 10z = 24$
21. Answers may vary:
$$\ell = \begin{cases} x = 14t \\ y = -1 - 10t \\ z = 2 - 8t \end{cases}$$
23. $(-3, -7, -5)$
25. No point of intersection; the plane and line are parallel.
27. $\sqrt{5/7}$
29. $1/\sqrt{3}$
31. If P is any point in the plane, and Q is also in the plane, then $\overrightarrow{PQ}$ lies parallel to the plane and is orthogonal to $\vec{n}$, the normal vector. Thus $\vec{n}\cdot\overrightarrow{PQ} = 0$, giving the distance as 0.

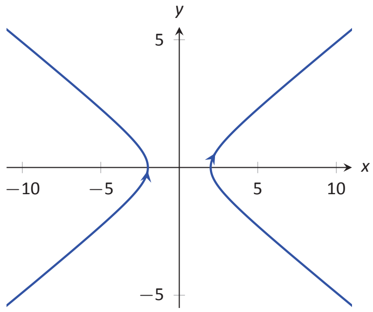
Chapter 11

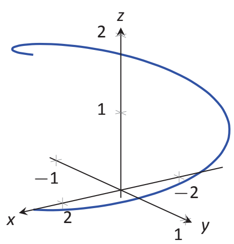
Section 11.1

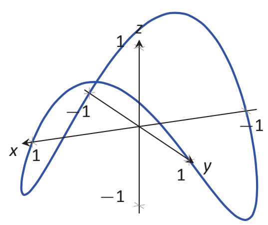
1. parametric equations
3. displacement
5. 

7. 

9. 

11. 

13. 

15. 

17. $\| \vec{r}(t) \| = \sqrt{25\cos^2 t + 9\sin^2 t}$.

19. $\| \vec{r}(t) \| = \sqrt{\cos^2 t + t^2 + t^4}$.

21. Answers may vary; three solutions are
$\vec{r}(t) = \langle 3\sin t + 5, 3\cos t + 5 \rangle$,
$\vec{r}(t) = \langle -3\cos t + 5, 3\sin t + 5 \rangle$ and
$\vec{r}(t) = \langle 3\cos t + 5, -3\sin t + 5 \rangle$.

23. Answers may vary, though most direct solutions are
$\vec{r}(t) = \langle -3\cos t + 3, 2\sin t - 2 \rangle$,
$\vec{r}(t) = \langle 3\cos t + 3, -2\sin t - 2 \rangle$ and
$\vec{r}(t) = \langle 3\sin t + 3, 2\cos t - 2 \rangle$.

25. Answers may vary, though most direct solutions are
$\vec{r}(t) = \langle t, -1/2(t-1) + 5 \rangle$,
$\vec{r}(t) = \langle t + 1, -1/2t + 5 \rangle$,
$\vec{r}(t) = \langle -2t + 1, t + 5 \rangle$ and
$\vec{r}(t) = \langle 2t + 1, -t + 5 \rangle$.

27. Answers may vary, though most direct solution is
$\vec{r}(t) = \langle 3\cos(4\pi t), 3\sin(4\pi t), 3t \rangle$.

29. $\langle 1, 1 \rangle$

31. $\langle 1, 2, 7 \rangle$

Section 11.2

1. component

3. It is difficult to identify the points on the graphs of $\vec{r}(t)$ and $\vec{r}\,'(t)$ that correspond to each other.

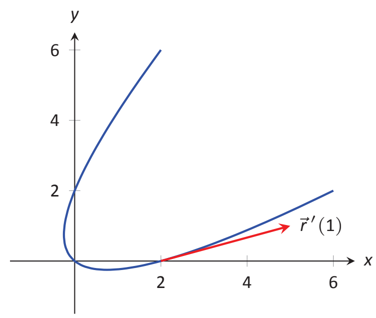
5. $\langle e^3, 0 \rangle$

7. $\langle 2t, 1, 0 \rangle$

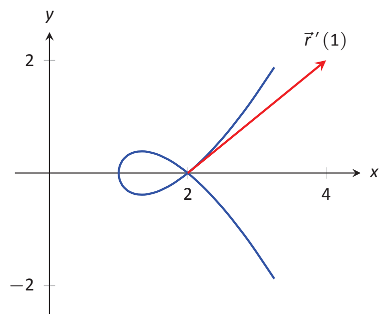
9. $(0, \infty)$

11. $\vec{r}\,'(t) = \langle -1/t^2, 5/(3t+1)^2, \sec^2 t \rangle$

13. $\vec{r}\,'(t) = \langle 2t, 1 \rangle \cdot \langle \sin t, 2t + 5 \rangle + \langle t^2 + 1, t - 1 \rangle \cdot \langle \cos t, 2 \rangle = (t^2 + 1)\cos t + 2t\sin t + 4t + 3$

15. 

$\vec{r}\,'(t) = \langle 2t + 1, 2t - 1 \rangle$

17. 

$\vec{r}\,'(t) = \langle 2t, 3t^2 - 1 \rangle$

19. $\ell(t) = \langle 2, 0 \rangle + t\langle 3, 1 \rangle$

21. $\ell(t) = \langle -3, 0, \pi \rangle + t\langle 0, -3, 1 \rangle$

23. $t = 2n\pi$, where n is an integer; so $t = \ldots - 4\pi, -2\pi, 0, 2\pi, 4\pi, \ldots$

25. $\vec{r}(t)$ is not smooth at $t = 3\pi/4 + n\pi$, where n is an integer

27. Both derivatives return $\langle 5t^4, 4t^3 - 3t^2, 3t^2 \rangle$.

29. Both derivatives return $\langle 2t - e^t - 1, \cos t - 3t^2, (t^2 + 2t)e^t - (t-1)\cos t - \sin t \rangle$.

31. $\langle \tan^{-1} t, \tan t \rangle + \vec{C}$

33. $\langle 4, -4 \rangle$

35. $\vec{r}(t) = \langle \ln|t + 1| + 1, -\ln|\cos t| + 2 \rangle$

37. $\vec{r}(t) = \langle -\cos t + 1, t - \sin t, e^t - t - 1 \rangle$

39. 10π

41. $\sqrt{2}(1 - e^{-1})$

Section 11.3

1. Velocity is a vector, indicating an objects direction of travel and its rate of distance change (i.e., its speed). Speed is a scalar.

3. The average velocity is found by dividing the displacement by the time traveled – it is a vector. The average speed is found by dividing the distance traveled by the time traveled – it is a scalar.

5. One example is traveling at a constant speed s in a circle, ending at the starting position. Since the displacement is $\vec{0}$, the average velocity is $\vec{0}$, hence $||\,\vec{0}\,|| = 0$. But traveling at constant speed s means the average speed is also $s > 0$.

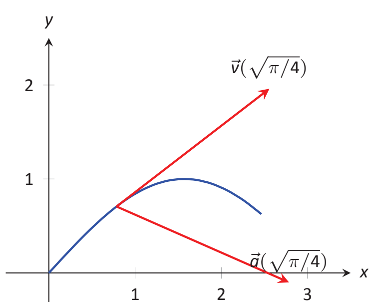
7. (a) $\vec{v}(t) = \langle 2, 5, 0\rangle$
   (b) $\vec{a}(t) = \langle 0, 0, 0\rangle$
   (c) $\vec{v}(t)\cdot\vec{a}(t) = 0$
   (d) The speed is constant. The speed is $\sqrt{29}$.

9. (a) $\vec{v}(t) = \langle -\sin t, \cos t\rangle$
   (b) $\vec{a}(t) = \langle -\cos t, -\sin t\rangle$
   (c) $\vec{v}(t)\cdot\vec{a}(t) = 0$
   (d) The speed is constant. The speed is 1.

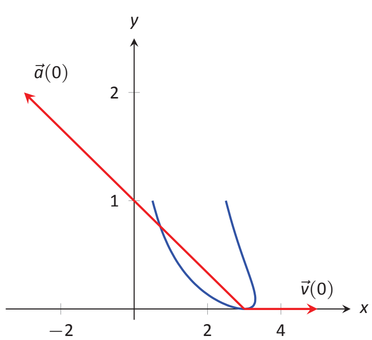
11. (a) $\vec{v}(t) = \langle 2t, \cos t, -\sin t\rangle$
   (b) $\vec{a}(t) = \langle 2, -\sin t, -\cos t\rangle$
   (c) $\vec{v}(t)\cdot\vec{a}(t) = 4t$
   (d) The speed is not constant.

13. (a) $\vec{v}(t) = \langle \cos t - \sin t, 3, \cos t + \sin t\rangle$
   (b) $\vec{a}(t) = \langle -\sin t - \cos t, 0, -\sin t + \cos t\rangle$
   (c) $\vec{v}(t)\cdot\vec{a}(t) = 0$
   (d) The speed is constant. The speed is $\sqrt{11}$.

15. $\vec{v}(t) = \left\langle 2t, 2t\cos\left(t^2\right)\right\rangle$, $\vec{a}(t) = \left\langle 2, 2\left(\cos\left(t^2\right) - 2t^2\sin\left(t^2\right)\right)\right\rangle$



17. $\vec{v}(t) = \left\langle -\frac{2(t^2+3t-1)}{(t^2+1)^2}, 2t\right\rangle$, $\vec{a}(t) = \left\langle \frac{2(2t^3+9t^2-6t-3)}{(t^2+1)^3}, 2\right\rangle$



19. $||\,\vec{v}(t)\,|| = |t|\sqrt{9t^2 - 12t + 8}$.
min: $t = 0$; max: $t = -1$

21. $||\,\vec{v}(t)\,|| = \sqrt{4\sin^2 t + 25\cos^2 t}$.
min: $t = \pi/2,\ 3\pi/2$; max: $t = 0,\ 2\pi$

23. $||\,\vec{v}(t)\,|| = \sqrt{2 - 2\sin t}$.
min: $t = \pi/2$; max: $t = 3\pi/2$

25. $||\,\vec{v}(t)\,|| = \sqrt{8t^2 + 3}$.
min: $t = 0$; max: $t = 1$

27. $||\,\vec{v}(t)\,|| = \sqrt{g^2t^2 - (2gv_0\sin\theta)t + v_0^2}$.
min: $t = (v_0\sin\theta)/g$; max: $t = 0$, $t = (2v_0\sin\theta)/g$

29. (a) $\vec{r}_1(\pi/2) = \langle 0, 3\rangle$; $\vec{r}_2(\pi/8) = \langle 0, 3\rangle$
   (b) $\vec{v}_1(\pi/2) = \langle -3, 0\rangle$; $||\,\vec{v}_1(\pi/2)\,|| = 3$; $\vec{a}_1(\pi/2) = \langle 0, -3\rangle$
   $\vec{v}_2(\pi/8) = \langle -12, 0\rangle$; $||\,\vec{v}_2(\pi/8)\,|| = 12$;
   $\vec{a}_2(\pi/8) = \langle 0, -48\rangle$

31. (a) $\vec{r}_1(1) = \langle 1, 1\rangle$; $\vec{r}_2(\pi/2) = \langle 1, 1\rangle$
   (b) $\vec{v}_1(1) = \langle 1, 1/2\rangle$; $||\,\vec{v}_1(1)\,|| = \sqrt{5}/2$; $\vec{a}_1(1) = \langle 0, -1/4\rangle$
   $\vec{v}_2(\pi/2) = \langle 0, 0\rangle$; $||\,\vec{v}_2(\pi/2)\,|| = 0$;
   $\vec{a}_2(\pi/2) = \langle -1, -1/2\rangle$

33. $\vec{v}(t) = \langle 2t - 1, 3t - 1\rangle$, $\vec{r}(t) = \left\langle t^2 - t + 5, 3t^2/2 - t - 5/2\right\rangle$

35. $\vec{v}(t) = \langle 10, -32t + 50\rangle$, $\vec{r}(t) = \left\langle 10t, -16t^2 + 50t\right\rangle$

37. Displacement: $\langle -10, 0\rangle$; distance traveled: $5\pi \approx 15.71$ft; average velocity: $\langle -10/\pi, 0\rangle \approx \langle -3.18, 0\rangle$; average speed: 5ft/s

39. Displacement: $\langle 10, 20, -20\rangle$; distance traveled: 30ft; average velocity: $\langle 1, 2, -2\rangle$; average speed: 3ft/s

41. The stone, while whirling, can be modeled by $\vec{r}(t) = \langle 3\cos(8\pi t), 3\sin(8\pi t)\rangle$.
   (a) For t-values $t = \sin^{-1}(3/20)/(8\pi) + n/4 \approx 0.006 + n/4$, where n is an integer.
   (b) $||\,\vec{r}\,'(t)\,|| = 24\pi \approx 51.4$ft/s
   (c) At $t = 0.006$, the stone is approximately 19.77ft from Goliath. Using the formula for projectile motion, we want the angle of elevation that lets a projectile starting at $\langle 0, 6\rangle$ with a initial velocity of 51.4ft/s arrive at $\langle 19.77, 9\rangle$. The desired angle is 0.27 radians, or 15.69°.

43. The position function of the ball is $\vec{r}(t) = \left\langle (146.67\cos\theta)t, -16t^2 + (146.67\sin\theta)t + 3\right\rangle$, where θ is the angle of elevation.
   (a) With $\theta = 20^\circ$, the ball reaches 310ft from home plate in 2.25 seconds; at this time, the height of the ball is 34.9ft, not enough to clear the Green Monster.
   (b) With $\theta = 21^\circ$, the ball reaches 310ft from home plate in 2.26s, with a height of 40ft, clearing the wall.

45. The position function of the ball is $\vec{r}(t) = \left\langle (v_0\cos\theta)t, -16t^2 + (v_0\sin\theta)t + 6\right\rangle$, where θ is the angle of elevation and v_0 is the initial ball speed.
   (a) With $v_0 = 73.33$ft/s, there are two angles of elevation possible. An angle of $\theta = 9.47^\circ$ delivers the ball in 0.83s, while an angle of 79.57° delivers the ball in 4.5s.
   (b) With $\theta = 8^\circ$, the initial speed must be 53.8mph$\approx$ 78.9ft/s.

Section 11.4

1. 1

3. $\vec{T}(t)$ and $\vec{N}(t)$.

5. $\vec{T}(t) = \left\langle \frac{4t}{\sqrt{20t^2-4t+1}}, \frac{2t-1}{\sqrt{20t^2-4t+1}}\right\rangle$; $\vec{T}(1) = \left\langle 4/\sqrt{17}, 1/\sqrt{17}\right\rangle$

7. $\vec{T}(t) = \frac{\cos t\sin t}{\sqrt{\cos^2 t\sin^2 t}}\langle -\cos t, \sin t\rangle$. (Be careful; this cannot be simplified as just $\langle -\cos t, \sin t\rangle$ as $\sqrt{\cos^2 t\sin^2 t} \neq \cos t\sin t$, but rather $|\cos t\sin t|$.) $\vec{T}(\pi/4) = \left\langle -\sqrt{2}/2, \sqrt{2}/2\right\rangle$

9. $\ell(t) = \langle 2, 0\rangle + t\left\langle 4/\sqrt{17}, 1/\sqrt{17}\right\rangle$; in parametric form,
$$\ell(t) = \begin{cases} x &= 2 + 4t/\sqrt{17} \\ y &= t/\sqrt{17}\end{cases}$$

11. $\ell(t) = \left\langle \sqrt{2}/4, \sqrt{2}/4\right\rangle + t\left\langle -\sqrt{2}/2, \sqrt{2}/2\right\rangle$; in parametric form,
$$\ell(t) = \begin{cases} x &= \sqrt{2}/4 - \sqrt{2}t/2 \\ y &= \sqrt{2}/4 + \sqrt{2}t/2\end{cases}$$

13. $\vec{T}(t)=\langle -\sin t, \cos t\rangle$; $\vec{N}(t)=\langle -\cos t, -\sin t\rangle$

15. $\vec{T}(t)=\left\langle -\frac{\sin t}{\sqrt{4\cos^2 t+\sin^2 t}}, \frac{2\cos t}{\sqrt{4\cos^2 t+\sin^2 t}}\right\rangle$;
$\vec{N}(t)=\left\langle -\frac{2\cos t}{\sqrt{4\cos^2 t+\sin^2 t}}, -\frac{\sin t}{\sqrt{4\cos^2 t+\sin^2 t}}\right\rangle$

17. (a) Be sure to show work
(b) $\vec{N}(\pi/4)=\langle -5/\sqrt{34}, -3/\sqrt{34}\rangle$

19. (a) Be sure to show work
(b) $\vec{N}(0)=\left\langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\rangle$

21. $\vec{T}(t)=\frac{1}{\sqrt{5}}\langle 2, \cos t, -\sin t\rangle$; $\vec{N}(t)=\langle 0, -\sin t, -\cos t\rangle$

23. $\vec{T}(t)=\frac{1}{\sqrt{a^2+b^2}}\langle -a\sin t, a\cos t, b\rangle$; $\vec{N}(t)=\langle -\cos t, -\sin t, 0\rangle$

25. $a_{\text{T}}=\frac{4t}{\sqrt{1+4t^2}}$ and $a_{\text{N}}=\sqrt{4-\frac{16t^2}{1+4t^2}}$
At $t=0$, $a_{\text{T}}=0$ and $a_{\text{N}}=2$;
At $t=1$, $a_{\text{T}}=4/\sqrt{5}$ and $a_{\text{N}}=2/\sqrt{5}$.
At $t=0$, all acceleration comes in the form of changing the direction of velocity and not the speed; at $t=1$, more acceleration comes in changing the speed than in changing direction.

27. $a_{\text{T}}=0$ and $a_{\text{N}}=2$
At $t=0$, $a_{\text{T}}=0$ and $a_{\text{N}}=2$;
At $t=\pi/2$, $a_{\text{T}}=0$ and $a_{\text{N}}=2$.
The object moves at constant speed, so all acceleration comes from changing direction, hence $a_{\text{T}}=0$. $\vec{a}(t)$ is always parallel to $\vec{N}(t)$, but twice as long, hence $a_{\text{N}}=2$.

29. $a_{\text{T}}=0$ and $a_{\text{N}}=a$
At $t=0$, $a_{\text{T}}=0$ and $a_{\text{N}}=a$;
At $t=\pi/2$, $a_{\text{T}}=0$ and $a_{\text{N}}=a$.
The object moves at constant speed, meaning that a_{T} is always 0. The object "rises" along the z-axis at a constant rate, so all acceleration comes in the form of changing direction circling the z-axis. The greater the radius of this circle the greater the acceleration, hence $a_{\text{N}}=a$.

Section 11.5

1. time and/or distance

3. Answers may include lines, circles, helixes

5. κ

7. $s=3t$, so $\vec{r}(s)=\langle 2s/3, s/3, -2s/3\rangle$

9. $s=\sqrt{13}t$, so $\vec{r}(s)=\langle 3\cos(s/\sqrt{13}), 3\sin(s/\sqrt{13}), 2s/\sqrt{13}\rangle$

11. $\kappa=\frac{|6x|}{\left(1+(3x^2-1)^2\right)^{3/2}}$;
$\kappa(0)=0$, $\kappa(1/2)=\frac{192}{17\sqrt{17}}\approx 2.74$.

13. $\kappa=\frac{|\cos x|}{\left(1+\sin^2 x\right)^{3/2}}$;
$\kappa(0)=1$, $\kappa(\pi/2)=0$

15. $\kappa=\frac{|2\cos t\cos(2t)+4\sin t\sin(2t)|}{\left(4\cos^2(2t)+\sin^2 t\right)^{3/2}}$;
$\kappa(0)=1/4$, $\kappa(\pi/4)=8$

17. $\kappa=\frac{|6t^2+2|}{\left(4t^2+(3t^2-1)^2\right)^{3/2}}$;
$\kappa(0)=2$, $\kappa(5)=\frac{19}{1394\sqrt{1394}}\approx 0.0004$

19. $\kappa=0$;
$\kappa(0)=0$, $\kappa(1)=0$

21. $\kappa=\frac{3}{13}$;
$\kappa(0)=3/13$, $\kappa(\pi/2)=3/13$

23. maximized at $x=\pm\frac{\sqrt{2}}{\sqrt[4]{5}}$

25. maximized at $t=1/4$

27. radius of curvature is $5\sqrt{5}/4$.

29. radius of curvature is 9.

31. $x^2+(y-1/2)^2=1/4$, or $\vec{c}(t)=\langle 1/2\cos t, 1/2\sin t+1/2\rangle$

33. $x^2+(y+8)^2=81$, or $\vec{c}(t)=\langle 9\cos t, 9\sin t-8\rangle$

Chapter 12

Section 12.1

1. Answers will vary.

3. topographical

5. surface

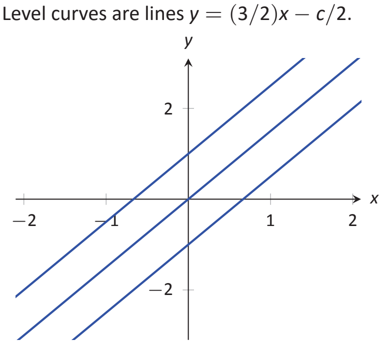
7. domain: $\mathbb{R}^2$
range: $z\geq 2$

9. domain: $\mathbb{R}^2$
range: $\mathbb{R}$

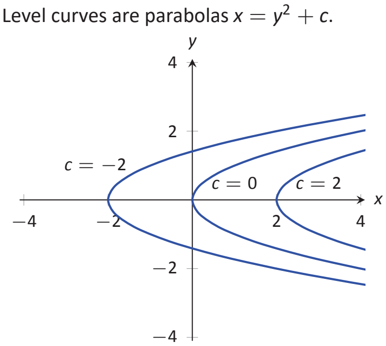
11. domain: $\mathbb{R}^2$
range: $0<z\leq 1$

13. domain: $\{(x,y)\mid x^2+y^2\leq 9\}$, i.e., the domain is the circle and interior of a circle centered at the origin with radius 3.
range: $0\leq z\leq 3$

15. Level curves are lines $y=(3/2)x-c/2$.



17. Level curves are parabolas $x=y^2+c$.



19. When $c\neq 0$, the level curves are circles, centered at $(1/c, -1/c)$ with radius $\sqrt{2/c^2-1}$. When $c=0$, the level curve is the line $y=x$.