19. 5 ft–lb

21. (a) 52,929.6 ft–lb
    (b) 18,525.3 ft–lb
    (c) When 3.83 ft of water have been pumped from the tank, leaving about 2.17 ft in the tank.

23. 212,135 ft–lb

25. 187,214 ft–lb

27. 4,917,150 J

Section 7.6

1. Answers will vary.

3. 499.2 lb

5. 6739.2 lb

7. 3920.7 lb

9. 2496 lb

11. 602.59 lb

13. (a) 2340 lb
    (b) 5625 lb

15. (a) 1597.44 lb
    (b) 3840 lb

17. (a) 56.42 lb
    (b) 135.62 lb

19. 5.1 ft

Section 7.7

1. $\left(\frac{6}{5}, \frac{15}{7}\right)$

3. $\left(0, -\frac{14}{3\pi+18}\right)$

5. $\left(\frac{\pi}{2} - 1, \frac{\pi}{8}\right)$

7. $\left(0, \frac{4\pi-3\sqrt{3}}{4\pi-6\ln\left(2+\sqrt{3}\right)}\right)$

9. $\left(\frac{b-a}{3}, \frac{c}{3}\right)$

Chapter 8

Section 8.1

1. Answers will vary.

3. Answers will vary.

5. $2, \frac{8}{3}, \frac{8}{3}, \frac{32}{15}, \frac{64}{45}$

7. $\frac{1}{3}, 2, \frac{81}{5}, \frac{512}{3}, \frac{15625}{7}$

9. $a_n = 3n + 1$

11. $a_n = 10 \cdot 2^{n-1}$

13. $1/7$

15. 0

17. diverges

19. converges to 0

21. diverges

23. converges to e

25. converges to 0

27. converges to 2

29. bounded

31. bounded

33. neither bounded above or below

35. monotonically increasing

37. never monotonic

39. Let $\{a_n\}$ be given such that $\lim\limits_{n\to\infty} |a_n| = 0$. By the definition of the limit of a sequence, given any $\varepsilon > 0$, there is a m such that for all $n > m$, $\left| |a_n| - 0 \right| < \varepsilon$. Since $\left| |a_n| - 0 \right| = |a_n - 0|$, this directly implies that for all $n > m$, $|a_n - 0| < \varepsilon$, meaning that $\lim\limits_{n\to\infty} a_n = 0$.

41. Left to reader

Section 8.2

1. Answers will vary.

3. One sequence is the sequence of terms $\{a_n\}$. The other is the sequence of n^{th} partial sums, $\{S_n\} = \{\sum_{i=1}^{n} a_i\}$.

5. F

7. (a) $1, \frac{5}{4}, \frac{49}{36}, \frac{205}{144}, \frac{5269}{3600}$
    (b) Plot omitted

9. (a) $1, 3, 6, 10, 15$
    (b) Plot omitted

11. (a) $\frac{1}{3}, \frac{4}{9}, \frac{13}{27}, \frac{40}{81}, \frac{121}{243}$
    (b) Plot omitted

13. (a) $0.1, 0.11, 0.111, 0.1111, 0.11111$
    (b) Plot omitted

15. $\lim\limits_{n\to\infty} a_n = \infty$; by Theorem 64 the series diverges.

17. $\lim\limits_{n\to\infty} a_n = 1$; by Theorem 64 the series diverges.

19. $\lim\limits_{n\to\infty} a_n = e$; by Theorem 64 the series diverges.

21. Diverges

23. Converges

25. (a) $S_n = \frac{1-(1/4)^n}{3/4}$
    (b) Converges to $4/3$.

27. (a) $S_n = \begin{cases} \frac{n+1}{2} & \text{n is odd} \\ -\frac{n}{2} & \text{n is even} \end{cases}$
    (b) Diverges

29. (a) $S_n = \frac{1-(1/e)^{n+1}}{1-1/e}$.
    (b) Converges to $1/(1 - 1/e) = e/(e - 1)$.

31. (a) With partial fractions, $a_n = \frac{1}{n} - \frac{1}{n+1}$. Thus $S_n = 1 - \frac{1}{n+1}$.
    (b) Converges to 1.

33. (a) Use partial fraction decomposition to recognize the telescoping series: $a_n = \frac{1}{2}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$. Then $S_n = \frac{n}{2n+1}$.
    (b) Converges to $1/2$.

35. (a) $S_n = 1 - \frac{1}{(n+1)^2}$
    (b) Converges to 1.

37. (a) $a_n = 1/2^n + 1/3^n$ for $n \geq 0$. Thus $S_n = \frac{1-1/2^2}{1/2} + \frac{1-1/3^n}{2/3}$.
    (b) Converges to $2 + 3/2 = 7/2$.

39. (a) $S_n = \frac{1-(\sin 1)^{n+1}}{1-\sin 1}$
    (b) Converges to $\frac{1}{1-\sin 1}$..

41. Using partial fractions, we can show that $a_n = \frac{1}{4}\left(\frac{1}{2n-1} + \frac{1}{2n+1}\right)$. The series is effectively twice the sum of the odd terms of the Harmonic Series which was shown to diverge in Exercise 40. Thus this series diverges.

Section 8.3

1. continuous, positive and decreasing
3. The Integral Test (we do not have a continuous definition of $n!$ yet) and the Limit Comparison Test (same as above, hence we cannot take its derivative).
5. Converges
7. Diverges
9. Diverges
11. Diverges
13. Diverges
15. Converges
17. Converges
19. Diverges
21. Converges; compare to $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}$, as $1/(n^2+3n-5) \le 1/n^2$ for all $n > 1$.
23. Diverges; compare to $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$, as $1/n \le \ln n/n$ for all $n \ge 2$.
25. Diverges; compare to $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$. Since $n = \sqrt{n^2} > \sqrt{n^2-1}$, $1/n \le 1/\sqrt{n^2-1}$ for all $n \ge 2$.
27. Diverges; compare to $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$:
$$\frac{1}{n} = \frac{n^2}{n^3} < \frac{n^2+n+1}{n^3} < \frac{n^2+n+1}{n^3-5},$$
for all $n \ge 1$.
29. Diverges; compare to $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$. Note that
$$\frac{n}{n^2-1} = \frac{n^2}{n^2-1}\cdot\frac{1}{n} > \frac{1}{n},$$
as $\frac{n^2}{n^2-1} > 1$, for all $n \ge 2$.
31. Converges; compare to $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}$.
33. Diverges; compare to $\displaystyle\sum_{n=1}^{\infty}\frac{\ln n}{n}$.
35. Diverges; compare to $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$.
37. Diverges; compare to $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$. Just as $\displaystyle\lim_{n\to 0}\frac{\sin n}{n} = 1$, $\displaystyle\lim_{n\to\infty}\frac{\sin(1/n)}{1/n} = 1$.
39. Converges; compare to $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$.
41. Converges; Integral Test
43. Diverges; the n^{th} Term Test and Direct Comparison Test can be used.
45. Converges; the Direct Comparison Test can be used with sequence $1/3^n$.
47. Diverges; the n^{th} Term Test can be used, along with the Integral Test.
49. (a) Converges; use Direct Comparison Test as $\frac{a_n}{n} < n$.
    (b) Converges; since original series converges, we know $\lim_{n\to\infty} a_n = 0$. Thus for large n, $a_n a_{n+1} < a_n$.
    (c) Converges; similar logic to part (b) so $(a_n)^2 < a_n$.
    (d) May converge; certainly $na_n > a_n$ but that does not mean it does not converge.
    (e) Does not converge, using logic from (b) and n^{th} Term Test.

Section 8.4

1. algebraic, or polynomial.
3. Integral Test, Limit Comparison Test, and Root Test
5. Converges
7. Converges
9. The Ratio Test is inconclusive; the p-Series Test states it diverges.
11. Converges
13. Converges; note the summation can be rewritten as $\displaystyle\sum_{n=1}^{\infty}\frac{2^n n!}{3^n n!}$, from which the Ratio Test can be applied.
15. Converges
17. Converges
19. Diverges
21. Diverges. The Root Test is inconclusive, but the n^{th}-Term Test shows divergence. (The terms of the sequence approach e^2, not 0, as $n \to \infty$.)
23. Converges
25. Diverges; Limit Comparison Test
27. Converges; Ratio Test or Limit Comparison Test with $1/3^n$.
29. Diverges; n^{th}-Term Test or Limit Comparison Test with 1.
31. Diverges; Direct Comparison Test with $1/n$
33. Converges; Root Test

Section 8.5

1. The signs of the terms do not alternate; in the given series, some terms are negative and the others positive, but they do not necessarily alternate.
3. Many examples exist; one common example is $a_n = (-1)^n/n$.
5. (a) converges
   (b) converges (p-Series)
   (c) absolute
7. (a) diverges (limit of terms is not 0)
   (b) diverges
   (c) n/a; diverges
9. (a) converges
   (b) diverges (Limit Comparison Test with $1/n$)
   (c) conditional
11. (a) diverges (limit of terms is not 0)
    (b) diverges
    (c) n/a; diverges

13. (a) diverges (terms oscillate between ± 1)
(b) diverges
(c) n/a; diverges

15. (a) converges
(b) converges (Geometric Series with $r = 2/3$)
(c) absolute

17. (a) converges
(b) converges (Ratio Test)
(c) absolute

19. (a) converges
(b) diverges (p-Series Test with $p = 1/2$)
(c) conditional

21. $S_5 = -1.1906$; $S_6 = -0.6767$;
$-1.1906 \leq \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)} \leq -0.6767$

23. $S_6 = 0.3681$; $S_7 = 0.3679$;
$0.3681 \leq \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \leq 0.3679$

25. $n = 5$

27. $n = 7$

29. $n = 5$ ($(2n)! > 10^8$ when $n \geq 6$)

Section 8.6

1. 1

3. 5

5. $1 + 2x + 4x^2 + 8x^3 + 16x^4$

7. $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$

9. (a) $R = \infty$
(b) $(-\infty, \infty)$

11. (a) $R = 1$
(b) $(2, 4]$

13. (a) $R = 2$
(b) $(-2, 2)$

15. (a) $R = 1/5$
(b) $(4/5, 6/5)$

17. (a) $R = 1$
(b) $(-1, 1)$

19. (a) $R = \infty$
(b) $(-\infty, \infty)$

21. (a) $R = 1$
(b) $[-1, 1]$

23. (a) $R = 1$
(b) $[-3, -1]$

25. (a) $R = 4$
(b) $x = (-8, 0)$

27. (a) $f'(x) = \displaystyle\sum_{n=1}^{\infty} x^{n-1}; \quad (-1, 1)$
(b) $\displaystyle\int f(x)\,dx = C + \sum_{n=1}^{\infty} \frac{1}{n(n+1)} x^{n+1}; \quad [-1, 1]$

29. (a) $f'(x) = \displaystyle\sum_{n=1}^{\infty} n(-3)^n x^{n-1}; \quad (-1/3, 1/3)$
(b) $\displaystyle\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{(-3)^n}{n+1} x^{n+1}; \quad (-1/3, 1/3]$

31. (a) $f'(x) = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^{n-1}}{(n-1)!} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^n}{n!}; \quad (-\infty, \infty)$
(b) $\displaystyle\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{(n+1)!}; \quad (-\infty, \infty)$

33. $5 + 25x + \frac{125}{2}x^2 + \frac{625}{6}x^3 + \frac{3125}{24}x^4$

35. $1 + 2x + x^2 + \frac{1}{3}x^3 + \frac{1}{12}x^4$

37. $1 + x + x^2 + \frac{1}{3}x^3 + \frac{1}{6}x^4$

Section 8.7

1. The Maclaurin polynomial is a special case of Taylor polynomials. Taylor polynomials are centered at a specific x-value; when that x-value is 0, it is a Maclauring polynomial.

3. $p_2(x) = 6 + 3x - 4x^2$.

5. $p_3(x) = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3$

7. $p_8(x) = x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4 + \frac{1}{24}x^5$

9. $p_4(x) = \frac{2x^4}{3} + \frac{4x^3}{3} + 2x^2 + 2x + 1$

11. $p_4(x) = x^4 - x^3 + x^2 - x + 1$

13. $p_4(x) = 1 + \frac{1}{2}(-1+x) - \frac{1}{8}(-1+x)^2 + \frac{1}{16}(-1+x)^3 - \frac{5}{128}(-1+x)^4$

15. $p_6(x) = \frac{1}{\sqrt{2}} - \frac{-\frac{\pi}{4}+x}{\sqrt{2}} - \frac{(-\frac{\pi}{4}+x)^2}{2\sqrt{2}} + \frac{(-\frac{\pi}{4}+x)^3}{6\sqrt{2}} + \frac{(-\frac{\pi}{4}+x)^4}{24\sqrt{2}} - \frac{(-\frac{\pi}{4}+x)^5}{120\sqrt{2}} - \frac{(-\frac{\pi}{4}+x)^6}{720\sqrt{2}}$

17. $p_5(x) = \frac{1}{2} - \frac{x-2}{4} + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3 + \frac{1}{32}(x-2)^4 - \frac{1}{64}(x-2)^5$

19. $p_3(x) = \frac{1}{2} + \frac{1+x}{2} + \frac{1}{4}(1+x)^2$

21. $p_3(x) = x - \frac{x^3}{6}$; $p_3(0.1) = 0.09983$. Error is bounded by $\pm\frac{1}{4!} \cdot 0.1^4 \approx \pm 0.000004167$.

23. $p_2(x) = 3 + \frac{1}{6}(-9+x) - \frac{1}{216}(-9+x)^2$; $p_2(10) = 3.16204$. The third derivative of $f(x) = \sqrt{x}$ is bounded on $(8, 11)$ by 0.003. Error is bounded by $\pm\frac{0.003}{3!} \cdot 1^3 = \pm 0.0005$.

25. The n^{th} derivative of $f(x) = e^x$ is bounded by 3 on intervals containing 0 and 1. Thus $|R_n(1)| \leq \frac{3}{(n+1)!}1^{(n+1)}$. When $n = 7$, this is less than 0.0001.

27. The n^{th} derivative of $f(x) = \cos x$ is bounded by 1 on intervals containing 0 and $\pi/3$. Thus $|R_n(\pi/3)| \leq \frac{1}{(n+1)!}(\pi/3)^{(n+1)}$. When $n = 7$, this is less than 0.0001. Since the Maclaurin polynomial of $\cos x$ only uses even powers, we can actually just use $n = 6$.

29. The n^{th} term is $\frac{1}{n!}x^n$.

31. The n^{th} term is x^n.

33. The n^{th} term is $(-1)^n \frac{(x-1)^n}{n}$.

35. $3 + 15x + \dfrac{75}{2}x^2 + \dfrac{375}{6}x^3 + \dfrac{1875}{24}x^4$

Section 8.8

1. A Taylor polynomial is a **polynomial**, containing a finite number of terms. A Taylor series is a **series**, the summation of an infinite number of terms.

3. All derivatives of e^x are e^x which evaluate to 1 at $x=0$.
The Taylor series starts $1+x+\frac{1}{2}x^2+\frac{1}{3!}x^3+\frac{1}{4!}x^4+\cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}\frac{x^n}{n!}$

5. The n^{th} derivative of $1/(1-x)$ is $f^{(n)}(x)=(n)!/(1-x)^{n+1}$, which evaluates to $n!$ at $x=0$.
The Taylor series starts $1+x+x^2+x^3+\cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}x^n$

7. The Taylor series starts
$0-(x-\pi/2)+0x^2+\frac{1}{6}(x-\pi/2)^3+0x^4-\frac{1}{120}(x-\pi/2)^5$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1}\frac{(x-\pi/2)^{2n+1}}{(2n+1)!}$

9. $f^{(n)}(x)=(-1)^ne^{-x}$; at $x=0$, $f^{(n)}(0)=-1$ when n is odd and $f^{(n)}(0)=1$ when n is even.
The Taylor series starts $1-x+\frac{1}{2}x^2-\frac{1}{3!}x^3+\cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^n}{n!}$.

11. $f^{(n)}(x)=(-1)^{n+1}\frac{n!}{(x+1)^{n+1}}$; at $x=1$, $f^{(n)}(1)=(-1)^{n+1}\frac{n!}{2^{n+1}}$
The Taylor series starts
$\frac{1}{2}+\frac{1}{4}(x-1)-\frac{1}{8}(x-1)^2+\frac{1}{16}(x-1)^3\cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{2^{n+1}}$.

13. Given a value x, the magnitude of the error term $R_n(x)$ is bounded by
$$\left|R_n(x)\right|\leq\frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}\left|x^{(n+1)}\right|,$$
where z is between 0 and x.
If $x>0$, then $z<x$ and $f^{(n+1)}(z)=e^z<e^x$. If $x<0$, then $x<z<0$ and $f^{(n+1)}(z)=e^z<1$. So given a fixed x value, let $M=\max\{e^x,1\}$; $f^{(n)}(z)<M$. This allows us to state
$$\left|R_n(x)\right|\leq\frac{M}{(n+1)!}\left|x^{(n+1)}\right|.$$
For any x, $\displaystyle\lim_{n\to\infty}\frac{M}{(n+1)!}\left|x^{(n+1)}\right|=0$. Thus by the Squeeze Theorem, we conclude that $\displaystyle\lim_{n\to\infty}R_n(x)=0$ for all x, and hence
$$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}\quad\text{for all } x.$$

15. Given a value x, the magnitude of the error term $R_n(x)$ is bounded by
$$\left|R_n(x)\right|\leq\frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}\left|(x-1)^{(n+1)}\right|,$$
where z is between 1 and x.
Note that $\left|f^{(n+1)}(x)\right|=\frac{n!}{x^{n+1}}$.
We consider the cases when $x>1$ and when $x<1$ separately.
If $x>1$, then $1<z<x$ and $f^{(n+1)}(z)=\frac{n!}{z^{n+1}}<n!$. Thus
$$\left|R_n(x)\right|\leq\frac{n!}{(n+1)!}\left|(x-1)^{(n+1)}\right|=\frac{(x-1)^{n+1}}{n+1}.$$
For a fixed x,
$$\lim_{n\to\infty}\frac{(x-1)^{n+1}}{n+1}=0.$$
If $0<x<1$, then $x<z<1$ and $f^{(n+1)}(z)=\frac{n!}{z^{n+1}}<\frac{n!}{x^{n+1}}$. Thus
$$\left|R_n(x)\right|\leq\frac{n!/x^{n+1}}{(n+1)!}\left|(x-1)^{(n+1)}\right|=\frac{x^{n+1}}{n+1}(1-x)^{n+1}.$$
Since $0<x<1$, $x^{n+1}<1$ and $(1-x)^{n+1}<1$. We can then extend the inequality from above to state
$$\left|R_n(x)\right|\leq\frac{x^{n+1}}{n+1}(1-x)^{n+1}<\frac{1}{n+1}.$$
As $n\to\infty$, $1/(n+1)\to 0$. Thus by the Squeeze Theorem, we conclude that $\displaystyle\lim_{n\to\infty}R_n(x)=0$ for all x, and hence
$$\ln x=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n}\quad\text{for all } 0<x\leq 2.$$

17. Given $\displaystyle\cos x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$,
$\displaystyle\cos(-x)=\sum_{n=0}^{\infty}(-1)^n\frac{(-x)^{2n}}{(2n)!}=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}=\cos x$, as all powers in the series are even.

19. Given $\displaystyle\sin x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$,
$\displaystyle\frac{d}{dx}\big(\sin x\big)=\frac{d}{dx}\left(\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}\right)=$
$\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(2n+1)x^{2n}}{(2n+1)!}=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}=\cos x$. (The summation still starts at $n=0$ as there was no constant term in the expansion of $\sin x$).

21. $1+\dfrac{x}{2}-\dfrac{x^2}{8}+\dfrac{x^3}{16}-\dfrac{5x^4}{128}$

23. $1+\dfrac{x}{3}-\dfrac{x^2}{9}+\dfrac{5x^3}{81}-\dfrac{10x^4}{243}$

25. $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(x^2)^{2n}}{(2n)!}=\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n}}{(2n)!}$

27. $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(2x+3)^{2n+1}}{(2n+1)!}$

29. $\displaystyle C+\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{n!(2n+1)}$

31. $x+x^2+\dfrac{x^3}{3}-\dfrac{x^5}{30}$

33. $\displaystyle\sum_{n=1}^{\infty}nx^n$

35. $\displaystyle\int_0^{\sqrt{\pi}}\sin\left(x^2\right)\,dx\approx\int_0^{\sqrt{\pi}}\left(x^2-\frac{x^6}{6}+\frac{x^{10}}{120}-\frac{x^{14}}{5040}\right)dx=0.8877$

37. $\displaystyle\sum_{n=1}^{\infty}\frac{x^n}{n^2}$

39. $\pi^2/6$

41. At $x=0$, both sides are 0. The derivative of the left side is $-\frac{\ln(1-x)}{x}-\frac{\ln(1+x)}{x}$. By algebra and rules of logarithms, this can be shown equivalent to the derivative of the right side, which is $-\frac{\ln\left(1-x^2\right)}{x}$.

43. $\frac{\pi^2}{12}-\frac{(\ln 2)^2}{2}$

45. -1

Chapter 9

Section 9.1