7. (a) $\frac{dy}{dx} = \frac{\theta\cos\theta + \sin\theta}{\cos\theta - \theta\sin\theta}$

   (b) tangent line: $y = -2/\pi x + \pi/2$; normal line: $y = \pi/2x + \pi/2$

9. (a) $\frac{dy}{dx} = \frac{4\sin(t)\cos(4t) + \sin(4t)\cos(t)}{4\cos(t)\cos(4t) - \sin(t)\sin(4t)}$

   (b) tangent line: $y = 5\sqrt{3}(x + \sqrt{3}/4) - 3/4$; normal line: $y = -1/5\sqrt{3}(x + \sqrt{3}/4) - 3/4$

11. horizontal: $\theta = \pi/2, 3\pi/2$;
    vertical: $\theta = 0, \pi, 2\pi$

13. horizontal: $\theta = \tan^{-1}(1/\sqrt{5}),\ \pi/2,\ \pi - \tan^{-1}(1/\sqrt{5}),\ \pi + \tan^{-1}(1/\sqrt{5}),\ 3\pi/2,\ 2\pi - \tan^{-1}(1/\sqrt{5})$;
    vertical: $\theta = 0,\ \tan^{-1}(\sqrt{5}),\ \pi - \tan^{-1}(\sqrt{5}),\ \pi,\ \pi + \tan^{-1}(\sqrt{5}),\ 2\pi - \tan^{-1}(\sqrt{5})$

15. In polar: $\theta = 0 \cong \theta = \pi$
    In rectangular: $y = 0$

17. area = 4π

19. area = $\pi/12$

21. area = $\pi - 3\sqrt{3}/2$

23. area = $\pi + 3\sqrt{3}$

25. area = $\displaystyle\int_{\pi/12}^{\pi/3} \frac{1}{2}\sin^2(3\theta)\,d\theta - \int_{\pi/12}^{\pi/6} \frac{1}{2}\cos^2(3\theta)\,d\theta = \frac{1}{12} + \frac{\pi}{24}$

27. area = $\displaystyle\int_{0}^{\pi/3} \frac{1}{2}(1-\cos\theta)^2\,d\theta + \int_{\pi/3}^{\pi/2} \frac{1}{2}(\cos\theta)^2\,d\theta = \frac{7\pi}{24} - \frac{\sqrt{3}}{2} \approx 0.0503$

29. 4π

31. $L \approx 2.2592$; (actual value $L = 2.22748$)

33. $SA = 16\pi$

35. $SA = 32\pi/5$

37. $SA = 36\pi$

Chapter 10

Section 10.1

1. right hand

3. curve (a parabola); surface (a cylinder)

5. a hyperboloid of two sheets

7. $||\overline{AB}|| = \sqrt{6}$; $||\overline{BC}|| = \sqrt{17}$; $||\overline{AC}|| = \sqrt{11}$. Yes, it is a right triangle as $||\overline{AB}||^2 + ||\overline{AC}||^2 = ||\overline{BC}||^2$.

9. Center at $(4, 0, -7)$; radius = $\sqrt{118}$

11. $(x-6)^2 + y^2 + z^2 = 25$

13. Center at $(-2, 1, 2)$; radius = $\sqrt{5}$

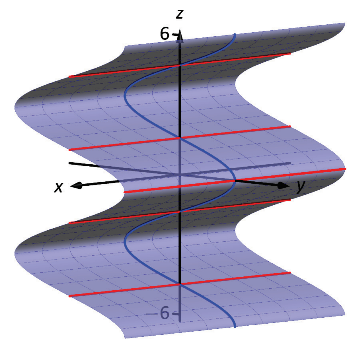
15. $\frac{(x-3)^2}{16} + \frac{(y+2)^2}{25} + z^2 = 1$

17. Circles

19. A single point.

21. Region bounded between the planes $x = 0$ (the $y - z$ coordinate plane) and $x = 3$.

23. All points in space where the y value is greater than 3; viewing space as often depicted in this text, this is the region "to the right" of the plane $y = 3$ (which is parallel to the $x - z$ coordinate plane.)

25.


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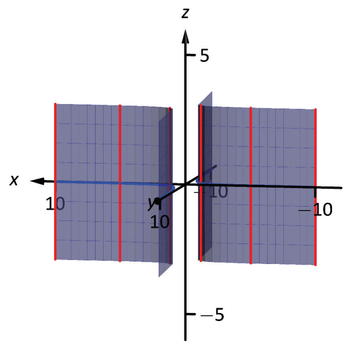
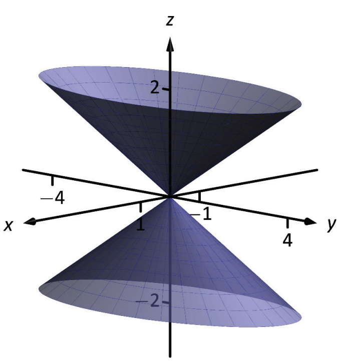


29. $y^2 + z^2 = x^4$

31. $z = \frac{1}{\sqrt{x^2+y^2}}$

33. (b) $x^2 - y^2 + z^2 = 0$

35. (a) $y^2 - x^2 - z^2 = 1$

37.


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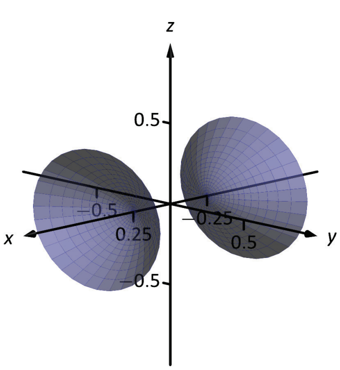
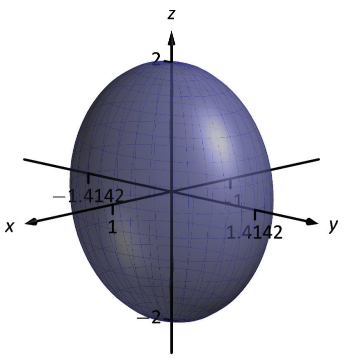
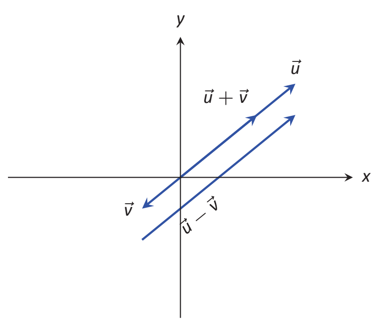
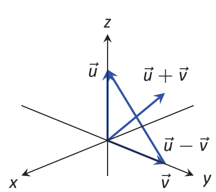


41. 

Section 10.2

1. Answers will vary.
3. A vector with magnitude 1.
5. It stretches the vector by a factor of 2, and points it in the opposite direction.
7. $\overrightarrow{PQ} = \langle -4, 4 \rangle = -4\vec{i} + 4\vec{j}$
9. $\overrightarrow{PQ} = \langle 2, 2, 0 \rangle = 2\vec{i} + 2\vec{j}$
11. (a) $\vec{u} + \vec{v} = \langle 3, 2, 1 \rangle$; $\vec{u} - \vec{v} = \langle -1, 0, -3 \rangle$; $\pi\vec{u} - \sqrt{2}\vec{v} = \langle \pi - 2\sqrt{2}, \pi - \sqrt{2}, -\pi - 2\sqrt{2} \rangle$.
    (c) $\vec{x} = \langle -1, 0, -3 \rangle$.
13. 

    Sketch of $\vec{u} - \vec{v}$ shifted for clarity.
15. 

17. $||\,\vec{u}\,|| = \sqrt{17}$, $||\,\vec{v}\,|| = \sqrt{3}$, $||\,\vec{u}+\vec{v}\,|| = \sqrt{14}$, $||\,\vec{u}-\vec{v}\,|| = \sqrt{26}$
19. $||\,\vec{u}\,|| = 7$, $||\,\vec{v}\,|| = 35$, $||\,\vec{u}+\vec{v}\,|| = 42$, $||\,\vec{u}-\vec{v}\,|| = 28$
21. $\vec{u} = \langle 3/\sqrt{58}, 7/\sqrt{58} \rangle$
23. $\vec{u} = \langle 1/3, -2/3, 2/3 \rangle$
25. $\vec{u} = \langle \cos 50^\circ, \sin 50^\circ \rangle \approx \langle 0.643, 0.766 \rangle$.
27.
$$\begin{aligned} ||\,\vec{u}\,|| &= \sqrt{\sin^2\varphi\cos^2\theta + \sin^2\varphi\sin^2\theta + \cos^2\varphi} \\ &= \sqrt{\sin^2\varphi(\cos^2\theta + \sin^2\theta) + \cos^2\varphi} \\ &= \sqrt{\sin^2\varphi + \cos^2\varphi} \\ &= 1. \end{aligned}$$
29. The force on each chain is 100lb.
31. The force on each chain is 50lb.
33. $\theta = 5.71^\circ$; the weight is lifted 0.005 ft (about 1/16th of an inch).
35. $\theta = 84.29^\circ$; the weight is lifted 9 ft.

Section 10.3

1. Scalar
3. By considering the sign of the dot product of the two vectors. If the dot product is positive, the angle is acute; if the dot product is negative, the angle is obtuse.
5. -22
7. 3
9. not defined
11. Answers will vary.
13. $\theta = 0.3218 \approx 18.43^\circ$
15. $\theta = \pi/4 = 45^\circ$
17. Answers will vary; two possible answers are $\langle -7, 4 \rangle$ and $\langle 14, -8 \rangle$.
19. Answers will vary; two possible answers are $\langle 1, 0, -1 \rangle$ and $\langle 4, 5, -9 \rangle$.
21. $\text{proj}_{\vec{v}}\,\vec{u} = \langle -1/2, 3/2 \rangle$.
23. $\text{proj}_{\vec{v}}\,\vec{u} = \langle -1/2, -1/2 \rangle$.
25. $\text{proj}_{\vec{v}}\,\vec{u} = \langle 1, 2, 3 \rangle$.
27. $\vec{u} = \langle -1/2, 3/2 \rangle + \langle 3/2, 1/2 \rangle$.
29. $\vec{u} = \langle -1/2, -1/2 \rangle + \langle -5/2, 5/2 \rangle$.
31. $\vec{u} = \langle 1, 2, 3 \rangle + \langle 0, 3, -2 \rangle$.
33. 1.96lb
35. 141.42ft–lb
37. 500ft–lb
39. 500ft–lb

Section 10.4

1. vector
3. “Perpendicular” is one answer.
5. Torque
7. $\vec{u} \times \vec{v} = \langle 11, 1, -17 \rangle$
9. $\vec{u} \times \vec{v} = \langle 47, -36, -44 \rangle$
11. $\vec{u} \times \vec{v} = \langle 0, 0, 0 \rangle$
13. $\vec{i} \times \vec{k} = -\vec{j}$
15. Answers will vary.
17. 5
19. 0
21. $\sqrt{14}$
23. 3
25. $5\sqrt{2}/2$
27. 1
29. 7
31. 2
33. $\pm\frac{1}{\sqrt{6}}\langle 1, 1, -2 \rangle$
35. $\langle 0, \pm 1, 0 \rangle$
37. 87.5ft–lb
39. $200/3 \approx 66.67$ft–lb

41. With $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$, we have
$$\begin{aligned}\vec{u} \cdot (\vec{u} \times \vec{v}) &= \langle u_1, u_2, u_3 \rangle \cdot (\langle u_2v_3 - u_3v_2, -(u_1v_3 - u_3v_1), u_1v_2 - u_2v_1 \rangle) \\ &= u_1(u_2v_3 - u_3v_2) - u2(u_1v_3 - u_3v_1) + u3(u_1v_2 - u_2v_1) \\ &= 0.\end{aligned}$$

43. 54

Section 10.5

1. A point on the line and the direction of the line.
3. parallel, skew
5. vector: $\ell(t) = \langle 2, -4, 1 \rangle + t\langle 9, 2, 5 \rangle$
parametric: $x = 2 + 9t$, $y = -4 + 2t$, $z = 1 + 5t$
symmetric: $(x - 2)/9 = (y + 4)/2 = (z - 1)/5$
7. vector: $\ell(t) = \langle -2, 5, 4 \rangle + t\langle 0, 1, 3 \rangle$
parametric: $x = -2$, $y = 5 + t$, $z = 4 + 3t$
symmetric: $x = -2$, $y - 5 = (z - 4)/3$
9. Answers can vary: vector: $\ell(t) = \langle 2, 1, 5 \rangle + t\langle 5, -3, -1 \rangle$
parametric: $x = 2 + 5t$, $y = 1 - 3t$, $z = 5 - t$
symmetric: $(x - 2)/5 = -(y - 1)/3 = -(z - 5)$
11. vector: $\ell(t) = \langle 1, 5, 5 \rangle + t\langle 1, -3, 0 \rangle$
parametric: $x = 1 + t$, $y = 5 - 3t$, $z = 5$
symmetric: $x - 1 = (y - 5)/(-3)$, $z = 5$
13. Answers can vary; here the direction is given by $\vec{d_1} \times \vec{d_2}$: vector: $\ell(t) = \langle 5, 1, 9 \rangle + t\langle 0, -1, 0 \rangle$
parametric: $x = 5$, $y = 1 - t$, $z = 9$
symmetric: $x = 5$, $z = 9$
15. Answers can vary; here the direction is given by $\vec{d_1} \times \vec{d_2}$: vector: $\ell(t) = \langle 2, 2, 3 \rangle + t\langle 5, -1, -3 \rangle$
parametric: $x = 2 + 5t$, $y = 2 - t$, $z = 3 - 3t$
symmetric: $(x - 2)/5 = -(y - 2) = -(z - 3)/3$
17. intersecting; $\ell_1(2) = \ell_2(-2) = \langle 12, 3, 7 \rangle$
19. same
21. parallel
23. skew
25. $3\sqrt{2}$
27. 5
29. 2
31. (Note: this solution is easier once one has studied Section 10.6.) Since the two lines intersect, we can state $P_2 = P_1 + a\vec{d_1} + b\vec{d_2}$ for some scalars a and b. (Here we abuse notation slightly and add points to vectors.) Thus $\overrightarrow{P_1P_2} = a\vec{d_1} + b\vec{d_2}$. Vector $\vec{c}$ is the cross product of $\vec{d_1}$ and $\vec{d_2}$, hence is orthogonal to both, and hence is orthogonal to $\overrightarrow{P_1P_2}$. Thus $\overrightarrow{P_1P_2} \cdot \vec{c} = 0$, and the distance between lines is 0.

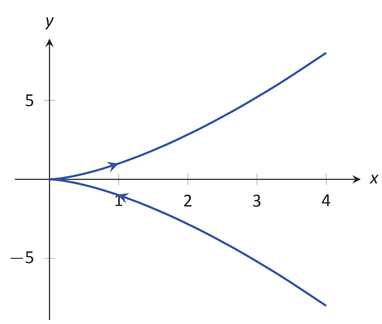
Section 10.6

1. A point in the plane and a normal vector (i.e., a direction orthogonal to the plane).
3. Answers will vary.
5. Answers will vary.
7. Standard form: $3(x - 2) - (y - 3) + 7(z - 4) = 0$
general form: $3x - y + 7z = 31$
9. Answers may vary;
Standard form: $8(x - 1) + 4(y - 2) - 4(z - 3) = 0$
general form: $8x + 4y - 4z = 4$
11. Answers may vary;
Standard form: $-7(x - 2) + 2(y - 1) + (z - 2) = 0$
general form: $-7x + 2y + z = -10$
13. Answers may vary;
Standard form: $2(x - 1) - (y - 1) = 0$
general form: $2x - y = 1$
15. Answers may vary;
Standard form: $2(x - 2) - (y + 6) - 4(z - 1) = 0$
general form: $2x - y - 4z = 6$
17. Answers may vary;
Standard form: $(x - 5) + (y - 7) + (z - 3) = 0$
general form: $x + y + z = 15$
19. Answers may vary;
Standard form: $3(x + 4) + 8(y - 7) - 10(z - 2) = 0$
general form: $3x + 8y - 10z = 24$
21. Answers may vary:
$$\ell = \begin{cases} x = 14t \\ y = -1 - 10t \\ z = 2 - 8t \end{cases}$$
23. $(-3, -7, -5)$
25. No point of intersection; the plane and line are parallel.
27. $\sqrt{5/7}$
29. $1/\sqrt{3}$
31. If P is any point in the plane, and Q is also in the plane, then $\overrightarrow{PQ}$ lies parallel to the plane and is orthogonal to $\vec{n}$, the normal vector. Thus $\vec{n} \cdot \overrightarrow{PQ} = 0$, giving the distance as 0.

Chapter 11

Section 11.1

1. parametric equations
3. displacement
5.



7.