Integrals of the form $\int \sin^m x \cos^n x \; dx$

$\int \sin x \cos x dx$

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$u = \sin x$

$u = \cos x$

$\int \sin^m x \cos^n x dx$

m, n

$\cos^2 x + \sin^2 x = 1$

$_i nt_1 Integrals Involving Powers of Sine and Cosine Consider \int \sin^m x \cos^n x dx$

m, n

m

$m = 2k + 1$

k

$$\sin^m x = \sin^{2k+1} x = \sin^{2k} x \sin x = (\sin^2 x)^k \sin x = (1 - \cos^2 x)^k \sin x.$$

$$\int \sin^m x \cos^n x dx = \int (1 - \cos^2 x)^k \sin x \cos^n x dx = -\int (1 - u^2)^k u^n du,$$

$u = \cos x$

$du = -\sin x dx$

n

$$\int \sin^m x \cos^n x dx = \int u^m (1 - u^2)^k du,$$

$u = \sin x$

$du = \cos x dx$

m

n

$$\cos^2 x = \frac{1 + \cos(2x)}{2} and \sin^2 x = \frac{1 - \cos(2x)}{2}$$

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$_t rigint1 Integrating powers of sine and cosine Evaluate \int \sin^5 x \cos^8 x dx$

$\sin^5 x$

$\sin^5 x = \sin^4 x \sin x = (\sin^2 x)^2 \sin x = (1 - \cos^2 x)^2 \sin x.$

$\int (1 - \cos^2 x)^2 \cos^8 x \sin x dx$

$u = \cos x$

$du = -\sin x dx$

$$\int (1 - \cos^2)^2 \cos^8 x \sin x dx = -\int (1 - u^2)^2 u^8 du = -\int \left(1 - 2u^2 + u^4\right) u^8 du = -\int \left(u^8 - 2u^{10} + u^{12}\right) du.$$

$\int (1 - \cos^2)^2 \cos^8 x \sin x dx = -\int (1 - u^2)^2 u^8 du = -\int \left(1 - 2u^2 + u^4\right) u^8 du = -\int \left(u^8 - 2u^{10} + u^{12}\right) du.$

$6\sin^4 x - 4\sin^6 x + \sin^8 x)\cos x.$

$$\int \sin^5 x\cos^9 x\,dx = \int \sin^5 x\big(1-4\sin^2 x+6\sin^4 x-4\sin^6 x+\sin^8 x\big)\cos x\,dx.$$

$u = \sin x$

$du = \cos x\,dx$

$\int \sin^5 x\big(1- 4\sin^2 x+ 6\sin^4 x- 4\sin^6 x+ \sin^8 x\big)\cos x\,dx =$

$\int u^5(1- 4u^2+ 6u^4- 4u^6+ u^8)du =$

$\int \big(u^5- 4u^7+ 6u^9- 4u^{11}+ u^{13}\big)du$

$= \frac{1}{6}u^6- \frac{1}{2}u^8+ \frac{3}{5}u^{10}- \frac{1}{3}u^{12}+ \frac{1}{14}u^{14}+ C$

$= \frac{1}{6}\sin^6 x- \frac{1}{2}\sin^8 x+ \frac{3}{5}\sin^{10} x- \frac{1}{3}\sin^{12} x+ \frac{1}{14}\sin^{14} x+ C.$

Technology Note:

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$\int \sin^5 x\cos^9 x\,dx$

$$f(x) = -\frac{45\cos(2x)}{16384}-\frac{5\cos(4x)}{8192}+\frac{19\cos(6x)}{49152}+\frac{\cos(8x)}{4096}-\frac{\cos(10x)}{81920}-\frac{\cos(12x)}{24576}-\frac{\cos(14x)}{114688},$$

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$$g(x) = \frac{1}{6}\sin^6 x-\frac{1}{2}\sin^8 x+\frac{3}{5}\sin^{10} x-\frac{1}{3}\sin^{12} x+\frac{1}{14}\sin^{14} x.$$

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f g *only by a constant*

$g(x) = f(x)+ C$

C

$f(x)$

$g(x)$

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$_t rigint3 Integrating powers of sine and cosine Evaluate \int \cos^4 x\sin^2 x\,dx$

$\int \cos^4 x\sin^2 x\,dx =$

$\int \left(\frac{1+\cos(2x)}{2}\right)^2\left(\frac{1-\cos(2x)}{2}\right)dx$

$= \int \frac{1+2\cos(2x)+\cos^2(2x)}{4}\cdot \frac{1-\cos(2x)}{2}dx$

$= \int \frac{1}{8}\big(1+ \cos(2x)- \cos^2(2x)- \cos^3(2x)\big)dx$

$\cos(2x)$

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$\cos^2(2x)$