The function f itself has no critical points. Checking the value of f at the corners $(1,1)$, $(-1,1)$ and the critical point $(5/14, 25/196)$, we find the absolute maximum is at $(5/14, 25/196, 25/28) \approx (0.357, 0.128, 0.893)$ and the absolute minimum is at $(-1,1,-12)$.

19. The region has two "corners" at $(-1,-1)$ and $(1,1)$.
Along the line $y = x$, $f(x,y)$ becomes $f(x) = 3x - 2x^2$. Along this line, we have a critical point at $(3/4, 3/4)$.
Along the curve $y = x^2 + x - 1$, $f(x,y)$ becomes $f(x) = x^2 + 3x - 3$. There is a critical point along this curve at $(-3/2, -1/4)$. Since $x = -3/2$ lies outside our bounded region, we ignore this critical point.
The function f itself has no critical points.
Checking the value of f at $(-1,-1)$, $(1,1)$, $(3/4,3/4)$, we find the absolute maximum is at $(3/4, 3/4, 9/8)$ and the absolute minimum is at $(-1,-1,-5)$.

21. $10\text{m} \times 16\text{m} \times 48\text{m}$

Section 12.9

1. perpendicular or orthogonal

3. T

5. $f_{\max} = \dfrac{2}{3\sqrt{3}}$ at $\left(\pm\sqrt{\dfrac{2}{3}}, \dfrac{1}{\sqrt{3}}\right)$, and $f_{\min} = -\dfrac{2}{3\sqrt{3}}$ at $\left(\pm\sqrt{\dfrac{2}{3}}, -\dfrac{1}{\sqrt{3}}\right)$

7. $f_{\max} = \dfrac{\sqrt{82}}{3}$ at $\left(\dfrac{9}{\sqrt{82}}, \dfrac{1}{3\sqrt{82}}\right)$, and $f_{\min} = -\dfrac{\sqrt{82}}{3}$ at $\left(-\dfrac{9}{\sqrt{82}}, -\dfrac{1}{3\sqrt{82}}\right)$

9. $f_{\max} = 1$ at $(1,1)$

11. $f_{\min} = \dfrac{1}{2}$ at the point $\left(\dfrac{1}{2}, 0, -\dfrac{1}{2}\right)$

Chapter 13

Section 13.1

1. $C(y)$, meaning that instead of being just a constant, like the number 5, it is a function of y, which acts like a constant when taking derivatives with respect to x.

3. curve to curve, then from point to point

5. (a) $18x^2 + 42x - 117$
   (b) -108

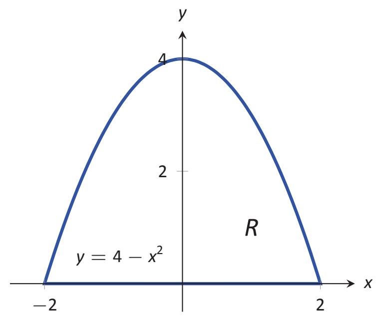
7. (a) $x^4/2 - x^2 + 2x - 3/2$
   (b) $23/15$

9. (a) $\sin^2 y$
   (b) $\pi/2$

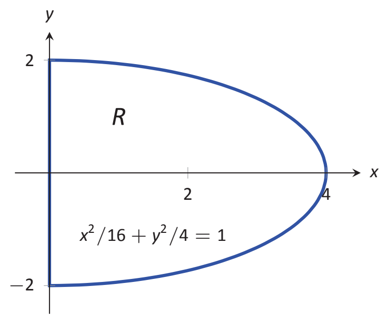
11. $\displaystyle\int_1^4 \int_{-2}^1 dy\, dx$ and $\displaystyle\int_{-2}^1 \int_1^4 dx\, dy$.
area of $R = 9u^2$

13. $\displaystyle\int_2^4 \int_{x-1}^{7-x} dy\, dx$. The order $dx\, dy$ needs two iterated integrals as x is bounded above by two different functions. This gives:
$$\int_1^3 \int_2^{y+1} dx\, dy + \int_3^5 \int_2^{7-y} dx\, dy.$$
area of $R = 4u^2$

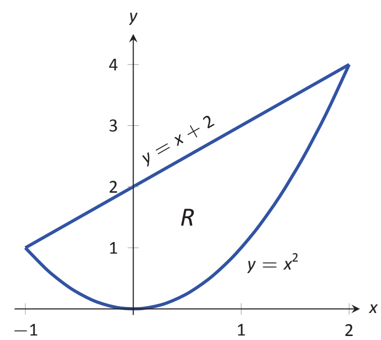
15. $\displaystyle\int_0^1 \int_{x^4}^{\sqrt{x}} dy\, dx$ and $\displaystyle\int_0^1 \int_{y^2}^{\sqrt[4]{y}} dx\, dy$
area of $R = 7/15u^2$

17. 

area of $R = \displaystyle\int_0^4 \int_{-\sqrt{4-y}}^{\sqrt{4-y}} dx\, dy$

19. 

area of $R = \displaystyle\int_0^4 \int_{-\sqrt{4-x^2/4}}^{\sqrt{4-x^2/4}} dy\, dx$

21. 

area of $R = \displaystyle\int_{-1}^2 \int_{x^2}^{x+2} dy\, dx$

23. 20

25. $2\sqrt{3} - \sqrt{6}$

27. 756

29. The integrand $\sqrt{x+y}$ cannot be written as $f(x)g(y)$ where f is a function of only x and g is a function of only y.

Section 13.2

1. volume

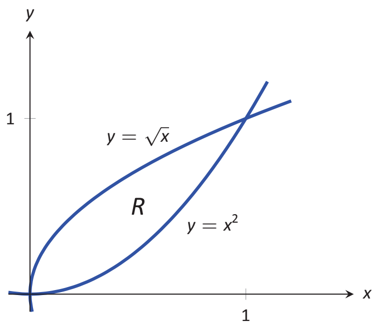
3. The double integral gives the signed volume under the surface. Since the surface is always positive, it is always above the x-y plane and hence produces only "positive" volume.

5. 6; $\displaystyle\int_{-1}^1 \int_1^2 \left(\frac{x}{y} + 3\right) dy\, dx$

7. 112/3; $\displaystyle\int_0^2 \int_0^{4-2y} \left(3x^2 - y + 2\right) dx\, dy$

9. 16/5; $\displaystyle\int_{-1}^1 \int_0^{1-x^2} (x + y + 2)\, dy\, dx$

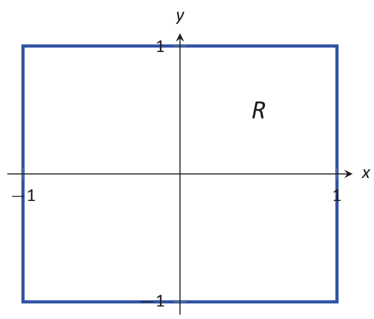
11. (a)



(b) $\int_0^1 \int_{x^2}^{\sqrt{x}} x^2 y \, dy \, dx = \int_0^1 \int_{y^2}^{\sqrt{y}} x^2 y \, dx \, dy.$

(c) $\frac{3}{56}$

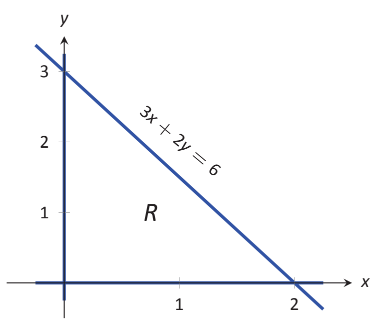
13. (a)



(b) $\int_{-1}^1 \int_{-1}^1 x^2 - y^2 \, dy \, dx = \int_{-1}^1 \int_{-1}^1 x^2 - y^2 \, dx \, dy.$

(c) 0

15. (a)

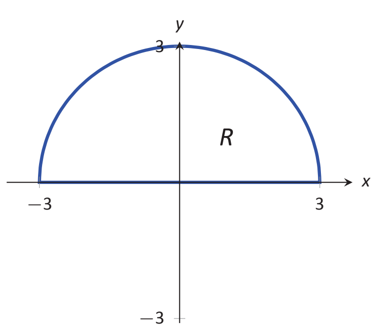


(b)

(c) $\int_0^2 \int_0^{3-3/2x} \big(6 - 3x - 2y\big) \, dy \, dx =$
$\int_0^3 \int_0^{2-2/3y} \big(6 - 3x - 2y\big) \, dx \, dy.$

(d) 6

17. (a)



(b) $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \big(x^3 y - x\big) \, dy \, dx =$
$\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \big(x^3 y - x\big) \, dx \, dy.$

(c) 0

19. Integrating e^{x^2} with respect to x is not possible in terms of elementary functions. $\int_0^2 \int_0^{2x} e^{x^2} \, dy \, dx = e^4 - 1.$

21. Integrating $\int_y^1 \frac{2y}{x^2+y^2} \, dx$ gives $\tan^{-1}(1/y) - \pi/4$; integrating $\tan^{-1}(1/y)$ is hard.
$\int_0^1 \int_0^x \frac{2y}{x^2+y^2} \, dy \, dx = \ln 2.$

23. average value of $f = 6/2 = 3$

25. average value of $f = \frac{112/3}{4} = 28/3$

Section 13.3

1. $f\big(r\cos\theta, r\sin\theta\big)$, $r \, dr \, d\theta$

3. $\int_0^{2\pi} \int_0^1 \big(3r\cos\theta - r\sin\theta + 4\big) r \, dr \, d\theta = 4\pi$

5. $\int_0^{\pi} \int_{\cos\theta}^{3\cos\theta} \big(8 - r\sin\theta\big) r \, dr \, d\theta = 16\pi$

7. $\int_0^{2\pi} \int_1^2 \big(\ln(r^2)\big) r \, dr \, d\theta = 2\pi\big(\ln 16 - 3/2\big)$

9. $\int_{-\pi/2}^{\pi/2} \int_0^6 \big(r^2\cos^2\theta - r^2\sin^2\theta\big) r \, dr \, d\theta =$
$\int_{-\pi/2}^{\pi/2} \int_0^6 \big(r^2\cos(2\theta)\big) r \, dr \, d\theta = 0$

11. $\int_{-\pi/2}^{\pi/2} \int_0^5 \big(r^2\big) \, dr \, d\theta = 125\pi/3$

13. $\int_0^{\pi/4} \int_0^{\sqrt{8}} \big(r\cos\theta + r\sin\theta\big) r \, dr \, d\theta = 16\sqrt{2}/3$

15. (a) This is impossible to integrate with rectangular coordinates as $e^{-(x^2+y^2)}$ does not have an antiderivative in terms of elementary functions.

(b) $\int_0^{2\pi} \int_0^a re^{r^2} \, dr \, d\theta = \pi\left(1 - e^{-a^2}\right).$

(c) $\lim_{a\to\infty} \pi(1 - e^{-a^2}) = \pi$. This implies that there is a finite volume under the surface $e^{-(x^2+y^2)}$ over the entire x-y plane.

Section 13.4

1. Because they are scalar multiples of each other.

3. "little masses"

5. M_x measures the moment about the x-axis, meaning we need to measure distance from the x-axis. Such measurements are measures in the y-direction.

7. $\bar{x} = 5.25$

9. $(\bar{x}, \bar{y}) = (0, 3)$

11. $M = 150$g;

13. $M = 2$lb

15. $M = 16\pi \approx 50.27$kg

17. $M = 54\pi \approx 169.65$lb

19. $M = 150$g; $M_y = 600$; $M_x = -75$; $(\bar{x}, \bar{y}) = (4, -0.5)$

21. $M = 2$lb; $M_y = 0$; $M_x = 2/3$; $(\bar{x}, \bar{y}) = (0, 1/3)$

23. $M = 16\pi \approx 50.27$kg; $M_y = 4\pi$; $M_x = 4\pi$; $(\bar{x}, \bar{y}) = (1/4, 1/4)$

25. $M = 54\pi \approx 169.65$lb; $M_y = 0$; $M_x = 504$; $(\bar{x}, \bar{y}) = (0, 2.97)$

27. $I_x = 64/3$; $I_y = 64/3$; $I_O = 128/3$

29. $I_x = 16/3$; $I_y = 64/3$; $I_O = 80/3$

Section 13.5

1. arc length

3. surface areas

5. Intuitively, adding h to f only shifts f up (i.e., parallel to the z-axis) and does not change its shape. Therefore it will not change the surface area over R.

   Analytically, $f_x = g_x$ and $f_y = g_y$; therefore, the surface area of each is computed with identical double integrals.

7. $SA = \displaystyle\int_0^{2\pi}\int_0^{2\pi} \sqrt{1+\cos^2 x\cos^2 y + \sin^2 x \sin^2 y}\, dx\, dy$

9. $SA = \displaystyle\int_{-1}^{1}\int_{-1}^{1} \sqrt{1+4x^2+4y^2}\, dx\, dy$

11. $SA = \displaystyle\int_0^3\int_{-1}^{1} \sqrt{1+9+49}\, dx\, dy = 6\sqrt{59} \approx 46.09$

13. This is easier in polar:

$$\begin{aligned} SA &= \int_0^{2\pi}\int_0^4 r\sqrt{1+4r^2\cos^2 t + 4r^2\sin^2 t}\, dr\, d\theta \\ &= \int_0^{2\pi}\int_0^4 r\sqrt{1+4r^2}\, dr\, d\theta \\ &= \frac{\pi}{6}\left(65\sqrt{65}-1\right) \approx 273.87 \end{aligned}$$

15.

$$\begin{aligned} SA &= \int_0^2\int_0^{2x} \sqrt{1+1+4x^2}\, dy\, dx \\ &= \int_0^2 \left(2x\sqrt{2+4x^2}\right) dx \\ &= \frac{26}{3}\sqrt{2} \approx 12.26 \end{aligned}$$

17. This is easier in polar:

$$\begin{aligned} SA &= \int_0^{2\pi}\int_0^5 r\sqrt{1+\frac{4r^2\cos^2 t + 4r^2\sin^2 t}{r^2\sin^2 t + r^2\cos^2 t}}\, dr\, d\theta \\ &= \int_0^{2\pi}\int_0^5 r\sqrt{5}\, dr\, d\theta \\ &= 25\pi\sqrt{5} \approx 175.62 \end{aligned}$$

19. Integrating in polar is easiest considering R:

$$\begin{aligned} SA &= \int_0^{2\pi}\int_0^1 r\sqrt{1+c^2+d^2}\, dr\, d\theta \\ &= \int_0^{2\pi} \frac{1}{2}\left(\sqrt{1+c^2+d^2}\right) dy \\ &= \pi\sqrt{1+c^2+d^2}. \end{aligned}$$

   The value of h does not matter as it only shifts the plane vertically (i.e., parallel to the z-axis). Different values of h do not create different ellipses in the plane.

Section 13.6

1. surface to surface, curve to curve and point to point

3. Answers can vary. From this section we used triple integration to find the volume of a solid region, the mass of a solid, and the center of mass of a solid.

5. $V = \int_{-1}^{1}\int_{-1}^{1}\left(8 - x^2 - y^2 - (2x+y)\right) dx\, dy = 88/3$

7. $V = \int_0^{\pi}\int_0^{x}\left(\cos x \sin y + 2 - \sin x \cos y\right) dy\, dx = \pi^2 - \pi \approx 6.728$

9. $dz\,dy\,dx$: $\displaystyle\int_0^3\int_0^{1-x/3}\int_0^{2-2x/3-2y} dz\,dy\,dx$

   $dz\,dx\,dy$: $\displaystyle\int_0^1\int_0^{3-3y}\int_0^{2-2x/3-2y} dz\,dx\,dy$

   $dy\,dz\,dx$: $\displaystyle\int_0^3\int_0^{2-2x/3}\int_0^{1-x/3-z/2} dy\,dz\,dx$

   $dy\,dx\,dz$: $\displaystyle\int_0^2\int_0^{3-3z/2}\int_0^{1-x/3-z/2} dy\,dx\,dz$

   $dx\,dz\,dy$: $\displaystyle\int_0^1\int_0^{2-2y}\int_0^{3-3y-3z/2} dx\,dz\,dy$

   $dx\,dy\,dz$: $\displaystyle\int_0^2\int_0^{1-z/2}\int_0^{3-3y-3z/2} dx\,dy\,dz$

   $V = \displaystyle\int_0^3\int_0^{1-x/3}\int_0^{2-2x/3-2y} dz\,dy\,dx = 1.$

11. $dz\,dy\,dx$: $\displaystyle\int_0^2\int_{-2}^{0}\int_{y^2/2}^{-y} dz\,dy\,dx$

   $dz\,dx\,dy$: $\displaystyle\int_{-2}^{0}\int_0^2\int_{y^2/2}^{-y} dz\,dx\,dy$

   $dy\,dz\,dx$: $\displaystyle\int_0^2\int_0^2\int_{-\sqrt{2z}}^{-z} dy\,dz\,dx$

   $dy\,dx\,dz$: $\displaystyle\int_0^2\int_0^2\int_{-\sqrt{2z}}^{-z} dy\,dx\,dz$

   $dx\,dz\,dy$: $\displaystyle\int_{-2}^{0}\int_{y^2/2}^{-y}\int_0^2 dx\,dz\,dy$

   $dx\,dy\,dz$: $\displaystyle\int_0^2\int_{-\sqrt{2z}}^{-z}\int_0^2 dx\,dy\,dz$

   $V = \displaystyle\int_0^2\int_0^2\int_{-\sqrt{2z}}^{-z} dy\,dz\,dx = 4/3.$

13. $dz\,dy\,dx$: $\displaystyle\int_0^2\int_{1-x/2}^{1}\int_0^{2x+4y-4} dz\,dy\,dx$

   $dz\,dx\,dy$: $\displaystyle\int_0^1\int_{2-2y}^{2}\int_0^{2x+4y-4} dz\,dx\,dy$

   $dy\,dz\,dx$: $\displaystyle\int_0^2\int_0^{2x}\int_{z/4-x/2+1}^{1} dy\,dz\,dx$

   $dy\,dx\,dz$: $\displaystyle\int_0^4\int_{z/2}^{2}\int_{z/4-x/2+1}^{1} dy\,dx\,dz$

   $dx\,dz\,dy$: $\displaystyle\int_0^1\int_0^{4y}\int_{z/2-2y+2}^{2} dx\,dz\,dy$

   $dx\,dy\,dz$: $\displaystyle\int_0^4\int_{z/4}^{1}\int_{z/2-2y+2}^{2} dx\,dy\,dz$

   $V = \displaystyle\int_0^4\int_{z/4}^{1}\int_{2y-z/2-2}^{2} dx\,dy\,dz = 4/3.$

15. $dz\,dy\,dx$: $\displaystyle\int_0^1\int_0^{1-x^2}\int_0^{\sqrt{1-y}} dz\,dy\,dx$

   $dz\,dx\,dy$: $\displaystyle\int_0^1\int_0^{\sqrt{1-y}}\int_0^{\sqrt{1-y}} dz\,dx\,dy$

   $dy\,dz\,dx$: $\displaystyle\int_0^1\int_0^{x}\int_0^{1-x^2} dy\,dz\,dx + \int_0^1\int_x^1\int_0^{1-z^2} dy\,dz\,dx$

   $dy\,dx\,dz$: $\displaystyle\int_0^1\int_0^{z}\int_0^{1-z^2} dy\,dx\,dz + \int_0^1\int_z^1\int_0^{1-x^2} dy\,dx\,dz$

   $dx\,dz\,dy$: $\displaystyle\int_0^1\int_0^{\sqrt{1-y}}\int_0^{\sqrt{1-y}} dx\,dz\,dy$

   $dx\,dy\,dz$: $\displaystyle\int_0^1\int_0^{1-z^2}\int_0^{\sqrt{1-y}} dx\,dy\,dz$

   Answers will vary. Neither order is particularly "hard." The order *dz dy dx* requires integrating a square root, so powers can be messy; the order *dy dz dx* requires two triple integrals, but each uses only polynomials.

17. 8

19. π

21. $M = 10$, $M_{yz} = 15/2$, $M_{xz} = 5/2$, $M_{xy} = 5$; $(\bar{x}, \bar{y}, \bar{z}) = (3/4, 1/4, 1/2)$

23. $M = 16/5$, $M_{yz} = 16/3$, $M_{xz} = 104/45$, $M_{xy} = 32/9$; $(\bar{x}, \bar{y}, \bar{z}) = (5/3, 13/18, 10/9) \approx (1.67, 0.72, 1.11)$

Section 13.7

1. 2π

3. (a) $(2\sqrt{2}, \pi/4, 1)$ and $(2, 5\pi/6, 0)$
   (b) $(\sqrt{2}, \sqrt{2}, 2)$ and $(0, -3, -4)$

5. A cylindrical surface or tube, centered along the z-axis of radius 1, extending from the x-y plane up to the plane $z = 1$ (i.e., the tube has a length of 1).

7. This is a curve, a circle of radius 2, centered at $(0, 0, 5)$, lying parallel to the x-y plane (i.e., in the plane $z = 5$).

9. $\frac{128\pi}{3}$

11. 0

13. Bounded between the planes $z = 0$ and $z = 5$, between the cylinders $x^2 + y^2 = 9$ and $x^2 + y^2 = 16$: describes a "pipe" or "tube" of length 5, an inner radius of 3 and outer radius of 4.

15. Bounded between $y \geq 0$, inside the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$ and below the cone $z = 2 - \sqrt{x^2 + y^2}$: describes cylindrical solid of height 1 and radius 2, topped with an inverted cone of height 1 and base radius 1 with point at $(0, 0, 2)$.

17. Bounded between the plane $z = 0$, inside the cylinder $x^2 + y^2 = a^2$, and below the upper hemisphere $z = \sqrt{a^2 - x^2 - y^2} + b$, with radius a and centered at $(0, 0, b)$: describes a cylindrical solid of radius a and height b, topped with the upper hemisphere of radius a.

19. In cylindrical coordinates, the density is $\delta(r, \theta, z) = z$. Thus mass is
$$\int_0^{2\pi} \int_2^3 \int_0^{10} zr\, dz\, dr\, d\theta = 250\pi.$$

21. In cylindrical coordinates, the density is $\delta(r, \theta, z) = 1$. Thus mass is
$$\int_0^{2\pi} \int_0^1 \int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} r\, dz\, dr\, d\theta = 4\pi/3.$$

23. In cylindrical coordinates, the density is $\delta(r, \theta, z) = z$. Thus mass is
$$M = \int_0^{2\pi} \int_2^3 \int_0^{10} zr\, dz\, dr\, d\theta = 250\pi.$$
We find $M_{yz} = 0$, $M_{xz} = 0$, and $M_{xy} = 5000\pi/3$, placing the center of mass at $(0, 0, 20/3)$.

25. In cylindrical coordinates, the density is $\delta(r, \theta, z) = 1$. Thus mass is
$$\int_0^{2\pi} \int_0^1 \int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} r\, dz\, dr\, d\theta = 2\pi/3.$$
We find $M_{yz} = 0$, $M_{xz} = 0$, and $M_{xy} = \pi/4$, placing the center of mass at $(0, 0, 3/8)$.

Section 13.8

1. In cylindrical, r determines how far from the origin one goes in the x-y plane before considering the z-component. Equivalently, if on projects a point in cylindrical coordinates onto the x-y plane, r will be the distance of this projection from the origin.
   In spherical, ρ is the distance from the origin to the point.

3. π

5. T

7. (a) Cylindrical: $(1, \pi/2, 1)$ and $(1, \pi, 1)$
   Spherical: $(\sqrt{2}, \pi/2, \pi/4)$ and $(\sqrt{2}, \pi, \pi/4)$
   (b) Rectangular: $(0, 0, 1)$ and $(-1, -\sqrt{3}, 0)$
   Spherical: $(1, \pi, 0)$ and $(2, 4\pi/3, \pi/2)$
   (c) Rectangular: $(\sqrt{3}, 1, 0)$ and $(0, 0, -3)$
   Cylindrical: $(2, \pi/6, 0)$ and $(0, \pi, -3)$

9. This is a region of space, a half of a solid cone with rounded top, where the rounded top is a portion of the ball of radius 2 centered at the origin and the sides of the cone make an angle of $\pi/4$ with the positive z-axis. The bounds on θ mean only the portion "above" the x-z plane are retained.

11. $\frac{128\pi}{5}$

13. $\dfrac{\pi}{3}\left(2 - \sqrt{2}\right)$

15. Describes half of a spherical shell (i.e., $y \geq 0$) with inner radius of 1 and outer radius of 1.1 centered at the origin.

17. It is the region is space bounded below by $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 4$, with the portion above the cone $z = \sqrt{3}\sqrt{x^2 + y^2}$ removed: it describes a cone, where the side makes an angle of $\pi/4$ with the positive z-axis, topped by the portion of the ball of radius 2, centered at the origin, with the inner cone with angle $\pi/6$ removed, along with corresponding portion of the ball of radius 2.

19. The region in space is bounded below by the cone $z = \sqrt{3}\sqrt{x^2 + y^2}$ and above by the plane $z = a$: it describes a cone, with point at the origin, centered along the positive z-axis, with height of a and base radius of $a\tan(\pi/6)$.

21. In spherical coordinates, the density is $\delta(\rho, \theta, \varphi) = 1$. Thus mass is
$$\int_0^{\pi/2} \int_0^{2\pi} \int_0^1 \rho^2 \sin(\varphi)\, d\rho\, d\theta\, d\varphi = 2\pi/3.$$

23. In spherical coordinates, the density is $\delta(\rho, \theta, \varphi) = \rho\cos\varphi$. Thus mass is
$$\int_0^{\pi/4} \int_0^{2\pi} \int_0^1 \big(\rho\cos(\varphi)\big)\rho^2 \sin(\varphi)\, d\rho\, d\theta\, d\varphi = \pi/8.$$

25. In spherical coordinates, the density is $\delta(\rho, \theta, \varphi) = 1$. Thus mass is
$$\int_0^{\pi/2} \int_0^{2\pi} \int_0^1 \rho^2 \sin(\varphi)\, d\rho\, d\theta\, d\varphi = 2\pi/3.$$
We find $M_{yz} = 0$, $M_{xz} = 0$, and $M_{xy} = \pi/4$, placing the center of mass at $(0, 0, 3/8)$.

27. In spherical coordinates, the density is $\delta(\rho, \theta, \varphi) = \rho\cos\varphi$. Thus mass is
$$\int_0^{\pi/4} \int_0^{2\pi} \int_0^1 \big(\rho\cos(\varphi)\big)\rho^2 \sin(\varphi)\, d\rho\, d\theta\, d\varphi = \pi/8.$$
We find $M_{yz} = 0$, $M_{xz} = 0$, and $M_{xy} = (4 - \sqrt{2})\pi/30$, placing the center of mass at $(0, 0, 4(4 - \sqrt{2})/15)$.

29. Rectangular: $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} dz\, dy\, dx$
    Cylindrical: $\int_0^{2\pi} \int_0^1 \int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} r\, dz\, dr\, d\theta$
    Spherical: $\int_0^{\pi} \int_0^{2\pi} \int_0^1 \rho^2 \sin(\varphi)\, d\rho\, d\theta\, d\varphi$
    Spherical appears simplest, avoiding the integration of square-roots and using techniques such as Substitution; all bounds are constants.

31. Rectangular: $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^1 dz\, dy\, dx$
    Cylindrical: $\int_0^{2\pi} \int_0^1 \int_r^1 r\, dz\, dr\, d\theta$
    Spherical: $\int_0^{\pi/4} \int_0^{2\pi} \int_0^{\sec\varphi} \rho^2 \sin(\varphi)\, d\rho\, d\theta\, d\varphi$
    Cylindrical appears simplest, avoiding the integration of square-roots that rectangular uses. Spherical is not difficult, though it requires Substitution, an extra step.

33. The solid is the full cylinder of radius 1 with a height of 4 units

35. The solid is a cone opening down with spherical end, part of the solid unit sphere

37. -3

39. The limit does not exist.

Chapter 14

Section 14.1

1. When C is a curve in the plane and f is a surface defined over C, then $\int_C f(s)\,ds$ describes the area under the spatial curve that lies on f, over C.

3. The variable s denotes the arc-length parameter, which is generally difficult to use. The Key Idea allows one to parameterize a curve using another, ideally easier-to-use, parameter.

5. $12\sqrt{2}$

7. 1

9. $\frac{\sqrt{2}}{3}+\frac{1}{2}$

11. $10\pi^2$

13. Over the first subcurve of C, the line integral has a value of $2\sqrt{2}/3$; over the second subcurve, the line integral has a value of $\pi-2$. The total value of the line integral is thus $\pi+2\sqrt{2}/3-2$.

15. $\int_0^\pi t\sqrt{1+\cos^2 t}\,dt \approx 6.001$

17. $\int_{-1}^1 \left(3t^3+2t+5\right)\sqrt{9t^4+1}\,dt \approx 15.479$

19. 2π

21. $5/2$

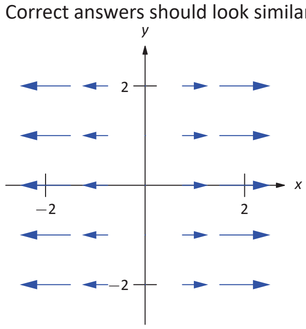
23. $M\approx 0.237$; center of mass is approximately $(0.173, 0.099, 0.065)$.

Section 14.2

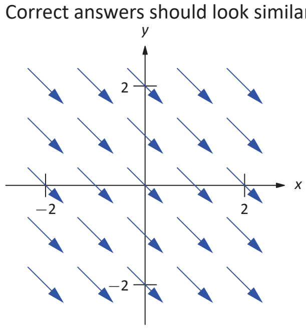
1. Answers will vary. Appropriate answers include velocities of moving particles (air, water, etc.); gravitational or electromagnetic forces.

3. Specific answers will vary, though should relate to the idea that the vector field is spinning clockwise at that point.

5. Correct answers should look similar to



7. Correct answers should look similar to



9. $\text{div}\,\vec{F}=1+2y$
   $\text{curl}\,\vec{F}=0$

11. $\text{div}\,\vec{F}=x\cos(xy)-y\sin(xy)$
    $\text{curl}\,\vec{F}=y\cos(xy)+x\sin(xy)$

13. $\text{div}\,\vec{F}=3$
    $\text{curl}\,\vec{F}=\langle -1,-1,-1\rangle$

15. $\text{div}\,\vec{F}=1+2y$
    $\text{curl}\,\vec{F}=0$

17. $\text{div}\,\vec{F}=2y-\sin z$
    $\text{curl}\,\vec{F}=\vec{0}$

Section 14.3

1. False. It is true for line integrals over scalar fields, though.

3. True.

5. We can conclude that $\vec{F}$ is conservative.

7. $\nabla f=\langle 3,1\rangle$

9. $\nabla f=\langle 1,2y\rangle$

11. $\nabla f=\langle 2x,-2y\rangle$

13. $\nabla f=\left\langle e^{x-y}-e^{x-y}\right\rangle$

15. $\nabla f=\left\langle y^x\ln y, xy^{x-1}\right\rangle$

17. Not conservative.

19. Not conservative.

21. Conservative. $f(x,y)=-4x^2+3xy+\frac{5y^2}{2}+C$

23. $5/3$. (One parameterization for C is $\vec{r}(t)=\langle t,t^2\rangle$ on $0\le t\le 1$.)

25. $2/5$. (One parameterization for C is $\vec{r}(t)=\langle t,t^3\rangle$ on $-1\le t\le 1$.)

27. 1.

29. $13/15$ joules. (One parameterization for C is $\vec{r}(t)=\langle t,\sqrt{t}\rangle$ on $0\le t\le 1$.)

31. 24 ft-lbs.

33. (a) $f(x,y)=x^2+xy+y^2$
    (b) $\text{curl}\,\vec{F}=0$.
    (c) 0.
    (d) 0 (with $A=(0,0)$ and $B=(0,0)$, $f(B)-f(A)=0$.)

35. (a) $f(x,y)=x^2+y^2+z^2$
    (b) $\text{curl}\,\vec{F}=\vec{0}$.
    (c) 0.
    (d) 0 (with $A=(1,0,0)$ and $B=(1,0,0)$, $f(B)-f(A)=250$.)

Section 14.4

1. along, across

3. the curl of $\vec{F}$, or $\text{curl}\,\vec{F}$

5. $\text{curl}\,\vec{F}$

7. 12

9. $-2/3$

11. $1/2$

13. 4π

15. 0

17. -4π