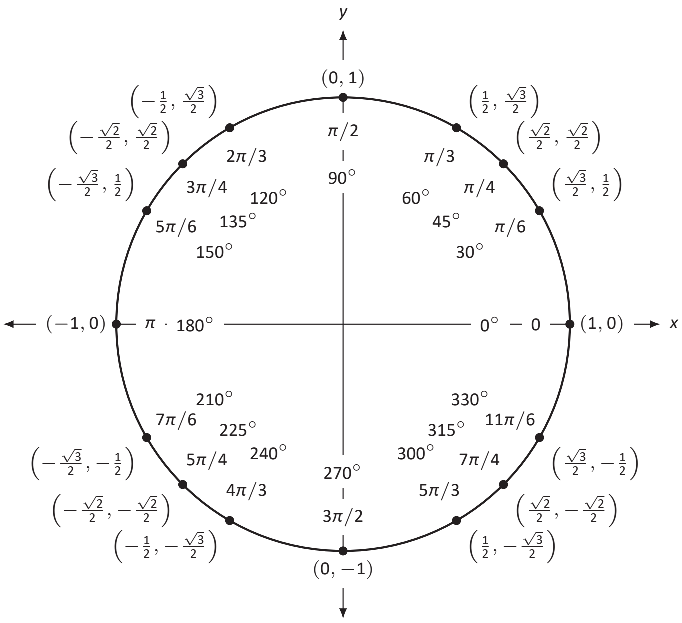
## Differentiation Rules

1. $\frac{d}{dx}(cx) = c$
2. $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$
3. $\frac{d}{dx}(f(x) \cdot g(x)) = f'(x)g(x) + f(x)g'(x)$
4. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$
5. $\frac{d}{dx}(f(g(x))) = f'(g(x)g'(x)$
6. $\frac{d}{dx}(c) = 0$
7. $\frac{d}{dx}(x) = 1$
8. $\frac{d}{dx}(x^n) = nx^{n-1}$
9. $\frac{d}{dx}(e^x) = e^x$
10. $\frac{d}{dx}(a^x) = \ln a \cdot a^x$
11. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
12. $\frac{d}{dx}(\log_a x) = \frac{1}{\ln(a)x}$
13. $\frac{d}{dx}(\sin x) = \cos x$
14. $\frac{d}{dx}(\cos x) = -\sin x$
15. $\frac{d}{dx}(\tan x) = \sec^2 x$
16. $\frac{d}{dx}(\cot x) = -\csc^2 x$
17. $\frac{d}{dx}(\sec x) = \sec x \tan x$
18. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
19. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
20. $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$
21. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
22. $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$
23. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$
24. $\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$
25. $\frac{d}{dx}(\sinh x) = \cosh x$
26. $\frac{d}{dx}(\cosh x) = \sinh x$
27. $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$
28. $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$
29. $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$
30. $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$
31. $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$
32. $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$
33. $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$
34. $\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$
35. $\frac{d}{dx}(\operatorname{sech}^{-1} x) = \frac{-1}{x\sqrt{1-x^2}}$
36. $\frac{d}{dx}(\operatorname{csch}^{-1} x) = \frac{-1}{|x|\sqrt{1+x^2}}$

## Integration Rules

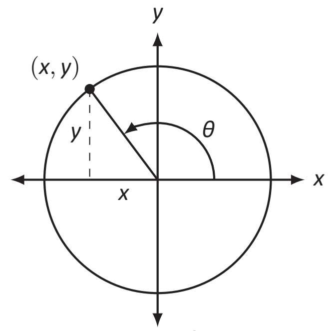
1. $\int c \cdot f(x)\,dx = c\int f(x)\,dx$
2. $\int f(x) \pm g(x)\,dx =$
   $\int f(x)\,dx \pm \int g(x)\,dx$
3. $\int 0\,dx = C$
4. $\int 1\,dx = x + C$
5. $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C,\ n \neq -1$
   $n \neq -1$
6. $\int e^x\,dx = e^x + C$
7. $\int a^x\,dx = \frac{1}{\ln a} \cdot a^x + C$
8. $\int \frac{1}{x}\,dx = \ln|x| + C$
9. $\int \sin x\,dx = -\cos x + C$
10. $\int \cos x\,dx = \sin x + C$
11. $\int \tan x\,dx = -\ln|\cos x| + C$
12. $\int \cot x\,dx = \ln|\sin x| + C$
13. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
14. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$
15. $\int \sec^2 x\,dx = \tan x + C$
16. $\int \csc^2 x\,dx = -\cot x + C$
17. $\int \sec x \tan x\,dx = \sec x + C$
18. $\int \csc x \cot x\,dx = -\csc x + C$
19. $\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$
20. $\int \cos^2 x\,dx = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$
21. $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
22. $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
23. $\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{|x|}{a}\right) + C$
24. $\int \sinh x\,dx = \cosh x + C$
25. $\int \cosh x\,dx = \sinh x + C$
26. $\int \tanh x\,dx = \ln(\cosh x) + C$
27. $\int \coth x\,dx = \ln|\sinh x| + C$
28. $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left|x + \sqrt{x^2-a^2}\right| + C$
29. $\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left|x + \sqrt{x^2+a^2}\right| + C$
30. $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$
31. $\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = \frac{1}{a}\ln\left(\frac{x}{a+\sqrt{a^2-x^2}}\right) + C$
32. $\int \frac{1}{x\sqrt{x^2+a^2}}\,dx = \frac{1}{a}\ln\left|\frac{x}{a+\sqrt{x^2+a^2}}\right| + C$

## The Unit Circle




## Definitions of the Trigonometric Functions

### Unit Circle Definition

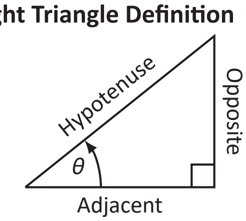



$$\sin\theta = y \qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = x \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

### Right Triangle Definition




$$\sin\theta = \frac{\text{O}}{\text{H}} \qquad \csc\theta = \frac{\text{H}}{\text{O}}$$

$$\cos\theta = \frac{\text{A}}{\text{H}} \qquad \sec\theta = \frac{\text{H}}{\text{A}}$$

$$\tan\theta = \frac{\text{O}}{\text{A}} \qquad \cot\theta = \frac{\text{A}}{\text{O}}$$

## Common Trigonometric Identities

### Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

### Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x \qquad \sec\left(\frac{\pi}{2} - x\right) = \csc x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

### Double Angle Formulas

$$\sin 2x = 2\sin x\cos x$$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x\end{aligned}$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

### Sum to Product Formulas

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

### Power–Reducing Formulas

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

### Even/Odd Identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

$$\cot(-x) = -\cot x$$

$$\sec(-x) = \sec x$$

$$\csc(-x) = -\csc x$$

### Product to Sum Formulas

$$\sin x\sin y = \frac{1}{2}\big(\cos(x-y) - \cos(x+y)\big)$$

$$\cos x\cos y = \frac{1}{2}\big(\cos(x-y) + \cos(x+y)\big)$$

$$\sin x\cos y = \frac{1}{2}\big(\sin(x+y) + \sin(x-y)\big)$$

### Angle Sum/Difference Formulas

$$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$$

$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$$

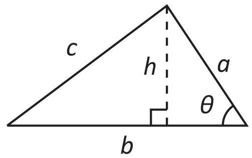
## Areas and Volumes

**Triangles**

$h = a \sin \theta$

Area = $\frac{1}{2}bh$

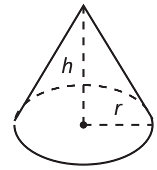
Law of Cosines:
$c^2 = a^2 + b^2 - 2ab\cos\theta$




**Right Circular Cone**

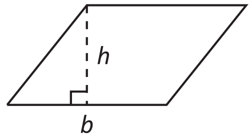
Volume = $\frac{1}{3}\pi r^2 h$

Surface Area =
$\pi r\sqrt{r^2 + h^2} + \pi r^2$




**Parallelograms**

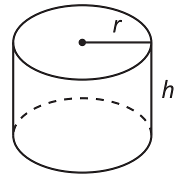
Area = $bh$




**Right Circular Cylinder**

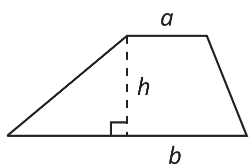
Volume = $\pi r^2 h$

Surface Area =
$2\pi rh + 2\pi r^2$




**Trapezoids**

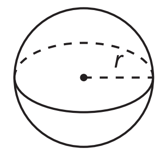
Area = $\frac{1}{2}(a+b)h$




**Sphere**

Volume = $\frac{4}{3}\pi r^3$

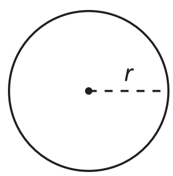
Surface Area =$4\pi r^2$




**Circles**

Area = $\pi r^2$

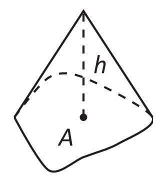
Circumference = $2\pi r$




**General Cone**

Area of Base = $A$

Volume = $\frac{1}{3}Ah$

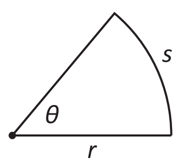



**Sectors of Circles**

$\theta$ in radians

Area = $\frac{1}{2}\theta r^2$

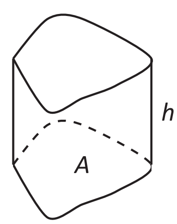
$s = r\theta$




**General Right Cylinder**

Area of Base = $A$

Volume = $Ah$



# Algebra

## Factors and Zeros of Polynomials

Let $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial. If $p(a) = 0$, then $a$ is a *zero* of the polynomial and a solution of the equation $p(x) = 0$. Furthermore, $(x - a)$ is a *factor* of the polynomial.

## Fundamental Theorem of Algebra

An $n$th degree polynomial has $n$ (not necessarily distinct) zeros. Although all of these zeros may be imaginary, a real polynomial of odd degree must have at least one real zero.

## Quadratic Formula

If $p(x) = ax^2 + bx + c$, and $0 \le b^2 - 4ac$, then the real zeros of $p$ are $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

## Special Factors

$x^2 - a^2 = (x - a)(x + a)$ $\qquad$ $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$ $\qquad$ $x^4 - a^4 = (x^2 - a^2)(x^2 + a^2)$

$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \cdots + nxy^{n-1} + y^n$

$(x - y)^n = x^n - nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 - \cdots \pm nxy^{n-1} \mp y^n$

## Binomial Theorem

$(x + y)^2 = x^2 + 2xy + y^2$ $\qquad$ $(x - y)^2 = x^2 - 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ $\qquad$ $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$ $\qquad$ $(x - y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

## Rational Zero Theorem

If $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has integer coefficients, then every *rational zero* of $p$ is of the form $x = r/s$, where $r$ is a factor of $a_0$ and $s$ is a factor of $a_n$.

## Factoring by Grouping

$acx^3 + adx^2 + bcx + bd = ax^2(cx + d) + b(cx + d) = (ax^2 + b)(cx + d)$

## Arithmetic Operations

$$ab + ac = a(b + c) \qquad \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \qquad \frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \left(\frac{a}{b}\right)\left(\frac{d}{c}\right) = \frac{ad}{bc} \qquad \frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc} \qquad \frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$a\left(\frac{b}{c}\right) = \frac{ab}{c} \qquad \frac{a - b}{c - d} = \frac{b - a}{d - c} \qquad \frac{ab + ac}{a} = b + c$$

## Exponents and Radicals

$$a^0 = 1,\ \ a \ne 0 \qquad (ab)^x = a^x b^x \qquad a^x a^y = a^{x+y} \qquad \sqrt{a} = a^{1/2} \qquad \frac{a^x}{a^y} = a^{x-y} \qquad \sqrt[n]{a} = a^{1/n}$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x} \qquad \sqrt[n]{a^m} = a^{m/n} \qquad a^{-x} = \frac{1}{a^x} \qquad \sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b} \qquad (a^x)^y = a^{xy} \qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

## Additional Formulas

**Summation Formulas:**

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

**Trapezoidal Rule:**

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left[f(x_1) + 2f(x_2) + 2f(x_3) + \ldots + 2f(x_n) + f(x_{n+1})\right]$$

with Error $\leq \dfrac{(b-a)^3}{12n^2}\left[\max\left|f''(x)\right|\right]$

**Simpson's Rule:**

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(x_1) + 4f(x_2) + 2f(x_3) + 4f(x_4) + \ldots + 2f(x_{n-1}) + 4f(x_n) + f(x_{n+1})\right]$$

with Error $\leq \dfrac{(b-a)^5}{180n^4}\left[\max\left|f^{(4)}(x)\right|\right]$

**Arc Length:**

$$L = \int_a^b \sqrt{1 + f'(x)^2}\ \ dx$$

**Surface of Revolution:**

$$S = 2\pi \int_a^b f(x)\sqrt{1 + f'(x)^2}\ \ dx$$

(where $f(x) \geq 0$)

$$S = 2\pi \int_a^b x\sqrt{1 + f'(x)^2}\ \ dx$$

(where $a, b \geq 0$)

**Work Done by a Variable Force:**

$$W = \int_a^b F(x)\ \ dx$$

**Force Exerted by a Fluid:**

$$F = \int_a^b w\, d(y)\, \ell(y)\ \ dy$$

**Taylor Series Expansion for $f(x)$:**

$$p_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

**Maclaurin Series Expansion for $f(x)$, where $c = 0$:**

$$p_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots + \frac{f^{(n)}(0)}{n!}x^n$$

## Summary of Tests for Series:

| Test | Series | Condition(s) of Convergence | Condition(s) of Divergence | Comment |
|---|---|---|---|---|
| $n$th-Term | $\sum_{n=1}^{\infty} a_n$ | | $\lim_{n\to\infty} a_n \neq 0$ | This test cannot be used to show convergence. |
| Geometric Series | $\sum_{n=0}^{\infty} r^n$ | $\lvert r\rvert < 1$ | $\lvert r\rvert \geq 1$ | Sum $= \dfrac{1}{1-r}$ |
| Telescoping Series | $\sum_{n=1}^{\infty} (b_n - b_{n+a})$ | $\lim_{n\to\infty} b_n = L$ | | Sum $= \left(\sum_{n=1}^{a} b_n\right) - L$ |
| $p$-Series | $\sum_{n=1}^{\infty} \dfrac{1}{(an+b)^p}$ | $p > 1$ | $p \leq 1$ | |
| Integral Test | $\sum_{n=0}^{\infty} a_n$ | $\int_1^{\infty} a(n)\,dn$ is convergent | $\int_1^{\infty} a(n)\,dn$ is divergent | $a_n = a(n)$ must be positive, continuous, and decreasing. |
| Direct Comparison | $\sum_{n=0}^{\infty} a_n$ | $\sum_{n=0}^{\infty} b_n$ converges and $0 \leq a_n \leq b_n$ | $\sum_{n=0}^{\infty} b_n$ diverges and $0 \leq b_n \leq a_n$ | $\{a_n\}$ must be positive |
| Limit Comparison | $\sum_{n=0}^{\infty} a_n$ | $\sum_{n=0}^{\infty} b_n$ converges and $\lim_{n\to\infty} a_n/b_n \geq 0$ (but not $\infty$) | $\sum_{n=0}^{\infty} b_n$ diverges and $\lim_{n\to\infty} a_n/b_n > 0$ (or $\infty$) | $\{a_n\}$ must be positive |
| Alternating Series | $\sum_{n=0}^{\infty} (-1)^n a_n$ or $\sum_{n=0}^{\infty} (-1)^{n+1} a_n$ | $a_n$ positive, decreasing, and $\lim_{n\to\infty} a_n = 0$ | | Can't be used to show divergence, though if $\lim_{n\to\infty} a_n \neq 0$, it diverges by the $n$th-Term test. |
| Ratio Test | $\sum_{n=0}^{\infty} a_n$ | $\lim_{n\to\infty} \dfrac{a_{n+1}}{a_n} < 1$ | $\lim_{n\to\infty} \dfrac{a_{n+1}}{a_n} > 1$ | $\{a_n\}$ must be positive Also diverges if $\lim_{n\to\infty} a_{n+1}/a_n = \infty$ |
| Root Test | $\sum_{n=0}^{\infty} a_n$ | $\lim_{n\to\infty} (a_n)^{1/n} < 1$ | $\lim_{n\to\infty} (a_n)^{1/n} > 1$ | $\{a_n\}$ must be positive Also diverges if $\lim_{n\to\infty} (a_n)^{1/n} = \infty$ |