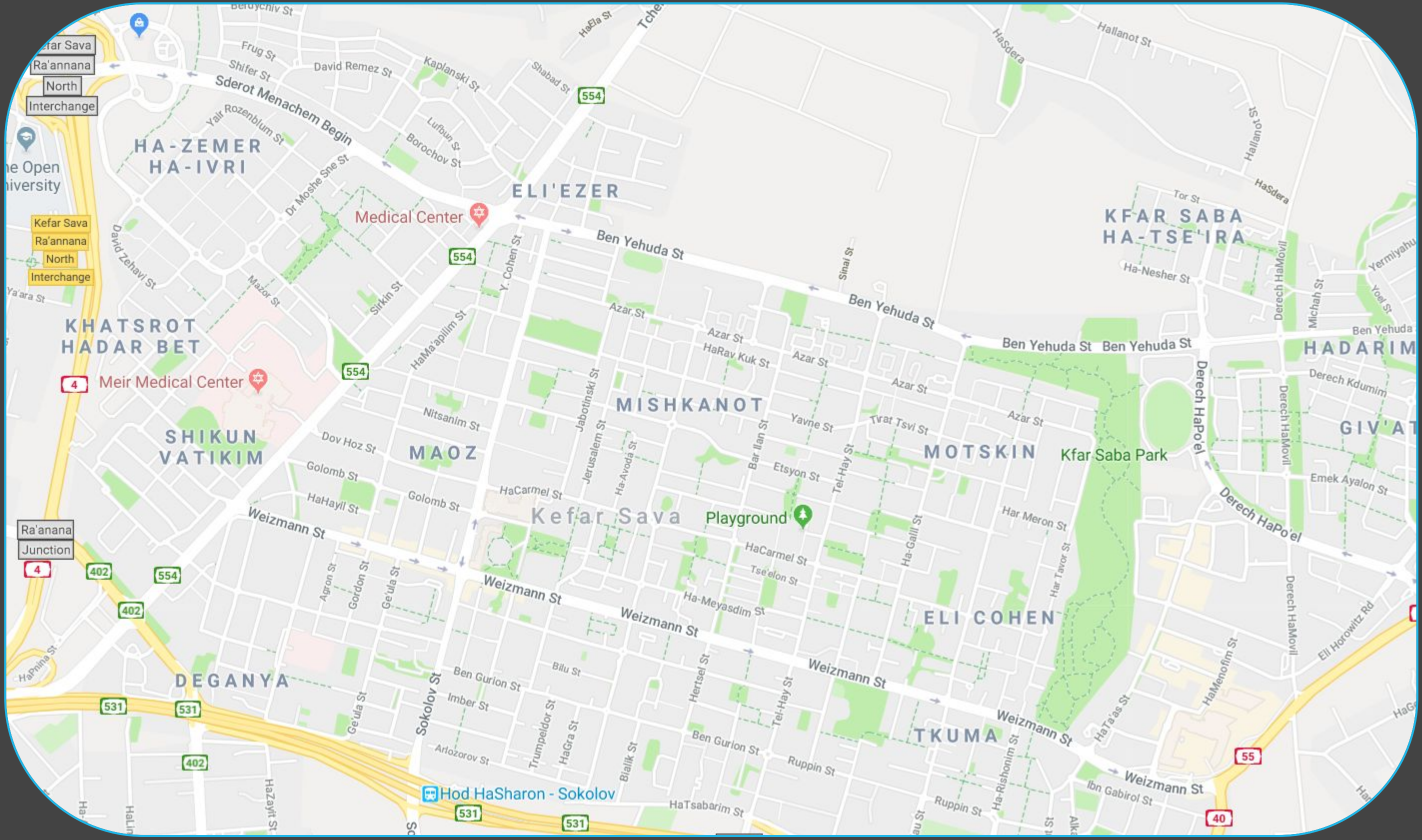
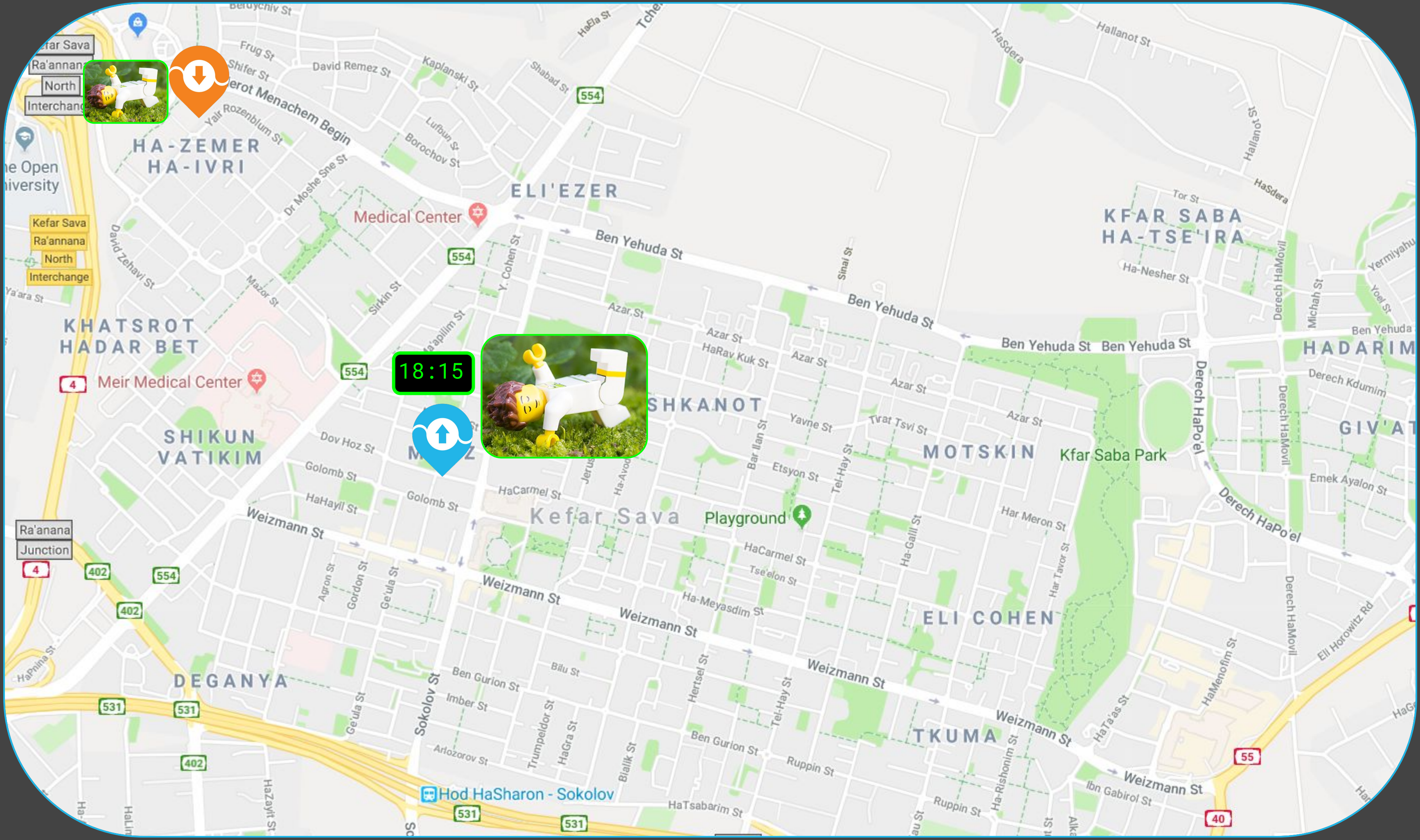
Sequence Alignment for Ride Sharing

Dalya Gartzman

via

PART I - Ride Sharing

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HA-ZEMER HA-IVRI
ELI'EZER
KFAR SABA HA-TSE'IRA
KHATSROT HADAR BET
SHIKUN VATIKIM
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HADARIM
DEGANYA
ELI COHEN
TKUMA
Kefar Sava
Medical Center
Meir Medical Center
Kfar Saba Park
Playground
Hod HaSharon - Sokolov
Kefar Sava Ra'annana North Interchange
Ra'anana Junction
Ben Yehuda St
Weizmann St
Derech HaPo'el
Sokolov St
Shderot Menachem Begin
Azar St
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Frug St
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Dr Moshe Sne St
Y. Cohen St
Bar Ilan St
Ha-Galil St
Har Tavor St
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Ibn Gabirol St
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HaGra St
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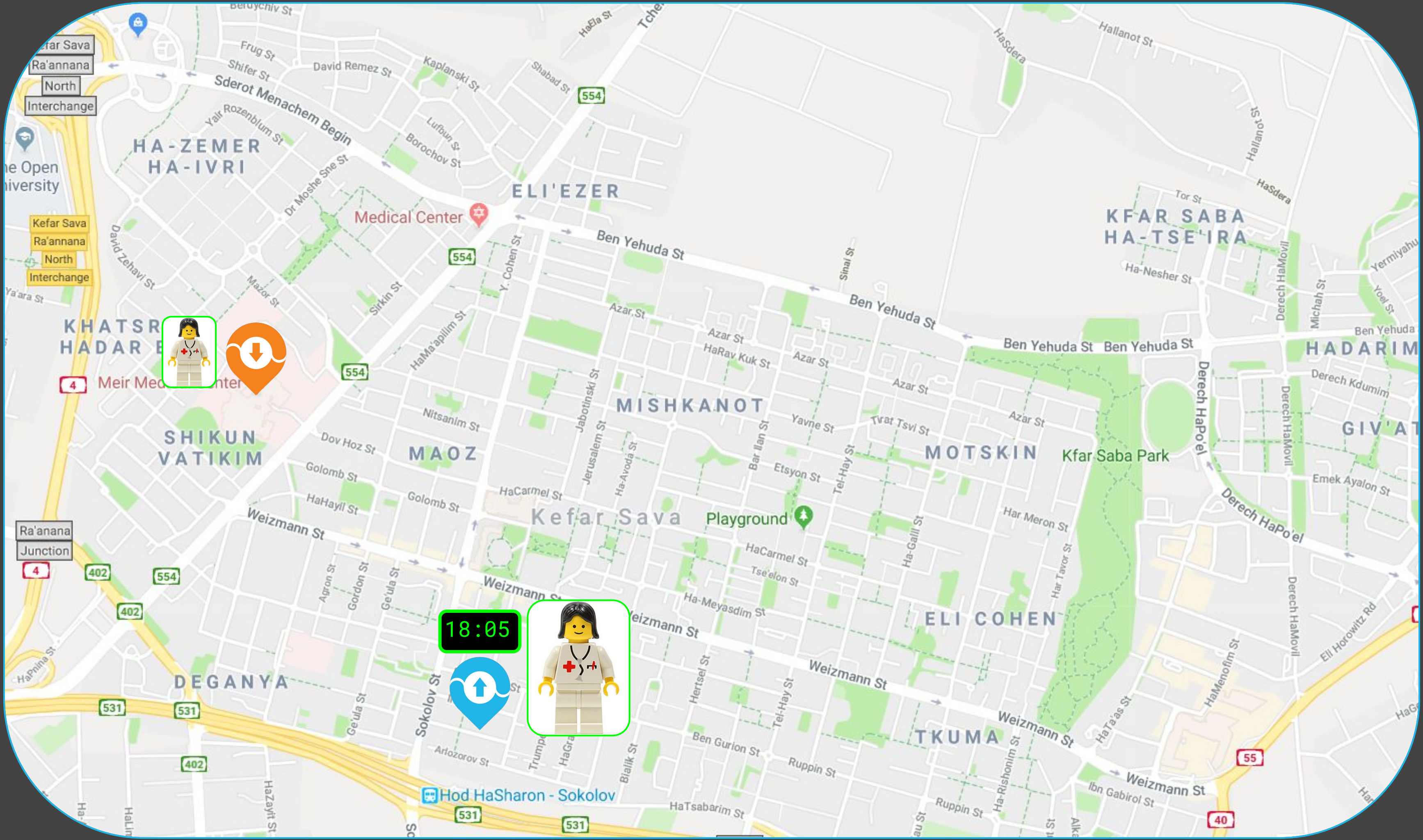
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Sharing is Caring
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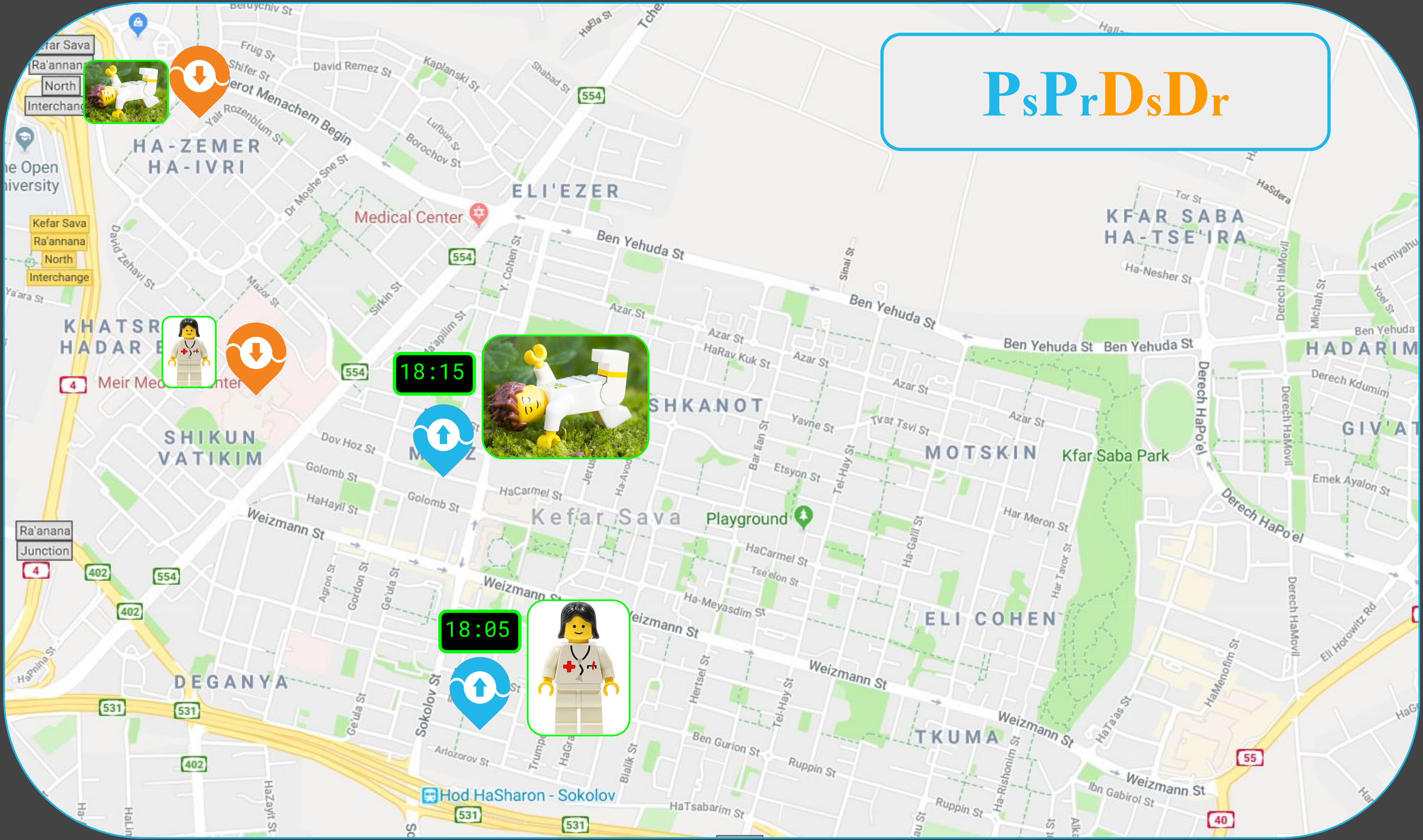
Question #1:

What is the best way
to perform
a given sequence of events?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18:15
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The Ride Sharing Machine
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Medical Center
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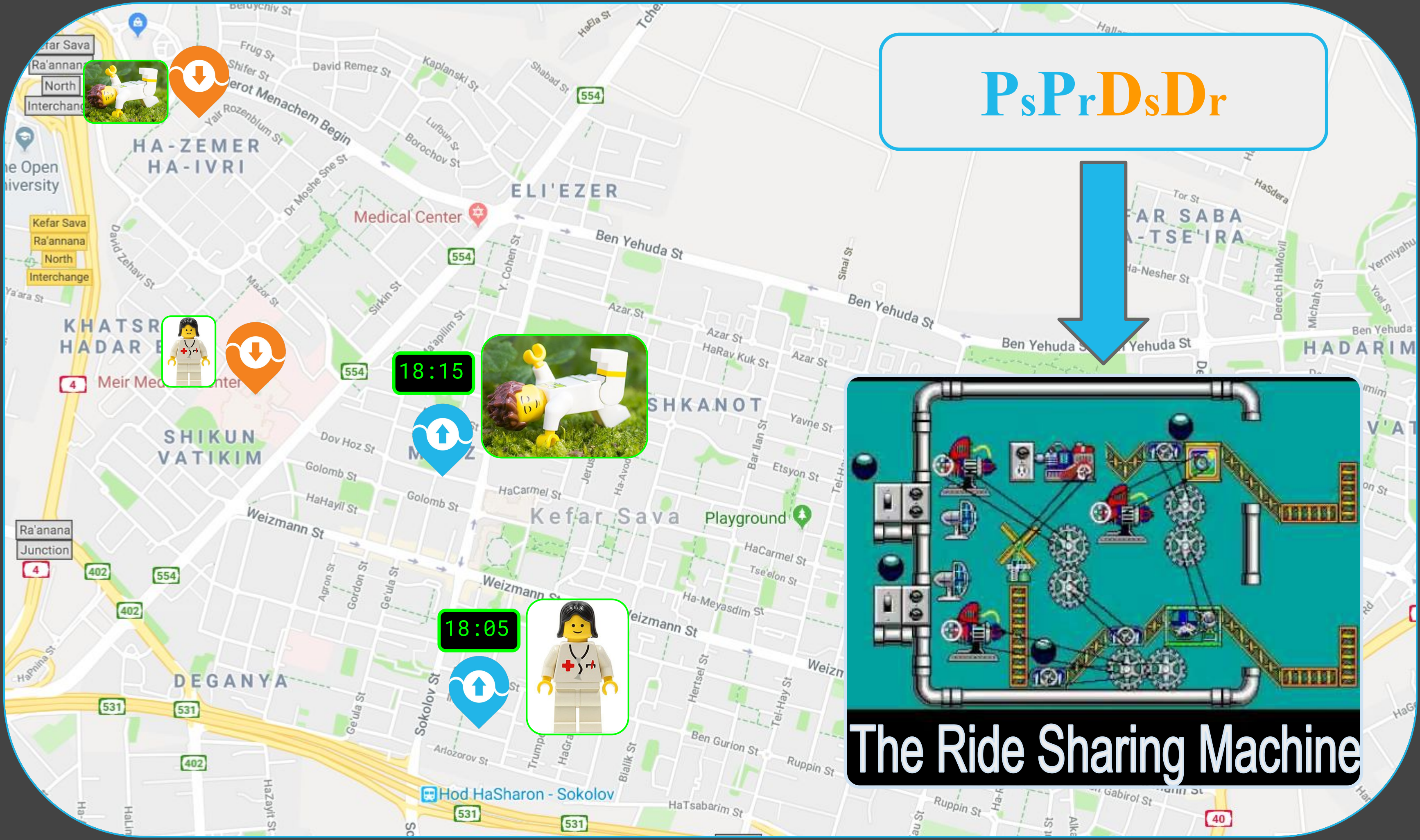
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The Ride Sharing Machine
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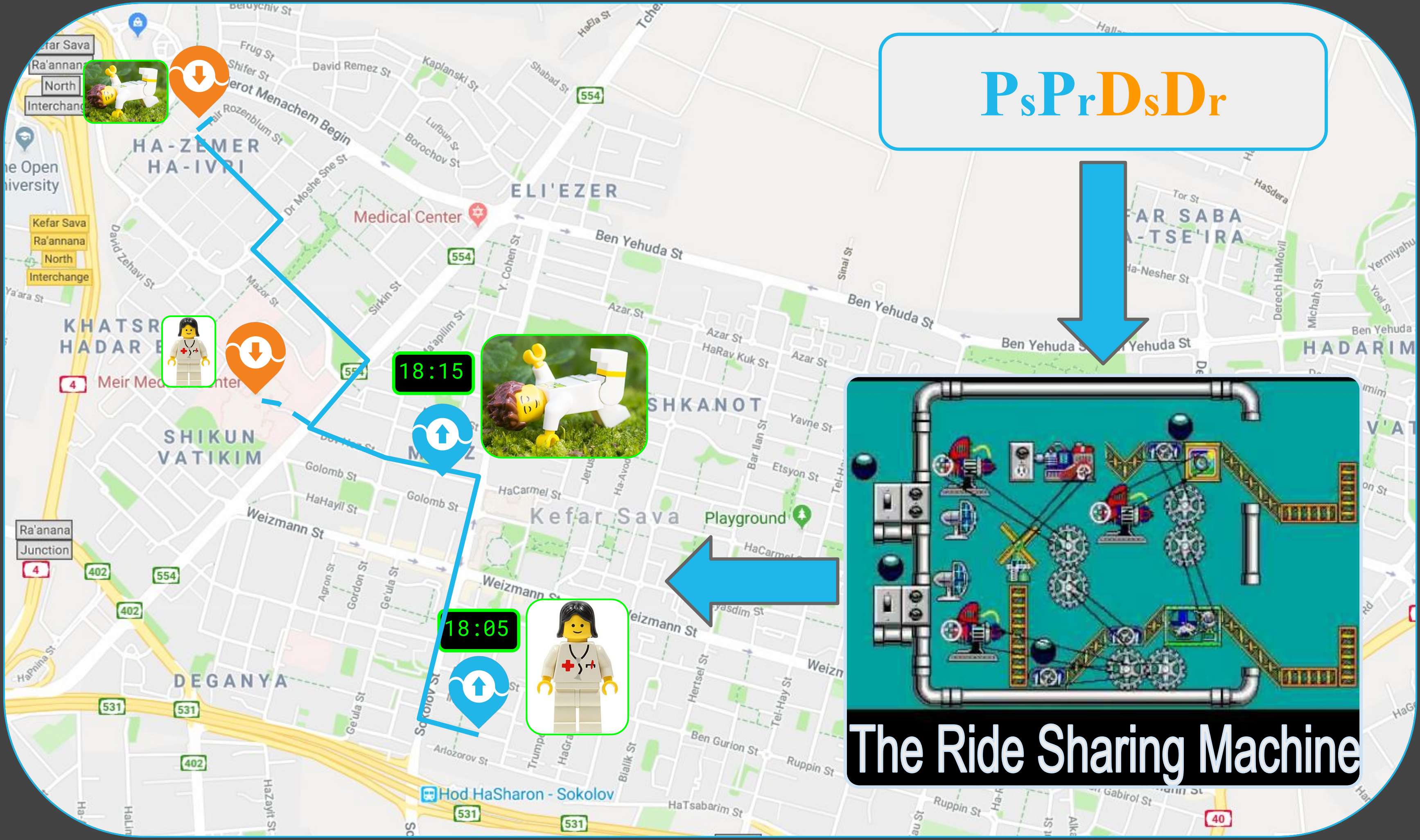
Hod HaSharon - Sokolov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The Ride Sharing Machine

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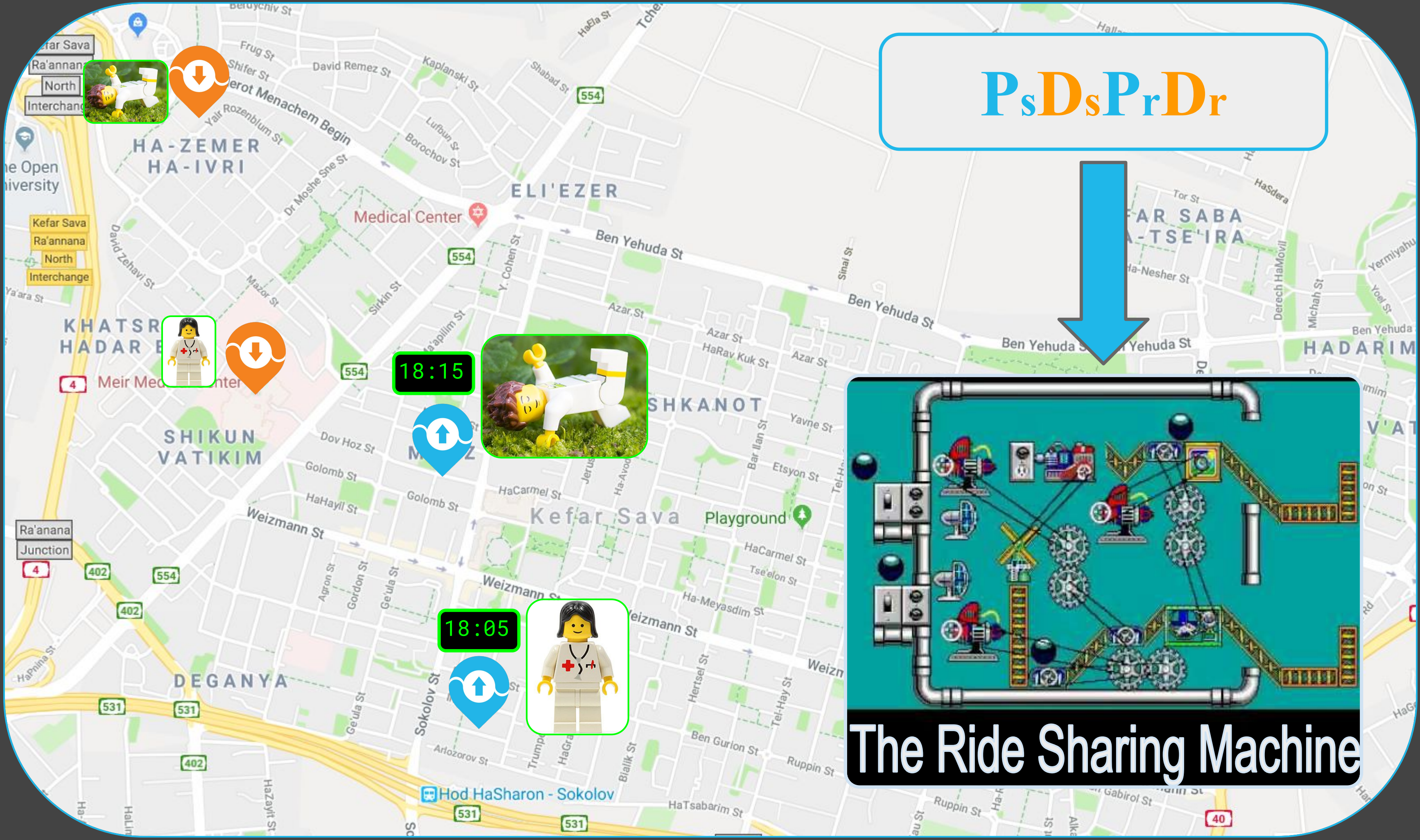
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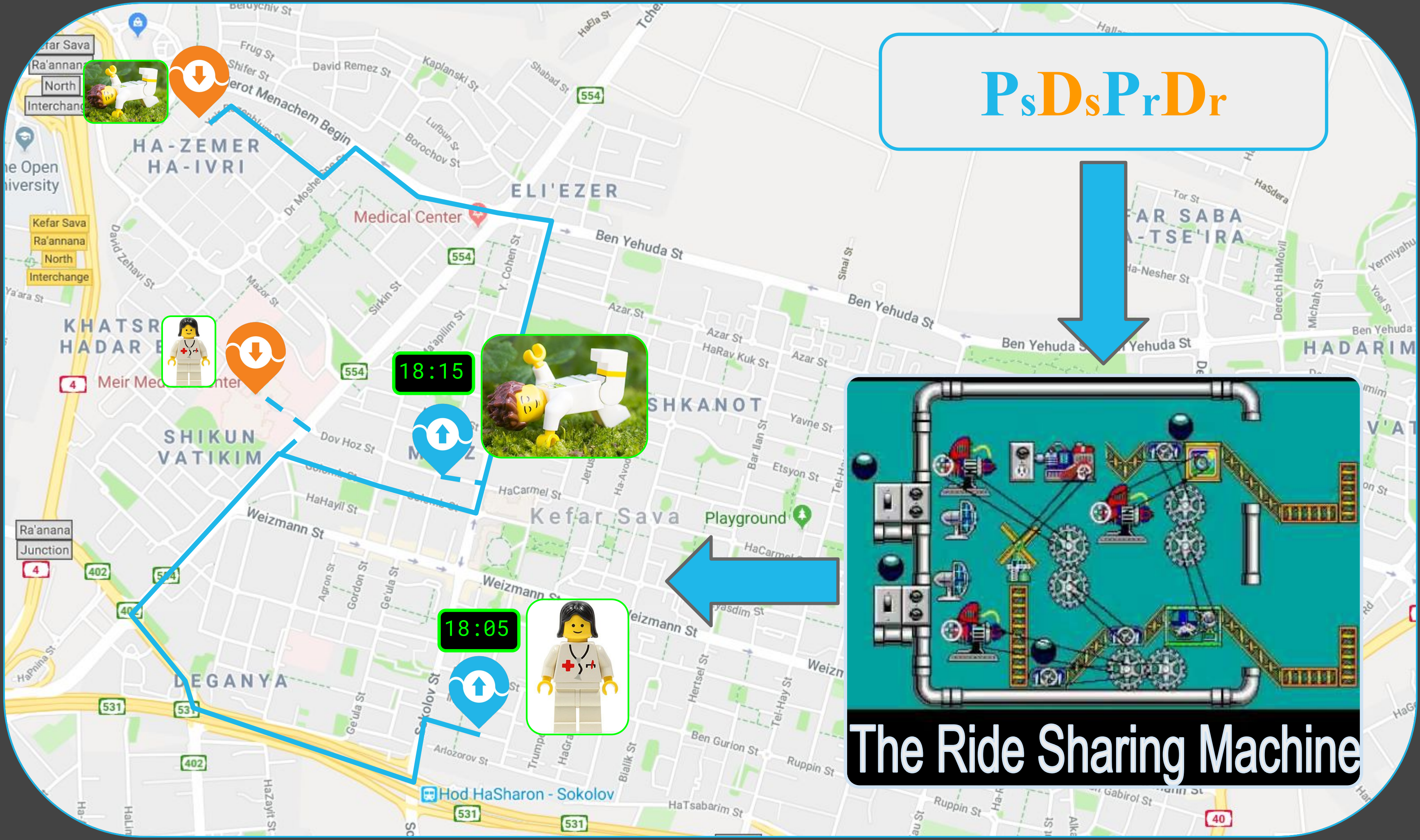
Kefar Sava Ra'annana North Interchange
Ra'anana Junction
The Ride Sharing Machine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The Ride Sharing Machine
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DEGANYA
Hod HaSharon - Sokolov
Playground

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Kefar Sava
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North
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Ra'anana
Junction
David Remez St
Kaplanski St
Derech HaPo'el
Sokolov St

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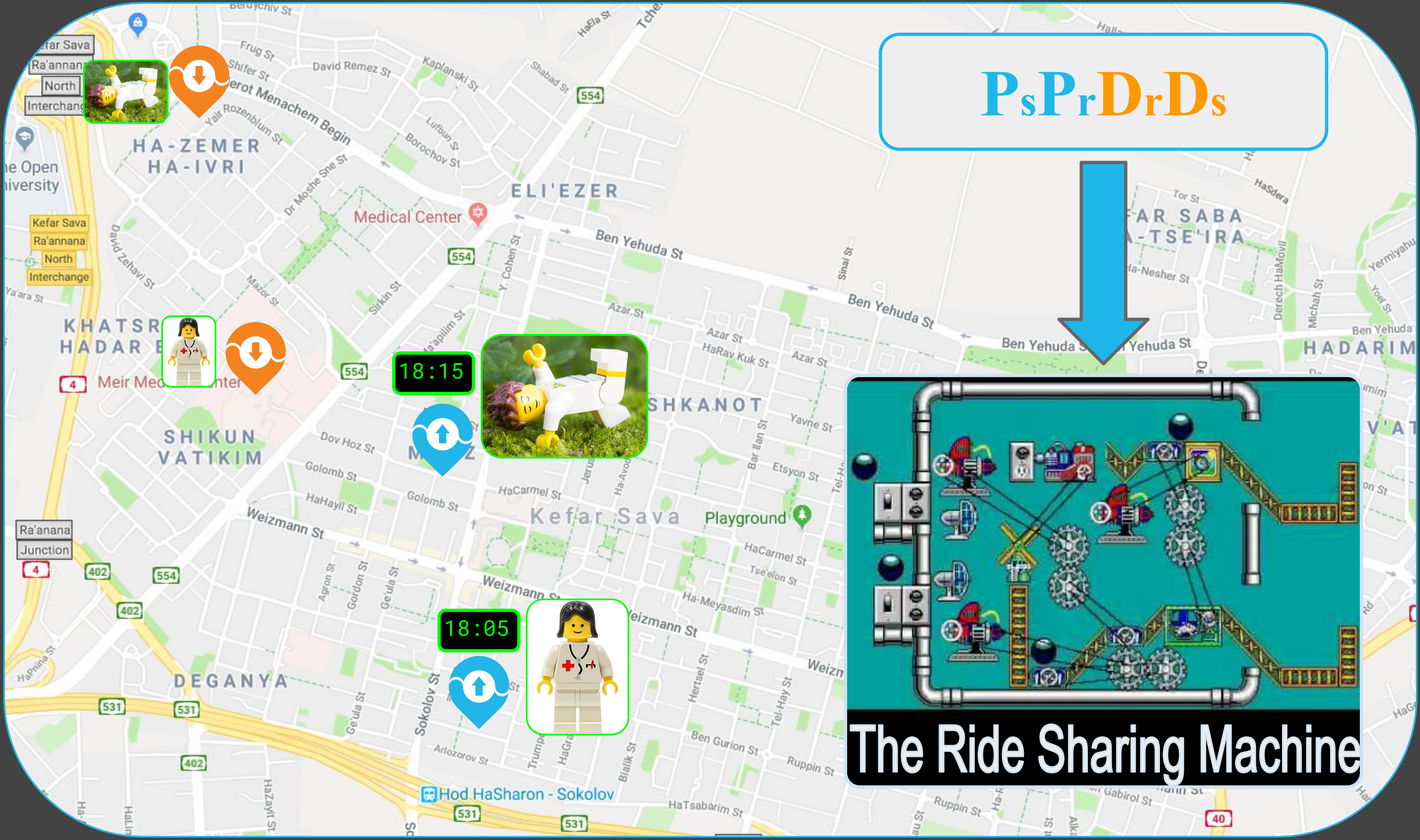

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Playground
Ben Yehuda St
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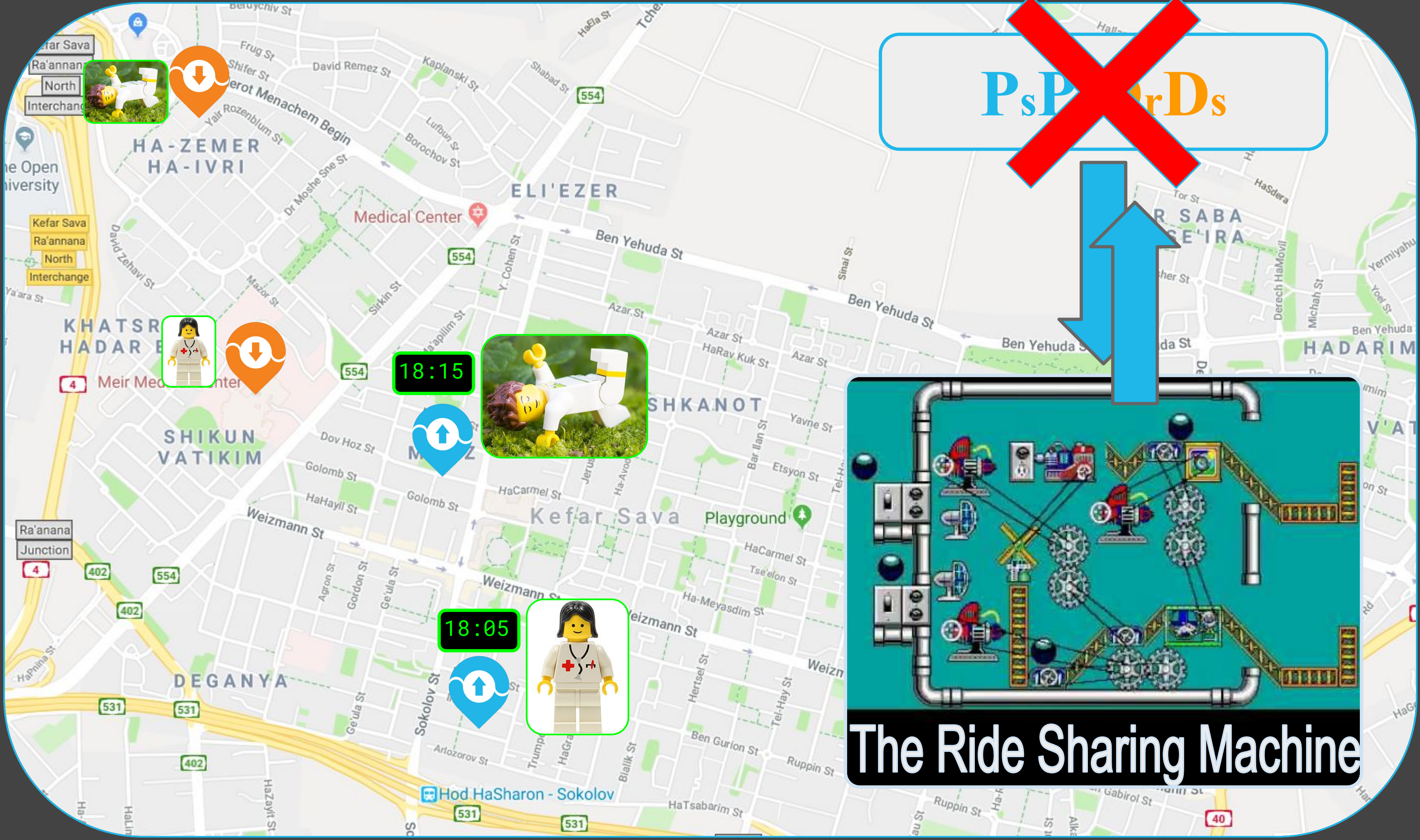
Hod HaSharon - Sokolov
The Ride Sharing Machine



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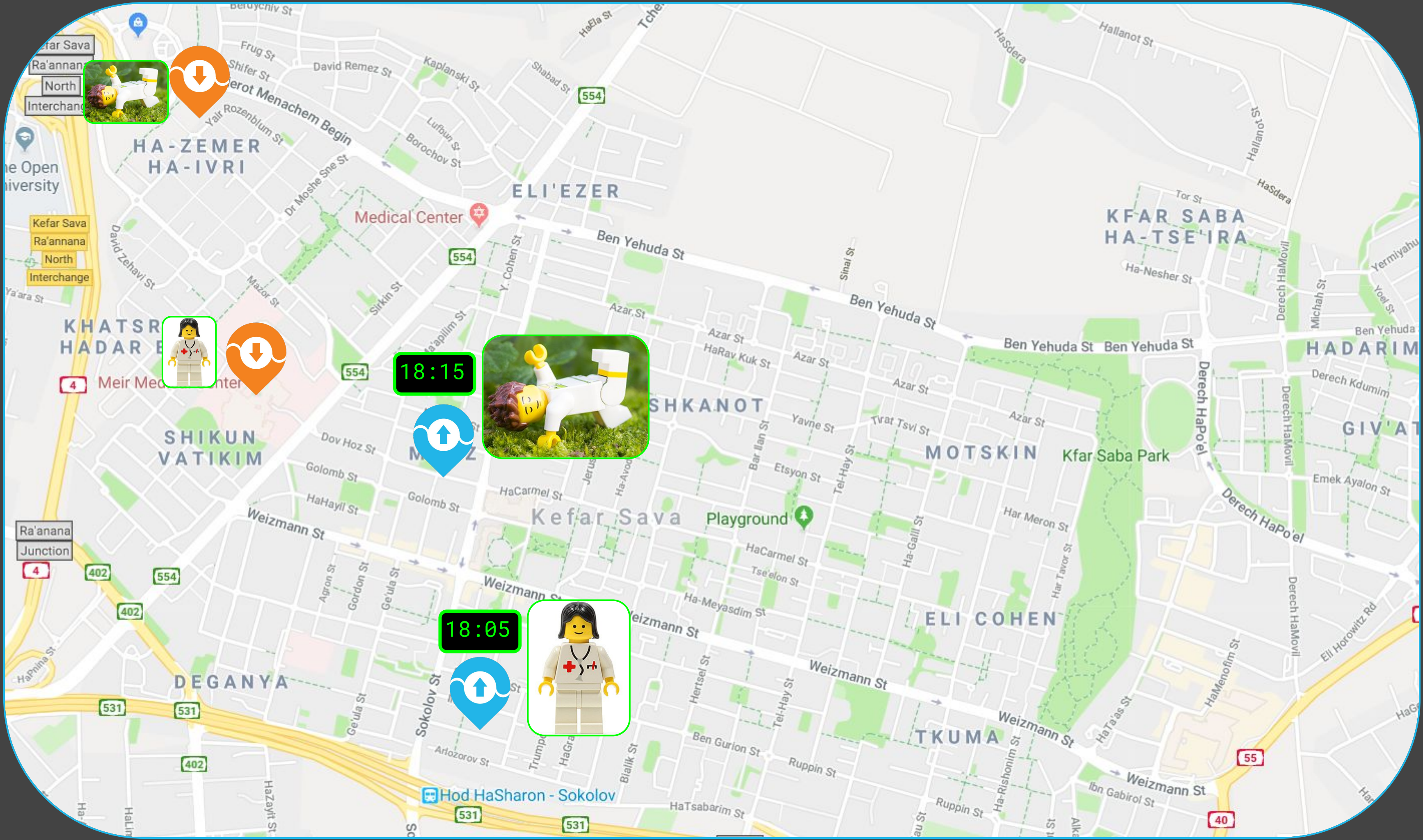
Hod HaSharon - Sokolov
Kefar Sava Ra'annana North Interchange
Ra'anana Junction
HADARIM

Question #1:

What is the best way to perform a given sequence of events?

Question #2:

What is the best permutation
to perform
a given set of events?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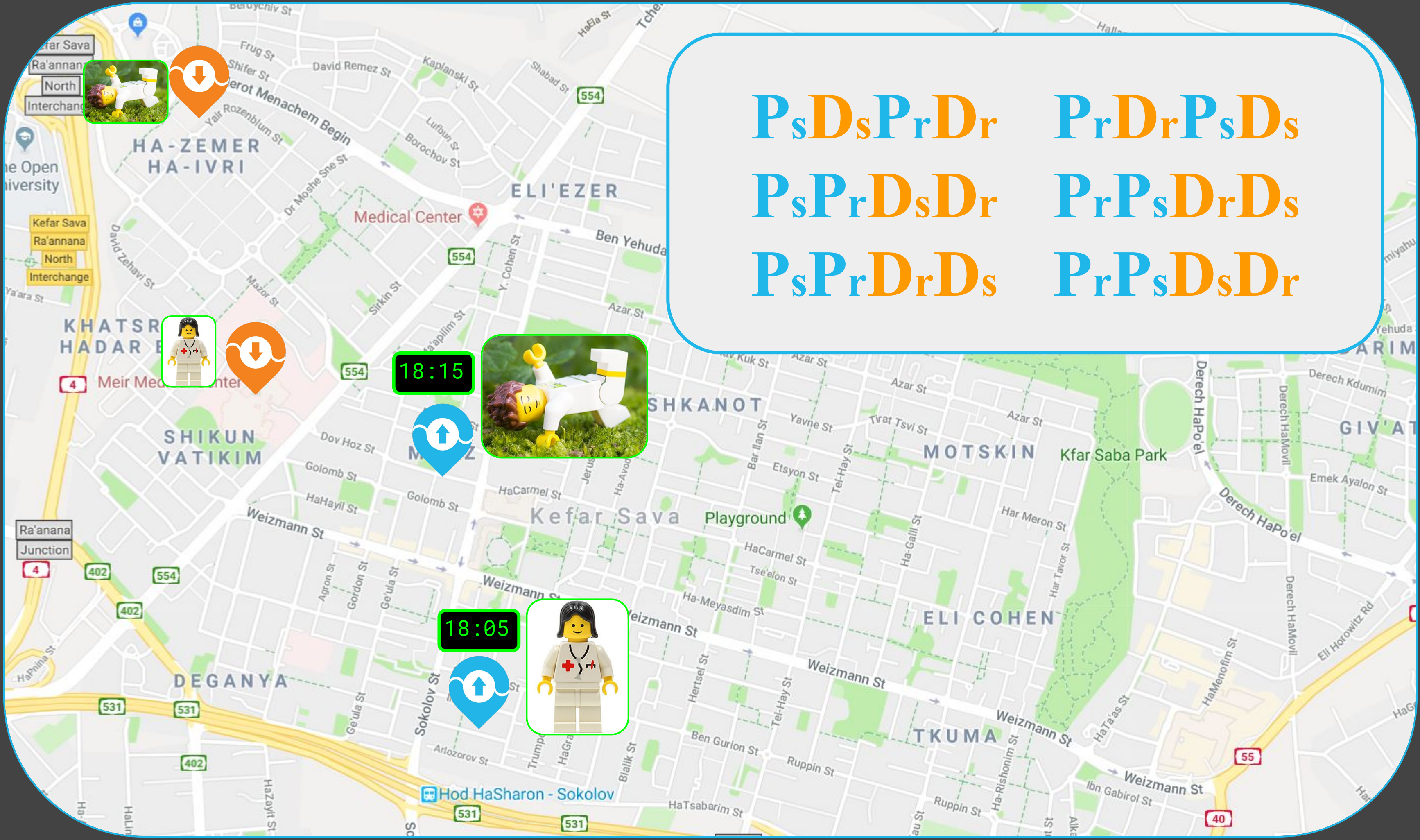

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Kefar Sava
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Meir Medical Center
Playground
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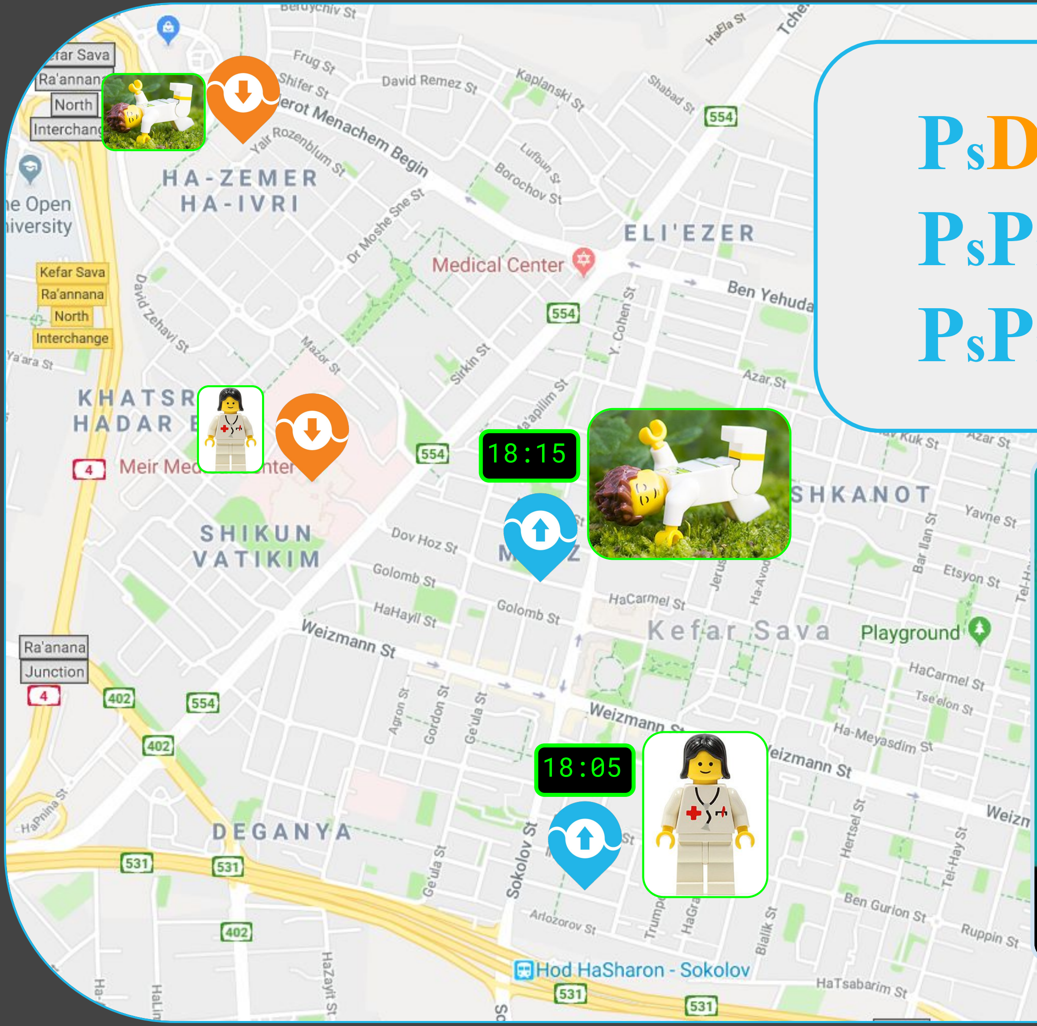

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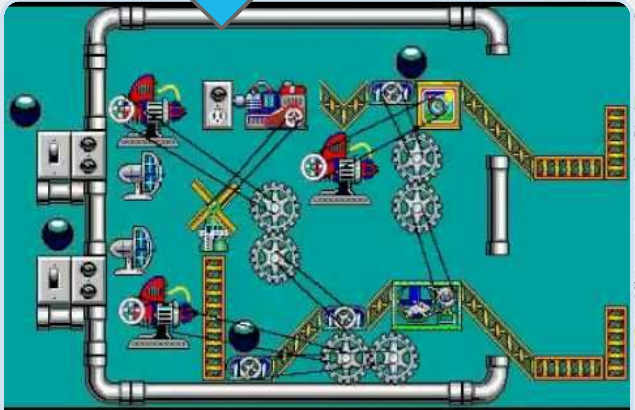
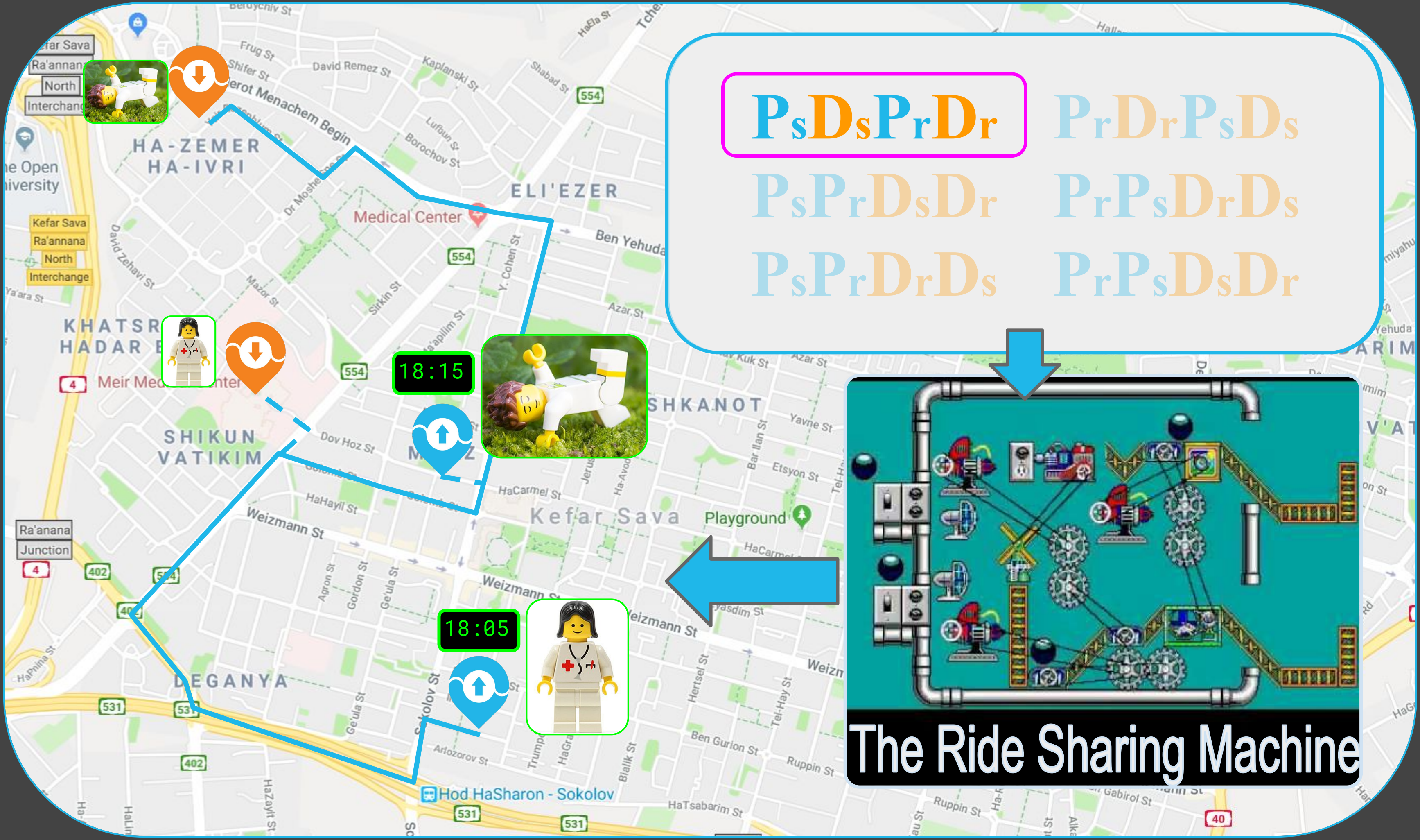


The Ride Sharing Machine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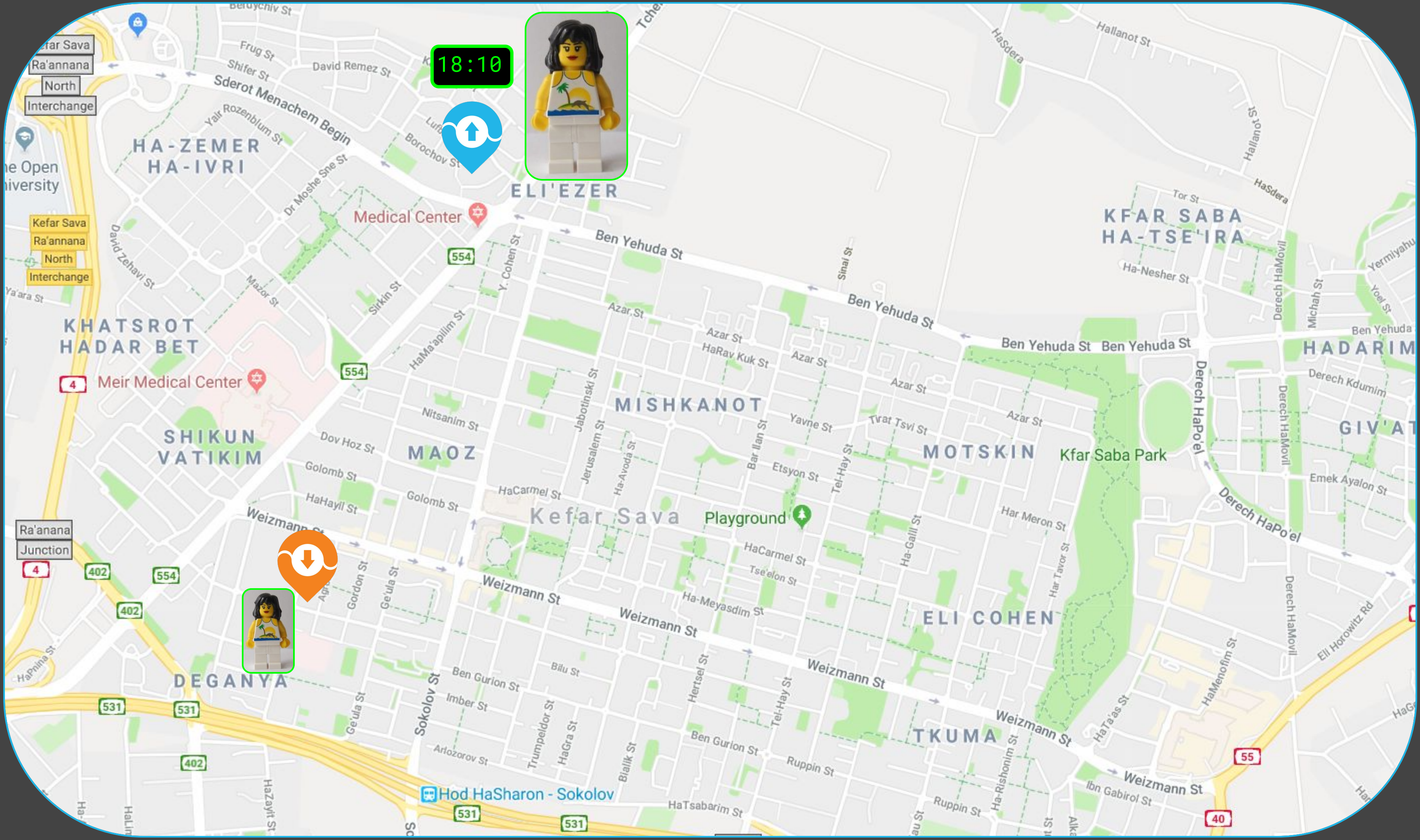
The Ride Sharing Machine



18:10
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Ra'anana Junction
Kefar Sava Ra'annana North Interchange

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Playground
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HADARIM
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Hod HaSharon - Sokolov
Sokolov St
David Remez St
Frug St
Shifer St
Borochov St
Dr Moshe Sne St
David Zehavi St
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Sirkin St
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Eli Horowitz Rd
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How many permutations?

How many permutations?

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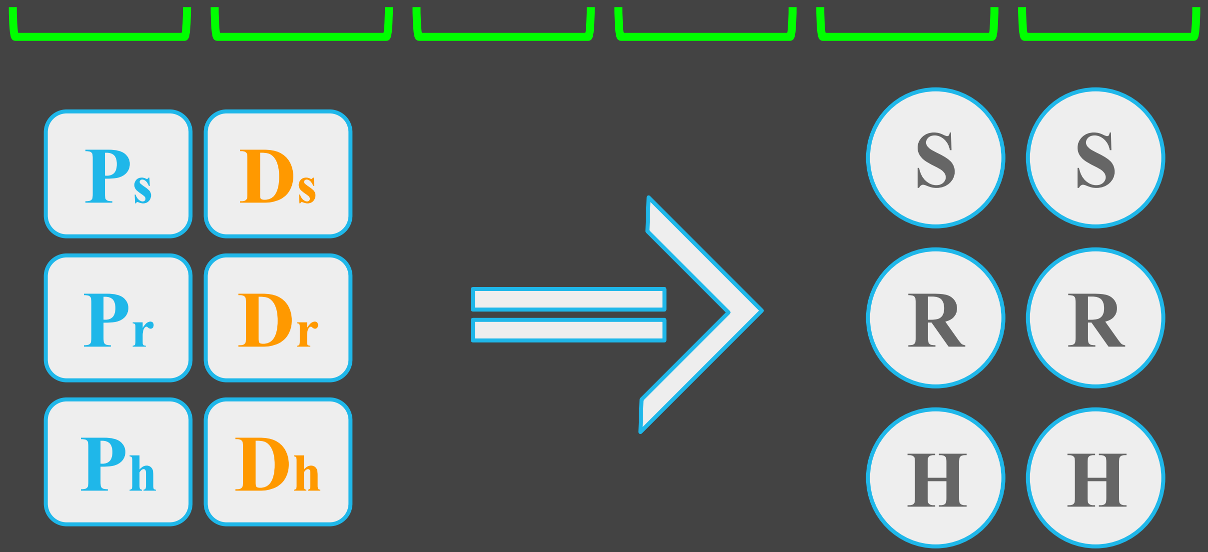
How many permutations?

P_s D_s

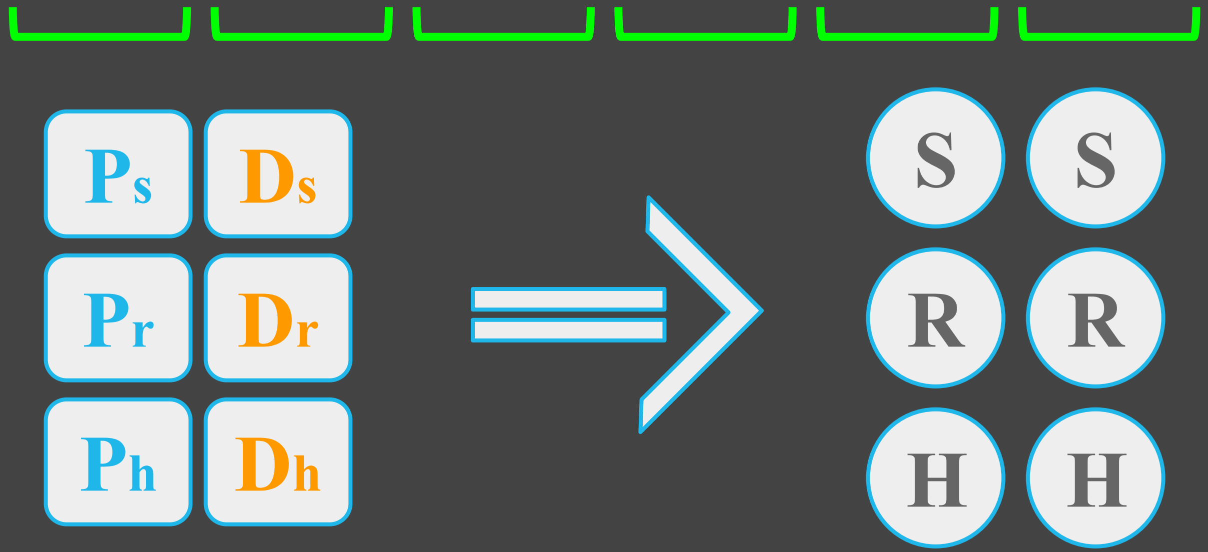
P_r D_r

P_h D_h

How many permutations?



How many permutations?



$$= \frac{6!}{(2!)^3}$$

How many permutations?

$$\frac{(2n)!}{2^n}$$

How many permutations?

$$\frac{(2n)!}{2^n}$$

Values:

n	1	2	3	4	5
$2^{-n}(2n)!$	1	6	90	2520	113400

How many permutations?

$$\frac{(2n)!}{2^n}$$

Values:

n	1	2	3
$2^{-n}\,(2\,n)!$	1	6	90

PART I - Conclusion

There's got to be a better way!

Question #3:

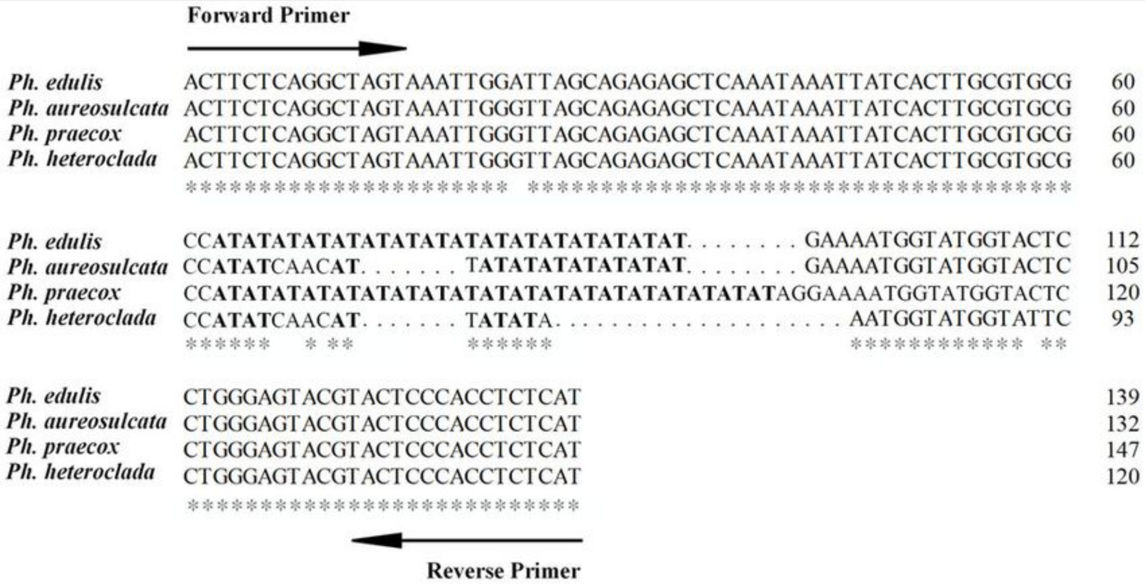
Given a set of events,
how can we scale down
the number of permutations?

PART II - Sequence Alignment

PART II - Sequence Alignment

PART II - Sequence Alignment

Motivation



PART II - Sequence Alignment

In The Wild

Sequence1	**-TCAGGA-TGAAC-G-**
Sequence2	**ATCACGA-TGAACC--**
Sequence3	**-TCACGATTGAACCGC**
Sequence4	**ATCACGAATGAATCC-**

PART II - Sequence Alignment

In The Wild

Sequence1	**-TCAGGA-TGAAC-G-**
Sequence2	**ATCACGA-TGAACC--**
Sequence3	**-TCAGGATTGAACCGC**
Sequence4	**ATCACGAATGAATCC-**

PART II - Sequence Alignment

In The Wild

```
Sequence1   -TCAGGA-TGAAC-G-
Sequence2   ATCACGA-TGAACC--
Sequence3   -TCAGGATTGAACCGC
Sequence4   ATCACGAATGAATCC-
```

PART II - Sequence Alignment

In OUR Wild

Sequence1	Ph	Ps	Dh	–	–	Ds
Sequence2	Ph	–	Dh	Pr	Dr	–

PART II - Sequence Alignment

In **OUR** Wild

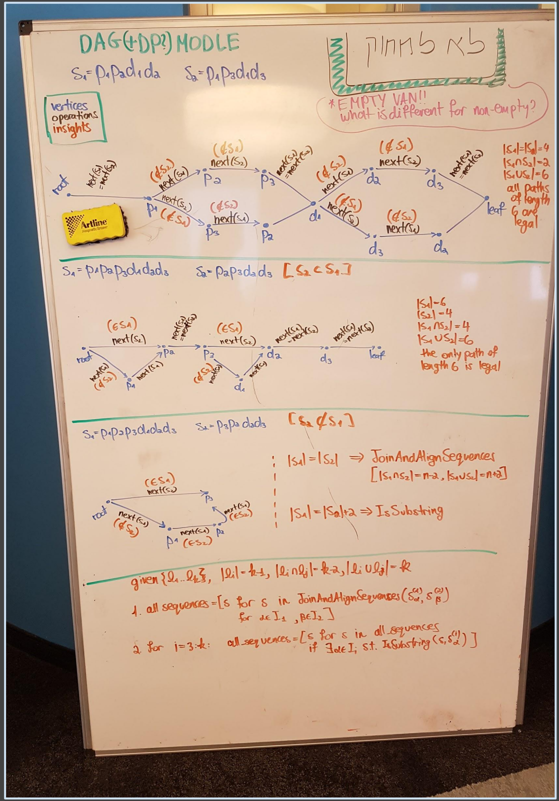
Sequence1	Ph	Ps	Dh	–	–	Ds
Sequence2	Ph	–	Dh	Pr	Dr	–

PART II - Sequence Alignment

In OUR Wild

Sequence1	Ph	Ps	Dh	-	-	Ds
Sequence2	Ph	-	Dh	Pr	Dr	-

PART II - Conclusion



PART III - Sequence Alignment for Ride Sharing

The Ride Sharing Problem

We are given:

- ▷ A set of n pairs of pickup and dropoff $T = \{p_i, d_i\}_{i \in [n]}$

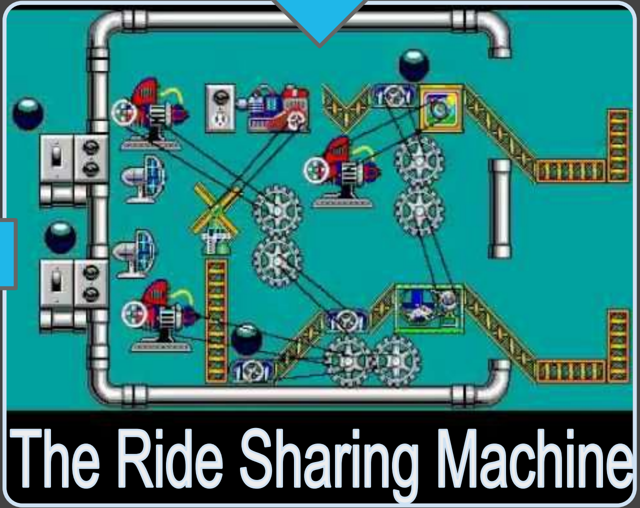
Our Goal:

Find the set of all possible permutations of T

The Ride Sharing Problem

We are given:

▷ A set of n pairs of pickup and dropoff $T = \{p_i, d_i\}_{i \in [n]}$



Our Goal:

Find the set of all possible permutations of T

The Ride Sharin

We are given:

- ▷ A set of n pairs of pickup and dropoff

Our Goal:

Find the set of all possible permutations of T

<u>Question #3:</u>

Given a set of events,

how can we scale down

the number of permutations?

Inductive Ride Sharing

We are given:

- ▷ A set of n pairs of pickup and dropoff $T = \{p_i, d_i\}_{i \in [n]}$

Our Goal:

Find the set of all possible permutations of T

Inductive Ride Sharing

We are given:

- ▷ A set of n pairs of pickup and dropoff $T = \{p_i, d_i\}_{i\in[n]}$
- ▷ For each subset of $n-1$ pairs $\left(T^{(j)} = \{p_i, d_i\}_{i\in[n], i\neq j}\right)$
  we are given a subset of all possible permutaions $S^{(j)} = \left\{s_k^{(j)}\right\}_{k\in K^{(j)}}$

Our Goal:

Find the set of all possible permutations of T that agree with all $\left\{S^{(j)}\right\}_{j\in[n]}$.

Inductive Ride Sharing

We are given:

- ▷ A set of $\boxed{n}$ pairs of pickup and dropoff $\boxed{T = \{p_i, d_i\}_{i\in[n]}}$
- ▷ For each subset of $\boxed{n-1}$ pairs $\boxed{T^{(j)} = \{p_i, d_i\}_{i\in[n], i\neq j}}$ we are given a subset of all possible permutaions $S^{(j)} = \left\{s_k^{(j)}\right\}$

Our Goal:

Find the set of all possil

Induction Assumption

Inductive Ride Sharing

Induction Step

$\ldots p_i, d_i\}_{i\in[n]}$

$\ldots\}_{i\in[n], i\neq j}\Big)$

we are given a subset of all possible permutaions $\boxed{S^{(j)} = \left\{s_k^{(j)}\right\}_{k\in K^{(j)}}}$

Our Goal:

Find the set of all possible permutations of T that agree with all $\boxed{\left\{S^{(j)}\right\}_{j\in[n]}}$

Inductive Ride Sharing

We are given:

- ▷ A set of n pai

- ▷ For each subs

  we are given

$k \in K^{(j)}$

Our Goal:

Find the set of $\boxed{\text{all possible permutations of } T}$ that agree with $\boxed{\text{all } \left\{S^{(j)}\right\}_{j\in[n]}}$.

Question #4:

How can we break down the induction step?

Induction Sub-Step

We are given:

- A set of n pairs of pickup and dropoff $T = \{p_i, d_i\}_{i \in [n]}$

Induction Sub-Step

We are given:

- ▷ A set of n pairs of pickup and dropoff $T = \{p_i, d_i\}_{i \in [n]}$
- ▷ Two subsets of $n-1$ pairs: $I_1 = \{2, ..., n\}, I_2 = \{1, 3, ..., n\}$

$$|I_1 \cap I_2| = n - 2$$
$$|I_1 \cup I_2| = n$$

Induction Sub-Step

We are given:

- ▷ A set of n pairs of pickup and dropoff $T = \{p_i, d_i\}_{i \in [n]}$
- ▷ Two subsets of $n-1$ pairs: $I_1 = \{2, ..., n\}, I_2 = \{1, 3, ..., n\}$
- ▷ One permutation from each subset: $\sigma_1 = \sigma\left(\{p_i, d_i\}_{i \in I_1}\right), \sigma_2 = \sigma\left(\{p_i, d_i\}_{i \in I_2}\right)$

Induction Sub-Step

We are given:

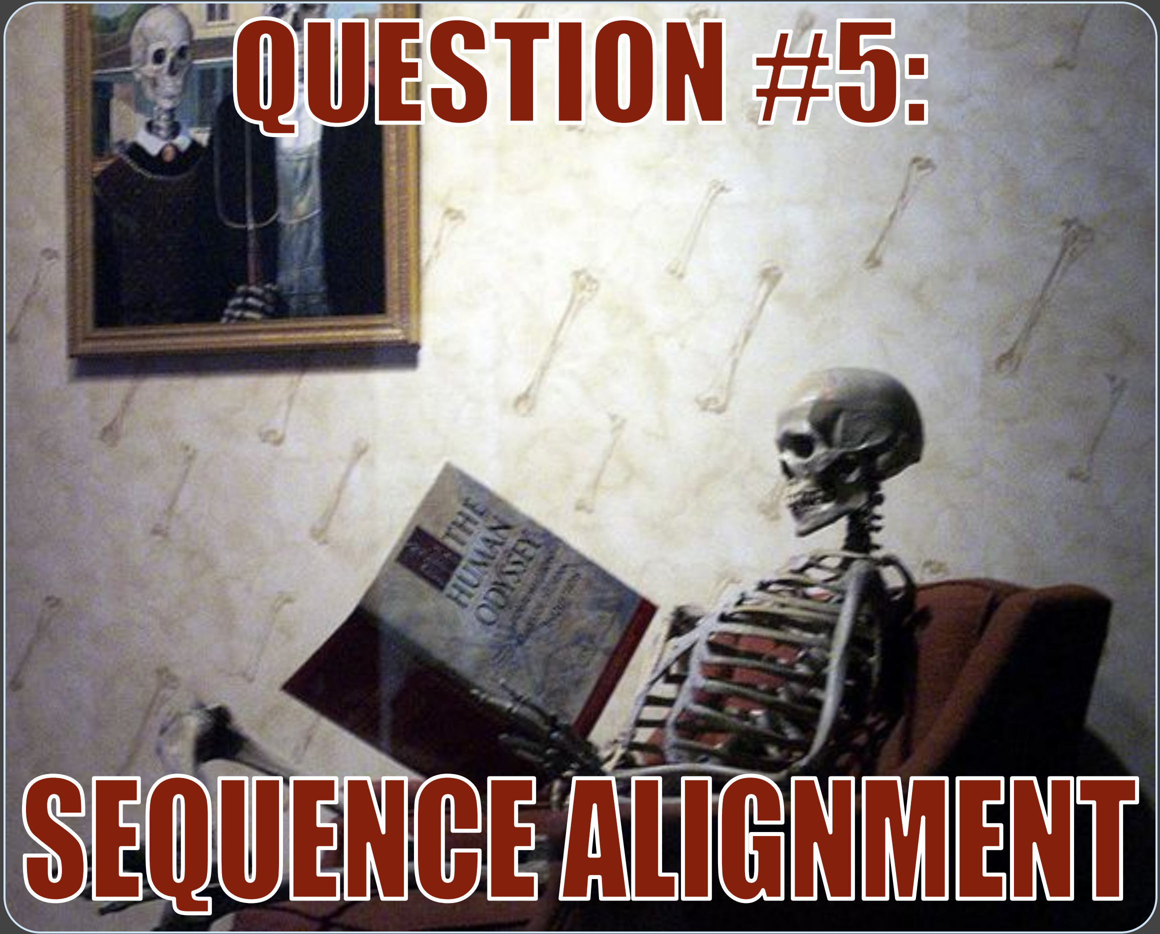
- ▷ A set of n pairs of pickup and dropoff $T = \{p_i, d_i\}_{i\in[n]}$
- ▷ Two subsets of $n-1$ pairs: $I_1 = \{2, ..., n\}, I_2 = \{1, 3, ..., n\}$
- ▷ One permutation from each subset: $\sigma_1 = \sigma\left(\{p_i, d_i\}_{i\in I_1}\right), \sigma_2 = \sigma\left(\{p_i, d_i\}_{i\in I_2}\right)$

Our Goal: Find the set of all possible permutations of T that agree with both σ_1 and σ_2.

<u>Question #5:</u>

How can we find all permutations that agree with two sub-permutations?

QUESTION #5:



SEQUENCE ALIGNMENT

Example: Induction Sub-Step

$T = \{P_r, P_s, P_h, D_h, D_r, D_s\}$

$\sigma_{hs} = P_h\, P_s\, D_h\, D_s$

$\sigma_{hr} = P_h\, D_h\, P_r\, D_r$

Example: Induction Sub-Step

$T = \{P_r, P_s, P_h, D_h, D_r, D_s\}$

$\sigma_{hs} = P_h\ P_s\ D_h\ D_s$

$\sigma_{hr} = P_h\ D_h\ P_r\ D_r$

Reminder:

Sequence1	`Ph`	`Ps`	`Dh`	`-`	`-`	`Ds`
Sequence2	`Ph`	`-`	`Dh`	`Pr`	`Dr`	`-`

$$\sigma_{hs} = P_h \, P_s \, D_h \, D_s$$

$$\sigma_{hr} = P_h \, D_h \, P_r \, D_r$$

$$\sigma_{hs} = \boxed{P_h} P_s \boxed{D_h} D_s$$

$$\sigma_{hr} = \boxed{P_h} \boxed{D_h} P_r D_r$$

mutual subsequence

$\sigma_{hs} = \boxed{P_h} P_s \boxed{D_h} D_s$

$\sigma_{hr} = \boxed{P_h} \boxed{D_h} P_r \, D_r$

mutual subsequence

START

P_h

D_h

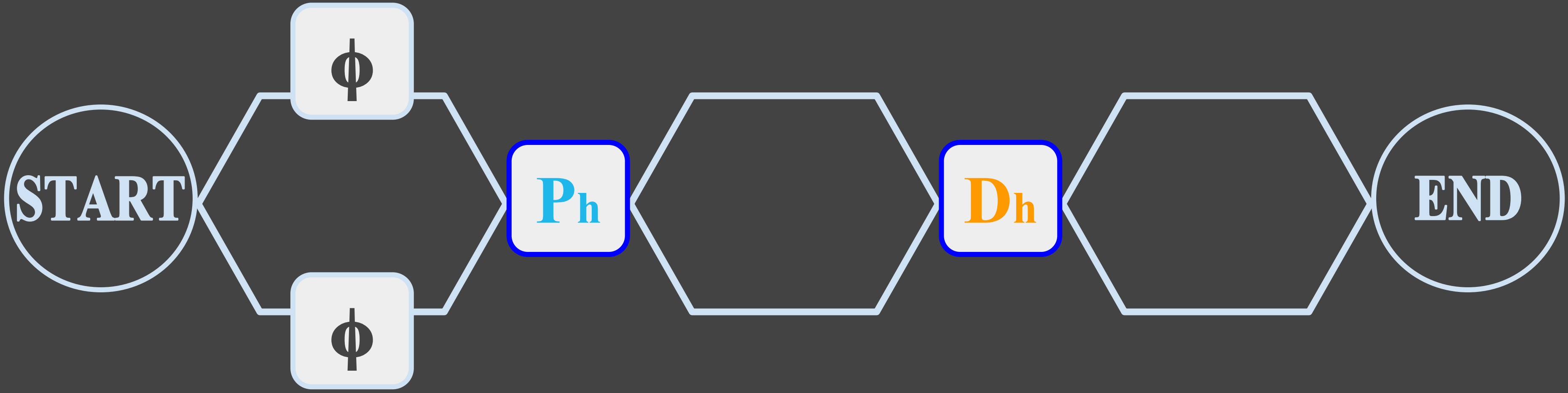
END

$$\sigma_{hs} = \boxed{P_h} P_s \boxed{D_h} D_s$$

$$\sigma_{hr} = \boxed{P_h} \boxed{D_h} P_r D_r$$

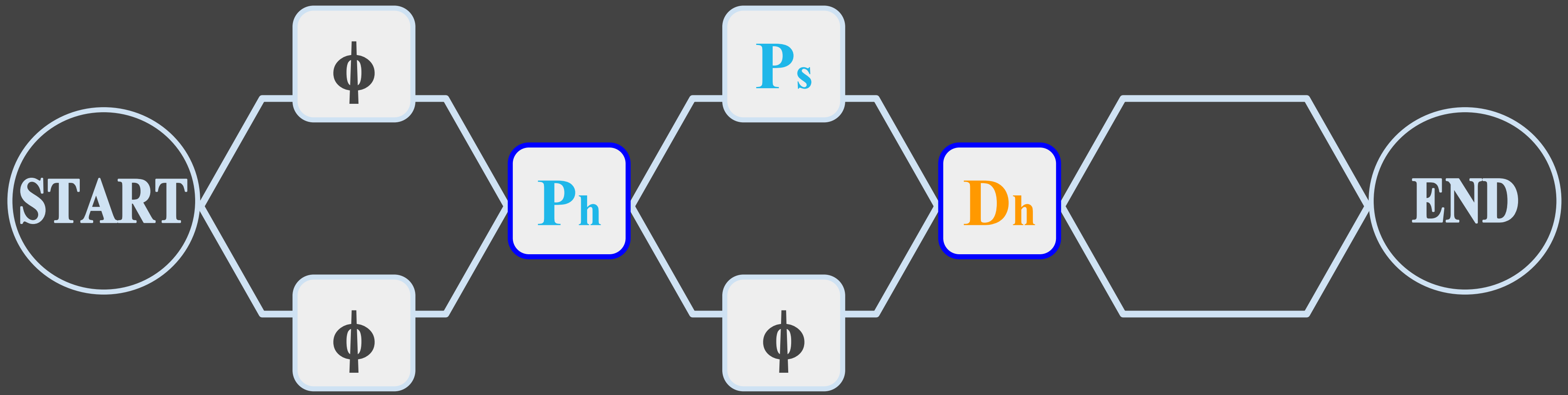
$$\sigma_{hs} = \boxed{P_h} P_s \boxed{D_h} D_s$$

$$\sigma_{hr} = \boxed{P_h} \boxed{D_h} P_r D_r$$



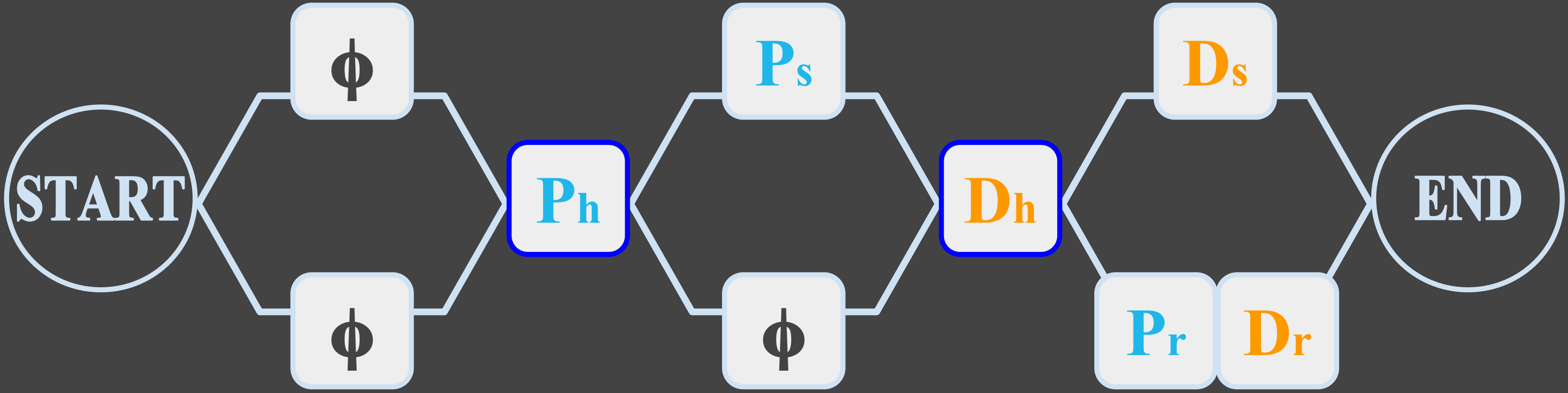
$$\sigma_{hs} = \boxed{P_h}\, P_s\, \boxed{D_h}\, D_s$$

$$\sigma_{hr} = \boxed{P_h}\, \boxed{D_h}\, P_r\, D_r$$



$$\sigma_{hs} = \boxed{P_h}\, P_s\, \boxed{D_h}\, D_s$$

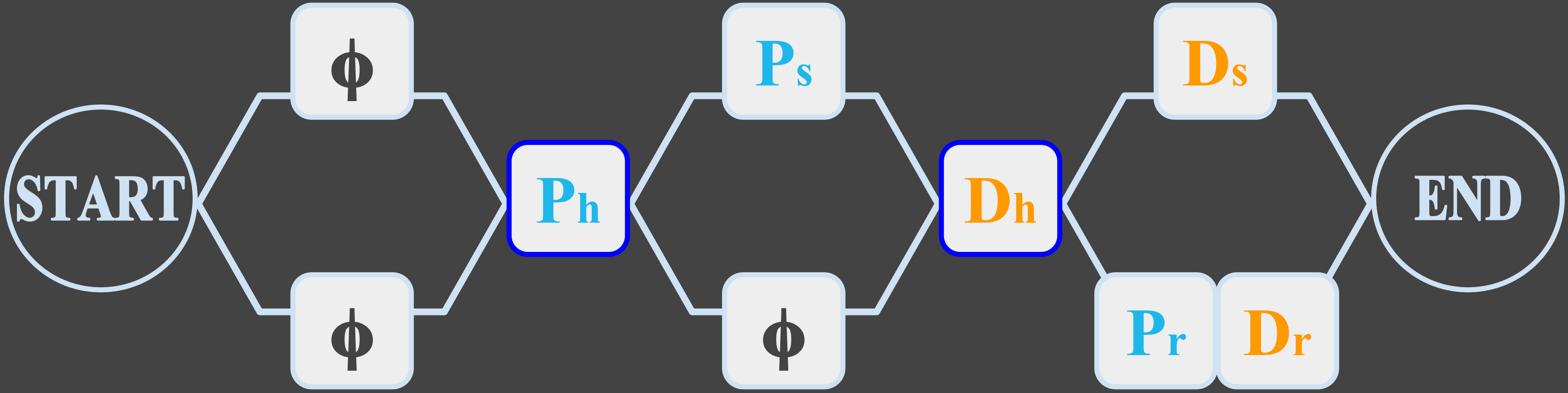
$$\sigma_{hr} = \boxed{P_h}\, \boxed{D_h}\, P_r\, D_r$$



$$\sigma_{hs} = \boxed{P_h}\, P_s\, \boxed{D_h}\, D_s$$

$$\sigma_{hr} = \boxed{P_h}\,\boxed{D_h}\, P_r\, D_r$$

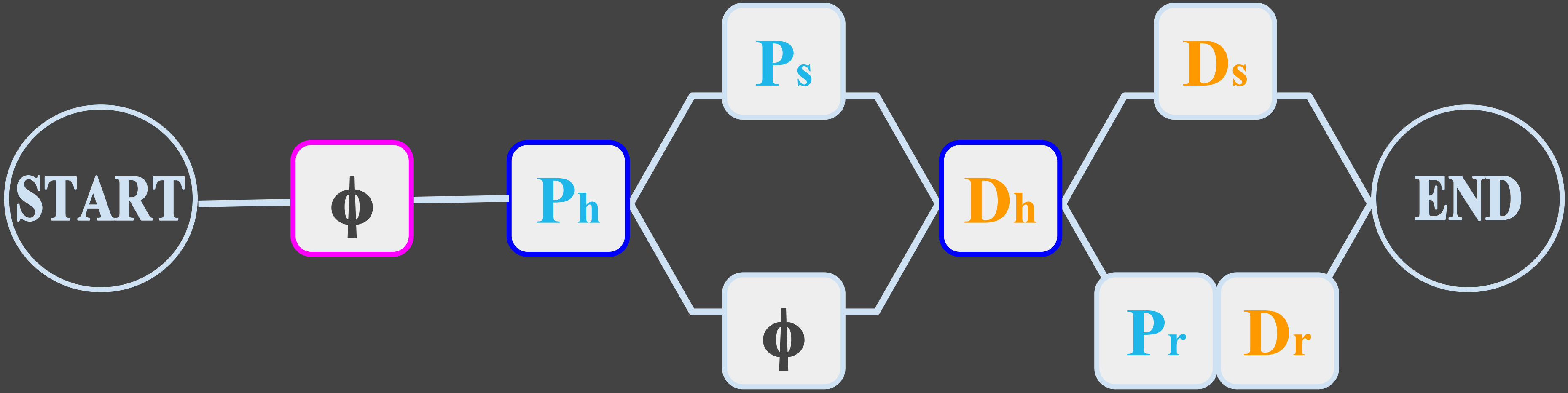
aligned insertions



$$\sigma_{hs} = \boxed{P_h}\, P_s\, \boxed{D_h}\, D_s$$

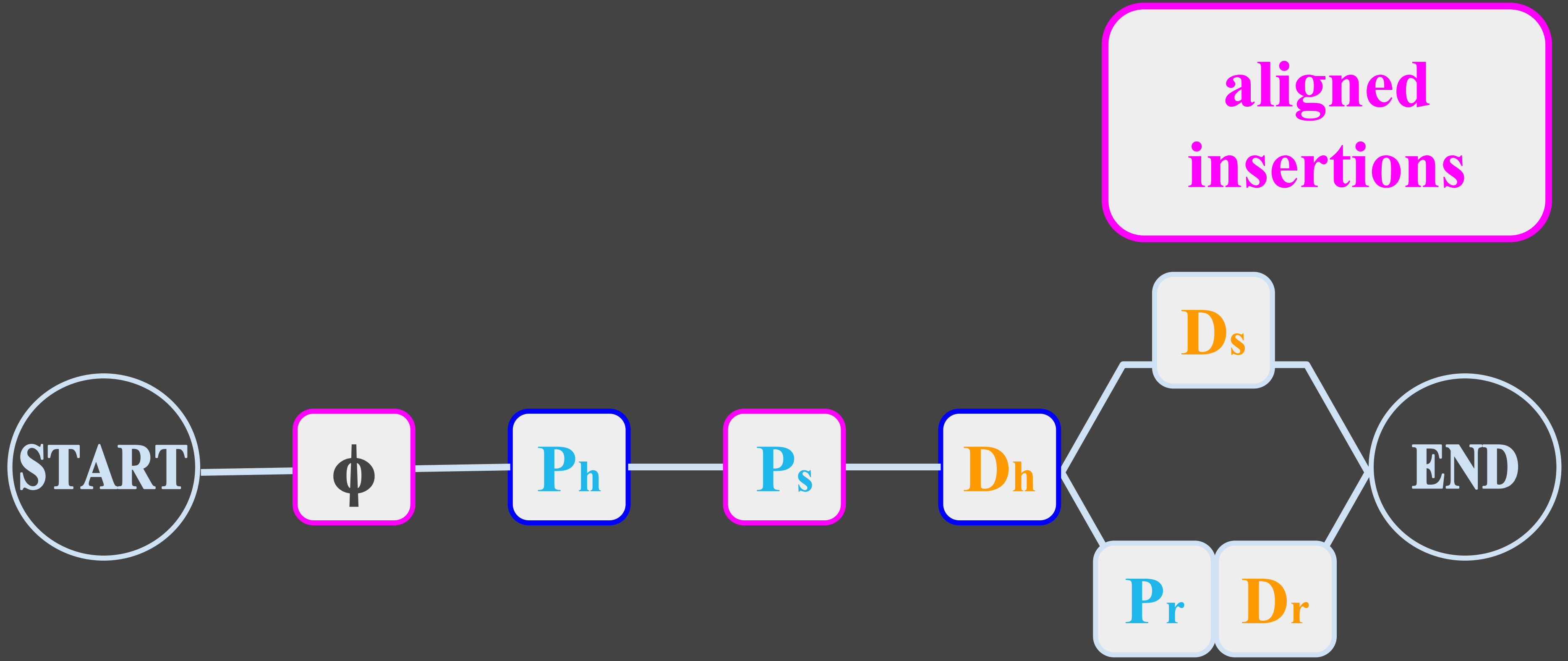
$$\sigma_{hr} = \boxed{P_h}\, \boxed{D_h}\, P_r\, D_r$$

aligned insertions



$$\sigma_{hs} = \boxed{P_h}\, P_s\, \boxed{D_h}\, D_s$$

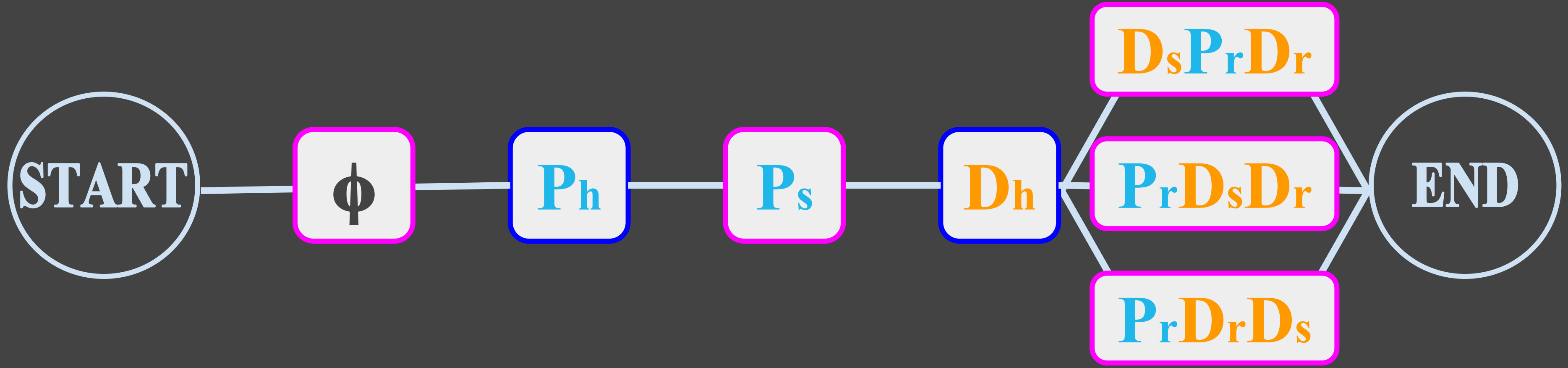
$$\sigma_{hr} = \boxed{P_h}\, \boxed{D_h}\, P_r\, D_r$$



$$\sigma_{hs} = \boxed{P_h}\, P_s \boxed{D_h}\, D_s$$

$$\sigma_{hr} = \boxed{P_h}\boxed{D_h}\, P_r\, D_r$$

aligned insertions



$\sigma_{hs} = P_h\ P_s\ D_h\ D_s$

$\sigma_{hr} = P_h\ D_h\ P_r\ D_r$

Induction sub-step result:

Ph Ps Dh Ds Pr Dr

Ph Ps Dh Pr Ds Dr

Ph Ps Dh Pr Dr Ds

Example: Full Induction Step

Example: Full Induction Step

hs

$P_h\ P_s\ D_h\ D_s$

$P_s\ D_s\ P_h\ D_h$

hr

$P_h\ D_h\ P_r\ D_r$

$P_h\ P_r\ D_h\ D_r$

Example: Full Induction Step

hs

$P_h\ P_s\ D_h\ D_s$

$P_s\ D_s\ P_h\ D_h$

hr

$P_h\ D_h\ P_r\ D_r$

$P_h\ P_r\ D_h\ D_r$

$P_h\ P_s\ D_h\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_s\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_r\ D_s$

Induction Sub-Step

Example: Full Induction Step

hs

P_h P_s D_h D_s

P_s D_s P_h D_h

hr

P_h D_h P_r D_r

P_h P_r D_h D_r

P_h P_s D_h D_s P_r D_r

P_h P_s D_h P_r D_s D_r

P_h P_s D_h P_r D_r D_s

P_h P_s P_r D_h D_s D_r

P_h P_r P_s D_h D_s D_r

P_h P_s P_r D_h D_r D_s

P_h P_r P_s D_h D_r D_s

Example: Full Induction Step

hs

$P_h\ P_s\ D_h\ D_s$

$\boxed{P_s\ D_s\ P_h\ D_h}$

hr

$\boxed{P_h\ D_h\ P_r\ D_r}$

$P_h\ P_r\ D_h\ D_r$

$P_h\ P_s\ D_h\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_s\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_r\ D_s$

$P_h\ P_s\ P_r\ D_h\ D_s\ D_r$

$P_h\ P_r\ P_s\ D_h\ D_s\ D_r$

$P_h\ P_s\ P_r\ D_h\ D_r\ D_s$

$P_h\ P_r\ P_s\ D_h\ D_r\ D_s$

$\boxed{P_s\ D_s\ P_h\ D_h\ P_r\ D_r}$

Example: Full Induction Step

hs

$P_h\ P_s\ D_h\ D_s$

$\boxed{P_s\ D_s\ P_h\ D_h}$

hr

$P_h\ D_h\ P_r\ D_r$

$\boxed{P_h\ P_r\ D_h\ D_r}$

$P_h\ P_s\ D_h\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_s\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_r\ D_s$

$P_h\ P_s\ P_r\ D_h\ D_s\ D_r$

$P_h\ P_r\ P_s\ D_h\ D_s\ D_r$

$P_h\ P_s\ P_r\ D_h\ D_r\ D_s$

$P_h\ P_r\ P_s\ D_h\ D_r\ D_s$

$P_s\ D_s\ P_h\ D_h\ P_r\ D_r$

$\boxed{P_s\ D_s\ P_h\ P_r\ D_h\ D_r}$

Example: Full Induction Step

hs

P_h P_s D_h D_s

P_s D_s P_h D_h

hr

P_h D_h P_r D_r

P_h P_r D_h D_r

P_h P_s D_h D_s P_r D_r

P_h P_s D_h P_r D_s D_r

P_h P_s D_h P_r D_r D_s

P_h P_s P_r D_h D_s D_r

P_h P_r P_s D_h D_s D_r

P_h P_s P_r D_h D_r D_s

P_h P_r P_s D_h D_r D_s

P_s D_s P_h D_h P_r D_r

P_s D_s P_h P_r D_h D_r

Example: Full Induction Step

hs

P_h P_s D_h D_s

P_s D_s P_h D_h

hr

P_h D_h P_r D_r

P_h P_r D_h D_r

sr

P_s P_r D_s D_r

P_s D_s P_r D_r

P_h P_s D_h D_s P_r D_r

P_h P_s D_h P_r D_s D_r

P_h P_s D_h P_r D_r D_s

P_h P_s P_r D_h D_s D_r

P_h P_r P_s D_h D_s D_r

P_h P_s P_r D_h D_r D_s

P_h P_r P_s D_h D_r D_s

P_s D_s P_h D_h P_r D_r

P_s D_s P_h P_r D_h D_r

Example: …tion Step

hs

P_h P_s D_h D_s

P_s D_s P_h D_h

hr

P_h D_h P_r D_r

P_h P_r D_h D_r

sr

P_s P_r D_s D_r

P_s D_s P_r D_r

P_s D_h D_s P_r D_r

P_s D_h P_r D_s D_r

P_s D_h P_r D_r D_s

P_s P_r D_h D_s D_r

P_r P_s D_h D_s D_r

P_s P_r D_h D_r D_s

P_r P_s D_h D_r D_s

D_s P_h D_h P_r D_r

D_s P_h P_r D_h D_r

Example: Full Induction Step

hs

$\boxed{P_h\ P_s\ D_h\ D_s}$

$P_s\ D_s\ P_h\ D_h$

hr

$\boxed{P_h\ D_h\ P_r\ D_r}$

$P_h\ P_r\ D_h\ D_r$

sr

$P_s\ P_r\ D_s\ D_r$?

$P_s\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_s\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_r\ D_s$

$P_h\ P_s\ P_r\ D_h\ D_s\ D_r$

$P_h\ P_r\ P_s\ D_h\ D_s\ D_r$

$P_h\ P_s\ P_r\ D_h\ D_r\ D_s$

$P_h\ P_r\ P_s\ D_h\ D_r\ D_s$

$P_s\ D_s\ P_h\ D_h\ P_r\ D_r$

$P_s\ D_s\ P_h\ P_r\ D_h\ D_r$

Example: Full Induction Step

hs

$\boxed{P_h\ P_s\ D_h\ D_s}$

$P_s\ D_s\ P_h\ D_h$

hr

$\boxed{P_h\ D_h\ P_r\ D_r}$

$P_h\ P_r\ D_h\ D_r$

sr

$P_s\ P_r\ D_s\ D_r$?

$\boxed{P_s}\ \boxed{D_s}\ \boxed{P_r}\ \boxed{D_r}$

$P_h\ \boxed{P_s}\ D_h\ \boxed{D_s}\ \boxed{P_r}\ \boxed{D_r}$

$P_h\ P_s\ D_h\ P_r\ D_s\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_r\ D_s$

$P_h\ P_s\ P_r\ D_h\ D_s\ D_r$

$P_h\ P_r\ P_s\ D_h\ D_s\ D_r$

$P_h\ P_s\ P_r\ D_h\ D_r\ D_s$

$P_h\ P_r\ P_s\ D_h\ D_r\ D_s$

$P_s\ D_s\ P_h\ D_h\ P_r\ D_r$

$P_s\ D_s\ P_h\ P_r\ D_h\ D_r$

Example: Full Induction Step

hs

$\boxed{P_h\ P_s\ D_h\ D_s}$

$P_s\ D_s\ P_h\ D_h$

hr

$\boxed{P_h\ D_h\ P_r\ D_r}$

$P_h\ P_r\ D_h\ D_r$

sr

$P_s\ P_r\ D_s\ D_r$

$P_s\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_s\ D_r$

$P_h\ \boxed{P_s}\ D_h\ \boxed{P_r}\ \boxed{D_r}\ \boxed{D_s}$

$P_h\ P_s\ P_r\ D_h\ D_s\ D_r$

$P_h\ P_r\ P_s\ D_h\ D_s\ D_r$

$P_h\ P_s\ P_r\ D_h\ D_r\ D_s$

$P_h\ P_r\ P_s\ D_h\ D_r\ D_s$

$P_s\ D_s\ P_h\ D_h\ P_r\ D_r$

$P_s\ D_s\ P_h\ P_r\ D_h\ D_r$

Example: Full Induction Step

sr

$P_s\ P_r\ D_s\ D_r$

$P_s\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_s\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_r\ D_s$

$P_h\ P_s\ P_r\ D_h\ D_s\ D_r$

$P_h\ P_r\ P_s\ D_h\ D_s\ D_r$

$P_h\ P_s\ P_r\ D_h\ D_r\ D_s$

$P_h\ P_r\ P_s\ D_h\ D_r\ D_s$

$P_s\ D_s\ P_h\ D_h\ P_r\ D_r$

$P_s\ D_s\ P_h\ P_r\ D_h\ D_r$

Example: Full Induction Step

sr

$P_s\ P_r\ D_s\ D_r$

$P_s\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_s\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_r\ D_s$

$P_h\ P_s\ P_r\ D_h\ D_s\ D_r$

$P_h\ P_r\ P_s\ D_h\ D_s\ D_r$

$P_h\ P_s\ P_r\ D_h\ D_r\ D_s$

$P_h\ P_r\ P_s\ D_h\ D_r\ D_s$

$P_s\ D_s\ P_h\ D_h\ P_r\ D_r$

$P_s\ D_s\ P_h\ P_r\ D_h\ D_r$

Example: Full Induction Step

sr

$P_s\ P_r\ D_s\ D_r$

$P_s\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_s\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_r\ D_s$

$P_h\ P_s\ P_r\ D_h\ D_s\ D_r$

$P_h\ P_r\ P_s\ D_h\ D_s\ D_r$

$P_h\ P_s\ P_r\ D_h\ D_r\ D_s$

$P_h\ P_r\ P_s\ D_h\ D_r\ D_s$

$P_s\ D_s\ P_h\ D_h\ P_r\ D_r$

$P_s\ D_s\ P_h\ P_r\ D_h\ D_r$

Example: Full Induction Step

$P_h\ P_s\ D_h\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_s\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_r\ D_s$

$P_h\ P_s\ P_r\ D_h\ D_s\ D_r$

$P_h\ P_r\ P_s\ D_h\ D_s\ D_r$

$P_h\ P_s\ P_r\ D_h\ D_r\ D_s$

$P_h\ P_r\ P_s\ D_h\ D_r\ D_s$

$P_s\ D_s\ P_h\ D_h\ P_r\ D_r$

$P_s\ D_s\ P_h\ P_r\ D_h\ D_r$

sr

$P_s\ P_r\ D_s\ D_r$

$P_s\ D_s\ P_r\ D_r$

Example: Full Induction Step

$P_h \; P_s \; D_h \; D_s \; P_r \; D_r$

$P_h \; P_s \; D_h \; P_r \; D_s \; D_r$

$P_h \; P_s \; P_r \; D_h \; D_s \; D_r$

$P_h \; P_r \; P_s \; D_h \; D_s \; D_r$

sr

$P_s \; P_r \; D_s \; D_r$

$P_s \; D_s \; P_r \; D_r$

$P_s \; D_s \; P_h \; D_h \; P_r \; D_r$

$P_s \; D_s \; P_h \; P_r \; D_h \; D_r$

Example: Full Induction Step

hs

P_h P_s D_h D_s

P_s D_s P_h D_h

hr

P_h D_h P_r D_r

P_h P_r D_h D_r

sr

P_s P_r D_s D_r

P_s D_s P_r D_r

P_h P_s D_h D_s P_r D_r

P_h P_s D_h P_r D_s D_r

P_h P_s P_r D_h D_s D_r

P_h P_r P_s D_h D_s D_r

P_s D_s P_h D_h P_r D_r

P_s D_s P_h P_r D_h D_r

Example: Full Induction Step

$P_h\ P_s\ D_h\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_s\ D_r$

$P_h\ P_s\ P_r\ D_h\ D_s\ D_r$

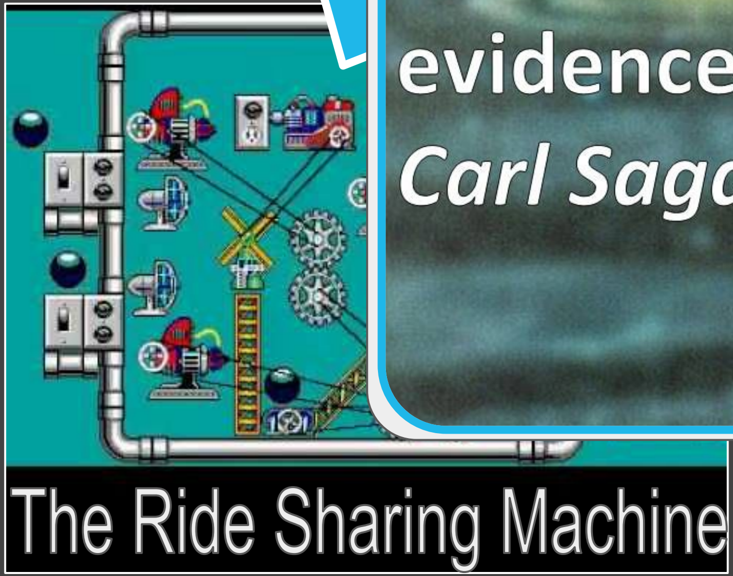
$P_h\ P_r\ P_s\ D_h\ D_s\ D_r$

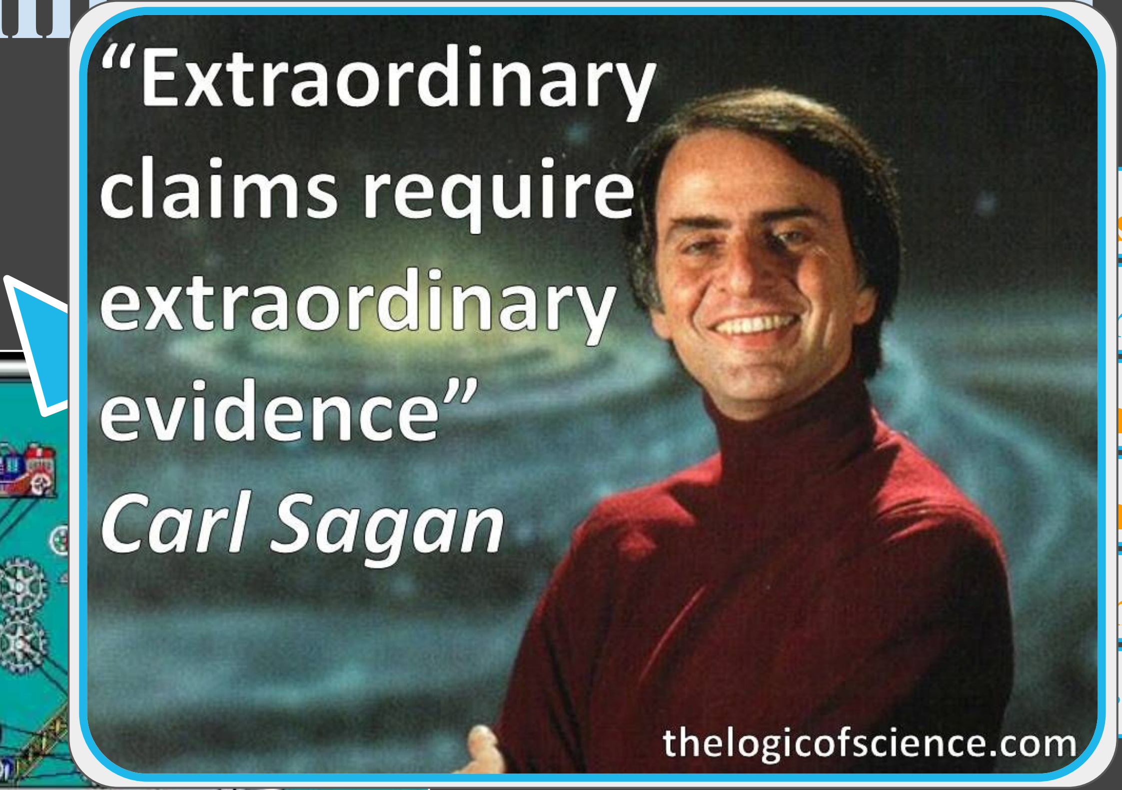
$P_s\ D_s\ P_h\ D_h\ P_r\ D_r$

$P_s\ D_s\ P_h\ P_r\ D_h\ D_r$

Example: Full Induction Step

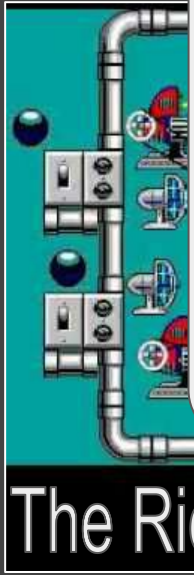
Example: Full Induction Step





- s P_r D_r
- r D_s D_r
- D_s D_r
- D_s D_r
- P_r D_r
- D_h D_r

Example: Full Induction Step



Complexity Analysis

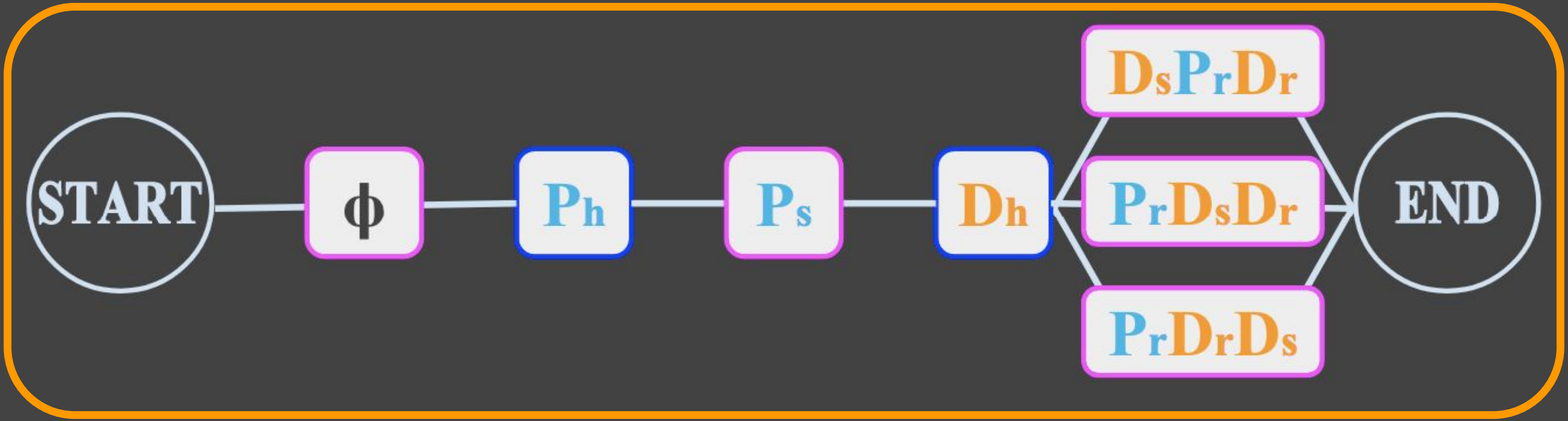
Complexity Analysis

n = number of people

k = number of permutations in the previous induction step

Complexity Analysis

1. Induction sub-step (Sequence Alignment): O(n)



Complexity Analysis

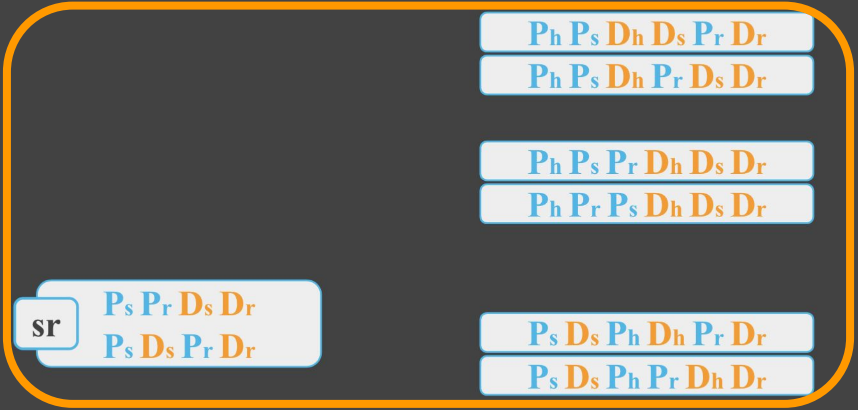
2. Construction: $O(k^2)$

(worst case, $k=|S_i|$ for all i)

Complexity Analysis

3. Filtering: $O(n \cdot k^3)$

(naive $\sim(n-2) \cdot k \cdot k^2$ comparissons)



Complexity Analysis

1. Induction sub-step: $O(n)$
2. Construction: $O(k^2)$
3. Filtering: $O(n \cdot k^3)$

Complexity Analysis

1. Induction sub-step: $O(n)$
2. Construction: $O(k^2)$
3. Filtering: $O(n \cdot k^3)$

=> Full Induction Step: $O(n^2k^3)$

Complexity Analysis

hs
$P_h\ P_s\ D_h\ D_s$
$P_s\ D_s\ P_h\ D_h$

hr
$P_h\ D_h\ P_r\ D_r$
$P_h\ P_r\ D_h\ D_r$

sr
$P_s\ P_r\ D_s\ D_r$
$P_s\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ D_s\ P_r\ D_r$
$P_h\ P_s\ D_h\ P_r\ D_s\ D_r$
$P_h\ P_s\ P_r\ D_h\ D_s\ D_r$
$P_h\ P_r\ P_s\ D_h\ D_s\ D_r$
$P_s\ D_s\ P_h\ D_h\ P_r\ D_r$
$P_s\ D_s\ P_h\ P_r\ D_h\ D_r$

=> Full Induction Step: $O(n^2k^3)$

Complexity Analysis

hs

$P_h\ P_s\ D_h\ D_s$

$P_s\ D_s\ P_h\ D_h$

hr

$P_h\ D_h\ P_r\ D_r$

$P_h\ P_r\ D_h\ D_r$

sr

$P_s\ P_r\ D_s\ D_r$

$P_s\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ D_s\ P_r\ D_r$

$P_h\ P_s\ D_h\ P_r\ D_s\ D_r$

$P_h\ P_s\ P_r\ D_h\ D_s\ D_r$

$P_h\ P_r\ P_s\ D_h\ D_s\ D_r$

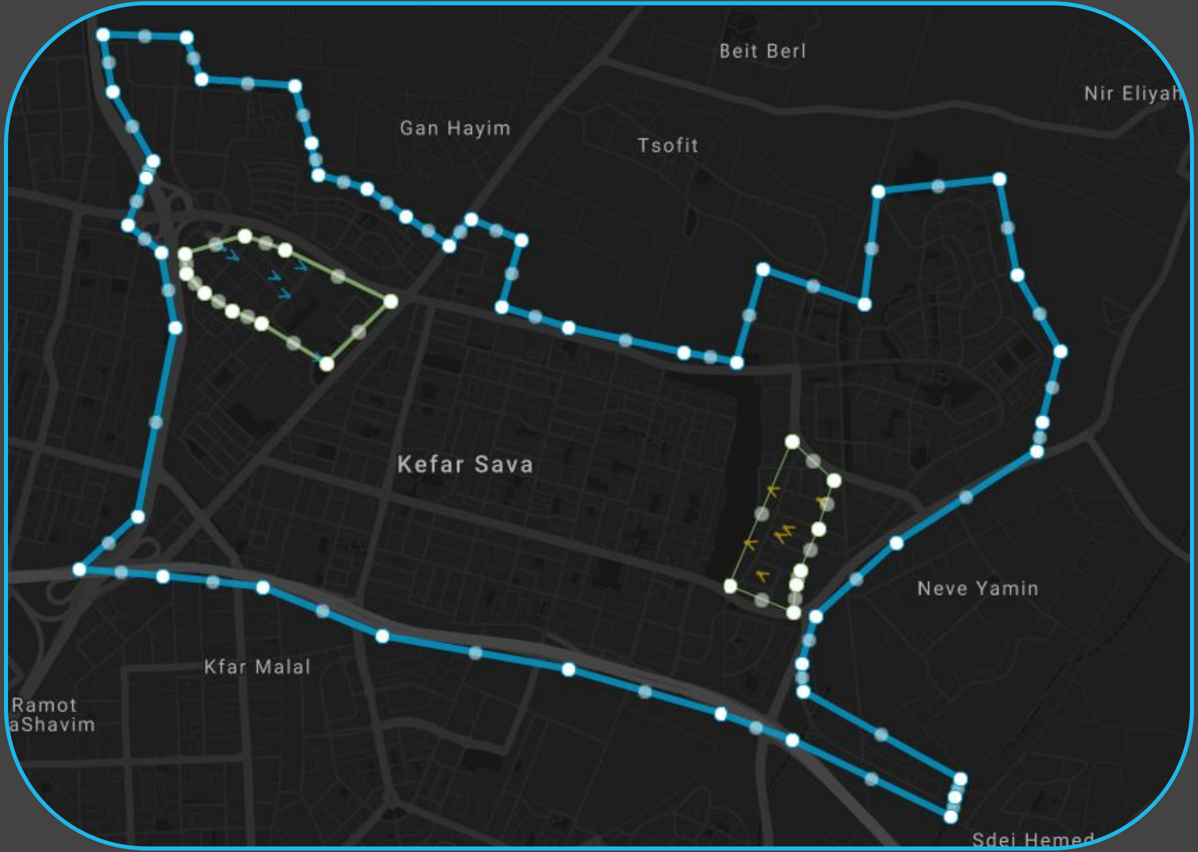
$P_s\ D_s\ P_h\ D_h\ P_r\ D_r$

$P_s\ D_s\ P_h\ P_r\ D_h\ D_r$

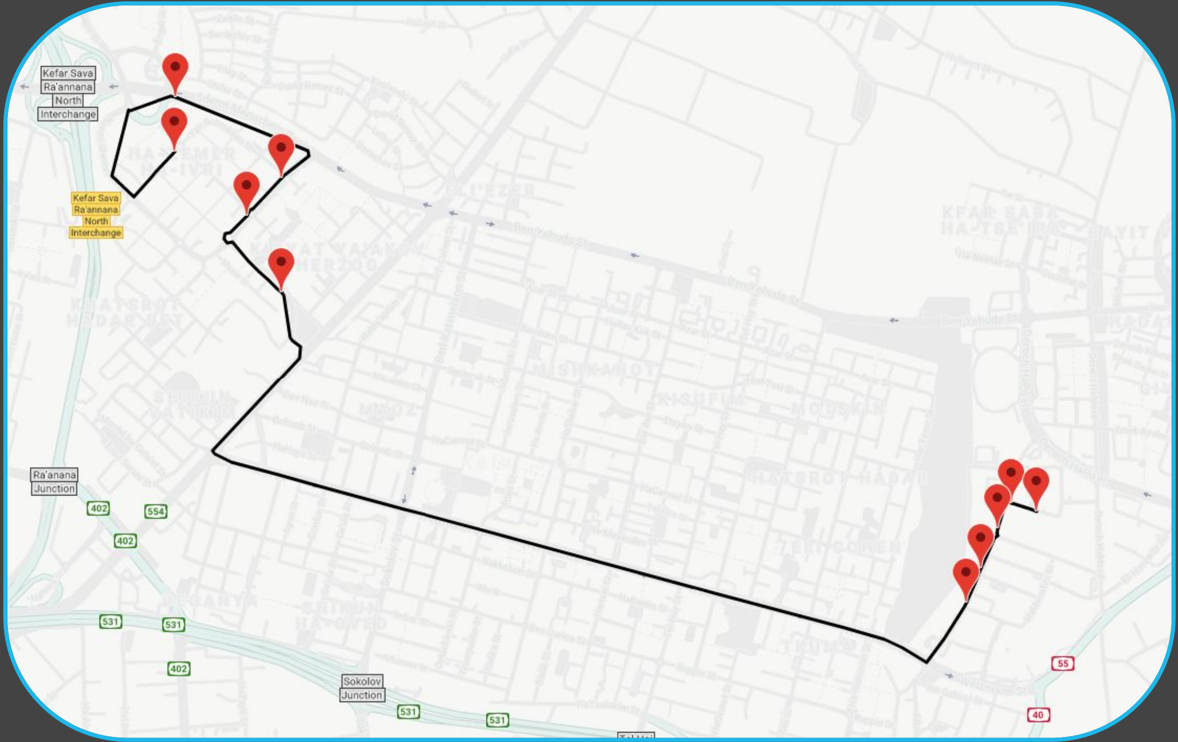
=> Full Induction Step: $O(n^2k^3)$

Maximal number of sequences: $O(k^2)$

Results in the Wild



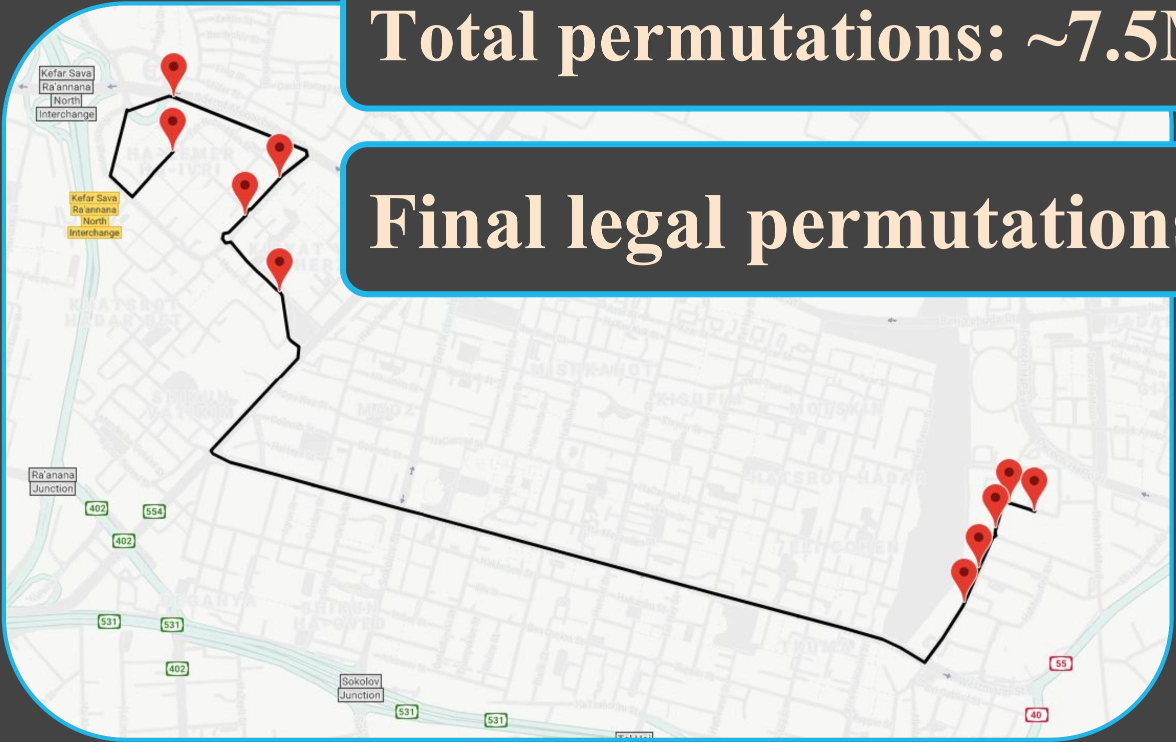
Results in the Wild



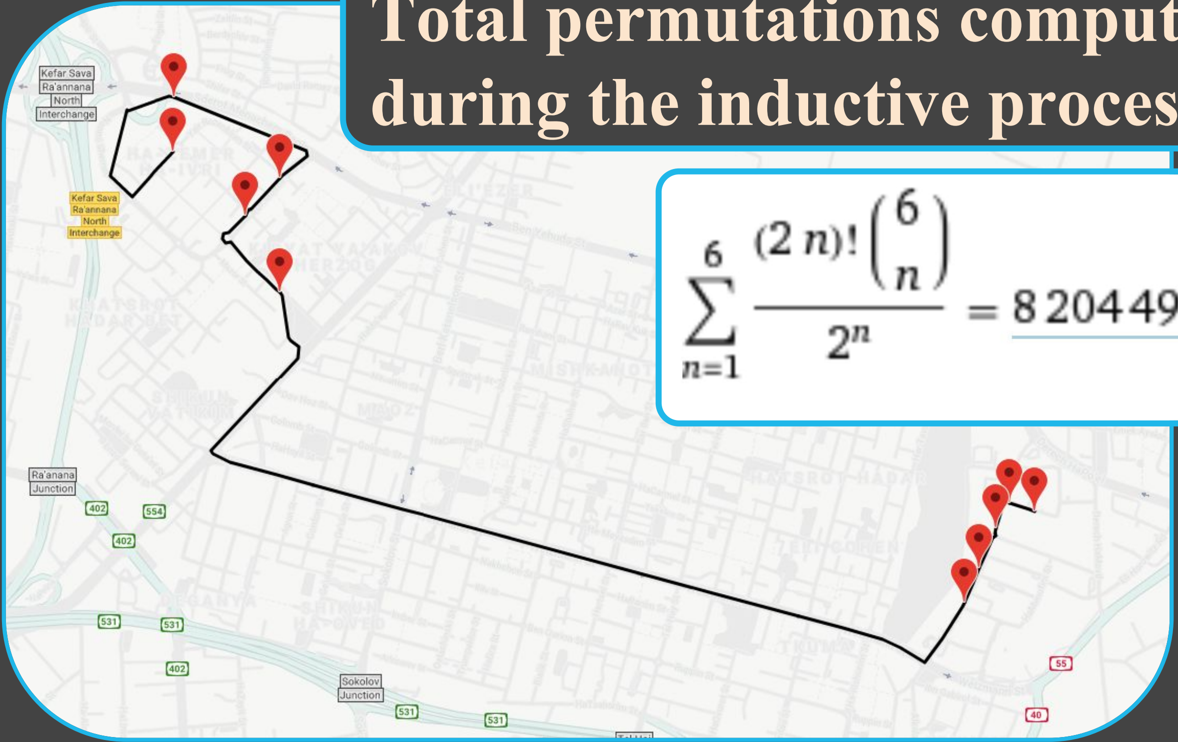
Results in the Wild

Total permutations: ~7.5M

Final legal permutations: 240



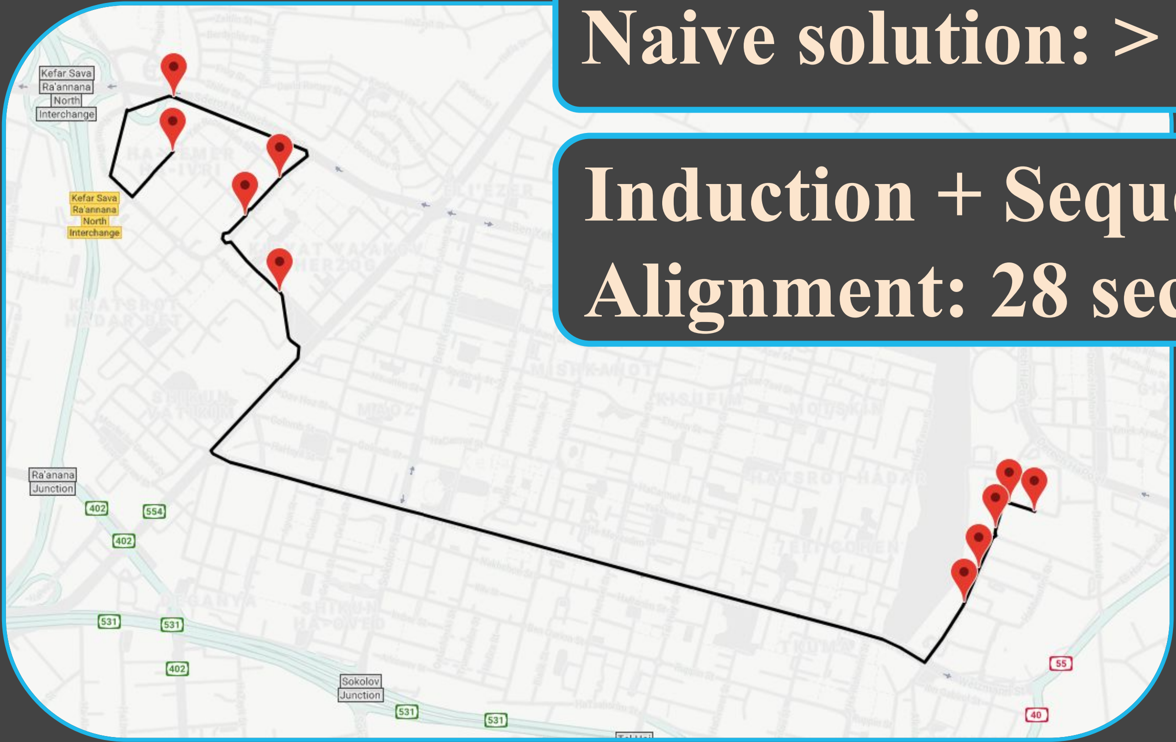
Results in the Wild



Total permutations computed during the inductive process: 3822

$$\sum_{n=1}^{6} \frac{(2\,n)!\binom{6}{n}}{2^n} = 8\,204\,496$$

Results in the Wild



Naive solution: > 1 hour

Induction + Sequence Alignment: 28 sec.

Take Home Message

Thank you :)

Questions?

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