

# BOZ780 Assignment 1

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## Question 1

## Question 2

### Question 2 (a) - General formulation

#### Sets

- $I$  set of facility types  $I \in (1, \dots, 4)$  as (Golf, Swimming, Gymnasium, Tennis)  
 $J$  set of sites  $J \in (1, \dots, 6)$

#### Parameters

- $d_{ij}$  user days for facility  $i$  on site  $j$   $i \in I, j \in J$   
 $a_j$  available land on site  $j$  in  $\text{ft}^2$   $j \in J$   
 $c_i$  construction cost for facility  $i$  in \$  $i \in I$   
 $r_i$  required land for facility  $i$  in  $\text{ft}^2$   $i \in I$

#### Variables

- $x_{ij} \begin{cases} 1 & \text{facility } i \text{ is built on site } j \\ 0 & \text{if else} \end{cases} \quad i \in I, j \in J$

#### Objectives

$$\min \sum_{i \in I} \sum_{j \in J} x_{ij} c_i \quad (\text{construction cost}) \quad (1)$$

$$\min \sum_{i \in I} \sum_{j \in J} x_{ij} a_j - x_{ij} r_i \quad (\text{unused land cost}) \quad (2)$$

$$\max \sum_{i \in I} \sum_{j \in J} x_{ij} d_{ij} \quad (\text{user days}) \quad (3)$$

#### Constraints

$$\sum_{i \in I} x_{ij} r_i \leq a_j \quad \forall j \in J \quad (\text{land capacity}) \quad (4)$$

$$x_{i1} + x_{i6} = 0 \quad \forall i \in (2, 3, 4) \quad (\text{no facilities on site 1 and 6}) \quad (5)$$

$$x_{1j} = 0 \quad \forall j \in (2, 3, 4, 5) \quad (\text{no golf on site 2 to 4}) \quad (6)$$

$$x_{11} + x_{16} \leq 1 \quad (\text{either site 1 or 6 can build golf}) \quad (7)$$

## Question 2 (b) - Basic goal programming

### Sets

- $K$  set of objectives  $K \in (1, 2, 3)$   
 $I$  set of facility types  $I \in (1, \dots, 4)$  as (Golf, Swimming, Gymnasium, Tennis)  
 $J$  set of sites  $J \in (1, \dots, 6)$

### Parameters

- $d_{ij}$  user days for facility  $i$  on site  $j$   $i \in I, j \in J$   
 $a_j$  available land on site  $j$  in  $\text{ft}^2$   $j \in J$   
 $c_i$  construction cost for facility  $i$  in \$  $i \in I$   
 $r_i$  required land for facility  $i$  in  $\text{ft}^2$   $i \in I$

### Variables

- $x_{ij} \begin{cases} 1 & \text{facility } i \text{ is built on site } j \\ 0 & \text{if else} \end{cases} \quad i \in I, j \in J$   
 $d_k$  deficiency variable for objective  $k$   $k \in K$

### Objectives

$$\min \sum_{k \in K} d_k \quad (\text{construction cost}) \quad (8)$$

### Constraints

$$\sum_{i \in I} \sum_{j \in J} x_{ij}(a_j - r_i) - d_1 \leq 40\,000 \text{ ft}^2 \quad (\text{P1: unused land cost}) \quad (9)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij}c_i - d_2 < \$1.2 \text{ million} \quad (\text{P2: construction cost}) \quad (10)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij}u_{ij} + d_3 \geq 200\,000 \text{ days} \quad (\text{P3: user days}) \quad (11)$$

$$\sum_{i \in I} x_{ij}r_i \leq a_j \quad \forall j \in J \quad (\text{land capacity}) \quad (12)$$

$$x_{i1} + x_{i6} = 0 \quad \forall i \in (2, 3, 4) \quad (\text{no facilities on site 1 and 6}) \quad (13)$$

$$x_{1j} = 0 \quad \forall j \in (2, 3, 4, 5) \quad (\text{no golf on site 2 to 4}) \quad (14)$$

$$x_{11} + x_{16} \leq 1 \quad (\text{either site 1 or 6 can build golf}) \quad (15)$$

### Solution

## Question 2 (c) - Pre-emptive goal programming

### Sets

- $K$  set of objectives  $K \in (1, 2, 3)$   
 $I$  set of facility types  $I \in (1, \dots, 4)$  as (Golf, Swimming, Gymnasium, Tennis)  
 $J$  set of sites  $J \in (1, \dots, 6)$

### Parameters

- $d_{ij}$  user days for facility  $i$  on site  $j$   $i \in I, j \in J$   
 $a_j$  available land on site  $j$  in  $\text{ft}^2$   $j \in J$   
 $c_i$  construction cost for facility  $i$  in \$  $i \in I$   
 $r_i$  required land for facility  $i$  in  $\text{ft}^2$   $i \in I$

### Variables

- $x_{ij} \begin{cases} 1 & \text{facility } i \text{ is built on site } j \\ 0 & \text{if else} \end{cases} \quad i \in I, j \in J$   
 $d_k$  deficiency variable for objective  $k$   $k \in K$

### Objectives

$$\min d_1 \quad (16)$$

$$\min d_2 \quad (\text{subject to } d_1) \quad (17)$$

$$\min d_3 \quad (\text{subject to } d_1 \text{ and } d_2) \quad (18)$$

### Constraints

$$d_1 = 0 \quad (\text{used in } d_2 \text{ and } d_3) \quad (19)$$

$$d_2 = 0 \quad (\text{used in } d_3) \quad (20)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij}(a_j - r_i) - d_1 \leq 40\,000 \text{ ft}^2 \quad (\text{P1: unused land cost}) \quad (21)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij}c_i - d_2 < \$1.2 \text{ million} \quad (\text{P2: construction cost}) \quad (22)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij}u_{ij} + d_3 \geq 200\,000 \text{ days} \quad (\text{P3: user days}) \quad (23)$$

$$\sum_{i \in I} x_{ij}r_i \leq a_j \quad \forall j \in J \quad (\text{land capacity}) \quad (24)$$

$$x_{i1} + x_{i6} = 0 \quad \forall i \in (2, 3, 4) \quad (\text{no facilities on site 1 and 6}) \quad (25)$$

$$x_{1j} = 0 \quad \forall j \in (2, 3, 4, 5) \quad (\text{no golf on site 2 to 4}) \quad (26)$$

$$x_{11} + x_{16} \leq 1 \quad (\text{either site 1 or 6 can build golf}) \quad (27)$$

### Solution

## Question 2 (d) - Weighted sum goal programming

### Sets

- $K$  set of objectives  $K \in (1, 2, 3)$   
 $I$  set of facility types  $I \in (1, \dots, 4)$  as (Golf, Swimming, Gymnasium, Tennis)  
 $J$  set of sites  $J \in (1, \dots, 6)$

### Parameters

- $d_{ij}$  user days for facility  $i$  on site  $j$   $i \in I, j \in J$   
 $a_j$  available land on site  $j$  in  $\text{ft}^2$   $j \in J$   
 $c_i$  construction cost for facility  $i$  in \$  $i \in I$   
 $r_i$  required land for facility  $i$  in  $\text{ft}^2$   $i \in I$

### Variables

- $x_{ij} \begin{cases} 1 & \text{facility } i \text{ is built on site } j \\ 0 & \text{if else} \end{cases} \quad i \in I, j \in J$   
 $d_k$  deficiency variable for objective  $k$   $k \in K$

### Objectives

$$\min \quad 1000d_1 + 10d_2 + d_3 \quad (28)$$

### Constraints

$$\sum_{i \in I} \sum_{j \in J} x_{ij}(a_j - r_i) - d_1 \leq 40\,000 \text{ ft}^2 \quad (\text{P1: unused land cost}) \quad (29)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij}c_i - d_2 < \$1.2 \text{ million} \quad (\text{P2: construction cost}) \quad (30)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij}u_{ij} + d_3 \geq 200\,000 \text{ days} \quad (\text{P3: user days}) \quad (31)$$

$$\sum_{i \in I} x_{ij}r_i \leq a_j \quad \forall j \in J \quad (\text{land capacity}) \quad (32)$$

$$x_{i1} + x_{i6} = 0 \quad \forall i \in (2, 3, 4) \quad (\text{no facilities on site 1 and 6}) \quad (33)$$

$$x_{1j} = 0 \quad \forall j \in (2, 3, 4, 5) \quad (\text{no golf on site 2 to 4}) \quad (34)$$

$$x_{11} + x_{16} \leq 1 \quad (\text{either site 1 or 6 can build golf}) \quad (35)$$

### Solution

## Question 2 (e) - Basic goal programming efficiency adjustment

### Sets

- $K$  set of objectives  $K \in (1, 2, 3)$   
 $I$  set of facility types  $I \in (1, \dots, 4)$  as (Golf, Swimming, Gymnasium, Tennis)  
 $J$  set of sites  $J \in (1, \dots, 6)$

### Parameters

- $d_{ij}$  user days for facility  $i$  on site  $j$   $i \in I, j \in J$   
 $a_j$  available land on site  $j$  in  $\text{ft}^2$   $j \in J$   
 $c_i$  construction cost for facility  $i$  in \$  $i \in I$   
 $r_i$  required land for facility  $i$  in  $\text{ft}^2$   $i \in I$

### Variables

- $x_{ij} \begin{cases} 1 & \text{facility } i \text{ is built on site } j \\ 0 & \text{if else} \end{cases} \quad i \in I, j \in J$   
 $d_k$  deficiency variable for objective  $k$   $k \in K$

### Objectives

A small number 0.001 has been selected as the adjustment factor with positive and negative signs applied for minimisation and maximisation respectively.

$$\begin{aligned}
 \min \quad & \sum_{k \in K} d_k + 0.001 \left( \sum_{i \in I} \sum_{j \in J} x_{ij} c_i \right) \\
 & + 0.001 \left( \sum_{i \in I} \sum_{j \in J} x_{ij} a_j - x_{ij} r_i \right) \\
 & - 0.001 \left( \sum_{i \in I} \sum_{j \in J} x_{ij} u_{ij} \right)
 \end{aligned} \tag{36}$$

### Constraints

$$\sum_{i \in I} \sum_{j \in J} x_{ij} (a_j - r_i) - d_1 \leq 40\,000 \text{ ft}^2 \quad (\text{P1: unused land cost}) \tag{37}$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij} c_i - d_2 < \$1.2 \text{ million} \quad (\text{P2: construction cost}) \tag{38}$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij} u_{ij} + d_3 \geq 200\,000 \text{ days} \quad (\text{P3: user days}) \tag{39}$$

$$\sum_{i \in I} x_{ij} r_i \leq a_j \quad \forall j \in J \quad (\text{land capacity}) \tag{40}$$

$$x_{i1} + x_{i6} = 0 \quad \forall i \in (2, 3, 4) \quad (\text{no facilities on site 1 and 6}) \tag{41}$$

$$x_{1j} = 0 \quad \forall j \in (2, 3, 4, 5) \quad (\text{no golf on site 2 to 4}) \tag{42}$$

$$x_{11} + x_{16} \leq 1 \quad (\text{either site 1 or 6 can build golf}) \tag{43}$$

### Solution