University of Pretoria
Department of Industrial and Systems Engineering

Operations Research BOZ 780

Lecturer: Dr. Wilna Bean

This is the BOZ 780 *Stochastic programming* semester assignment of 2020.

Please complete all the assignment questions and submit your answers electronically in a **single *.pdf file** at the ClickUP link **before midnight on 18 October 2020**.

No late or hand written submission will be accepted.

Complete 2 questions for 93 marks

Questions

1. Venture capital firm *JD* is trying to determine in which 10 projects it should invest. It knows how much money is available for investment each of the next N years, the NPV of each project, and the expected cash required by each project during each of the next N years (see Table 1). For example, if *JD* chooses to invest in project 1, the company must commit to the entire cash flow, that is \$6-million in year 1, \$3-million in year 2, and \$5-million in year 3. The values given in the table for the cash requirements are the *expected*

Table 1: Investment details for the *JD* problem

Investment	**Project**									
(\$ Million)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Year 1	6	9	12	15	18	21	24	27	30	35
Year 2	3	5	7	9	11	13	15	17	19	21
Year 3	5	7	9	12	12	14	16	11	20	24
NPV	20	30	40	50	60	70	80	90	100	130

values, but are in actual fact random uniformly distributed variables that can vary 10% above and below the given mean. For example, the expected cash required for project 1 in year 1, denoted by $\tilde{c_{11}}$, is \$6-million, but is actually a random variable from the uniform distribution, $\tilde{c_{11}} \sim U[5.4, 6.6]$.

12 (a) Formulate a stochastic program that will assist *JD* to determine in which projects they should invest in. For each year $t \in \boldsymbol{T} = \{1, 2, 3\}$, there is a limited amount of cash (in millions of dollars) available for investment, denoted by a_t.

9 (b) Use chance-constraint programming and the scenario approach to formulate the deterministic equivalent if it is given that *JD* must adhere to the limited cash availability 90% of the time. Show all your calculations/steps (including the chance-constraint, adjusted deterministic equivalent formluation and your sample size calculation).

9 (c) Given the uncertainty, reformulate the stochastic program of (a) into a deterministic equivalent if *JD* can make up a shortfall in the available cash they need for investment by using an (unlimited) line of credit. The credit, however, is available at an effective interest rate of 15% per annum. Show all the required steps (including the recourse model, its dual decomposition structure and the process you used to derive instances/realisations for random variables).

2. In treating a brain tumour with radiation, physicians want the maximum amount of radiation possible to bombard the tissue containing the tumours. The constraint is, however, that there is a maximum amount of radiation that normal tissue can handle without suffering tissue damage. Physicians must therefore decide how to aim the radiation so as to maximise the radiation that hits the tumour tissue subject to the constraint of not damaging the normal tissue. As a simple example of this situation, suppose six types of radiation beams (beams differ in where they are aimed and their intensity) can be aimed at a tumour. The region containing the tumour has been divided into six regions: three regions contain tumours and three contain normal tissue. The amount of radiation delivered to each region by each type of beam is shown in Table 2.

Table 2: Radiation data

	Normal			**Tumor**		
Beam	**1**	**2**	**3**	**1**	**2**	**3**
1	16	12	8	20	12	6
2	12	10	6	18	15	8
3	$\tilde{\zeta}_1$	$\tilde{\zeta}_2$	$\tilde{\zeta}_3$	13	10	17
4	$\tilde{\eta}_1$	$\tilde{\eta}_2$	$\tilde{\eta}_3$	6	18	16
5	9	4	11	13	5	14
6	8	7	7	10	10	10

The nature of beams 3 and 4, however, is slightly more erratic in efficiency, resulting in stochastic radiation delivery to normal tissue. The radiation amounts indicated in the attached file `RadiationData.txt` have been determined through 100 experimental observations.

15 (a) If each region of normal tissue can handle at most 40 units of radiation, formulate a stochastic model that can help physicians to decide which beams should be used to maximize the total amount of radiation received by the tumors. Show all your steps (including model formulation and the probability distributions of $\tilde{\zeta}_1$, $\tilde{\zeta}_2$, $\tilde{\zeta}_3$, $\tilde{\eta}_1$, $\tilde{\eta}_2$ and $\tilde{\eta}_3$).

9 (b) Solve the expected value model. Show all your steps (including adjusted model formulation, results and interpretation).

12 (c) Apply the scenario approach if it is given that the *at-most-40-units-of-radiation* constraint on normal tissue must hold 99% of the times. Show all your steps (including adjusted model formulation, results and interpretation).

9 (d) Apply the approximation approach if it is given that the *at-most-40-units-of-radiation* constraint on normal tissue must hold 99% of the times. Show all your steps (including adjusted model formulation, results and interpretation).

12 (e) Apply the fixed recourse approach taking into account that the over-radiation of normal tissue is penalized at twice the weight of radiation received by tumor regions. Show all your steps (including adjusted model formulation, results and interpretation).

6 (f) Compare the reliability of the chance-constrained scenario approach, chance-constrained approximation approach, and recourse programming approach to that of the expected-value model.