

BOZ780 Assignment 1

DS de Gouveia
15003079

October 10, 2020

Contents

1	Question 1	2
1.1	Question 1 (a): Graphical solution	2
1.2	Question 1 (b): Define points	3
1.3	Question 1 (c): Pre-emptive	4
1.3.1	Minimise z_1	4
1.3.2	Minimise z_2	5
1.4	Question 1 (d): Efficient frontier	6
2	Question 2	7
2.1	Question 2 (a) - General formulation	7
2.1.1	Sets	7
2.1.2	Parameters	7
2.1.3	Variables	7
2.1.4	Objectives	7
2.1.5	Constraints	7
2.2	Question 2 (b) - Basic goal programming	8
2.2.1	Sets	8
2.2.2	Parameters	8
2.2.3	Variables	8
2.2.4	Objectives	8
2.2.5	Constraints	8
2.2.6	Solution	9
2.3	Question 2 (c) - Pre-emptive goal programming	10
2.3.1	Sets	10
2.3.2	Parameters	10
2.3.3	Variables	10
2.3.4	Objectives	10
2.3.5	Constraints	10
2.3.6	Solution: P1	11
2.3.7	Solution: P2	11
2.3.8	Solution: P3	11
2.4	Question 2 (d) - Weighted sum goal programming	12
2.4.1	Sets	12
2.4.2	Parameters	12
2.4.3	Variables	12
2.4.4	Objectives	12

2.4.5	Constraints	12
2.4.6	Solution	12
2.5	Question 2 (e) - Basic goal programming efficiency adjustment	14
2.5.1	Sets	14
2.5.2	Parameters	14
2.5.3	Variables	14
2.5.4	Objectives	14
2.5.5	Constraints	14
2.5.6	Solution	15
3	Question 1 R Code	16
4	Question 2 R Code	20

Question 1

Dashed lines represent $z_1 = 5x_1 - x_2$, dotted lines represent $z_2 = x_1 + 4x_2$ and the feasible region, bounded by solid black lines is shaded gold. The co-ordinate grid in proceeding figures is represented as (x_2, x_1) , please take note of the axis.

Question 1 (a): Graphical solution

The solution to z_2 occurs at $(5,0)$ where the solution for z_1 occurs at $(0,4)$ both indicated in turquoise. These points do not coincide.

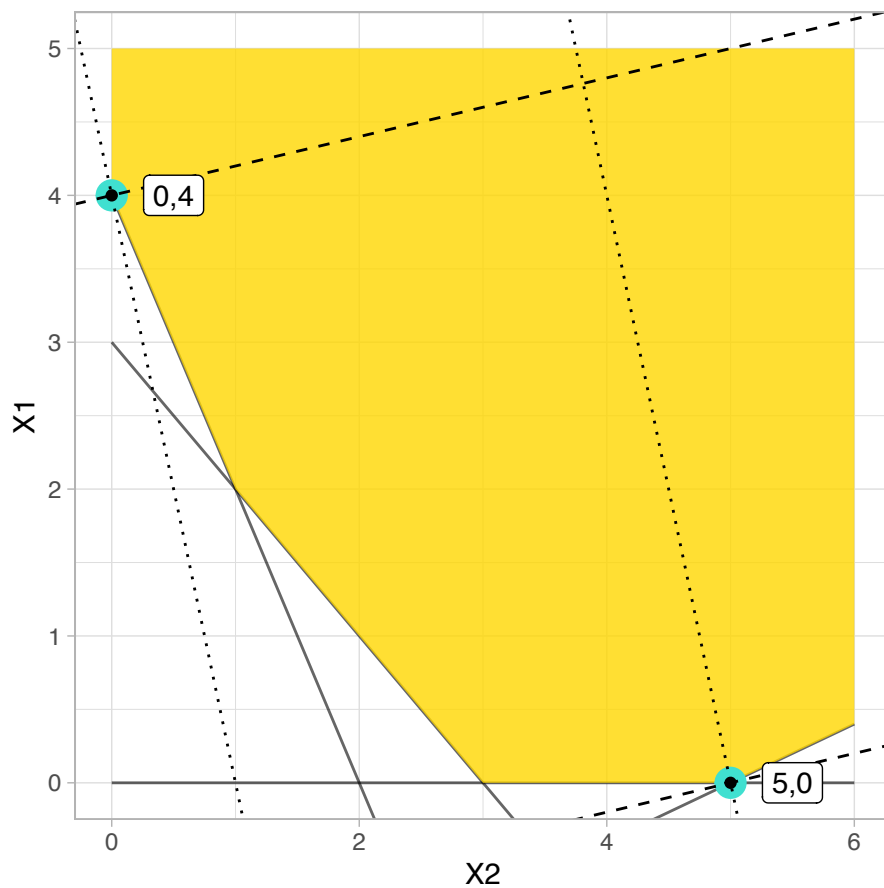


Figure 1: Optimal points of z_1 and z_2 are not located at the same position as the gradient of their respective objective functions are tangential to each other.

Question 1 (b): Define points

Points are defined as efficient if there is no area below the intersection of z_1 (dashed) and z_2 (dotted) that lies within the feasible region.

1. (5,0) Efficient
2. (2,1) Efficient
3. (1,1) Infeasible
4. (1,2) Efficient
5. (0,5) Dominated

Point (0,5) is dominated as point (0,4) improves on the solution without degrading either objective.

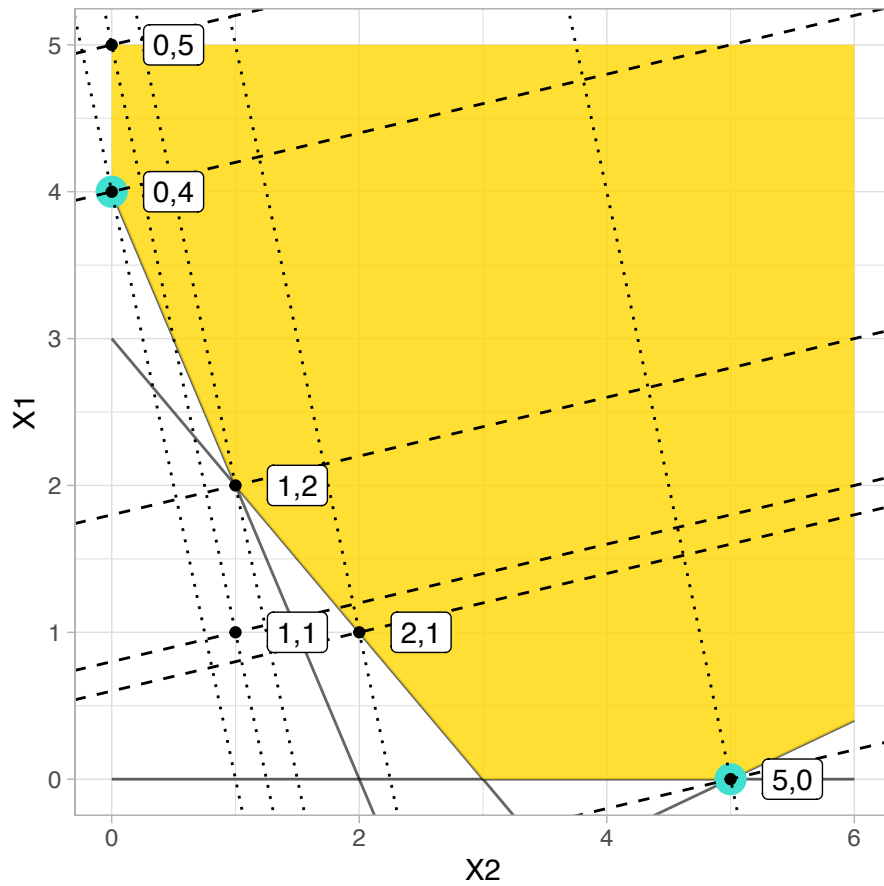


Figure 2: Efficient, dominated and other points

Question 1 (c): Pre-emptive

Minimise z_1

The solution found for z_1 is introduced as a constraint reducing the solution in which z_2 will attempt to find an efficient point in.

$$\min z_1 = 5x_1 - x_2 \quad (1)$$

Subject to:

$$-5x_1 + 2x_2 \leq 10 \quad (2)$$

$$x_1 + x_2 \geq 3 \quad (3)$$

$$x_1 + 2x_2 \geq 4 \quad (4)$$

$$x_1, x_2 \geq 0 \quad (5)$$

$$z_1 = -5 \quad (6)$$

$$x_1 = 0 \quad (7)$$

$$x_2 = 5 \quad (8)$$

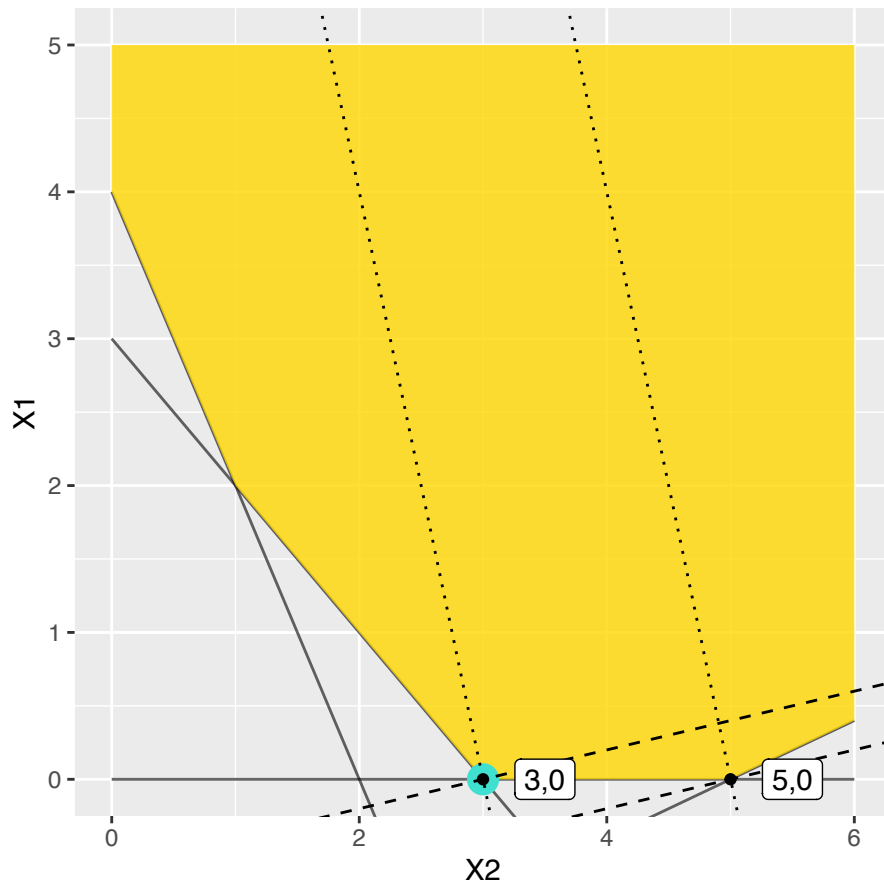


Figure 3: Pre-emptive feasible region with z_1 optimal point at $(5,0)$.

Minimise z_2

$$\min z_2 = x_1 + 4x_2 \quad (9)$$

Subject to:

$$5x_1 - x_2 \leq -5 \quad (10)$$

$$-5x_1 + 2x_2 \leq 10 \quad (11)$$

$$x_1 + x_2 \geq 3 \quad (12)$$

$$x_1 + 2x_2 \geq 4 \quad (13)$$

$$x_1, x_2 \geq 0 \quad (14)$$

$$z_2 = 12 \quad (15)$$

$$x_1 = 0 \quad (16)$$

$$x_2 = 3 \quad (17)$$

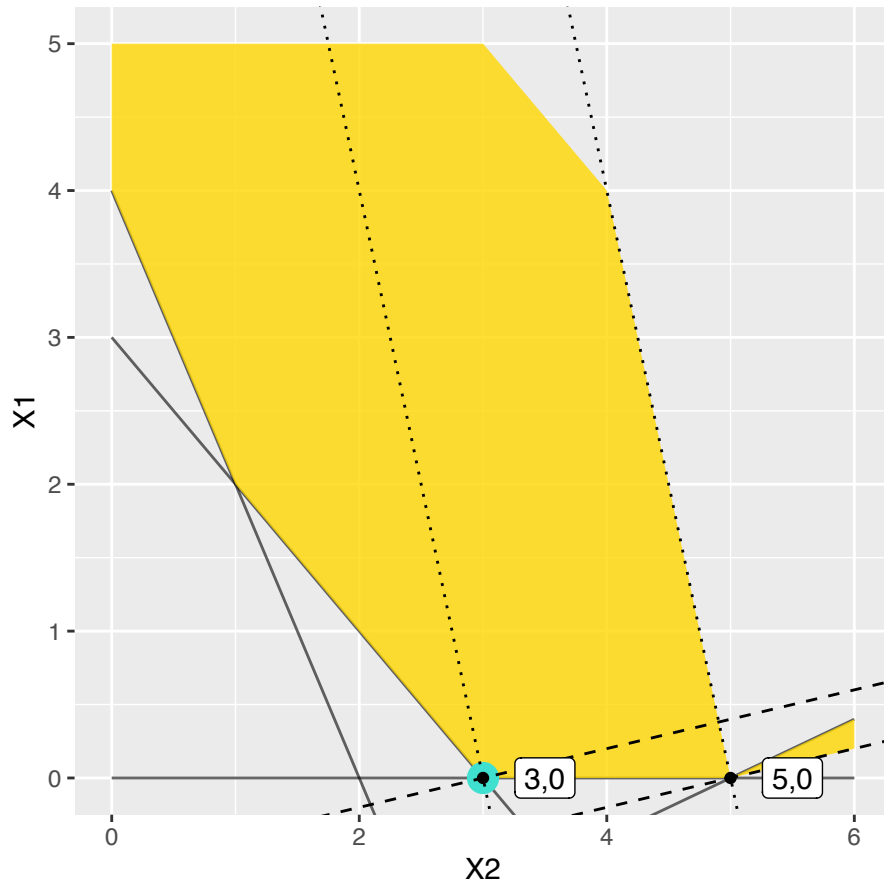


Figure 4: Pre-emptive feasible region including z_1 constraint with z_2 efficient point at $(3,0)$

Question 1 (d): Efficient frontier

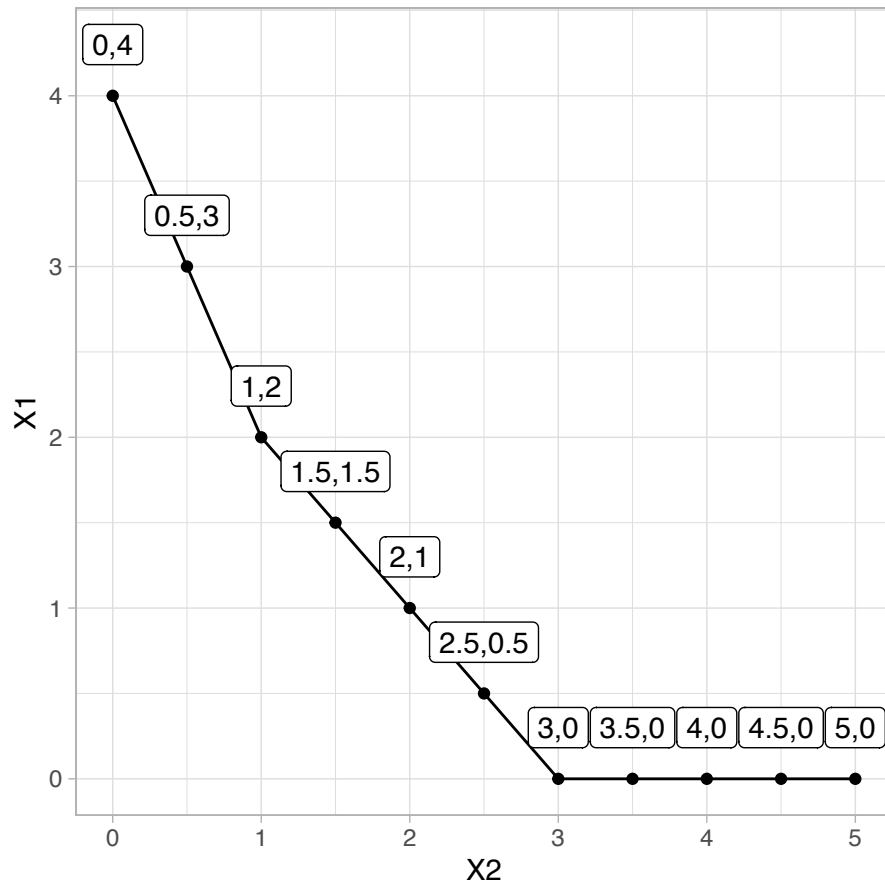


Figure 5: Efficient frontier for x_1 and x_2 in increments of 0.5

Question 2

The following assumptions apply to Question 2:

- The area required for a golf facility is stated as 62 0000 which exceeds the maximum area for site 1 and site 6. The lack of a thousand separator (620 000) and a large deviation from other sites indicates this may be a potential typo. A value of 62 000 has been used instead.
- Weights have been stated as 1000:10:1 which is assumed to reference P1:P2:P3 as priority one to three.

Question 2 (a) - General formulation

Sets

I set of facility types $I \in (1, \dots, 4)$ as (Golf, Swimming, Gymnasium, Tennis)
 J set of sites $J \in (1, \dots, 6)$

Parameters

d_{ij} user days for facility i on site j $i \in I, j \in J$
 a_j available land on site j in ft^2 $j \in J$
 c_i construction cost for facility i in \$ $i \in I$
 r_i required land for facility i in ft^2 $i \in I$

Variables

$x_{ij} \begin{cases} 1 & \text{facility } i \text{ is built on site } j \\ 0 & \text{if else} \end{cases} \quad i \in I, j \in J$

Objectives

$$\min \sum_{i \in I} \sum_{j \in J} x_{ij} c_i \quad (\text{construction cost}) \quad (1)$$

$$\min \sum_{j \in J} a_j - \sum_{i \in I} \sum_{j \in J} x_{ij} r_i \quad (\text{unused land cost}) \quad (2)$$

$$\max \sum_{i \in I} \sum_{j \in J} x_{ij} d_{ij} \quad (\text{user days}) \quad (3)$$

Constraints

$$\sum_{i \in I} x_{ij} r_i \leq a_j \quad \forall j \in J \quad (\text{land capacity}) \quad (4)$$

$$x_{i1} + x_{i6} = 0 \quad \forall i \in (2, 3, 4) \quad (\text{no facilities on site 1 and 6}) \quad (5)$$

$$x_{1j} = 0 \quad \forall j \in (2, 3, 4, 5) \quad (\text{no golf on site 2 to 4}) \quad (6)$$

$$x_{11} + x_{16} \leq 1 \quad (\text{either site 1 or 6 can build golf}) \quad (7)$$

Question 2 (b) - Basic goal programming

Sets

- K set of objectives $K \in (1, 2, 3)$
 I set of facility types $I \in (1, \dots, 4)$ as (Golf, Swimming, Gymnasium, Tennis)
 J set of sites $J \in (1, \dots, 6)$

Parameters

- d_{ij} user days for facility i on site j $i \in I, j \in J$
 a_j available land on site j in ft^2 $j \in J$
 c_i construction cost for facility i in \$ $i \in I$
 r_i required land for facility i in ft^2 $i \in I$

Variables

- $x_{ij} \begin{cases} 1 & \text{facility } i \text{ is built on site } j \\ 0 & \text{if else} \end{cases} \quad i \in I, j \in J$
 d_k deficiency variable for objective k $k \in K$

Objectives

$$\min \sum_{k \in K} d_k \quad (\text{deficiencies}) \quad (8)$$

Constraints

$$\begin{aligned} \sum_{j \in J} a_j - \sum_{i \in I} \sum_{j \in J} x_{ij} r_i - d_1 &\leq 40\,000 \text{ ft}^2 \\ \sum_{i \in I} \sum_{j \in J} x_{ij} r_i + d_1 &\geq \sum_{j \in J} a_j - 40\,000 \text{ ft}^2 \quad (\text{P1: unused land cost}) \end{aligned} \quad (9)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij} c_i - d_2 < \$1.2 \text{ million} \quad (\text{P2: construction cost}) \quad (10)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij} u_{ij} + d_3 \geq 200\,000 \text{ days} \quad (\text{P3: user days}) \quad (11)$$

$$\sum_{i \in I} x_{ij} r_i \leq a_j \quad \forall j \in J \quad (\text{land capacity}) \quad (12)$$

$$x_{i1} + x_{i6} = 0 \quad \forall i \in (2, 3, 4) \quad (\text{no facilities on site 1 and 6}) \quad (13)$$

$$x_{1j} = 0 \quad \forall j \in (2, 3, 4, 5) \quad (\text{no golf on site 2 to 4}) \quad (14)$$

$$x_{11} + x_{16} \leq 1 \quad (\text{either site 1 or 6 can build golf}) \quad (15)$$

Solution

Metric	Value	Target
Unused land	234 000 ft ²	40 000 ft ²
Used land	271 000 ft ²	-
Construction cost	\$ 980 000	\$ 1 200 000
User days	148 000 days	200 000 days

$$x_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \quad d_k = \begin{bmatrix} 194\,000 \\ 0 \\ 52\,000 \end{bmatrix} \quad \min(z) = 246\,000 \quad (16)$$

Question 2 (c) - Pre-emptive goal programming

Sets

- K set of objectives $K \in (1, 2, 3)$
 I set of facility types $I \in (1, \dots, 4)$ as (Golf, Swimming, Gymnasium, Tennis)
 J set of sites $J \in (1, \dots, 6)$

Parameters

- d_{ij} user days for facility i on site j $i \in I, j \in J$
 a_j available land on site j in ft^2 $j \in J$
 c_i construction cost for facility i in \$ $i \in I$
 r_i required land for facility i in ft^2 $i \in I$

Variables

- $x_{ij} \begin{cases} 1 & \text{facility } i \text{ is built on site } j \\ 0 & \text{if else} \end{cases} \quad i \in I, j \in J$
 d_k deficiency variable for objective k $k \in K$

Objectives

$$\min d_1 \quad (17)$$

$$\min d_2 \quad (\text{subject to } d_1) \quad (18)$$

$$\min d_3 \quad (\text{subject to } d_1 \text{ and } d_2) \quad (19)$$

Constraints

$$d_1 = 0 \quad (\text{used in } d_2 \text{ and } d_3) \quad (20)$$

$$d_2 = 0 \quad (\text{used in } d_3) \quad (21)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij} r_i + d_1 \geq \sum_{j \in J} a_j - 40\,000 \text{ ft}^2 \quad (\text{P1: unused land cost}) \quad (22)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij} c_i - d_2 < \$1.2 \text{ million} \quad (\text{P2: construction cost}) \quad (23)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij} u_{ij} + d_3 \geq 200\,000 \text{ days} \quad (\text{P3: user days}) \quad (24)$$

$$\sum_{i \in I} x_{ij} r_i \leq a_j \quad \forall j \in J \quad (\text{land capacity}) \quad (25)$$

$$x_{i1} + x_{i6} = 0 \quad \forall i \in (2, 3, 4) \quad (\text{no facilities on site 1 and 6}) \quad (26)$$

$$x_{1j} = 0 \quad \forall j \in (2, 3, 4, 5) \quad (\text{no golf on site 2 to 4}) \quad (27)$$

$$x_{11} + x_{16} \leq 1 \quad (\text{either site 1 or 6 can build golf}) \quad (28)$$

Solution: P1

Metric	Value	Target
Unused land	107 000 ft ²	40 000 ft ²
Used land	398 000 ft ²	-
Construction cost	\$ 4 015 000	\$ 1 200 000
User days	277 000 days	200 000 days

$$x_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \quad d_1 = [67\ 000] \quad \min(z) = 67\ 000 \quad (29)$$

Solution: P2

A new constraint was included whereby $d_1 \leq 67\ 000$.

Metric	Value	Target
Unused land	107 000 ft ²	40 000 ft ²
Used land	398 000 ft ²	-
Construction cost	\$ 4 015 000	\$ 1 200 000
User days	277 000 days	200 000 days

$$x_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \quad d_2 = [2\ 815\ 000] \quad \min(z) = 2\ 815\ 000 \quad (30)$$

Solution: P3

Two new constraint were included whereby $d_1 \leq 67\ 000$ and $d_2 \leq 2\ 815\ 000$.

Metric	Value	Target
Unused land	107 000 ft ²	40 000 ft ²
Used land	398 000 ft ²	-
Construction cost	\$ 4 015 000	\$ 1 200 000
User days	277 000 days	200 000 days

$$x_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \quad d_3 = [0] \quad \min(z) = 0 \quad (31)$$

The first priority ensured that area was maximised, inadvertently ensuring more user days (than targeted) could occur. However, the construction cost greatly increased and with the imposed d_1 constraint could not decrease after the second minimisation attempt.

Question 2 (d) - Weighted sum goal programming

Sets

- K set of objectives $K \in (1, 2, 3)$
 I set of facility types $I \in (1, \dots, 4)$ as (Golf, Swimming, Gymnasium, Tennis)
 J set of sites $J \in (1, \dots, 6)$

Parameters

- d_{ij} user days for facility i on site j $i \in I, j \in J$
 a_j available land on site j in ft^2 $j \in J$
 c_i construction cost for facility i in \$ $i \in I$
 r_i required land for facility i in ft^2 $i \in I$

Variables

- $x_{ij} \begin{cases} 1 & \text{facility } i \text{ is built on site } j \\ 0 & \text{if else} \end{cases} \quad i \in I, j \in J$
 d_k deficiency variable for objective k $k \in K$

Objectives

$$\min \quad 1000d_1 + 10d_2 + d_3 \quad (32)$$

Constraints

$$\sum_{i \in I} \sum_{j \in J} x_{ij} r_i + d_1 \geq \sum_{j \in J} a_j - 40\,000 \text{ ft}^2 \quad (\text{P1: unused land cost}) \quad (33)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij} c_i - d_2 < \$1.2 \text{ million} \quad (\text{P2: construction cost}) \quad (34)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij} d_{ij} + d_3 \geq 200\,000 \text{ days} \quad (\text{P3: user days}) \quad (35)$$

$$\sum_{i \in I} x_{ij} r_i \leq a_j \quad \forall j \in J \quad (\text{land capacity}) \quad (36)$$

$$x_{i1} + x_{i6} = 0 \quad \forall i \in (2, 3, 4) \quad (\text{no facilities on site 1 and 6}) \quad (37)$$

$$x_{1j} = 0 \quad \forall j \in (2, 3, 4, 5) \quad (\text{no golf on site 2 to 4}) \quad (38)$$

$$x_{11} + x_{16} \leq 1 \quad (\text{either site 1 or 6 can build golf}) \quad (39)$$

Solution

Metric	Value	Target
Unused land	107 000 ft^2	40 000 ft^2
Used land	398 000 ft^2	-
Construction cost	\$ 4 015 000	\$ 1 200 000
User days	277 000 days	200 000 days

$$x_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \quad d_k = \begin{bmatrix} 67\,000 \\ 2\,814\,999 \\ 0 \end{bmatrix} \quad \min(z) = 95\,150\,000 \quad (40)$$

Question 2 (e) - Basic goal programming efficiency adjustment

Sets

- K set of objectives $K \in (1, 2, 3)$
 I set of facility types $I \in (1, \dots, 4)$ as (Golf, Swimming, Gymnasium, Tennis)
 J set of sites $J \in (1, \dots, 6)$

Parameters

- d_{ij} user days for facility i on site j $i \in I, j \in J$
 a_j available land on site j in ft^2 $j \in J$
 c_i construction cost for facility i in \$ $i \in I$
 r_i required land for facility i in ft^2 $i \in I$

Variables

- $x_{ij} \begin{cases} 1 & \text{facility } i \text{ is built on site } j \\ 0 & \text{if else} \end{cases} \quad i \in I, j \in J$
 d_k deficiency variable for objective k $k \in K$

Objectives

A small number 0.26 has been selected as the adjustment factor with positive and negative signs applied for minimisation and maximisation respectively.

$$\begin{aligned}
 \min \quad & \sum_{k \in K} d_k + 0.001 \left(\sum_{i \in I} \sum_{j \in J} x_{ij} c_i \right) \\
 & - 0.26 \left(\sum_{i \in I} \sum_{j \in J} x_{ij} r_i \right) \\
 & - 0.26 \left(\sum_{i \in I} \sum_{j \in J} x_{ij} u_{ij} \right)
 \end{aligned} \tag{41}$$

The unused area objective function (2) has been converted from a minimisation of unused area to a maximisation of used area resulting in a -0.26 factor. The outcome of the objective is unchained and the constant a_j is removed whilst the land capacity constraint prevents the area of used land from surpassing what is available.

Constraints

$$\sum_{i \in I} \sum_{j \in J} x_{ij} r_i + d_1 \geq \sum_{j \in J} a_j - 40\,000 \text{ ft}^2 \quad (\text{P1: unused land cost}) \tag{42}$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij} c_i - d_2 < \$1.2 \text{ million} \quad (\text{P2: construction cost}) \tag{43}$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij} u_{ij} + d_3 \geq 200\,000 \text{ days} \quad (\text{P3: user days}) \tag{44}$$

$$\sum_{i \in I} x_{ij} r_i \leq a_j \quad \forall j \in J \quad (\text{land capacity}) \quad (45)$$

$$x_{i1} + x_{i6} = 0 \quad \forall i \in (2, 3, 4) \quad (\text{no facilities on site 1 and 6}) \quad (46)$$

$$x_{1j} = 0 \quad \forall j \in (2, 3, 4, 5) \quad (\text{no golf on site 2 to 4}) \quad (47)$$

$$x_{11} + x_{16} \leq 1 \quad (\text{either site 1 or 6 can build golf}) \quad (48)$$

Solution

Metric	Value	Target
Unused land	263 000 ft ²	40 000 ft ²
Used land	242 000 ft ²	-
Construction cost	\$ 680 000	\$ 1 200 000
User days	116 000 days	200 000 days

$$x_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \quad d_k = \begin{bmatrix} 223\,000 \\ 0 \\ 84\,000 \end{bmatrix} \quad \min(z) = 390\,720 \quad (49)$$

Including the efficiency adjustment results in the cheaper facility being built over others. The co-efficient values attributed to cost far exceed that of area and user days giving it a greater relative weight.

Question 1 R Code

```
library(tidyverse)
library(gridExtra)
library(lpSolve)
library(extrafont)

### ===== Question 1
w <- 12
h <- 12

lp(direction = "min",
  objective.in = c(1,4),
  const.mat = matrix(c(-5,1,1,0,1,
                        2,1,2,1,0),ncol = 2,byrow = FALSE),
  const.dir = c("<=", ">=", ">=", ">=", ">="),
  const.rhs = c(10,3,4,0,0))$solution

lengthX <- 6
lengthY <- 5

x <- 0:lengthX
c1 <- (x)*2/5-2
c2 <- -x +3
c3 <- -2*x+4
c4 <- rep(0,lengthX+1)

df.points <- data.frame(x=c(0,1,1,2,5,0),
                        y=c(5,2,1,1,0,4))
df.pointsOptimal <- data.frame(x=c(0,5),
                               y=c(4,0))

point.coords = paste(df.points$x,df.points$y,sep=",")
z1.intercept <- df.points$y -1/5*df.points$x
z2.intercept <- df.points$y +4 *df.points$x

# ==== Optimal points
pointOptimal.coords = paste(df.pointsOptimal$x,df.pointsOptimal$y,sep=",")
data.frame(x, c1,c2,c3,c4) %>%
  ggplot(aes(x = x))+
  geom_line(aes(y=c1), alpha = 0.6)+
  geom_line(aes(y=c2), alpha = 0.6)+
  geom_line(aes(y=c3), alpha = 0.6)+
  geom_line(aes(y=c4), alpha = 0.6)+
  geom_ribbon(aes(ymin = pmax(c1,c2,c3,c4),
                  ymax = lengthY),
            fill = 'gold',
            alpha = 0.8)+
  geom_point(data = df.pointsOptimal, aes(x=x, y=y), color = "turquoise", size = 5)+
```

```

geom_point(data = df.pointsOptimal, aes(x=x, y=y))+
coord_cartesian(xlim = c(0,lengthX), ylim = c(0,lengthY))+
labs(x="X2", y="X1")+
geom_abline(slope = 1/5, intercept = c(4,-1), linetype = "dashed", color = "black")+
geom_abline(slope = -4, intercept = c(4,20), linetype = "dotted", color = "black")+
geom_label(data = df.pointsOptimal,aes(x+.5,y,label=pointOptimal.coords))+
theme_light()

ggsave("Optimal_points_Z1_Z2.pdf",dpi = "retina", width = w, height = h, units = "cm")

# === Which points are eff

data.frame(x, c1,c2,c3,c4) %>%
  ggplot(aes(x = x))+
  geom_line(aes(y=c1), alpha = 0.6)+
  geom_line(aes(y=c2), alpha = 0.6)+
  geom_line(aes(y=c3), alpha = 0.6)+
  geom_line(aes(y=c4), alpha = 0.6)+
  geom_ribbon(aes(ymin = pmax(c1,c2,c3,c4),
                      ymax = lengthY),
            fill = 'gold',
            alpha = 0.8)+
  geom_point(data = df.pointsOptimal, aes(x=x, y=y), color = "turquoise", size = 5)+
  geom_point(data = df.points, aes(x=x, y=y))+
  coord_cartesian(xlim = c(0,lengthX), ylim = c(0,lengthY))+
  labs(x="X2", y="X1")+
  geom_abline(slope = 1/5, intercept = z1.intercept, linetype = "dashed", color = "black")+
  geom_abline(slope = -4, intercept = z2.intercept, linetype = "dotted", color = "black")+
  geom_label(data = df.points,aes(x+.5,y,label=point.coords))+
  theme_light()

ggsave("Efficient_dominated_other_points.pdf",dpi = "retina", width = w, height = h, unit

# Question 1 c - pre-emptive

z1 <- lp(direction = "min",
  objective.in = c(5,-1),
  const.mat = matrix(c(-5,1,1,0,1,
                      2,1,2,1,0),ncol = 2,byrow = FALSE),
  const.dir = c("<=", ">=", ">=", ">=", ">="),
  const.rhs = c(10,3,4,0,0))

z2 <- lp(direction = "min",
  objective.in = c(1,4),
  const.mat = matrix(c(-5,1,1,0,1,1,0,
                      2,1,2,1,0,0,1),ncol = 2,byrow = FALSE),
  const.dir = c("<=", ">=", ">=", ">=", ">=", "<=", "<="),
  const.rhs = c(10,3,4,0,0,z1$solution[1],z1$solution[2]))

z1

```

```

z2
z1$solution
z2$solution

df.pointsOptimal <- data.frame(y=c(z1$solution[1],z2$solution[1]),
                               x=c(z1$solution[2],z2$solution[2]))
point.coords = paste(df.pointsOptimal$x,df.pointsOptimal$y,sep=",")
z1.intercept <- df.pointsOptimal$y -1/5*df.pointsOptimal$x
z2.intercept <- df.pointsOptimal$y +4 *df.pointsOptimal$x

data.frame(x,c1,c2,c3,c4) %>%
  ggplot(aes(x = x))+
  geom_line(aes(y=c1), alpha = 0.6)+
  geom_line(aes(y=c2), alpha = 0.6)+
  geom_line(aes(y=c3), alpha = 0.6)+
  geom_line(aes(y=c4), alpha = 0.6)+
  geom_ribbon(aes(ymin = pmax(c1,c2,c3,c4),
                    ymax = lengthY),
            fill = 'gold',
            alpha = 0.8)+
  geom_point(data = df.pointsOptimal[2,], aes(x=x, y=y), color = "turquoise", size = 5)+
  geom_point(data = df.pointsOptimal, aes(x=x, y=y))+
  coord_cartesian(xlim = c(0,lengthX), ylim = c(0,lengthY))+
  labs(x="X2", y="X1")+
  geom_abline(slope = 1/5, intercept = z1.intercept, linetype = "dashed", color = "black")
  geom_abline(slope = -4, intercept = z2.intercept, linetype = "dotted", color = "black")
  geom_label(data = df.pointsOptimal,aes(x+.5,y,label=point.coords))

ggsave("Pre_emptive.pdf",dpi = "retina", width = w, height = h, units = "cm")

#Pre-emptive with constraint from past solution
c5 <- 1/5*(x)-1
upper <- rep(lengthY, lengthX+1)

data.frame(x,c1,c2,c3,c4,c5) %>%
  ggplot(aes(x = x))+
  geom_line(aes(y=c1), alpha = 0.6)+
  geom_line(aes(y=c2), alpha = 0.6)+
  geom_line(aes(y=c3), alpha = 0.6)+
  geom_line(aes(y=c4), alpha = 0.6)+
  geom_ribbon(aes(ymin = pmax(c1,c2,c3,c4),
                    ymax = c(5,5,5,5,4,0,0.2)),
            fill = 'gold',
            alpha = 0.8)+
  geom_point(data = df.pointsOptimal[2,], aes(x=x, y=y), color = "turquoise", size = 5)+

```

```

geom_point(data = df.pointsOptimal, aes(x=x, y=y))+
coord_cartesian(xlim = c(0,lengthX), ylim = c(0,lengthY))+
labs(x="X2", y="X1")+
geom_abline(slope = 1/5, intercept = z1.intercept, linetype = "dashed", color = "black")
geom_abline(slope = -4, intercept = z2.intercept, linetype = "dotted", color = "black")
geom_label(data = df.pointsOptimal,aes(x+.5,y,label=point.coords))

ggsave("Pre_emptive2.pdf",dpi = "retina", width = w, height = h, units = "cm")

# Eff fronteir

z2.seq <- seq(0,5,0.5)
df.eff <- data.frame(Obj=z2.seq, x=z2.seq, y=z2.seq)
n = 1
for (i in z2.seq){

  ef.front <- lp(direction = "min",
    objective.in = c(5,-1),
    const.mat = matrix(c(-5,1,1,0,1,0,
                        2,1,2,1,0,1),ncol = 2,byrow = FALSE),
    const.dir = c("<=", ">=", ">=", ">=", ">=", "<="),
    const.rhs = c(10,3,4,0,0,i))
  df.eff$Obj[n] <- ef.front$objval
  df.eff$x[n] <- ef.front$solution[2]
  df.eff$y[n] <- ef.front$solution[1]
  n <- n+1
}

df.eff %>%
  ggplot(aes(x=x, y=y))+
  geom_path()+
  geom_point()+
  labs(x="X2", y="X1")+
  geom_label(aes(x,y+.3,label=paste(x,y,sep=",")))+
  theme_light()

ggsave("Efficient_frontier.pdf",dpi = "retina", width = w, height = h, units = "cm")

```

Question 2 R Code

```
library(tidyverse)
library(lpSolve)
matmaker <- function(index,length){
  vec <- rep(0,length)
  vec[index] <- 1
  return(vec)
}

#===== Q2
#===== Parameters
i <- 4
j <- 6
k <- 3

uij <- 1000* c(31, 0, 0, 0, 0,27,
               0,25,21,32,32, 0,
               0,37,29,38,28, 0,
               0,20,23,22,20, 0)
aj <- 1000* rep(c(80,70,80,90,120,65),i)
ci <- rep(c(340000, 300000, 840000, 85000),j) %>% matrix(nrow = i, byrow = F) %>% t() %>%
ri <- rep(c(62000 ,29000 ,38000 ,45000),j) %>% matrix(nrow = i, byrow = F) %>% t() %>% a
dk <- c(0,0,0)

dk.k <- c(rep(0,j*i),c(1,1,1))
uij.k <- c(dij, dk)
aj.k <- c(aj, dk)
ci.k <- c(ci, dk)
ri.k <- c(ri, dk)

len <- length(ri.k)

fn.cost <- function(lpsolve.out){
  cost.landused <- sum(lpsolve.out[1:24]*ri)
  cost.landunused <- -sum(lpsolve.out[1:24]*ri) + sum(1000*c(80,70,80,90,120,65))
  cost.construction <- sum(lpsolve.out[1:24]*ci)
  cost.userdays <- sum(lpsolve.out[1:24]*uij)
  return(paste(paste("Used land",cost.landused,sep = ": "),
               paste("Unused land",cost.landunused,sep = ": "),
               paste("Construction cost",cost.construction,sep = ": "),
               paste("User days",cost.userdays,sep = ": "),sep = "
"))
}

#===== Constraints
cons1.lhs <- c(ri.k*matmaker(c(1,7,13,19),len),
              ri.k*matmaker(c(1+1,7+1,13+1,19+1),len),
              ri.k*matmaker(c(1+2,7+2,13+2,19+2),len),
              ri.k*matmaker(c(1+3,7+3,13+3,19+3),len),
```

```

ri.k*matmaker(c(1+4,7+4,13+4,19+4),len),
ri.k*matmaker(c(1+5,7+5,13+5,19+5),len)) %>% matrix(nrow = 6, byrow = T)
cons1.rhs <- 1000*c(80,70,80,90,120,65)

cons2.lhs <- (rep(1,len)*matmaker(c(7,13,19,12,18,24,2,3,4,5),len))#[1:24] %>% matrix(nro
cons2.rhs <- 0

cons3.lhs <- (rep(1,len)*matmaker(c(1,6),len))#[1:24] %>% matrix(nrow = 4, byrow = T)
cons3.rhs <- 1

base.cons.lhs <- rbind(cons1.lhs,cons2.lhs,cons3.lhs)
base.cons.rhs <- c(cons1.rhs, cons2.rhs, cons3.rhs)
base.cons.dir <- c(rep("<=",j), "=", ">=")
#===== Q2B
q2b.obj <- rep(1,len)*matmaker(c(27,26,25),len)

cons1.lhs <- (ri.k+dk.k*matmaker(25,len))#[1:24] %>% matrix(nrow = 4, byrow = T)
cons1.rhs <- -40000 + (1000*sum(80,70,80,90,120,65))
cons2.lhs <- ci.k-dk.k*matmaker(26,len)
cons2.rhs <- 1200000
cons3.lhs <- uij.k+dk.k*matmaker(27,len)
cons3.rhs <- 200000

q2b.cons.lhs <- rbind(base.cons.lhs, cons1.lhs, cons2.lhs, cons3.lhs)
q2b.cons.rhs <- c(base.cons.rhs, cons1.rhs, cons2.rhs, cons3.rhs)
q2b.cons.dir <- c(base.cons.dir, ">=", "<=", ">=")

q2b.lp <- lp(direction = "min",
  objective.in = q2b.obj,
  const.mat = q2b.cons.lhs,
  const.rhs = q2b.cons.rhs,
  const.dir = q2b.cons.dir,
  binary.vec = 1:24)
q2b.lp
q2b.lp$solution[1:24] %>% matrix(nrow = i, byrow = T)
q2b.lp$solution[25:27]
q2b.lp$solution %>% fn.cost()
#===== Q2C
#===== P1
q2c.obj.p1 <- rep(1,len)*matmaker(c(25),len)

q2c.lp <- lp(direction = "min",
  objective.in = q2c.obj.p1,
  const.mat = q2b.cons.lhs,
  const.rhs = q2b.cons.rhs,
  const.dir = q2b.cons.dir,
  binary.vec = 1:24)
q2c.lp
q2c.lp$solution[1:24] %>% matrix(nrow = i, byrow = T)

```

```

q2c.lp$solution[25:27] %>% as.integer()
q2c.lp$solution %>% fn.cost
#===== P2
q2c.obj.p2 <- rep(1,len)*matmaker(c(26),len)

cons1.lhs <- rep(1,len)*matmaker(25,len)
cons1.rhs <- q2c.lp$solution[25]
cons1.dir <- "<="

q2c.cons.lhs.p2 <- rbind(q2b.cons.lhs, cons1.lhs)
q2c.cons.rhs.p2 <- c(q2b.cons.rhs,cons1.rhs)
q2c.cons.dir.p2 <- c(q2b.cons.dir,cons1.dir)

q2c.lp.p2 <- lp(direction = "min",
               objective.in = q2c.obj.p2,
               const.mat = q2c.cons.lhs.p2,
               const.rhs = q2c.cons.rhs.p2,
               const.dir = q2c.cons.dir.p2,
               binary.vec = 1:24)
q2c.lp.p2
q2c.lp.p2$solution[1:24] %>% matrix(nrow = i, byrow = T)
q2c.lp.p2$solution[25:27] %>% as.integer()
q2c.lp.p2$solution %>% fn.cost()
#===== P3
q2c.obj.p3 <- rep(1,len)*matmaker(c(27),len)

cons1.lhs <- rep(1,len)*matmaker(26,len)
cons1.rhs <- q2c.lp.p2$solution[26]
cons1.dir <- "<="

q2c.cons.lhs.p3 <- rbind(q2c.cons.lhs.p2, cons1.lhs)
q2c.cons.rhs.p3 <- c(q2c.cons.rhs.p2,cons1.rhs)
q2c.cons.dir.p3 <- c(q2c.cons.dir.p2,cons1.dir)

q2c.lp.p3 <- lp(direction = "min",
               objective.in = q2c.obj.p3,
               const.mat = q2c.cons.lhs.p3,
               const.rhs = q2c.cons.rhs.p3,
               const.dir = q2c.cons.dir.p3,
               binary.vec = 1:24)
q2c.lp.p3
q2c.lp.p3$solution[1:24] %>% matrix(nrow = i, byrow = T)
q2c.lp.p3$solution[25:27] %>% as.integer()
q2c.lp.p3$solution %>% fn.cost()
#===== Q2D
q2d.obj <- rep(1,len)*matmaker(c(27,26,25),len)
q2d.obj[25] <- 1000
q2d.obj[26] <- 10
q2d.obj[27] <- 1

```

```

q2d.lp <- lp(direction = "min",
             objective.in = q2d.obj,
             const.mat = q2b.cons.lhs,
             const.rhs = q2b.cons.rhs,
             const.dir = q2b.cons.dir,
             binary.vec = 1:24)

q2d.lp
q2d.lp$solution[1:24] %>% matrix(nrow = i, byrow = T)
q2d.lp$solution[25:27]
q2d.lp$solution %>% fn.cost()

#===== Q2E
q2e.obj <- 0.26*(ci.k - uij.k - ri.k)      #ri is treated as maximization
q2e.obj[25:27] <- c(1,1,1)
q2e.lp <- lp(direction = "min",
             objective.in = q2e.obj,
             const.mat = q2b.cons.lhs,
             const.rhs = q2b.cons.rhs,
             const.dir = q2b.cons.dir,
             binary.vec = 1:24)

q2e.lp
q2e.lp$solution[1:24] %>% matrix(nrow = i, byrow = T)
q2e.lp$solution[25:27]
q2e.lp$solution %>% fn.cost()

# Results
q2b.lp$solution %>% fn.cost()
q2b.lp
q2b.lp$solution[1:24] %>% matrix(nrow = i, byrow = T)
q2b.lp$solution[25:27] %>% as.integer()

q2c.lp$solution %>% fn.cost()
q2c.lp
q2c.lp$solution[1:24] %>% matrix(nrow = i, byrow = T)
q2c.lp$solution[25:27] %>% as.integer()

q2c.lp.p2$solution %>% fn.cost()
q2c.lp.p2
q2c.lp.p2$solution[1:24] %>% matrix(nrow = i, byrow = T)
q2c.lp.p2$solution[25:27] %>% as.integer()

q2c.lp.p3$solution %>% fn.cost()
q2c.lp.p3
q2c.lp.p3$solution[1:24] %>% matrix(nrow = i, byrow = T)
q2c.lp.p3$solution[25:27] %>% as.integer()

```



```
q2d.lp$solution %>% fn.cost()
q2d.lp
q2d.lp$solution[1:24] %>% matrix(nrow = i, byrow = T)
q2d.lp$solution[25:27] %>% as.integer()

q2e.lp$solution %>% fn.cost()
q2e.lp
q2e.lp$solution[1:24] %>% matrix(nrow = i, byrow = T)
q2e.lp$solution[25:27] %>% as.integer()
```