BOZ780 Assignment 1

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Question 2 (a) - General formulation

Sets

I set of facility types $I \in (1, \ldots, 4)$ as (Golf, Swimming, Gymnasium, Tennis)
J set of sites $J \in (1, \ldots, 6)$

Parameters

d_{ij}	user days for facility i on site j	$i \in I, j \in J$
a_j	available land on site j in ft^2	$j \in J$
c_i	construction cost for facility i in \$	$i \in I$
r_i	required land for facility i in ft^2	$i \in I$

Variables

$$x_{ij} \quad \begin{cases} 1 & \text{facility } i \text{ is built on site } j \\ 0 & \text{if else} \end{cases} \qquad i \in I, j \in J$$

Objectives

$$\min \quad \sum_{i \in I} \sum_{j \in J} x_{ij} c_i \qquad \text{(construction cost)} \tag{1}$$

$$\min \quad \sum_{i \in I} \sum_{j \in J} x_{ij} a_j - x_{ij} r_i \qquad \text{(unused land cost)} \tag{2}$$

$$\max \quad \sum_{i \in I} \sum_{j \in J} x_{ij} u_{ij} \qquad \text{(user days)} \tag{3}$$

Constraints

$$\sum_{i \in I} x_{ij} r_i \leq a_j \qquad \forall j \in J \qquad \text{(land capacity)} \tag{4}$$

$$x_{i1} + x_{i6} = 0 \qquad \forall i \in (2, 3, 4) \qquad \text{(no facilities on site 1 and 6)} \tag{5}$$

$$x_{1j} = 0 \qquad \forall j \in (2, 3, 4, 5) \qquad \text{(no golf on site 2 to 4)} \tag{6}$$

$$x_{11} + x_{16} \leq 1 \qquad \text{(either site 1 or 6 can build golf)} \tag{7}$$

Question 2 (b) - Basic goal programming

Sets

K set of objectives $K \in (1, 2, 3)$
I set of facility types $I \in (1, \ldots, 4)$ as (Golf, Swimming, Gymnasium, Tennis)
J set of sites $J \in (1, \ldots, 6)$

Parameters

d_{ij}	user days for facility i on site j	$i \in I, j \in J$
a_j	available land on site j in ft^2	$j \in J$
c_i	construction cost for facility i in \$	$i \in I$
r_i	required land for facility i in ft^2	$i \in I$

Variables

x_{ij} $\begin{cases} 1 & \text{facility } i \text{ is built on site } j \\ 0 & \text{if else} \end{cases}$ $i \in I, j \in J$

d_k deficiency variable for objective k $k \in K$

Objectives

$$\min \quad \sum_{k \in K} d_k \qquad \text{(construction cost)} \tag{8}$$

Constraints

$$\sum_{i \in I} \sum_{j \in J} x_{ij}(a_j - r_i) - d_1 \leq 40\ 000 \text{ ft}^2 \qquad \text{(P1: unused land cost)} \tag{9}$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij} c_i - d_2 < \$1.2 \text{ million} \qquad \text{(P2: construction cost)} \tag{10}$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij} u_{ij} + d_3 \geq 200\ 000 \text{ days} \qquad \text{(P3: user days)} \tag{11}$$

$$\sum_{i \in I} x_{ij} r_i \leq a_j \qquad \forall j \in J \qquad \text{(land capacity)} \tag{12}$$

$$x_{i1} + x_{i6} = 0 \qquad \forall i \in (2, 3, 4) \qquad \text{(no facilities on site 1 and 6)} \tag{13}$$

$$x_{1j} = 0 \qquad \forall j \in (2, 3, 4, 5) \qquad \text{(no golf on site 2 to 4)} \tag{14}$$

$$x_{11} + x_{16} \leq 1 \qquad \text{(either site 1 or 6 can build golf)} \tag{15}$$

Solution

Question 2 (c) - Pre-emptive goal programming

Sets

K set of objectives $K \in (1, 2, 3)$
I set of facility types $I \in (1, \ldots, 4)$ as (Golf, Swimming, Gymnasium, Tennis)
J set of sites $J \in (1, \ldots, 6)$

Parameters

d_{ij}	user days for facility i on site j	$i \in I, j \in J$
a_j	available land on site j in ft^2	$j \in J$
c_i	construction cost for facility i in \$	$i \in I$
r_i	required land for facility i in ft^2	$i \in I$

Variables

x_{ij} $\begin{cases} 1 & \text{facility } i \text{ is built on site } j \\ 0 & \text{if else} \end{cases}$ $i \in I, j \in J$

d_k deficiency variable for objective k $k \in K$

Objectives

$$\min \quad d_1 \tag{16}$$
$$\min \quad d_2 \qquad \text{(subject to } d_1\text{)} \tag{17}$$
$$\min \quad d_3 \qquad \text{(subject to } d_1 \text{ and } d_2\text{)} \tag{18}$$

Constraints

$$d_1 = 0 \qquad \text{(used in } d_2 \text{ and } d_3\text{)} \tag{19}$$
$$d_2 = 0 \qquad \text{(used in } d_3\text{)} \tag{20}$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij}(a_j - r_i) - d_1 \leq 40\ 000 \text{ ft}^2 \qquad \text{(P1: unused land cost)} \tag{21}$$
$$\sum_{i \in I} \sum_{j \in J} x_{ij} c_i - d_2 < \$1.2 \text{ million} \qquad \text{(P2: construction cost)} \tag{22}$$
$$\sum_{i \in I} \sum_{j \in J} x_{ij} u_{ij} + d_3 \geq 200\ 000 \text{ days} \qquad \text{(P3: user days)} \tag{23}$$

$$\sum_{i \in I} x_{ij} r_i \leq a_j \qquad \forall j \in J \qquad \text{(land capacity)} \tag{24}$$
$$x_{i1} + x_{i6} = 0 \qquad \forall i \in (2, 3, 4) \qquad \text{(no facilities on site 1 and 6)} \tag{25}$$
$$x_{1j} = 0 \qquad \forall j \in (2, 3, 4, 5) \qquad \text{(no golf on site 2 to 4)} \tag{26}$$
$$x_{11} + x_{16} \leq 1 \qquad \text{(either site 1 or 6 can build golf)} \tag{27}$$

Solution

Question 2 (d) - Weighted sum goal programming

Sets

K set of objectives $K \in (1, 2, 3)$
I set of facility types $I \in (1, \ldots, 4)$ as (Golf, Swimming, Gymnasium, Tennis)
J set of sites $J \in (1, \ldots, 6)$

Parameters

d_{ij}	user days for facility i on site j	$i \in I, j \in J$
a_j	available land on site j in ft^2	$j \in J$
c_i	construction cost for facility i in \$	$i \in I$
r_i	required land for facility i in ft^2	$i \in I$

Variables

x_{ij} $\begin{cases} 1 & \text{facility } i \text{ is built on site } j \\ 0 & \text{if else} \end{cases}$ $i \in I, j \in J$

d_k deficiency variable for objective k $k \in K$

Objectives

$$\min \quad 1000d_1 + 10d_2 + d_3 \tag{28}$$

Constraints

$$\sum_{i \in I} \sum_{j \in J} x_{ij}(a_j - r_i) - d_1 \leq 40\ 000 \text{ ft}^2 \qquad \text{(P1: unused land cost)} \tag{29}$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij} c_i - d_2 < \$1.2 \text{ million} \qquad \text{(P2: construction cost)} \tag{30}$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij} u_{ij} + d_3 \geq 200\ 000 \text{ days} \qquad \text{(P3: user days)} \tag{31}$$

$$\sum_{i \in I} x_{ij} r_i \leq a_j \qquad \forall j \in J \qquad \text{(land capacity)} \tag{32}$$

$$x_{i1} + x_{i6} = 0 \qquad \forall i \in (2, 3, 4) \qquad \text{(no facilities on site 1 and 6)} \tag{33}$$

$$x_{1j} = 0 \qquad \forall j \in (2, 3, 4, 5) \qquad \text{(no golf on site 2 to 4)} \tag{34}$$

$$x_{11} + x_{16} \leq 1 \qquad \text{(either site 1 or 6 can build golf)} \tag{35}$$

Solution

Question 2 (e) - Basic goal programming efficiency adjustment

Sets

K set of objectives $K \in (1, 2, 3)$
I set of facility types $I \in (1, \ldots, 4)$ as (Golf, Swimming, Gymnasium, Tennis)
J set of sites $J \in (1, \ldots, 6)$

Parameters

d_{ij}	user days for facility i on site j	$i \in I, j \in J$
a_j	available land on site j in ft^2	$j \in J$
c_i	construction cost for facility i in \$	$i \in I$
r_i	required land for facility i in ft^2	$i \in I$

Variables

x_{ij} $\begin{cases} 1 & \text{facility } i \text{ is built on site } j \\ 0 & \text{if else} \end{cases}$ $\quad i \in I, j \in J$

d_k deficiency variable for objective k $\quad k \in K$

Objectives

A small number 0.001 has been selected as the adjustment factor with positive and negative signs applied for minimisation and maximisation respectively.

$$\begin{aligned} \min \quad & \sum_{k \in K} d_k + 0.001\Big(\sum_{i \in I}\sum_{j \in J} x_{ij} c_i\Big) \\ & + 0.001\Big(\sum_{i \in I}\sum_{j \in J} x_{ij} a_j - x_{ij} r_i\Big) \\ & - 0.001\Big(\sum_{i \in I}\sum_{j \in J} x_{ij} u_{ij}\Big) \end{aligned} \tag{36}$$

Constraints

$$\sum_{i \in I}\sum_{j \in J} x_{ij}(a_j - r_i) - d_1 \leq 40\ 000 \text{ ft}^2 \qquad \text{(P1: unused land cost)} \tag{37}$$

$$\sum_{i \in I}\sum_{j \in J} x_{ij} c_i - d_2 < \$1.2 \text{ million} \qquad \text{(P2: construction cost)} \tag{38}$$

$$\sum_{i \in I}\sum_{j \in J} x_{ij} u_{ij} + d_3 \geq 200\ 000 \text{ days} \qquad \text{(P3: user days)} \tag{39}$$

$$\sum_{i \in I} x_{ij} r_i \leq a_j \qquad \forall j \in J \qquad \text{(land capacity)} \tag{40}$$

$$x_{i1} + x_{i6} = 0 \qquad \forall i \in (2, 3, 4) \qquad \text{(no facilities on site 1 and 6)} \tag{41}$$

$$x_{1j} = 0 \qquad \forall j \in (2, 3, 4, 5) \qquad \text{(no golf on site 2 to 4)} \tag{42}$$

$$x_{11} + x_{16} \leq 1 \qquad \text{(either site 1 or 6 can build golf)} \tag{43}$$

Solution