General topology parametrization & resolution for 2 cells

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- Incompressibility :

$$V_0 = f(\alpha_1) \cdot a \cdot h - R_2^2 \cdot \left(\alpha_2 - \frac{\sin(2\alpha_2)}{2}\right) \tag{1}$$

$$V_1 = -f(\alpha_1) \cdot a \cdot h + R_3^2 \cdot \left(\alpha_3 - \frac{\sin(2\alpha_3)}{2}\right) \tag{2}$$

with f the area of the circular contact segment per unit height and chord :

$$f(\alpha) = \frac{\left(1 + \tan\left(\frac{\alpha}{2}\right)^2\right)^2 \cdot \alpha - 2\tan\left(\frac{\alpha}{2}\right) + 2\tan\left(\frac{\alpha}{2}\right)^3}{4\tan\left(\frac{\alpha}{2}\right)^3}$$

- Chord lengths from arc lengths and angles :

$$L_1 \cdot \mathrm{sinc}(\alpha_1) = 2a \tag{3}$$

$$L_2 \cdot \mathrm{sinc}(\alpha_2) = 2a \tag{4}$$

$$L_3 \cdot \mathrm{sinc}(\alpha_3) = 2a \tag{5}$$

- Other geometrical constraints :

$$h \cdot \cos\left(\frac{\alpha_1}{2}\right) = a \cdot \sin\left(\frac{\alpha_1}{2}\right) \tag{6}$$

$$L_2 = -2R_2 \cdot \alpha_2 \tag{7}$$

$$L_3 = 2R_3 \cdot \alpha_3 \tag{8}$$

- Vertex coordinates from chord length and inclination angle, projected on x and y :

$$x_1 - x_0 = -2a \cdot \sin(\beta) \tag{9}$$

$$y_1 - y_0 = 2a \cdot \cos(\beta) \tag{10}$$

- Fixed vertex ”center of mass” at the origin, projected on x and y :

$$x_0 + x_1 = 0 \tag{11}$$
$$y_0 + y_1 = 0 \tag{12}$$

- Vectorial Plateau relation, projected on the chord and its normal :

$$(\sigma_0 + \sigma_1) \cdot \cos(\alpha_1) + \sigma_2 \cdot \cos(\alpha_2) + \sigma_3 \cdot \cos(\alpha_3) = 0 \tag{13}$$
$$(\sigma_0 + \sigma_1) \cdot \sin(\alpha_1) + \sigma_2 \cdot \sin(\alpha_2) + \sigma_3 \cdot \sin(\alpha_3) = 0 \tag{14}$$

- Equal elongation rates for both sides of a contact zone :

$$\frac{\dot{\sigma_0}}{k_0} - \frac{\dot{\sigma_1}}{k_1} + \frac{\sigma_0}{\eta_0} - \frac{\sigma_1}{\eta_1} = m_0 - m_1 \tag{15}$$

- Rheology relations for each independant side :

$$\dot{L_1} = L_1 \cdot \left(\frac{\dot{\sigma_1}}{k_1} + \frac{\sigma_1}{\eta_1} - m_1 \right) + 2v_{ca} \tag{16}$$
$$\dot{L_2} = L_2 \cdot \left(\frac{\dot{\sigma_2}}{k_0} + \frac{\sigma_2}{\eta_0} - m_0 \right) - 2v_{ca} \tag{17}$$
$$\dot{L_3} = L_3 \cdot \left(\frac{\dot{\sigma_3}}{k_1} + \frac{\sigma_3}{\eta_1} - m_1 \right) - 2v_{ca} \tag{18}$$

- Laplace overpressures at each free interface :

$$P_0 = \frac{\sigma_2}{R_2} \tag{19}$$
$$P_1 = \frac{\sigma_3}{R_3} \tag{20}$$

$\longrightarrow$ 20 equations, 20 variables :

$$\dot{h}, \dot{R_2}, \dot{R_3}, \dot{L_1}, \dot{L_2}, \dot{L_3}, \dot{a}, \dot{x_0}, \dot{x_1}, \dot{y_0}, \dot{y_1}, \dot{\alpha_1}, \dot{\alpha_2}, \dot{\alpha_3}, \dot{\sigma_0}, \dot{\sigma_1}, \dot{\sigma_2}, \dot{\sigma_3}, \dot{P_0}, \dot{P_1}$$