Table 1: Discrete models

	Parameters	**Support**	**PMF** $p_X(x)$	**Mean**	**Variance**
Bernoulli	$p \in [0,1]$	$x \in [0,1]$	$p^x(1-p)^{1-x}$	p	$p(1-p)$
Binomial	$p \in [0,1]$	$x \in \{0,1,\dots,n\}$	$\binom{n}{x} p^x(1-p)^{n-x}$	np	$np(1-p)$
Negative Binomial	$r \in \mathbb{Z}^+, p \in [0,1]$	$k \in \mathbb{N}$	$\binom{k+r-1}{k}(1-p)^r p^k$	$\frac{pr}{1-p}$	$\frac{pr}{(1-p)^2}$
Geometric	$p \in [0,1]$	$x \in \{1,2,\dots\}$	$p(1-p)^{x-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson	$\mu \in \mathbb{R}^+$	$x \in \{0,1,2,\dots\}$	$\frac{\mu^x}{x!}e^{-\mu}$	μ	μ

Table 2: Continuous models

	Parameters	**Support**	**PDF** $p_X(x)$	**Mean**	**Variance**
Uniform	$a < b \in \mathbb{R}$	$x \in [a,b]$	$\frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential	$\lambda > 0$	$x \in \mathbb{R}^+_{\not 0}$	$\lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Normal	$\begin{cases} \mu \in \mathbb{R} & \text{(mean)} \\ \sigma^2 > 0 & \text{(variance)} \end{cases}$	$x \in \mathbb{R}$	$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2 \right\}$	μ	σ^2
Beta	$a > 0,\ b > 0$	$x \in [0,1]$	$\frac{1}{\mathrm{B}(a,b)} x^{a-1}(1-x)^{b-1}$	$\frac{a}{a+b}$	$\frac{ab}{(a+b)^2(a+b+1)}$
Gamma	$a > 0, b > 0$	$x \in (0,\infty)$	$\frac{1}{\Gamma(a)} b^a x^{a-1} e^{-bx}$	$\frac{a}{b}$	$\frac{a}{b^2}$
Inverse Gamma	$a > 0, b > 0$	$x \in (0,\infty)$	$\frac{b^a}{\Gamma(a)} x^{-(a+1)} e^{-\frac{b}{x}}$	$\frac{b}{a-1}, a > 1$	$\frac{b^2}{(a-1)^2(a-2)}, a > 2$
Beta-Binomial	$n \in \mathbb{N}, a > 0, b > 0$	$k \in \{0,\dots,n\}$	$\binom{n}{k} \frac{B(k+a,\, n-k+b)}{B(a,b)}$	$\frac{na}{a+b}$	$\frac{nab(a+b+n)}{(a+b)^2(a+b+1)}$

Table 3: Conjugate analysis

Likelihood	**Conjugate prior distribution**	**Posterior hyperparameters** ($y = \sum Y_i$)	**Posterior predictive**
Binomial(n, θ)	$\theta \sim$ Beta$(1,1)$ (uniform)	Beta$(1+y,\ 1+n-y)$	Bernoulli$\Big(\frac{a'}{a'+b'}\Big)$
Binomial(n, θ)	$\theta \sim$ Beta(a,b)	Beta$(a+y,\ b+n-y)$	Bernoulli$\Big(\frac{a'}{a'+b'}\Big)$
Poisson(θ)	$\theta \sim$ Gamma(a,b)	Gamma$(a+y, b+n)$	Negative-Binomial$\Big(a', \frac{1}{1+b'}\Big)$
Normal(θ, σ^2) (known σ^2)	$\theta \sim$ Normal(μ_0, τ_0^2)	Normal$\Big(\frac{1}{\kappa}\Big(\frac{\mu_0}{\tau_0^2} + \frac{\sum_i y_i}{\sigma^2}\Big),\ \frac{1}{\kappa}\Big)$ $\kappa = \frac{1}{\tau_0^2} + \frac{n}{\sigma^2}$	Normal$\big(\mu_n, \tau_n^2 + \sigma^2\big)$
Normal(θ, σ^2) (known μ)	$\sigma^2 \sim$ Inv-Gamma(a,b)	InvGamma$\Big(a + \frac{n}{2},\ b + \frac{\sum_{i=1}^n (x_i - \mu)^2}{2}\Big)$	
Normal(θ, σ^2) (both unknown)	$\begin{cases} \frac{1}{\sigma^2} \sim \text{Gamma}\Big(\frac{\nu_0}{2},\ \frac{\nu_0 \sigma_0^2}{2}\Big) \\ \theta \mid \sigma^2 \sim \text{Normal}\Big(\mu_0,\ \frac{\sigma^2}{\kappa_0}\Big) \end{cases}$	$\begin{cases} \theta \sim \text{Normal}\Big(\mu_n,\ \frac{\sigma^2}{\kappa_n}\Big) \\ \frac{1}{\sigma^2} \sim \text{Gamma}\Big(\frac{\nu_n}{2},\ \frac{\nu_n \sigma_n^2}{2}\Big) \end{cases}$ $\kappa_n = \kappa_0 + n$ $\mu_n = \frac{\kappa_0 \mu_0 + n\bar{y}}{\kappa_n}$ $\nu_n = \nu_0 + n$ $\sigma_n^2 = \frac{1}{\nu_n}[\nu_0 \sigma_0^2 + \sum_i (\bar{y} - y_i)^2 + \frac{\kappa_0 n}{\kappa_n}(\bar{y} - \mu_0)^2]$	

Bayes factor for an event E s.t. $\Pr(E) = p$:

$$\text{odds}(E) = \frac{p}{1-p}$$

Given prior model M_1 and posterior model M_2 we compute:

$$\text{BF} = \frac{\text{odds}(E|M_2)}{\text{odds}(E|M_1)}$$