EVALUATING METHODS FOR ESTIMATING THE HURST EXPONENT IN TIME SERIES A COMPARATIVE ANALYSIS OF ACCURACY AND APPLICATION

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Abstract

The Hurst exponent (H) is a statistical measure used to understand the long-term memory of time series, characterized by persistence and anti-persistence phenomena. Accurate estimation of H is crucial in diverse fields, such as finance [2], geophysics, and telecommunications due to its ability to indicate the future tendency of a system based on its historical behavior. Numerous methods have been developed for estimating H, each with varying degrees of complexity and accuracy. This research reviews these methods, focusing on popular ones like box-counting, Katz, detrended fluctuation analysis, mean squared displacement (MSD), and Higuchi. To evaluate the reliability of these methods, fractional Brownian motion was used as a benchmark due to its well-defined Hurst parameter.

The impact of sample size on estimation efficiency

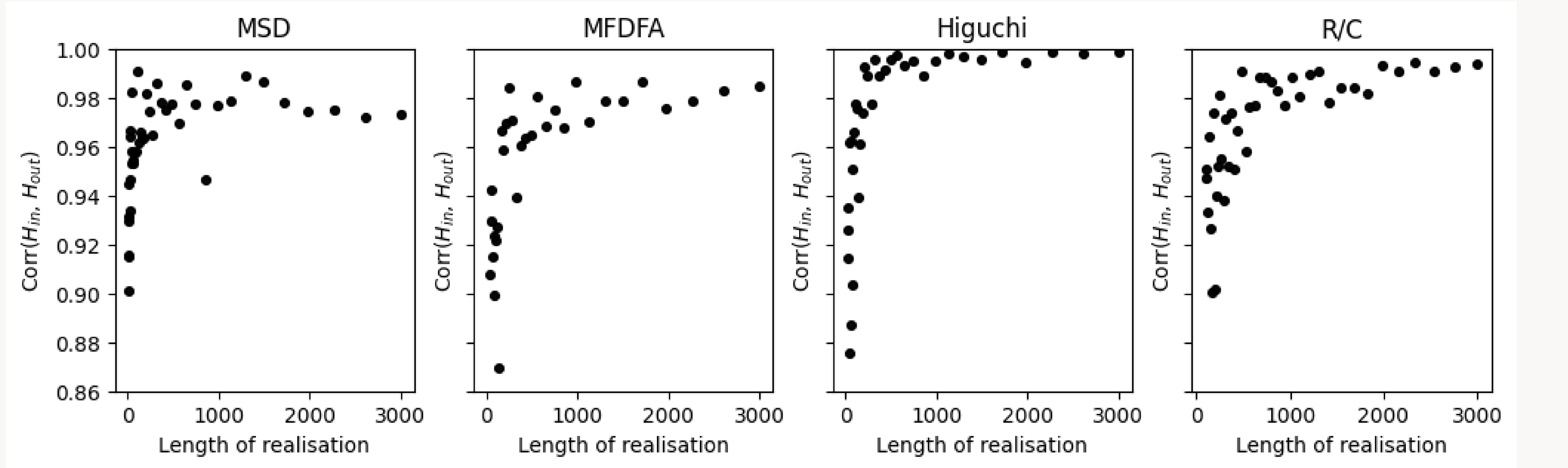

Figure 1: The accuracy of algorithms such as Higuchi and MSD improves with increased realization lengths, as evidenced by Pearson Correlation among 16 different H values they generate. It's noted that the relationship between the size of realization and correlation accuracy tends to be non-linear and concave.

Self-consistency and accuracy testing

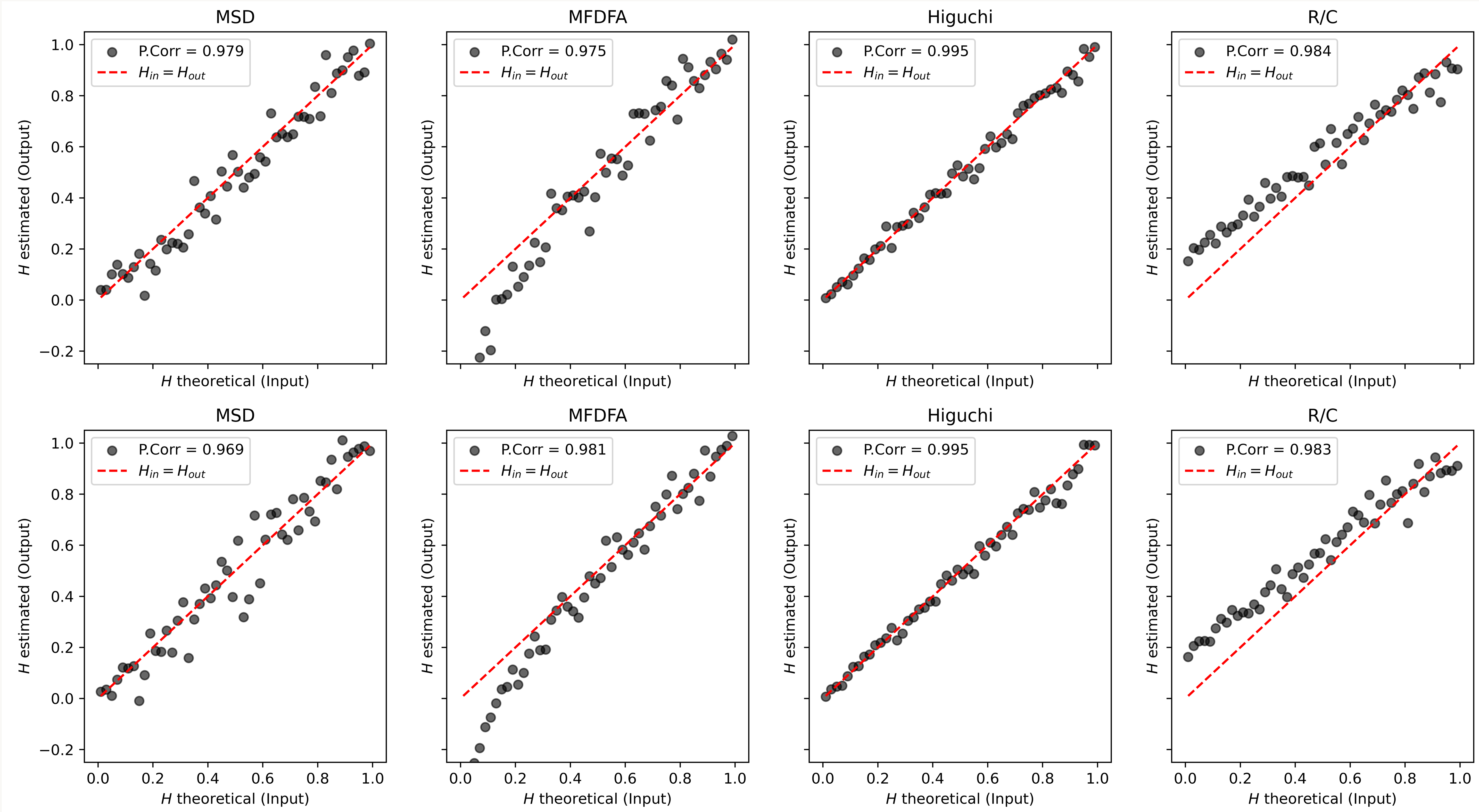

Figure 2: Self-consistency check: Each dot represents an FBM 1000 point realization. Generated with [1]. The X-axis shows the initial H used to generate the realization, while the Y-axis shows H calculated from that realization. Ideally, they should coincide completely. The upper (lower) row of graphs is generated using Hosking (Cholesky) algorithm. No differences are observed in generating algorithms. However, significant differences are seen in calculating algorithms: MSD has a large variance, while MFDFA and R/C exhibit systematic errors.

Fractional Brownian Motion (FBM) and Hurst exponent

FBM is defined as a centered gaussian process with covariance:

$$Cov[X(t), X(s)] = \frac{1}{2}\left[t^{2H} + s^{2H} - |t-s|^{2H}\right] \quad (1)$$

Where represents $X(t)$ value of time series at time t.

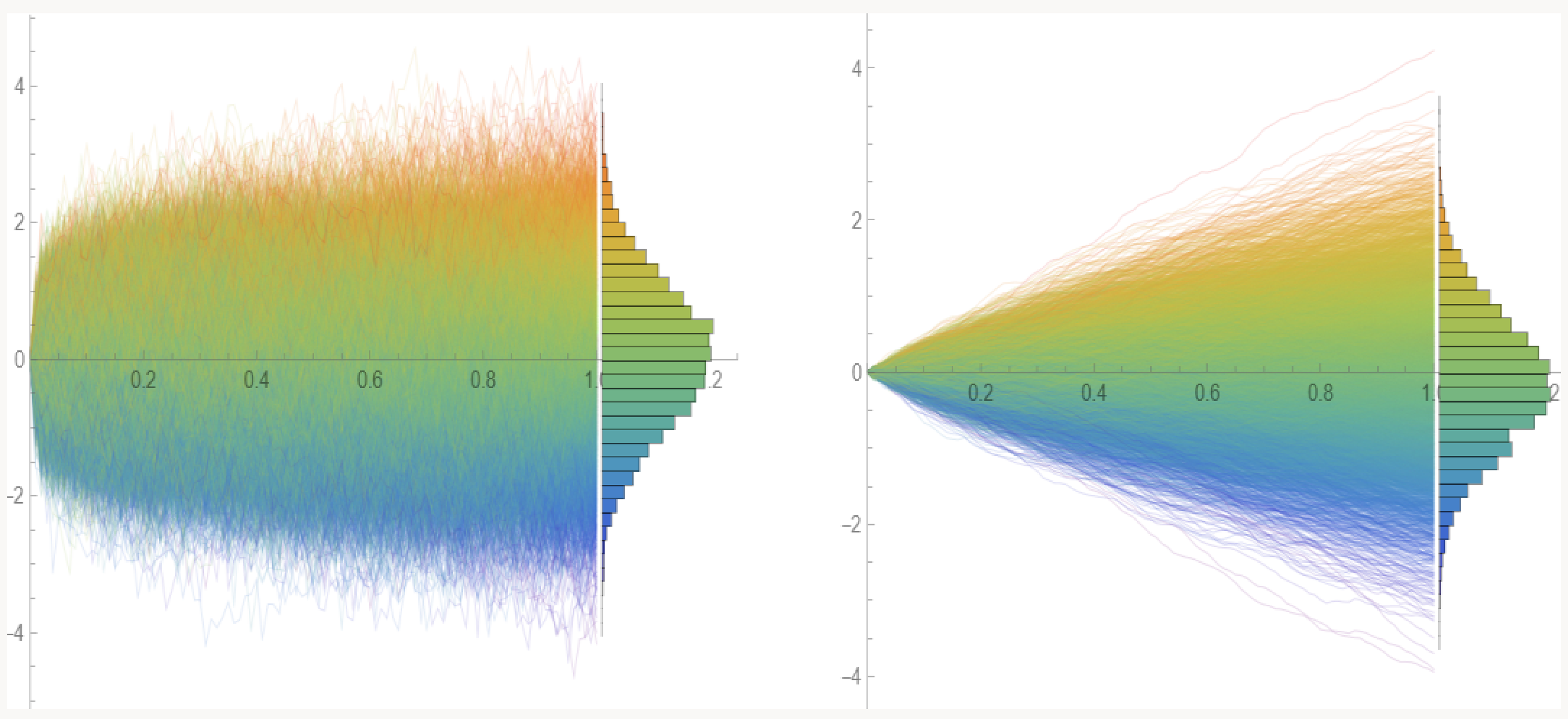

Figure 3: Many FBM realisations; On the end of each figure there's a concatinated histogram. The left with the input $H = 0.2$ has anti-persistent motion i.e. strongly revert to the mean, while the right with the input of $H = 0.8$ displays persistent motion i.e. maintain consistent trend.

Originally, H is derived from rescaled range analysis (R/S), a method for analyzing time series data:

$$\mathbb{E}\left[\frac{R(t)}{S(t)}\right] = Ct^{H} \text{ as } t \to \infty \quad (2)$$

Where $R(t)$ is range of data points of up to t, $S(t)$ is standard deviation up to t and C is constant [3].

Conclusions

To verify self-consistency, methodologies such as the Higuchi method, Mean Squared Displacement (MSD), Multifractal Detrended Fluctuation Analysis (MFDFA), and Rescaled Range (R/S) Analysis were employed.

- The Higuchi method has been most effective in the estimation of the Hurst parameter across the entire domain of H, compared to other methods
- The Higuchi method shows a rapid increase in accuracy with longer realization lengths
- MSD effectively estimates the Hurst parameter across the H domain but shows larger deviations compared to the Higuchi method.
- MFDFA underperformed for $H < 0.3$, with underestimation increasing as H decreases.
- (R/S) underperformed for $H < 0.5$, with overestimation increasing as H decreases.

Bibliography

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