

# Study of Nonsingular Fast Terminal Sliding-Mode Fault-Tolerant Control

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**Abstract**—This paper studies fault-tolerant control (FTC) designs based on nonsingular terminal sliding-mode control and nonsingular fast terminal sliding-mode control (NFTSMC). The proposed active FTC laws are shown to be able to achieve fault-tolerant objectives and maintain stabilization performance even when some of the actuators fail to operate. In comparison to existing sliding-mode control (SMC) fault-tolerant designs, the proposed schemes not only can retain the advantages of traditional SMC, including fast response, easy implementation, and robustness to disturbances/uncertainties, but also make the system states reach the control objective point in a finite amount of time. Moreover, they also resolve the potential singularity phenomena in traditional terminal and faster terminal SMC designs; meanwhile, the proposed NFTSMC fault-tolerant scheme also possesses the benefit of faster state convergence speed of NFTSMC. Finally, the proposed analytical results are also applied to the attitude control of a spacecraft. Simulation results demonstrate the benefits of the proposed schemes.

**Index Terms**—Fault detection, fault-tolerant control (FTC), nonlinear systems, nonsingular fast terminal sliding-mode control (NFTSMC), nonsingular terminal sliding-mode control (NTSMC), sliding-mode control (SMC).

## I. INTRODUCTION

WITH THE rapid development of advanced technologies and complex industrial systems, it is necessary to maintain ever increasing requirements for reliability and safety of control systems. The issues concerning fault-tolerant control (FTC) are thus widely discussed and applied in various areas (see, e.g., [1]–[43], and the references therein). It is worth noting that the Fault Detection and Diagnosis (FDD) (e.g., see [5], [8], [9], [12]–[18], [23]–[26], [31], [35]–[37], [41]–[43]) module is a cornerstone of an FTC control scheme [9], [12]. The module should be able to provide: 1) Fault Detection and Isolation (FDI) or 2) Fault Detection, Isolation, and Estimation, which is referred to as an FDD module. The performance of

the overall FTC scheme mainly depends on the performance of this FDD (or FDI) module. The effect of the detection delay and the gyroscopic effect on the performance of the overall FTC scheme are also discussed and explored [23]–[26]. Some researchers studied the FDD module and the overall FTC in the presence of different kinds of faults, e.g., sinusoidal fault [5], [8], [12], [23]–[26]. Ossmann and Varga [37] discussed the detection and identification of loss of efficiency faults in flight actuators. Moreover, the dynamic disturbances can mask small faults. Therefore, the size of the fault [12] will be discussed in FDD (or FDI). Persis and Isidori [1] presented the theoretical development of FDI with disturbance decoupling by means of the nonlinear geometric approach. Baldi *et al.* [42] discussed the application and development of such theory in the case of FDI with aerodynamic disturbance decoupling for satellite reaction wheels.

The main objective of FTC is to design an appropriate control law, such that the closed-loop system can tolerate some faults in specific control components and maintain the whole system stability with acceptable performance. In some critical systems, e.g., spacecraft, the actuator faults may make the overall performance deteriorate severely and become unstable and, thus, may terminate the control mission. This is also why researchers and scholars are continuously interested in developing the reliable control with fault-tolerant methodologies for spacecraft missions. With the aim of tackling these challenges, the main goal of this study is to develop proper FTC laws that possess the advantages of high control precision, robustness, and rapid response, even in the presence of actuator failures, such that the control mission of the system can be maintained.

Traditional sliding-mode control (SMC) has been widely applied in nonlinear systems [23]–[26], [44]–[50] because of its benefits, including fast response, easy implementation, and robustness to disturbances/uncertainties. Recently, Liang *et al.* [2], [4], [10] have applied the SMC techniques to the FTC design for spacecraft attitude stabilization control. In their application, the Takagi–Sugeno fuzzy systems are also incorporated to the nonlinear systems, so that the online computational burden, which has been widely discussed in [48], can be reduced. In general, the system under traditional SMC design has only asymptotic stable performance. To enhance the asymptotic convergence to a finite-time state convergence, Zak [51], [52] first proposed the so-called terminal SMC (TSMC) [52]. In comparison to traditional SMC, one nonlinear term is added to the sliding surface TSMC to improve the convergence performance. By suitably designing the parameters of TSMC, the system states can reach the control objective point on the sliding surface in a finite amount of time. Therefore, TSMC not

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only has the advantages of rapid response and robustness, but also offers the benefit of finite-time convergence [53]. However, it was reported that common TSMC design suffers a singular problem, and the control magnitude may become unbounded whenever the state reaches zero. This potential drawback has been resolved by so-called nonsingular TSMC (NTSMC) [54]–[56]. More recently, by utilizing new-type sliding surface design, some researchers have extended the results of NTSMC to the new-type NTSMC with faster convergence time [57]–[64], i.e., nonsingular fast TSMC (NFTSMC). The NFTSMC not only retains the advantages of NTSMC but also provides a faster state convergence than that of NTSMC. In light of the remarkable importance and advantages previously mentioned, this paper will investigate FTC design by using NTSMC and NFTSMC, respectively.

The organization of this paper is organized as follows: Section II states the problem formulation and main goal of this paper. Section III describes the FTC design by using NTSMC and NFTSMC, respectively. The analytical results are then applied to the attitude control of a spacecraft in Section IV. Finally, Section V gives the conclusions.

## II. PROBLEM STATEMENT

Consider a class of second-order nonlinear control systems

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \quad (1)$$

$$\dot{\mathbf{x}}_2 = \mathbf{f}(\mathbf{x}) + G(\mathbf{x})\mathbf{u} + \mathbf{d} \quad (2)$$

where  $\mathbf{x}_1 = (x_1, \dots, x_n)^T \in \mathbb{R}^n$ ,  $\mathbf{x}_2 = (x_{n+1}, \dots, x_{2n})^T \in \mathbb{R}^n$ , and  $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2)^T$  represent the system states;  $\mathbf{u} = (u_1, \dots, u_m)^T \in \mathbb{R}^m$  with  $m \geq n$  are the control inputs;  $\mathbf{d} = (d_1, \dots, d_n)^T \in \mathbb{R}^n$  represent the possible model uncertainties and/or disturbances;  $\mathbf{f}(\mathbf{x}) \in \mathbb{R}^n$  and  $G(\mathbf{x}) \in \mathbb{R}^{n \times m}$  are smooth functions with  $\mathbf{f}(\mathbf{0}) = \mathbf{0}$ ; and  $(\cdot)^T$  denotes the transpose of a vector or a matrix.

In this paper, we assume that the actuators' faults have been successfully detected and diagnosed by a suitable FDD scheme. From the FDD information, the overall actuators can be divided into two groups, i.e.,  $\mathcal{H}$  and  $\mathcal{F}$ , in which we assume that all of the actuators in  $\mathcal{H}$  are healthy, while those in  $\mathcal{F}$  experience faults. Thus, System (1) and (2) can be rewritten as

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \quad (3)$$

$$\dot{\mathbf{x}}_2 = \mathbf{f}(\mathbf{x}) + G_{\mathcal{H}}(\mathbf{x})\mathbf{u}_{\mathcal{H}} + G_{\mathcal{F}}(\mathbf{x})\mathbf{u}_{\mathcal{F}} + \mathbf{d} \quad (4)$$

where  $G(\mathbf{x}) = (G_{\mathcal{H}}(\mathbf{x}); G_{\mathcal{F}}(\mathbf{x}))$  and  $\mathbf{u} = (\mathbf{u}_{\mathcal{H}}^T; \mathbf{u}_{\mathcal{F}}^T)^T$ . In the rest of this paper, we assume that  $\mathbf{u}_{\mathcal{H}} \in \mathbb{R}^k$ ,  $\mathbf{u}_{\mathcal{F}} \in \mathbb{R}^{m-k}$ , and  $m \geq k \geq n$ . For the succeeding FTC law design, we impose the following assumptions.

**Assumption 1:** For any system state  $\mathbf{x} \in \mathbb{R}^{2n}$ , the matrix  $G_{\mathcal{H}}(\mathbf{x}) \in \mathbb{R}^{n \times k}$  has full row rank, i.e.,  $\text{rank}(G_{\mathcal{H}}(\mathbf{x})) = n$ .

Note that Assumption 1 implies that there exist sufficient healthy actuators to perform the FTC task. In addition, we assume that the control inputs in the set of  $\mathcal{F}$  are diagnosed as

$$\mathbf{u}_{\mathcal{F}} = \hat{\mathbf{u}}_{\mathcal{F}} + \Delta\mathbf{u}_{\mathcal{F}} \quad (5)$$

where  $\hat{\mathbf{u}}_{\mathcal{F}}$  and  $\Delta\mathbf{u}_{\mathcal{F}}$  represent the estimated value and estimated error of the faulty actuator  $\mathbf{u}_{\mathcal{F}}$ , respectively. Therefore, System (3) and (4) can be described as

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \quad (6)$$

$$\dot{\mathbf{x}}_2 = \mathbf{f}(\mathbf{x}) + G_{\mathcal{H}}(\mathbf{x})\mathbf{u}_{\mathcal{H}} + G_{\mathcal{F}}(\mathbf{x})(\hat{\mathbf{u}}_{\mathcal{F}} + \Delta\mathbf{u}_{\mathcal{F}}) + \mathbf{d}. \quad (7)$$

Based on System (6) and (7), an appropriate  $\mathbf{u}_{\mathcal{H}}$  will be organized, so that the states of the closed-loop system converge to the origin in a finite amount of time, even when the actuators in  $\mathcal{F}$  are detected and diagnosed as experiencing faults by an FDD mechanism.

## III. DESIGN OF FAULT-TOLERANT CONTROLLERS

### A. Nonsingular Terminal Sliding-Mode Fault-Tolerant Control (NTSM-FTC) Design

Considering System (6) and (7), here, we will employ the NTSMC technique to design the FTC law, so that the closed-loop system trajectory can reach the origin in a finite amount of time. According to the concepts of NTSMC, we design the sliding surface of NTSMC as

$$\mathbf{s} = \mathbf{x}_1 + \beta^{-1} \text{sig}^{\frac{p}{q}}(\mathbf{x}_2) = \mathbf{0} \quad (8)$$

where  $\mathbf{s} = (s_1, \dots, s_n)^T \in \mathbb{R}^n$  is the sliding variable,  $\beta^{-1} = \text{diag}\{\beta_1^{-1}, \dots, \beta_n^{-1}\} \in \mathbb{R}^{n \times n}$  is a positive definite matrix,  $p$  and  $q$  are positive odd numbers satisfying the relation  $1 < p/q < 2$ , and  $\text{sig}^{p/q}(\mathbf{x}_2) := (|\dot{x}_1|^{p/q} \text{sign}(\dot{x}_1), \dots, |\dot{x}_n|^{p/q} \text{sign}(\dot{x}_n))^T \in \mathbb{R}^n$ . Then, the time derivative of sliding variable  $\mathbf{s}$  along the trajectories of System (6) and (7) is

$$\dot{\mathbf{s}} = \mathbf{x}_2 + \beta^{-1} \frac{p}{q} \text{sig}^{(\frac{p}{q})-1}(\mathbf{x}_2) \cdot [\mathbf{f}(\mathbf{x}) + G_{\mathcal{H}}(\mathbf{x})\mathbf{u}_{\mathcal{H}} + G_{\mathcal{F}}(\mathbf{x}) \cdot (\hat{\mathbf{u}}_{\mathcal{F}} + \Delta\mathbf{u}_{\mathcal{F}}) + \mathbf{d}] \quad (9)$$

where

$$\text{sig}^{(\frac{p}{q})-1}(\mathbf{x}_2) := \text{diag} \left\{ |\dot{x}_1|^{(\frac{p}{q})-1}, \dots, |\dot{x}_n|^{(\frac{p}{q})-1} \right\} \in \mathbb{R}^{n \times n}.$$

It is worth noting from (9) that the estimated value  $\hat{\mathbf{u}}_{\mathcal{F}}$  is available from FDD information. Hence, the total uncertainties take the form of  $G_{\mathcal{F}}(\mathbf{x})\Delta\mathbf{u}_{\mathcal{F}} + \mathbf{d}$ . Moreover, due to the success of fault diagnosis, the estimation error  $\Delta\mathbf{u}_{\mathcal{F}}$  can be assumed to be bounded. This can be described by the assumption as follows.

**Assumption 2:** There exists a nonnegative function  $\rho(\mathbf{x}, t)$ , such that

$$\|G_{\mathcal{F}}(\mathbf{x})\Delta\mathbf{u}_{\mathcal{F}} + \mathbf{d}\| \leq \rho(\mathbf{x}, t) \text{ for all } \mathbf{x} \in \mathbb{R}^{2n} \text{ and } t \in \mathbb{R}^+.$$

Under Assumptions 1 and 2, we design the NTSMC-FTC law as follows:

$$\mathbf{u}_{\mathcal{H}} = \mathbf{u}_{\mathcal{H}eq} + \mathbf{u}_{\mathcal{H}re} \quad (10)$$

$$\mathbf{u}_{\mathcal{H}eq} = -G_{\mathcal{H}}^+(\mathbf{x}) \left[ \mathbf{f}(\mathbf{x}) + \beta \frac{q}{p} \text{sig}^{2-(\frac{p}{q})}(\mathbf{x}_2) + G_{\mathcal{F}}(\mathbf{x})\hat{\mathbf{u}}_{\mathcal{F}} \right] \quad (11)$$

$$\mathbf{u}_{\mathcal{H}re} = -[\eta + \rho(\mathbf{x}, t)] \cdot G_{\mathcal{H}}^+(\mathbf{x}) \text{sign}(\mathbf{s}) \quad (12)$$

where  $G_{\mathcal{H}}^+(\mathbf{x}) = G_{\mathcal{H}}^T(\mathbf{x})[G_{\mathcal{H}}(\mathbf{x})G_{\mathcal{H}}^T(\mathbf{x})]^{-1} \in \mathbb{R}^{k \times n}$ ,  $\eta > 0$ ,  $\beta := (\beta^{-1})^{-1} \in \mathbb{R}^{n \times n}$ ,  $\rho(\mathbf{x}, t)$  is given by Assumption 2,  $\mathbf{sign}(\mathbf{s}) = (\text{sign}(s_1), \dots, \text{sign}(s_n))^T \in \mathbb{R}^n$ , and

$$\mathbf{sig}^{2-(\frac{p}{q})}(\mathbf{x}_2) := \left( |\dot{x}_1|^{2-(\frac{p}{q})} \text{sign}(\dot{x}_1), \dots, |\dot{x}_n|^{2-(\frac{p}{q})} \text{sign}(\dot{x}_n) \right)^T \in \mathbb{R}^n.$$

Here, we summarize our main result in the following theorem.

**Theorem 1:** Suppose that System (1) and (2) experiences actuator faults at the control channel denoted by  $\mathcal{F}$  with estimated value  $\hat{\mathbf{u}}_{\mathcal{F}}$  and estimation error  $\Delta \mathbf{u}_{\mathcal{F}}$  given by (5). Then, under Assumptions 1 and 2, and controller defined by (10)–(12), the system states of System (1) and (2) can converge to the origin, i.e., the equilibrium point, in a finite amount of time.

*Proof:* Denote  $(\cdot)_i$  as the  $(i, i)$  element of a matrix. From Assumptions 1 and 2, (9)–(12), and the fact that  $\mathbf{x}_2 = (|\dot{x}_1| \text{sign}(\dot{x}_1), \dots, |\dot{x}_n| \text{sign}(\dot{x}_n))^T$ , we have

$$\begin{aligned} \mathbf{s}^T \dot{\mathbf{s}} &= \mathbf{s}^T \cdot \beta^{-1} \frac{p}{q} \text{sig}^{(\frac{p}{q})-1}(\mathbf{x}_2) \{G_{\mathcal{F}}(\mathbf{x}) \Delta \mathbf{u}_{\mathcal{F}} + \mathbf{d} \\ &\quad - [\eta + \rho(\mathbf{x}, t)] \cdot \mathbf{sign}(\mathbf{s})\} \\ &\leq \sum_{i=1}^n \left\{ \left( \beta^{-1} \frac{p}{q} \text{sig}^{(\frac{p}{q})-1}(\mathbf{x}_2) \right)_i \right. \\ &\quad \cdot |s_i| \cdot \|G_{\mathcal{F}}(\mathbf{x}) \Delta \mathbf{u}_{\mathcal{F}} + \mathbf{d}\|_{\infty} \\ &\quad \left. - [\eta + \rho(\mathbf{x}, t)] \cdot \left( \beta^{-1} \frac{p}{q} \text{sig}^{(\frac{p}{q})-1}(\mathbf{x}_2) \right)_i \cdot |s_i| \right\} \\ &\leq -\eta \cdot \Pi(\mathbf{x}_2) \cdot \|\mathbf{s}\| \end{aligned} \quad (13)$$

where

$$\Pi(\mathbf{x}_2) := \inf_{1 \leq i \leq n} \left\{ \left( \beta^{-1} \frac{p}{q} \text{sig}^{(\frac{p}{q})-1}(\mathbf{x}_2) \right)_i \right\}. \quad (14)$$

Then, from Inequality (13) and the fact that  $\mathbf{x}_2 = \mathbf{0}$  is not an attractor [54], we can conclude that the sliding mode can be reached in a finite amount of time, i.e.,  $\mathbf{s}(t) = \mathbf{0}$  for all  $t \geq t_0 + t_r$ , where  $t_0$  is the initial time and  $t_r$  is the time horizon of reaching phase. Next, when the sliding mode is achieved, it can be found from (8) that the reduced model takes the form  $\dot{\mathbf{x}}_1 = -\beta' \mathbf{sig}^{q/p}(\mathbf{x}_1)$  for all  $t \geq t_0 + t_r$ , where  $\beta' = \text{diag}\{\beta_1^{q/p}, \dots, \beta_n^{q/p}\}$  and  $\mathbf{sig}^{q/p}(\mathbf{x}_1) = (|x_1|^{q/p} \text{sign}(x_1), \dots, |x_n|^{q/p} \text{sign}(x_n))^T \in \mathbb{R}^n$ ; moreover, the largest finite time  $t_s$  that is taken to force  $\mathbf{x}_1(t_0 + t_r)$  to  $\mathbf{x}_1(t_0 + t_r + t_s) = \mathbf{0}$  is easily determined as

$$t_s = \max_{1 \leq i \leq n} \left\{ \frac{p}{(\beta_i)^{\frac{q}{p}}(p-q)} |x_i(t_0 + t_r)|^{\frac{(p-q)}{p}} \right\}. \quad (15)$$

Once  $\mathbf{x}_1 = \mathbf{0}$  is achieved, from (8), one can find that  $\mathbf{x}_2 = \mathbf{0}$  is also achieved. From the analysis previously given, we can conclude that the system state can converge to zero in a finite amount of time, i.e.,  $t_0 + t_r + t_s$ . This completes the proof. ■

## B. Nonsingular Fast Terminal Sliding-Mode Fault-Tolerant Control (NFTSM-FTC) Design

Here, the NFTSMC [60] will be employed to design the FTC law. Following the idea of NFTSMC, we introduce the sliding surface of NFTSMC as follows:

$$\mathbf{s} = \mathbf{x}_1 + \alpha^{-1} \mathbf{sig}^{\gamma}(\mathbf{x}_1) + \beta^{-1} \mathbf{sig}^{\frac{p}{q}}(\mathbf{x}_2) = \mathbf{0} \quad (16)$$

where  $\mathbf{s} \in \mathbb{R}^n$  is the sliding variable;  $\alpha^{-1} = \text{diag}\{\alpha_1^{-1}, \dots, \alpha_n^{-1}\} \in \mathbb{R}^{n \times n}$  and  $\beta^{-1} = \text{diag}\{\beta_1^{-1}, \dots, \beta_n^{-1}\} \in \mathbb{R}^{n \times n}$  are positive definite matrices, respectively;  $p$  and  $q$  are positive odd numbers satisfying the relation  $1 < p/q < 2$  and  $\gamma > p/q$ ;  $\mathbf{sig}^{p/q}(\mathbf{x}_2) := (|\dot{x}_1|^{p/q} \text{sign}(\dot{x}_1), \dots, |\dot{x}_n|^{p/q} \text{sign}(\dot{x}_n))^T \in \mathbb{R}^n$ ; and  $\mathbf{sig}^{\gamma}(\mathbf{x}_1) := (|x_1|^{\gamma} \text{sign}(x_1), \dots, |x_n|^{\gamma} \text{sign}(x_n))^T \in \mathbb{R}^n$ . In this case, the time derivative of sliding variable along the trajectories of System (6) and (7) is

$$\begin{aligned} \dot{\mathbf{s}} &= \mathbf{x}_2 + \alpha^{-1} \gamma \mathbf{sig}^{\gamma-1}(\mathbf{x}_1) \cdot \mathbf{x}_2 \\ &\quad + \beta^{-1} \frac{p}{q} \text{sig}^{(\frac{p}{q})-1}(\mathbf{x}_2) \cdot [\mathbf{f}(\mathbf{x}) + G_{\mathcal{H}}(\mathbf{x}) \mathbf{u}_{\mathcal{H}} \\ &\quad + G_{\mathcal{F}}(\mathbf{x})(\hat{\mathbf{u}}_{\mathcal{F}} + \Delta \mathbf{u}_{\mathcal{F}}) + \mathbf{d}] \end{aligned} \quad (17)$$

where

$$\begin{aligned} \mathbf{sig}^{\gamma-1}(\mathbf{x}_1) &:= \text{diag}\{|x_1|^{\gamma-1}, \dots, |x_n|^{\gamma-1}\} \in \mathbb{R}^{n \times n} \\ \text{sig}^{(\frac{p}{q})-1}(\mathbf{x}_2) &:= \text{diag}\left\{|\dot{x}_1|^{(\frac{p}{q})-1}, \dots, |\dot{x}_n|^{(\frac{p}{q})-1}\right\} \in \mathbb{R}^{n \times n}. \end{aligned}$$

Again, the total uncertainties have the form of  $G_{\mathcal{F}}(\mathbf{x}) \Delta \mathbf{u}_{\mathcal{F}} + \mathbf{d}$ ; thus, Assumption 2 is also imposed in this case because of the success of fault diagnosed. Under Assumptions 1 and 2, we design the NFTSMC-FTC law as follows:

$$\mathbf{u}_{\mathcal{H}} = \mathbf{u}_{\mathcal{H}eq} + \mathbf{u}_{\mathcal{H}re} \quad (18)$$

$$\begin{aligned} \mathbf{u}_{\mathcal{H}eq} &= -G_{\mathcal{H}}^+(\mathbf{x}) \left[ \mathbf{f}(\mathbf{x}) + G_{\mathcal{F}}(\mathbf{x}) \hat{\mathbf{u}}_{\mathcal{F}} + \beta \frac{q}{p} \mathbf{sig}^{2-(\frac{p}{q})}(\mathbf{x}_2) \right. \\ &\quad \left. + \alpha^{-1} \gamma \mathbf{sig}^{\gamma-1}(\mathbf{x}_1) \cdot \beta \frac{q}{p} \mathbf{sig}^{2-(\frac{p}{q})}(\mathbf{x}_2) \right] \end{aligned} \quad (19)$$

$$\mathbf{u}_{\mathcal{H}re} = -[\eta + \rho(\mathbf{x}, t)] \cdot G_{\mathcal{H}}^+(\mathbf{x}) \mathbf{sign}(\mathbf{s}) \quad (20)$$

where  $G_{\mathcal{H}}^+(\mathbf{x}) = G_{\mathcal{H}}^T(\mathbf{x})[G_{\mathcal{H}}(\mathbf{x})G_{\mathcal{H}}^T(\mathbf{x})]^{-1} \in \mathbb{R}^{k \times n}$ ,  $\eta > 0$ ,  $\beta := (\beta^{-1})^{-1} \in \mathbb{R}^{n \times n}$ ,  $\rho(\mathbf{x}, t)$  is given by Assumption 2,  $\mathbf{sign}(\mathbf{s}) = (\text{sign}(s_1), \dots, \text{sign}(s_n))^T \in \mathbb{R}^n$ , and

$$\mathbf{sig}^{2-(\frac{p}{q})}(\mathbf{x}_2) := \left( |\dot{x}_1|^{2-(\frac{p}{q})} \text{sign}(\dot{x}_1), \dots, |\dot{x}_n|^{2-(\frac{p}{q})} \text{sign}(\dot{x}_n) \right)^T \in \mathbb{R}^n.$$

From the aforementioned design, we have the following result.

**Theorem 2:** Suppose that System (1) and (2) experiences actuator faults at the control channel denoted by  $\mathcal{F}$  with estimated value  $\hat{\mathbf{u}}_{\mathcal{F}}$  and estimation error  $\Delta \mathbf{u}_{\mathcal{F}}$  given by (5). Then, under Assumptions 1 and 2, and controller defined by (18)–(20), the system states of System (1) and (2) can converge to the origin, i.e., the equilibrium point, in a finite amount of time.

*Proof:* The proof is similar to that of Theorem 1, except that the largest finite time  $t_s$  is taken to force  $\mathbf{x}_1(t_0 + t_r)$  to  $\mathbf{x}_1(t_0 + t_r + t_s)$  having the value of (for details, please refer to [60])

$$t_s = \max_{1 \leq i \leq n} \left\{ \frac{1}{\beta_i^{\frac{q}{p}}} \int_0^{|x_i(t_0+t_r)|} \frac{1}{(\tau - \alpha_i^{-1} \tau^{\gamma})^{\frac{q}{p}}} d\tau \right\}. \quad (21)$$

Thus, as the result of Theorem 1, one can conclude that the system state will converge to zero in a finite amount of time  $t_0 + t_r + t_s$ . The proof is completed. ■

**Remark 1:** In the NTSMC and NFTSMC design, the selection of  $p$ ,  $q$ , and  $\gamma$  should satisfy the rules  $1 < p/q < 2$  and  $\gamma > p/q$ . In general, the relative parameters in controller design can be tuned by some adaptive methods [57], [60].

#### IV. SIMULATION AND DISCUSSION

Consider the following dynamics for satellite attitude stabilization control in [2], [10], and [34], as described in the same form as (1) and (2) with  $n = 3$ , in which  $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x}))^T$ ,  $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2)^T$ ,  $\mathbf{x}_1 = (x_1, x_2, x_3)^T = (\phi, \theta, \psi)^T$ ,  $\mathbf{x}_2 = (x_4, x_5, x_6)^T = (\dot{\phi}, \dot{\theta}, \dot{\psi})^T$ ,  $\mathbf{u} = (u_1, u_2, u_3, u_4)^T$ , and  $\mathbf{d} = (d_1, d_2, d_3)^T$ . Here,  $\phi$ ,  $\theta$ , and  $\psi$  are Euler's angles for the  $x$ -,  $y$ -, and  $z$ -axes, respectively;  $\mathbf{u}$  denote actuators, providing the force torques by reaction wheels or thrusters in four directions; and  $\mathbf{f}(\mathbf{x})$  and  $G(\mathbf{x})$  are described as follows:

$$\begin{aligned} f_1(\mathbf{x}) = & \omega_0 x_6 c x_3 c x_2 - \omega_0 x_5 s x_3 s x_2 \\ & + \frac{I_y - I_z}{I_x} \left[ x_5 x_6 + \omega_0 x_5 c x_1 s x_3 s x_2 + \omega_0 x_5 c x_3 s x_1 \right. \\ & + \omega_0 x_6 c x_3 c x_1 + \frac{1}{2} \omega_0^2 s(2x_3) c^2 x_1 s x_2 \\ & + \frac{1}{2} \omega_0^2 c^2 x_3 s(2x_1) - \omega_0 x_6 s x_3 s x_2 s x_1 \\ & - \frac{1}{2} \omega_0^2 s^2 x_2 s^2 x_3 s(2x_1) \\ & - \frac{1}{2} \omega_0^2 s(2x_3) s x_2 s^2 x_1 \\ & \left. - \frac{3}{2} \omega_0^2 c^2 x_2 s(2x_1) \right] \end{aligned} \quad (22)$$

$$\begin{aligned} f_2(\mathbf{x}) = & \omega_0 x_6 s x_3 c x_1 + \omega_0 x_4 c x_3 s x_1 + \omega_0 x_6 c x_3 s x_2 s x_1 \\ & + \omega_0 x_5 s x_3 c x_2 s x_1 + \omega_0 x_4 s x_3 s x_2 c x_1 \\ & + \frac{I_z - I_x}{I_y} \left[ x_4 x_6 + \omega_0 x_4 c x_1 s x_3 s x_2 \right. \\ & + \omega_0 x_4 c x_3 s x_1 - \omega_0 x_6 s x_3 c x_2 \\ & - \frac{1}{2} \omega_0^2 s(2x_2) s^2 x_3 c x_1 \\ & - \frac{1}{2} \omega_0^2 c x_2 s x_1 s(2x_3) \\ & \left. + \frac{3}{2} \omega_0^2 s(2x_2) c x_1 \right] \end{aligned} \quad (23)$$

$$\begin{aligned} f_3(\mathbf{x}) = & \omega_0 x_4 s x_1 s x_3 s x_2 - \omega_0 x_6 c x_1 c x_3 s x_2 \\ & - \omega_0 x_5 c x_1 s x_3 c x_2 + \omega_0 x_6 s x_3 s x_1 - \omega_0 x_4 c x_3 c x_1 \\ & + \frac{I_x - I_y}{I_z} \left[ x_4 x_5 + \omega_0 x_4 c x_3 c x_1 \right. \\ & - \omega_0 x_4 s x_3 s x_2 s x_1 - \omega_0 x_5 s x_3 c x_2 \\ & - \frac{1}{2} \omega_0^2 s(2x_3) c x_2 c x_1 \\ & + \frac{1}{2} \omega_0^2 s^2 x_3 s x_1 s(2x_2) \\ & \left. - \frac{3}{2} \omega_0^2 s(2x_2) s x_1 \right] \end{aligned} \quad (24)$$

$$G(\mathbf{x}) = \begin{pmatrix} 0.67 & 0.67 & 0.67 & 0.67 \\ 0.69 & -0.69 & -0.69 & 0.69 \\ 0.28 & 0.28 & -0.28 & -0.28 \end{pmatrix}. \quad (25)$$

Here,  $I_x$ ,  $I_y$ , and  $I_z$  are the inertia with respect to the three body coordinate axes,  $\omega_0$  denotes the constant orbital rate, and  $c$  and  $s$  denote the cos and sin functions, respectively. Details of these parameters can be referred to [2]. Moreover, we adopt an FDD mechanism from [2, eq. (10) and eq. (11), p. 335] for active FTC mission. For completeness, we briefly recall the mechanism as follows. The main idea of the FDD mechanism is to decouple the control input through a coordinate transformation, so that any fault associated with a channel can be diagnosed. Following this idea, the coordinate transformation is chosen to have the form  $\mathbf{z}_1 = \mathbf{x}_1$  and  $\mathbf{z}_2 = P\mathbf{x}_2$ , where  $P := (g_1, g_2, g_3)^{-1}$ , and  $g_i$  denotes the  $i$ th column of  $G(\mathbf{x})$ . With the new state variables, (1) will be transformed as

$$\dot{\mathbf{z}}_1 = P^{-1} \mathbf{z}_2, \quad \dot{\mathbf{z}}_2 = \mathbf{f}^{\text{new}}(\mathbf{z}) + G^{\text{new}}(\mathbf{z})\mathbf{u} + P\mathbf{d} \quad (26)$$

where

$$\mathbf{f}^{\text{new}}(\mathbf{z}) = P\mathbf{f}(\mathbf{z}_1, P^{-1}\mathbf{z}_2) \quad (27)$$

$$\begin{aligned} G^{\text{new}}(\mathbf{z}) = & PG(\mathbf{z}_1, P^{-1}\mathbf{z}_2) \\ = & \begin{pmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & l_3 \end{pmatrix}. \end{aligned} \quad (28)$$

Since every three columns taken out from  $G(\mathbf{x})$  are linearly independent, all constants  $l_1$ ,  $l_2$ , and  $l_3$  are nonzero. Based on the transformed system (27), the observer and the associated residual signals  $\gamma_i$  are chosen as follows:

$$\dot{\xi}_i = f_i^{\text{new}}(\mathbf{z}) + u_i + l_i u_4 + k_i \cdot (z_{i+3} - \xi_i) \quad (29)$$

$$\gamma_i = z_{i+3} - \xi_i \quad (30)$$

for  $1 \leq i \leq 3$ , where  $k_i > 0$  for all  $1 \leq i \leq 3$ , and  $f_i^{\text{new}}(\mathbf{z})$  represents the  $i$ th entry of  $\mathbf{f}^{\text{new}}(\mathbf{z})$ . It was shown in [2] that the observer presented in (29) and (30) can achieve FDD for any single actuator fault and can estimate the output value of the faulty actuator, e.g., the  $i$ th actuator, to be  $u_i + k_i \cdot \gamma_i$ , where  $u_i$  is the designed output value of the  $i$ th actuator. With these settings, an alarm will be triggered if the magnitude of the  $i$ th residual signal  $r_i$  from the observer is greater than a threshold set by the designer. In this example, we assume that the actuator  $u_2$  fails to work at time  $t = 4$  s. In order to alleviate the chattering phenomena in the presented FTC, we



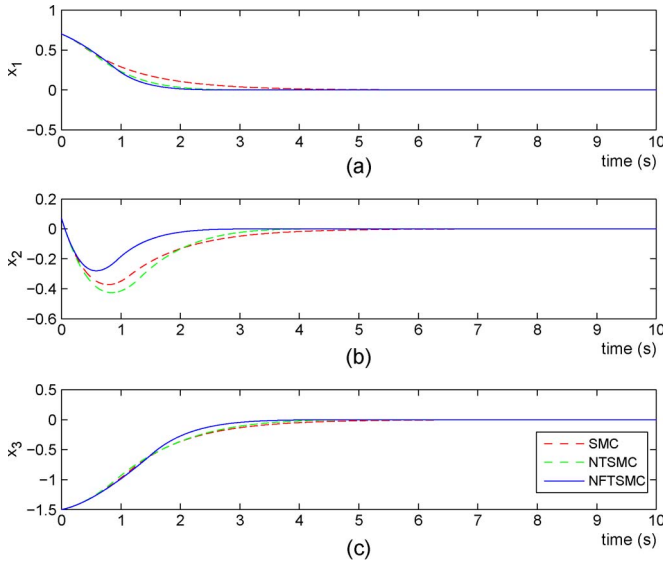


Fig. 1. Time history of system states  $x_1$ ,  $x_2$ , and  $x_3$ .

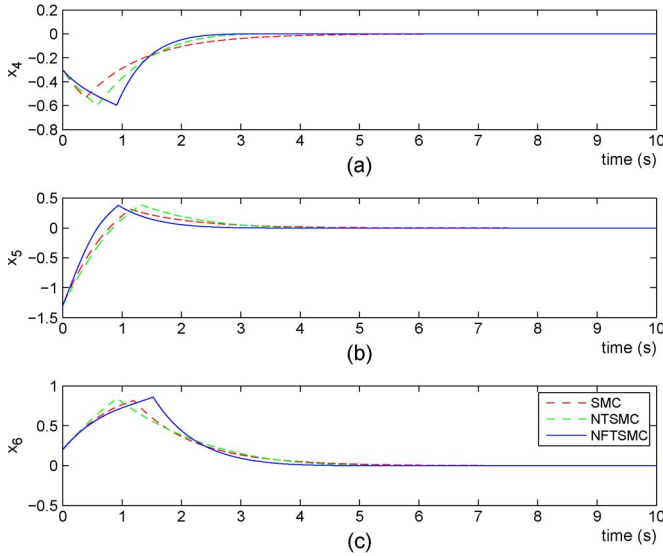


Fig. 2. Time history of system states  $x_4$ ,  $x_5$ , and  $x_6$ .

use the saturation function to replace the original ideal switching function with boundary layer to be  $\varepsilon = 0.002$ . Moreover, for demonstration, the initial condition and disturbance are set as  $\mathbf{x}(0) = (0.8, 0.15, -1.2, 0.15, -1.2, -0.3)^T$  and  $\mathbf{d} = 0.05 \cdot (\sin(t), \cos(2t), \sin(3t))^T$ , respectively. The alarm threshold for each actuator is chosen as 0.015. Other parameters are set as follows:  $p = 9$ ,  $q = 7$ ,  $\gamma = 1.4$ ,  $\alpha = \text{diag}\{1, 1, 1\}$ ,  $\beta = \text{diag}\{1.2, 0.9, 0.8\}$ ,  $\rho(\mathbf{x}, t) = \|\mathbf{d}\|_\infty := \sup_t \|\mathbf{d}\|$ , and  $\eta = 1$ .

Simulation results are shown in Figs. 1–7. In this example, we use three types of active FTC laws, including sliding-mode FTC law (labeled SMC) (which can be referred to [2]) and presented NTSMC-FTC (labeled NTSMC) and NFTSMC-FTC laws (labeled NFTSMC). In this example, Figs. 1 and 2 show the time history of six system states, while Figs. 4 and 6 show the time history of the three sliding variables and the control inputs, respectively. In Fig. 1(a)–(c), we can see that the proposed FTC laws can still make the closed-loop system achieve stabilization task under the allowable disturbances and actuator fault; moreover, Fig. 2(a)–(c) indicates that the angular

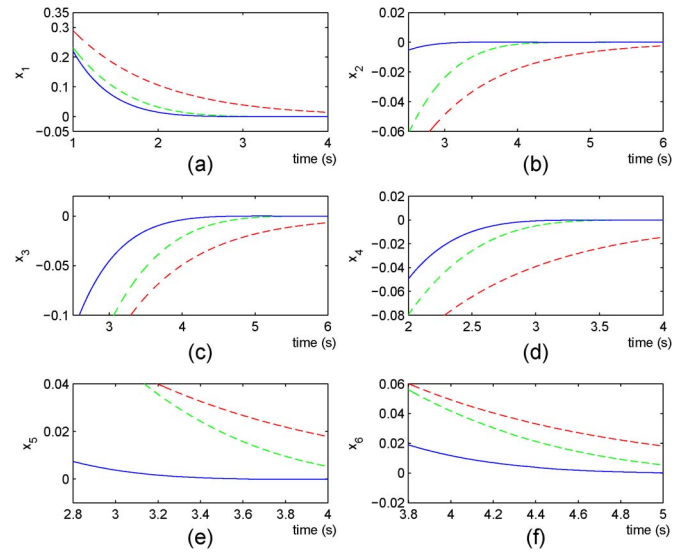


Fig. 3. Zoom in on time history of system states  $x_1$  to  $x_6$ .

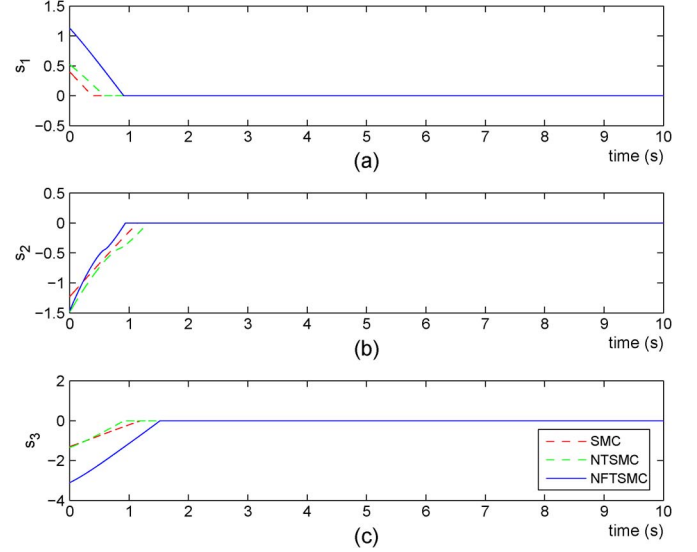


Fig. 4. Time history of the three sliding variables.

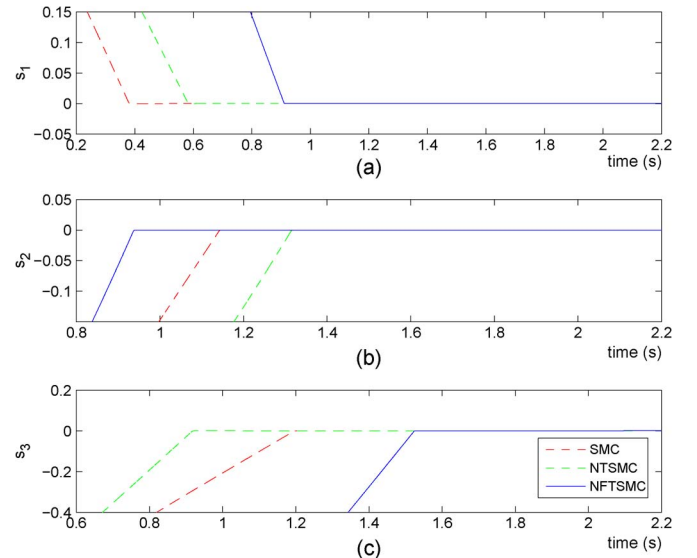


Fig. 5. Zoom in on time history of the three sliding variables.

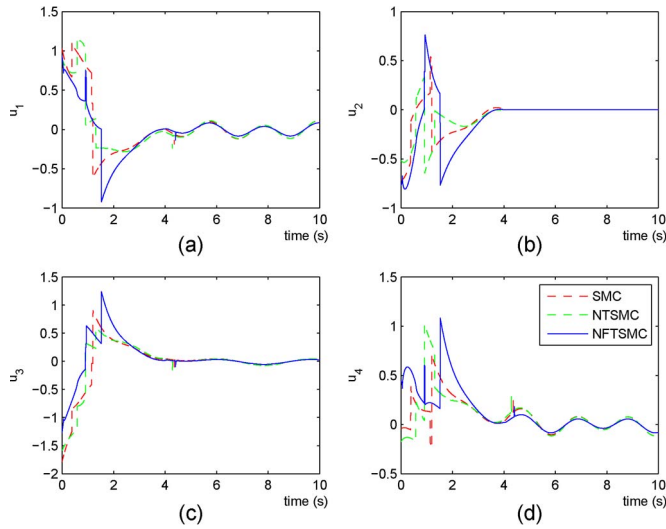


Fig. 6. Time history of the four control inputs.

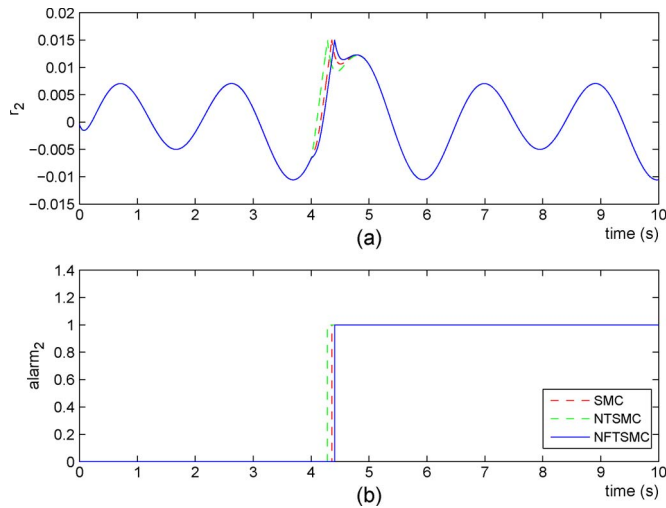


Fig. 7. (a) Residual signal  $r_2$ . (b) Alarm signal  $alarm_2$ .

speeds can also converge to zero. Fig. 3, i.e., a zoom in on Figs. 1(a)–(c) and 2(a)–(c), clearly shows that NFTSMC will result in faster state convergence time in comparison with NTSMC and SMC. This also demonstrates that NFTSMC possesses the characteristic of fast state convergence speed after the closed-loop system is in sliding mode. In Fig. 4, three FTC laws can make the system states reach the sliding surface in a finite amount of time. In Fig. 6, we can observe that  $u_2$  fails at  $t = 4$  s and its output value becomes 0; moreover, the value of the residual signal  $\gamma_2$  reaches the threshold value 0.015, which results in FDD triggering the  $alarm_2$  near  $t = 4$  s (as shown in Fig. 7). After the success of FDD, the FTC law is reconfigured, so that the control mission is continuously performed. This action corresponds to the jumping of control inputs' curves near  $t = 4$  s, as shown in Fig. 6(a), (c), and (d). From the simulation results, we can verify the effectiveness of the proposed methods. It is worth noting that the time delay of the FDD mechanism is included in this example. A more detailed discussion concerning possible effects in FTC with FDD, such as time delay effect, gyroscopic effect, and the effect by different kinds of input forms, can be found in [5], [8], [12], and [23]–[26].

## V. CONCLUSION

In this paper, new-type nonsingular terminal sliding-mode fault-tolerant control (NTSM-FTC) and nonsingular fast terminal sliding-mode fault-tolerant control (NFTSM-FTC) laws have been presented for a set of second-order nonlinear control systems. It was shown that the closed-loop system can achieve the stabilization mission, even when some of the actuators fail to operate while retaining the advantages of the traditional SMC approach. Moreover, the system states of the closed-loop system were shown to be able to converge to the equivalent point in a finite amount of time; meanwhile, the potential singularity phenomena that exist in both the traditional terminal and the faster terminal sliding-mode designs are resolved. Through application to spacecraft attitude control, the simulation results demonstrate the effectiveness of the proposed schemes.

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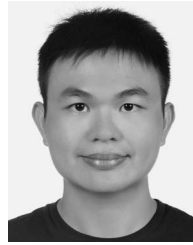
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