# A Generalized Solution to Infiltration[1]

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## ABSTRACT

**A generalized solution to the moisture flow (Richards') equation is developed for infiltration. The equation is expressed in terms of dimensionless time, depth, and water content. The reduced form applies for the hydraulic functions of van Genuchten or of Brooks and Corey as well as to other scaled forms and the solution utilizes the procedure of Philip. Answers are presented in concise tables which, in principle, allow for finding moisture profiles, wetting front, intake rate and cumulative intake for a variety of soils and for varying initial water contents. For application, specifically measured hydraulic conductivities can be best-fitted to the forms used here or generalized empirical values can be used from the literature. Thus, either specific or generic parameters may be used, depending on availability for a given problem.**

***Additional Index Words:*** **Richards equation, hydraulic conductivity, moisture flow.**

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Soil water movement for unsaturated conditions is most often modeled by Richards' equation. Due to the difficulty of solving this highly nonlinear equation, emphasis has been on techniques and algorithms. Actual calculations have been applied only for specific cases. The prevailing logic too often is that the tedious steps of the numerical solutions must always be repeated for each specific case. A contributing factor is that the necessary hydraulic functions, namely the soil water characteristic curve and the unsaturated hydraulic conductivity, are often developed in a pragmatic fashion.

The objective of this study is to find comprrehensive, numerical solutions that may be directly applied with minimal further calculations. Specifically, we will assume scaled forms applicable for similar media as well as the hydraulic functions of van Genuchten (1980) and Brooks and Corey (1964). The approach is analogous but more general than that of Warrick and Amoozegar-Fard (1979). Possible uses include repeated calculations with little effort to describe spatial variability or provide answers for general situations with minimal input data. We will address infiltration into homogeneous soils, which is initially at a uniform water content. The surface is maintained at a constant moisture level and hysteresis is ignored.

## THEORY

We consider first Richards' equation for unsaturated water flow expressed as

$$\partial\theta/\partial t = (\partial/\partial x)(D\partial\theta/\partial x) - \partial K/\partial x \qquad [1]$$

with $\theta$ the volumetric water content, $t$ time, $x$ depth, $D$ soil water diffusivity, and $K$ unsaturated hydraulic conductivity. (In the original reference, Richards used a pressure head formulation for the righthand side, but pointed out if $h$ is a single valued function of $\theta$, the choice between $h$ and $\theta$ is "simply a matter of mathematical expediency", (see p. 324, Richards, 1931.) Reduced forms $W$, $T$, $X$, $K^*$ and $h^*$ are defined without dimensions:

$$W = (\theta - \theta_r)/(\theta_s - \theta_r) \qquad [2]$$

$$T = \alpha K_s t/(\theta_s - \theta_r) \qquad [3]$$

$$X = \alpha x \qquad [4]$$

$$K^* = K/K_s \qquad [5]$$

$$h^* = \alpha h \qquad [6]$$

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where $\alpha$ is a positive scaling factor for length (units $L^{-1}$), $h$ pressure head, and $\theta_r$ the residual water content. The $\theta_s$ and $K_s$ are usually taken as saturated $\theta$ and $K$, although this is not a requirement.

Substitution of the reduced variables in Eq. [1] results in

$$\partial W/\partial T = (\partial/\partial X)(D^*\partial W/\partial X) - \partial K^*/\partial X \qquad [7]$$

with the reduced $D^*$ as

$$D^* = K^*dh^*/dW = \alpha D(\theta_s - \theta_r)/K_s. \qquad [8]$$

We will consider a large class of hydraulic functions for which reduced conductivity and pressure head are of the functional forms $K^*(W)$ and $h^*(W)$, respectively. Examples of such relationships are extensive including the three interrelated classes in Table 1: generalized similar media, a form from van Genuchten (1978) and a form due to Brooks and Corey (1964). For similar media, $h^*$ and $K^*$ are determined and site specific $h$ and $K$ related to the average relationships by the scaling coefficient $\alpha$ (for each value of $W$) by Eq. [5] and 6. The "$K_s$" at each site is equal to $\alpha^2$ times an invariant average value. Examples of recent applications are given in Miller (1980) and Warrick and Nielsen (1980).

van Genuchten (1978) assumed Form 2 of $h^*$ given in Table 1 and then used results of Mualem (1976) to calculate corresponding $K$ functions. He took

$$W = (1 + |h^*|^n)^{-m} \qquad [9]$$

and used convenient forms for which Mualem's results are in closed form. The easiest, nontrivial case is for

$$m = 1 - 1/n \qquad [10]$$

which adequately approximates many "real" soils. The connection of these forms to those of Brooks and Corey as well as to the theory of Burdine (1953) are discussed by van Genuchten.

The Brooks and Corey (1964) forms have been used extensively including recent applications by Russo and Bresler (1980, 1981) for quantifying field variability. A disadvantage of Form 3 is that the water content is taken to be constant until the air entry value is exceeded, that is, until $|h|$ is greater than the so-called bubbling pressure. This leads to nonunique functions when expressing $h$ as a function of $\theta$ (or $h^*$ as a function of $W$) i.e. for $\theta = \theta_s$, $h$ has a range of values. The usual forms are

$$h^* = W^{-n} \qquad [11]$$

$$K^* = W^{nm} \qquad [12]$$

In Table 1, we assume $m = 2 + 2/n$ an approximation discussed by Russo and Bresler and others.

Appropriate initial and boundary conditions for infiltration are

$$W(X,O) = W_i \qquad [13]$$

$$W(O,T) = W_f \qquad [14a]$$

$$\lim_{X\to\infty} W(X,T) = W_i \qquad [14b]$$

In Eq. [14] the "$f$" refers to the "final" water content, as well as the boundary value.

Following a scheme similar to that of Warrick and Amoozegar-Fard (1979), we seek a single solution of Eq. [7] which is applicable to infiltration for many soil conditions. Specific solutions are then found by simple algebra (or interpolation).

Philip's infiltration solution to Eq. [7] is of the form (dimensionless)

$$X = \lambda(W)T^{1/2} + \chi(W)T + \psi(W)T^{3/2} + \ldots \qquad [15]$$

where $\lambda(W)$, $\chi(W)$ and $\psi(W)$ and any additional terms can be determined numerically as by Philip (1968) (also, see Kirkham and Powers, 1972).

The cumulative intake $I$ is the change in storage plus the integrated deep flow rate

$$I = \int_{\theta_i}^{\theta_f} x\, d\theta + K_i t \qquad [16]$$

where subscripts $i$ and $f$ refer to initial and final value of water content and conductivity. The appropriate dimensionless intake $I^*$ is

$$I^* = \alpha I/(\theta_s - \theta_r) \qquad [17]$$

Choosing the first three terms of the series of Eq. [15], the scaled intake is approximated by

$$I^* \approx AT^{1/2} + BT + CT^{3/2} \qquad [18]$$

with

$$A = \int_{W_i}^{W_f} \lambda(W)dW \qquad [19]$$

$$B = \int_{W_i}^{W_f} \chi(W)dW + K(W_i)/K_s \qquad [20]$$

$$C = \int_{W_i}^{W_f} \psi(W)dW \qquad [21]$$

The surface intake rate is

$$v_o = dI/dt \qquad [22]$$

or

$$v_o = K_s v_o^* \qquad [23]$$

with $v_o^*$ a dimensionless surface intake rate defined as

$$v_o^* = dI^*/dT \qquad [24]$$

For $t$ large, the intake process is dominated by gravitational forces and the series of Eq. [15] will diverge. However, $v_o$ and the wetting profile are physically defined. The surface intake approaches a constant value and the moisture front attains a limiting "profile-at-infinity" which moves downward at a rate of $[K(\theta_f) - K(\theta_i)]/(\theta_f - \theta_i)$. (Values of $\theta_f$ and $\theta_i$ correspond to the "final" and "initial" water contents.) Philip (1968, p. 250), suggested the series is valid for $T < T_g$ where

$$T_g = [A/(1 - K_i/K_f)]^2 \qquad [25]$$

For $T$ greater than $T_g$, $v_o^*$ will be $K_f/K_s$, and $I^*$ will be

$$I^* = I_g^* + (T - T_g)K_f/K_s, \quad T > T_g \qquad [26]$$

with $I_g^*$ the dimensionless form of the cumulative intake at $T_g^*$. By continuity, the wetting front will correspond to

$$X_{\mathrm{front}}(T) = X_{\mathrm{front}}(T_g) + (T - T_g)(K_f - K_i)/[K_s(W_f - W_i)] \qquad [27]$$

## RESULTS

The solution of Eq. [7] subject to constant initial and boundary conditions is by Philip (1968) as described in detail by Kirkham and Powers (1972). Results are presented here for three examples using Forms 2 and 3 of Table 1. The more general Form 1 is illustrated for a specific 150-ha field by Warrick and Amoozegar (1979) both for infiltration and drainage examples. Table 2 contains results for Eq. [19]-[21] and [25] using van Genuchten's (1978) relationships from Philip's solution for $n$ = 1.1, 1.25, 1.5, 1.75, 2.0, and 2.5 and $W_i$ = 0, 0.1, 0.2 and 0.3. Calculations are for 50 steps (i.e. 50 increments between $W_i$ and $W_f$) chosen after preliminary runs made in Example 1. In order to get reliable results for as few as 25 to 50 points, it was necessary to recognize the singularity of $D^*$ as

**Table 1. Three classes of hydraulic functions considered. Forms 2 and 3 are subclasses of 1.**

| | $\lvert h^*(W)\rvert$ | $K^*(W)$ |
|---|---|---|
| 1. Similar media† | Arbitrary | Arbitrary |
| 2. van Genuchten (1978) | $(W^{-n/(n-1)}-1)^{1/n}$ | $W^{0.5}[1-(1-W^{n/(n-1)})^{1-1/n}]^2$ |
| 3. Brooks and Corey (1964) | $W^{-n}$, $\lvert h^*\rvert > 1$ | $W^{2+2n}$ |
| | 1, $\lvert h^*\rvert < 1$ | |

† Eq. [5] and [6] are satisfied for each medium by $K$ and $h$. Site specificity is through $\alpha$ and by $K_s$ which is proportional to $\alpha^2$.

$W$ approaches 1 (see van Genuchten, 1978, esp. Eq. [11]):

$$D^*(W) = [(1 - m)/m]\, W^{0.5-1/m}\, [(1 - W^{1/m})^{-m} + (1 - W^{1/m})^{m} - 2]\,. \qquad [28]$$

At first we used as an approximation for the average $D$ over the "wettest" interval simply as the midpoint value. The results were dependent on number of steps up to at least 1600. This problem was alleviated by expanding Eq. [28] and integrating to approximate the average $D^*$ between $W = 1 - \Delta W$ and 1 as

$$(1/\Delta W) \int_{1-\Delta W}^{1} D^* dW \approx (1/my) + (1/m - 1)\{y/(1 + m) - 2 + (2 - m)\Delta W\, [y/(m + 2) - 1]/2m + \Delta W/(2my)\} \qquad [29]$$

where $y = (\Delta W/m)^m$.

Dimensioned forms $A'$, $B'$ and $C'$ (shown by primes) are

$$A'/A = [(\theta_s - \theta_r)K_s/\alpha]^{0.5} \qquad [30]$$

$$B'/B = K_s \qquad [31]$$

$$C'/C = [\alpha K_s^3/(\theta_s - \theta_r)]^{0.5} \qquad [32]$$

where $A$, $B$, and $C$ are from the table.

Also included in Table 3 are values of $\lambda$, $\chi$, and $\Psi$ for $W^* = 0.25$, 0.5 and 0.75 for several $n$ and $W_i$, where $W^* = (W - W_i)/(1 - W_i)$. The dimensional equivalents $\lambda'$, $\chi'$ and $\psi'$ satisfy

$$\lambda'/\lambda = [K_s/\alpha(\theta_s - \theta_r)]^{0.5} \qquad [33]$$

$$\chi'/\chi = K_s/(\theta_s - \theta_r) \qquad [34]$$

$$\psi'/\psi = \alpha^{0.5}[K_s/(\theta_s - \theta_r)]^{1.5} \qquad [35]$$

The use of Tables 2 and 3 will be illustrated in Examples 1 and 2. Example 3 will make use of the Brooks and Corey form and utilize values of $A$, $B$ and $C$ from Table 4.

### Example 1—Hypothetical Loam Soil

As the first example we compare results with a finite element solution by van Genuchten of the U.S. Salinity Laboratory (Personal communication). The step-by-step procedure necessary to determine the moisture front for $t < t_g$ is

1. Choose values of $K_s$, $\theta_s$, $\theta_r$, $\theta_i$, $\alpha$ and $n$.
2. Calculate $W_i$ from Eq. [2] and dimensionless $T$ of interest from Eq. [3].
3. From Table 3, find the appropriate $n$ and $W_i$ and read three values for $\lambda$, three for $\chi$ and three for $\psi$. The first set of values for $\lambda$, $\chi$ and $\psi$ defines the $X$ coordinate for $W^* = 0.25$ by Eq. [15]; the second set defines $X$ for $W^* = 0.5$ and the third for $W^* = 0.75$.
4. Calculate three $X$-coordinates for each moisture content by Eq. [15]. (For $t > t_g$, use Eq. [27] rather than [15].)
5. Convert the three pairs of values of $W^*$ and $X$ from Step 3 and 4 to real $\theta$ and $x$ by Eq. [4]. Use $W^* = (W - W_i)/(1 - W_i)$ along with Eq. [2] and [4].

**Table 2. Infiltration coefficients based on the van Genuchten (1980) forms for the unsaturated hydraulic functions.**

| $n$ | $W_i$ | $A$ | $B$ | $C$ | $T_g$ | $K_i^*$ |
|---|---|---|---|---|---|---|
| 1.1 | 0.0 | 0.306 | 0.272 | 0.193 | 0.094 | 0 |
| | 0.1 | 0.276 | 0.261 | 0.225 | 0.076 | $10^{-25}$ |
| | 0.2 | 0.248 | 0.249 | 0.264 | 0.061 | $10^{-18}$ |
| | 0.3 | 0.221 | 0.234 | 0.309 | 0.049 | $10^{-14}$ |
| 1.25 | 0.0 | 0.443 | 0.204 | 0.172 | 0.196 | 0 |
| | 0.1 | 0.418 | 0.199 | 0.181 | 0.174 | $10^{-12}$ |
| | 0.2 | 0.391 | 0.194 | 0.191 | 0.153 | $10^{-9}$ |
| | 0.3 | 0.364 | 0.189 | 0.203 | 0.133 | $10^{-7}$ |
| 1.5 | 0.0 | 0.656 | 0.257 | 0.133 | 0.430 | 0 |
| | 0.1 | 0.621 | 0.257 | 0.140 | 0.385 | $10^{-8}$ |
| | 0.2 | 0.584 | 0.257 | 0.148 | 0.341 | $10^{-6}$ |
| | 0.3 | 0.544 | 0.258 | 0.158 | 0.296 | $10^{-5}$ |
| 1.75 | 0.0 | 0.791 | 0.318 | 0.124 | 0.625 | 0 |
| | 0.1 | 0.749 | 0.320 | 0.131 | 0.561 | $10^{-6}$ |
| | 0.2 | 0.704 | 0.322 | 0.139 | 0.496 | $10^{-5}$ |
| | 0.3 | 0.656 | 0.326 | 0.148 | 0.431 | 0.0004 |
| 2.0 | 0.0 | 0.883 | 0.367 | 0.119 | 0.780 | $10^{-36}$ |
| | 0.1 | 0.836 | 0.370 | 0.126 | 0.699 | $10^{-5}$ |
| | 0.2 | 0.786 | 0.374 | 0.134 | 0.618 | $10^{-4}$ |
| | 0.3 | 0.733 | 0.378 | 0.143 | 0.538 | 0.0012 |
| 2.5 | 0.0 | 1.002 | 0.435 | 0.113 | 1.000 | $10^{-35}$ |
| | 0.1 | 0.949 | 0.439 | 0.119 | 0.900 | $10^{-4}$ |
| | 0.2 | 0.892 | 0.444 | 0.126 | 0.797 | 0.0078 |
| | 0.3 | 0.831 | 0.448 | 0.133 | 0.697 | 0.0038 |

For the loam soil chosen, values for $K_s$, $\theta_s$, $\theta_r$, $\theta_i$, $\alpha$, and $n$ are $6(10)^{-4}$ cm/s, 0.45, 0.1, 0.17, 0.01, and 2, respectively. For Step 2 above $W_i$ is $(0.17 - 0.1)/(0.45 - 0.1) = 0.2$. For $t = 1.5$ and 5 $h$, the corresponding $T$ are 0.093 and 0.309.

For Step 3 with $n = 2$ and $W_i = 0.2$, we read 9 values from Table 3. These include 3 values of $\lambda$, 3 values of $\chi$ and 3 for $\psi$: 1.128, 1.058, 0.909, 0.419, . . . 0.165. For Step 5, the dimensionless $X$ for $W^* = 0.25$ (or $W = 0.4$) is by Eq. [15]:

$$X(W) = X(0.4) = 1.128\, T^{0.5} + 0.419\, T + 0.146\, T^{1.5}\,.$$

For $T = 0.093$, $X$ is 0.387 and for $T = 0.309$, $X$ is 0.782. Similarly $X(0.6)$ and $X(0.8)$ are calculated giving a total of three points for each time. The corresponding real $\theta$'s and $x$'s are then calculated using Eq. [2] and [4] (Step 5).

The values obtained from the above procedure are plotted as triangles in Fig. 1A. Also shown are points using data not contained in Table 3 (dots) and a curve showing results by van Genuchten using a finite element solution (personal communication).

To calculate infiltration curves, Steps 1 and 2 above are repeated, followed by

3′. From Table 2, find the appropriate $n$ and $W_i$ and read the corresponding $A$, $B$, $C$ and $T_g$.
4′. Calculate dimensionless $I^*$ by Eq. [18] and/or $v_o^*$ by Eq. [24]. (If $T > T_g$, use $v_o^* = 1$ and/or [Eq.] 26 for $I^*$).
5′. Calculate dimensional I or $v_o$ by Eq. [17] or [23].

**Table 3. Values of $\lambda$, $\chi$ and $\Psi$ defining moisture profiles for $W^* = 0.25, 0.5$, and $0.75$ where $W^* = (W - W_i)/(1 - W_i)$. These are based on the van Genuchten relationships.**

| | | $\lambda$ | | | $\chi$ | | | $\Psi$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | $W_i$ | $W^* = 0.25$ | 0.50 | 0.75 | $W^* = 0.25$ | 0.50 | 0.75 | $W^* = 0.25$ | 0.50 | 0.75 |
| 1.1 | 0.0 | 0.314 | 0.314 | 0.312 | 0.266 | 0.266 | 0.269 | 0.200 | 0.200 | 0.200 |
| | 0.1 | 0.316 | 0.316 | 0.313 | 0.282 | 0.282 | 0.288 | 0.258 | 0.258 | 0.255 |
| | 0.2 | 0.321 | 0.320 | 0.316 | 0.299 | 0.300 | 0.309 | 0.336 | 0.336 | 0.334 |
| | 0.3 | 0.329 | 0.328 | 0.320 | 0.317 | 0.319 | 0.334 | 0.443 | 0.443 | 0.445 |
| 1.25 | 0.0 | 0.475 | 0.472 | 0.446 | 0.185 | 0.187 | 0.208 | 0.160 | 0.161 | 0.171 |
| | 0.1 | 0.503 | 0.498 | 0.463 | 0.198 | 0.202 | 0.229 | 0.183 | 0.186 | 0.202 |
| | 0.2 | 0.537 | 0.527 | 0.483 | 0.213 | 0.220 | 0.256 | 0.213 | 0.218 | 0.244 |
| | 0.3 | 0.579 | 0.562 | 0.504 | 0.232 | 0.245 | 0.292 | 0.250 | 0.261 | 0.301 |
| 1.5 | 0.0 | 0.725 | 0.710 | 0.642 | 0.229 | 0.239 | 0.280 | 0.119 | 0.121 | 0.130 |
| | 0.1 | 0.772 | 0.748 | 0.665 | 0.250 | 0.266 | 0.317 | 0.137 | 0.140 | 0.154 |
| | 0.2 | 0.828 | 0.791 | 0.691 | 0.278 | 0.302 | 0.364 | 0.159 | 0.165 | 0.186 |
| | 0.3 | 0.895 | 0.840 | 0.720 | 0.316 | 0.352 | 0.426 | 0.188 | 0.198 | 0.232 |
| 1.75 | 0.0 | 0.884 | 0.857 | 0.763 | 0.284 | 0.301 | 0.355 | 0.113 | 0.112 | 0.118 |
| | 0.1 | 0.941 | 0.901 | 0.790 | 0.315 | 0.340 | 0.404 | 0.129 | 0.129 | 0.140 |
| | 0.2 | 1.008 | 0.951 | 0.819 | 0.355 | 0.392 | 0.465 | 0.150 | 0.152 | 0.171 |
| | 0.3 | 1.088 | 1.008 | 0.853 | 0.409 | 0.461 | 0.544 | 0.177 | 0.183 | 0.217 |
| 2.0 | 0.0 | 0.990 | 0.955 | 0.846 | 0.331 | 0.354 | 0.413 | 0.110 | 0.107 | 0.111 |
| | 0.1 | 1.054 | 1.003 | 0.876 | 0.369 | 0.402 | 0.469 | 0.127 | 0.124 | 0.134 |
| | 0.2 | 1.128 | 1.058 | 0.909 | 0.419 | 0.464 | 0.539 | 0.146 | 0.145 | 0.165 |
| | 0.3 | 1.215 | 1.120 | 0.947 | 0.487 | 0.546 | 0.628 | 0.171 | 0.174 | 0.210 |
| 2.5 | 0.0 | 1.120 | 1.077 | 0.957 | 0.401 | 0.429 | 0.489 | 0.107 | 0.100 | 0.105 |
| | 0.1 | 1.191 | 1.130 | 0.992 | 0.449 | 0.488 | 0.552 | 0.122 | 0.115 | 0.127 |
| | 0.2 | 1.273 | 1.191 | 1.031 | 0.513 | 0.564 | 0.631 | 0.139 | 0.134 | 0.158 |
| | 0.3 | 1.370 | 1.260 | 1.077 | 0.598 | 0.661 | 0.728 | 0.160 | 0.161 | 0.202 |

For Step 3′, the $A$, $B$, and $C$ from Table 2 are 0.786, 0.374 and 0.134. (The dimensionalized $A'$, $B'$, and $C'$ are not required, but can be evaluated by Eq. [33]-[35] to give 6.83 cm $h^{-0.5}$, 0.808 cm $h^{-1}$ and 0.0719 cm $h^{-1.5}$). The infiltration rate $v_o$ is plotted as Fig. 1B and in agreement with the finite element and conventional, semianalytical (Philip's) results provided by van Genuchten. On the basis of the Fig. 1 comparison, we assumed the Philip algorithm was correctly programmed.

## Example 2—Yolo Light Clay

Moore (1939) determined soil water characteristics and unsaturated K for the Yolo light clay (and 3 other soils). Philip (cf. 1968) and Haverkamp et al. (1977) made calculations based on Moore's measurements.

A suitable $n$ value was chosen by plotting log $(K/K_s)$ for Moore's data (his Table 5) as a function of $W$ and comparing to van Genuchten's form from Table 1. A value of $n = 2$ is taken based on this comparison. A single quantitative judgment, based for example on least squares, is not obvious as the values match exactly at $K/K_s = 1$. Intuitively the "wet" end for simulation of infiltration is expected to be more critical than for dry conditions. In addition, the value of $\alpha$ chosen after assuming a value of $n$ somewhat compensates for the choice of $n$. The value of $\alpha = 0.015$ $cm^{-1}$ was chosen after assuming $n = 2$.

**Table 4—Infiltration coefficients based on Brooks and Corey relationships.**

| | | $n$ (dimensionless) | | | | | |
|---|---|---|---|---|---|---|---|
| $W_i$ | | 2 | 4 | 6 | 8 | 10 | 12 |
| 0.001 | $A$ | 0.939 | 1.11 | 1.18 | 1.23 | 1.26 | 1.28 |
| ($K_i^* < 10^{-5}$) | $B$ | 0.313 | 0.282 | 0.268 | 0.260 | 0.257 | 0.255 |
| | $C$ | 0.118 | 0.0963 | 0.0868 | 0.0809 | 0.0765 | 0.0734 |
| | $T_g$ | 0.784 | 0.906 | 0.956 | 0.983 | 0.998 | 1.01 |
| | $I_g^*$ | 1.16 | 1.39 | 1.40 | 1.55 | 1.59 | 1.61 |
| 0.1 | $A$ | 0.884 | 1.04 | 1.12 | 1.16 | 1.19 | 1.21 |
| ($K_i^* < 10^{-5}$) | $B$ | 0.319 | 0.285 | 0.270 | 0.262 | 0.257 | 0.255 |
| | $C$ | 0.125 | 0.103 | 0.0928 | 0.0866 | 0.0821 | 0.0785 |
| | $T_g$ | 0.720 | 0.826 | 0.870 | 0.892 | 0.906 | 0.914 |
| | $I_g^*$ | 1.06 | 1.26 | 1.35 | 1.41 | 1.44 | 1.46 |
| 0.2 | $A$ | 0.825 | 0.979 | 1.05 | 1.09 | 1.12 | 1.14 |
| ($K_i^* < 10^{-4}$) | $B$ | 0.327 | 0.290 | 0.273 | 0.264 | 0.258 | 0.255 |
| | $C$ | 0.134 | 0.110 | 0.0999 | 0.0935 | 0.0888 | 0.0852 |
| | $T_g$ | 0.656 | 0.744 | 0.781 | 0.801 | 0.812 | 0.819 |
| | $I_g^*$ | 0.955 | 1.13 | 1.21 | 1.26 | 1.28 | 1.30 |
| 0.5 | $A$ | 0.605 | 0.742 | 0.809 | 0.847 | 0.873 | 0.890 |
| ($K_i^*$ as | $B$ | 0.366 | 0.319 | 0.293 | 0.278 | 0.269 | 0.262 |
| specified) | $C$ | 0.174 | 0.147 | 0.134 | 0.125 | 0.120 | 0.116 |
| | $T_g$ | 0.480 | 0.502 | 0.515 | 0.523 | 0.527 | 0.532 |
| | $I_g^*$ | 0.653 | 0.738 | 0.781 | 0.805 | 0.821 | 0.832 |
| | $K_i^*$ | 0.0156 | <0.001 | <0.0001 | <0.0001 | <0.0001 | <0.0001 |

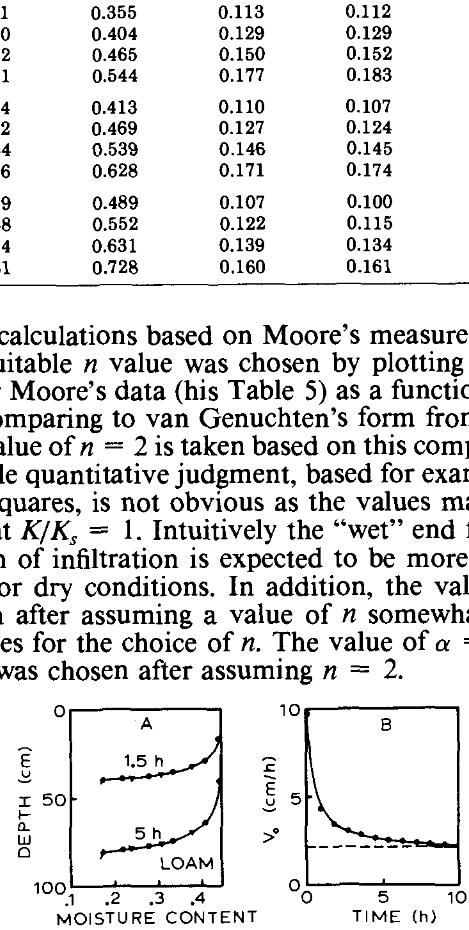


**Fig. 1—Moisture profile and infiltration rate for hypothetical loam soil comparing results from the van Genuchten functions with finite element model (personal communication from M.Th. van Genuchten). In part B, the dashed line shows the steady-state value.**

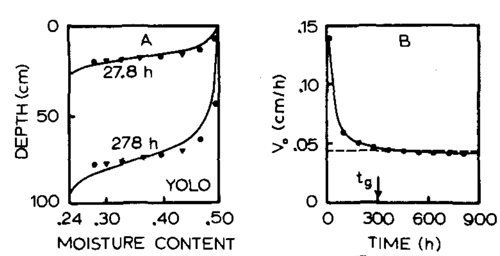


**Fig. 2—Moisture profile and infiltration rate for Yolo light clay. The solid line calculations are by Haverkamp et al. (1977), the individual points are based on the van Genuchten functions. In part B, the dashed line shows the steady-state value.**

The moisture profiles are given as Fig. 2A. The three triangles on each curve are values derived from Table 3. The appropriate values of $\lambda$, $\psi$, and $\chi$ are 1.215, 0.487, 0.171 for $W$ = 0.475 (i.e. $W^*$ = 0.25); 1.12, 0.546, and 0.174 for W = 0.65; 0.947, 0.628, and 0.210 for $W$ = 0.825. The calculated values are slightly "flatter" than the Haverkamp et al. results and the positions are slightly behind the earlier results. The infiltration relationship is shown as Fig. 2B and shows close agreement with the Haverkamp et al., results.

### Example 3—Sand

Haverkamp et al. (1977) experimentally determined values for the soil moisture release and unsaturated hydraulic conductivity curves for a sand used in a laboratory experiment. Using the Brooks-Corey (BC) method (Table 4), as an approximation for the soil properties, we found values of $n$ = 4.0 and $\alpha$ = 0.07 $cm^{-1}$ are consistent with the experimental data as well as the general values given by Clapp and Hornberger (1978). The value for $W_i$ is 0.13 which corresponds to $\theta_r$ = 0.075 and $\theta_s$ = 0.267.

Moisture profiles are given in Fig. 3A. The calculated profiles have approximately the same shape in the wetter region but become "flatter" when at the front. The intake rate calculated from Eq. [22] is given in Fig. 3B. The values are slightly larger for all times than those of Haverkamp et al., (1977). The time, $t_g$, where gravity effects dominate, calculated from Eq. [3] and [25], is 0.21 h. At lower moisture content, the diffusivity calculated with the BC method is smaller than the value calculated by Haverkamp et al., (1977) (see Figure 3C). At higher moisture content, the BC form chosen gives larger values which result in the larger infiltration rates and extended profile. This explains, in part, the inconsistencies between the results.

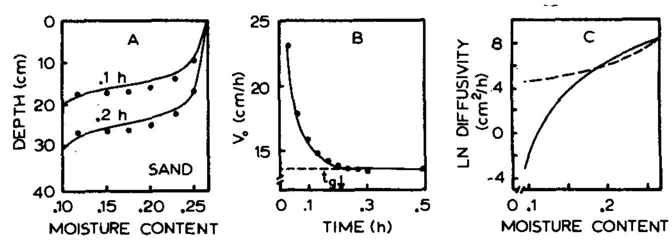


**Fig. 3—Infiltration rate, moisture profile and soil water diffusivity for the sand of Haverkamp et al. (1977). The individual points are based on the Brooks and Corey functions of Table 1. In part B, the dashed line shows the steady-state value. For part C, the solid line is for the Brooks-Corey approximation.**

## DISCUSSION AND CONCLUSIONS

The generalized results are based on Richards' equation and are exact for the specified parameters. The approach could be used for methods of solution other than that of Philip, but his algebraic forms are particularly easy to apply. Possibly, the same approach could also prove useful for describing redistribution or cyclic water inputs even though the associated numerical calculations would, by necessity, be nonanalytical.

For field situations where only scant information is available, textural classes (or any supplemental information such as sorptivity or $\theta$ vs. $h$) may be used to estimate infiltration and wetting depths. Although limited with respect to uncertainty of input, the solutions would remain based on sound physical principles. If a great deal of information is known regarding soil properties, the results may be applied to systematically describe heterogeneity. Sources of useful information may include data and collations by Holtan et al. (1968), Clapp and Hornberger (1978), Brakensiek (1979), McCuen et al. (1981), Nielsen et al. (1983) and Rawls and Brakensiek (1983). The Brooks-Corey form of the hydraulic functions were used to describe heterogeneity over 2 m by 2 m and over 0.8 ha by Russo and Bresler (1980, 1981).

In addition, results should prove useful for sufficiency checks of numerical solutions. The easy-to-use tabulated results of Table 2, 3, and 4 can be used to conveniently check complex computer algorithms for special cases.


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