1. With years counted since 1900, US census data indicates the following population for the US.

0	76.0
20	105.7
40	131.7
60	179.7
80	226.5
100	281.4

Here the figures in the right column are reported in millions. Use this data to "predict" what the population was in 2020 by two means: **(a)** fitting the data with an interpolating polynomial and **(a)** fitting with the best line in the least squares sense. Which "prediction" compares best the actual population in 2020 (about 331 million)?

3. Let $A \in \mathbb{R}^{10\times n}$ be a 10-by-n matrix, with $n \leq 10$, formed as follows. The n columns of A are the first n columns of the 10-by-10 Hilbert matrix $H =$ `hilb(10)`. That is, `A = H(:,1:n)` in MATLAB. Consider the vector $\mathbf{c} = [1, \dots, 1]^T$ of n ones, and set $\mathbf{b} = A\mathbf{c}$. With this $\mathbf{b}$ we know that the *overdetermined* system $A\mathbf{x} = \mathbf{b}$ has a unique solution, namely $\mathbf{x} = \mathbf{c}$.

(a) Use the normal equations to solve the least squares problem $A\mathbf{x} = \mathbf{b}$ for (i) $n = 6$ and (ii) $n = 8$ and compare the (numerically generated) least squares solution with the exact solution. How many correct decimal places can be computed? Use the condition number of $A^T A$ to explain the results.

(b) Repeat part **(a)**, but now solve by QR-factorization. Use the thick factorization from MATLAB's `qr`, also with `UTriSol` available from `UNM Learn`. You will have to truncate the upper triangular matrix from `qr` to get a square system. Also list the condition number of this truncated matrix. Compare your results with those of **(a)**.

3. Use the Givens rotation approach to compute (by hand) the QR factorization of the following matrix.

$$\begin{pmatrix} -4 & -4 \\ -2 & 7 \\ 4 & -5 \end{pmatrix}$$