1. The nonlinear system

$$f_1(u,v) = 6u^3 + uv - 3v^3 - 4 = 0, \qquad f_2(u,v) = u^2 - 18uv^2 + 16v^3 + 1 = 0$$

has $(u,v) = (1,1)$ as one solution. Use Newton's method to find two other solutions, reporting each to 8 digit accuracy.

2. The three-point Gauss-Legendre formula for approximating integrals has the form

$$\int_a^b f(x)dx \simeq h\left[w_1 f(a+c_1h) + w_2 f(a+c_2h) + w_3 f(a+c_3h)\right], \qquad h = b-a.$$

The vector $(w_1, w_2, w_3, c_1, c_2, c_3)^T$ of weights and points satisfies the nonlinear system

$$\begin{aligned}
1 &= w_1 + w_2 + w_3 \\
\frac{1}{2} &= w_1c_1 + w_2c_2 + w_3c_3 \\
\frac{1}{3} &= w_1c_1^2 + w_2c_2^2 + w_3c_3^2 \\
\frac{1}{4} &= w_1c_1^3 + w_2c_2^3 + w_3c_3^3 \\
\frac{1}{5} &= w_1c_1^4 + w_2c_2^4 + w_3c_3^4 \\
\frac{1}{6} &= w_1c_1^5 + w_2c_2^5 + w_3c_3^5.
\end{aligned}$$

These conditions ensure that over the interval $[0,1]$ the rule integrates exactly all polynomials of degree at most 5.

(a) Numerically solve the system using Newton's method. *Hint:* for the initial guess choose $w_1 = w_2 = w_3 = \frac{1}{3}$ and $c_1 = 0, c_2 = \frac{1}{2}, c_3 = 1$.

(b) Using your results from **(a)**, approximate the integral

$$\int_0^{\pi/4} e^x \cos(5x)dx.$$

Compare with the exact answer.