1. Find the multiplicity of the root $r = 0$ for $f(x) = \sin 2x - 2x$. Find the forward and backward errors of the approximate root $r_A = 0.005$.

2. Use the sensitivity formula to approximate the root nearest to 4 of

$$f(x) = (x-1)(x-2)(x-3)(x-4) - 10^{-6}x^6.$$

Find the error magnification factor. Use `fzero` to check your approximation.

3. (a) Use `fzero` to find the root of $f(x) = 2x\cos x - 2x + \sin x^3$ on $[-0.1, 0.2]$. Report the forward and backward errors. **(b)** Run the Bisection Method with starting interval $[-0.1, 0.2]$ to find as many correct digits as possible, and report your conclusion. Can you explain the results?

4. Big-O notation. By definition, $f(\varepsilon) = O(\psi(\varepsilon))$ as $\varepsilon \to 0$ means there exists positive constants K and η such that $|\varepsilon| < \eta \implies |f(\varepsilon)| \le K|\psi(\varepsilon)|$. [Note: in numerical analysis ε is $\varepsilon_{\text{mach}}$, and we are interested only in the one-sided limit $\varepsilon \to 0^+$. The definition above is easily adjusted to this case.] Similarly, $g(x) = O(\varphi(x))$ as $x \to \infty$ means there exists constants C and ρ such that $x > \rho \implies |g(x)| \le C|\varphi(x)|$. Are the following statements **true** or **false**? Justify your answers.

(a) $\sin(\varepsilon) = O(1)$, as $\varepsilon \to 0$

(b) $\sin(\varepsilon) = O(\varepsilon)$, as $\varepsilon \to 0$

(c) $\sin(\varepsilon) = O(\varepsilon^2)$, as $\varepsilon \to 0$

(d) $100x = O(x^{1.1})$, as $x \to \infty$.

Hint: You may use the fact that $|\sin(\varepsilon)/\varepsilon| \le 1$.

5. Consider the problem $y = h(x_1, x_2) = x_1x_2$ of multiplying two *nonzero* real numbers. On a computer the algorithm for approximating the product is $y_A = h_A(x_1, x_2) = \text{fl}(x_1) \odot \text{fl}(x_2)$, where $\odot$ is the inexact floating point multiplication. Show that the algorithm is backwards stable. That is, show $h_A(x_1, x_2) = h(x_1 + \delta x_1, x_2 + \delta x_2)$, where $|\delta x_1/x_1| = O(\varepsilon_{\text{mach}})$ and $|\delta x_2/x_2| = O(\varepsilon_{\text{mach}})$. Is the perturbation of the inputs unique?