1. Find the machine representation for the floating point number fl(100.2), and express the answer in hex format. Give an exact decimal expression for fl(100.2) and verify that

$$\frac{|\mathrm{fl}(100.2) - 100.2|}{|100.2|} < \varepsilon_{\mathrm{mach}},$$

where $\varepsilon_{\mathrm{mach}} = 2^{-52}$ is machine epsilon.

2. Explain how to most accurately compute the two roots of the equation $x^2 + 201x - 10^{-12} = 0$, and use MATLAB to compute the roots using your method.

3. The zeroth order cylindrical Bessel function has the Maclaurin series

$$J_0(x) = \sum_{k=0}^{\infty} (-1)^k \frac{1}{(k!)^2} \left(\frac{x}{2}\right)^{2k}.$$

To observe a Bessel function, excite a wave on the surface of your morning cup of coffee. $J_0(x)$ models the wave height as a function of radius.

(a) Write a MATLAB function `ApproxBesselJ0(x)` to approximate $J_0(x)$ as a Maclaurin-Taylor polynomial,

$$J_0(x) \simeq \sum_{k=0}^{N} (-1)^k \frac{1}{(k!)^2} \left(\frac{x}{2}\right)^{2k},$$

where x may be a scalar argument in your function. Your function should determine the minimum truncation N to ensure that the size $(N!)^{-2}|x/2|^{2N}$ of the last term is less than 10^{-8}. For full credit your function must be efficient and use recursion (but no need to use Horner's method).

(b) Prepare a table which for $x = 5, 20, 45$ compares your approximation of $J_0(x)$ with the value returned by MATLAB's built-in `besselJ(0,x)`. Is your function as accurate as you would expect based on the truncation error?

(c) Given an explanation for what you observe in **(b)**. *Hint:* For each $x = 5, 20, 45$ plot the terms

$$a_k = \frac{1}{(k!)^2} \left(\frac{x}{2}\right)^{2k}$$

which alternate in the series as a function of the index k. Note that the truncated sum can be expressed as the difference of two positive numbers.