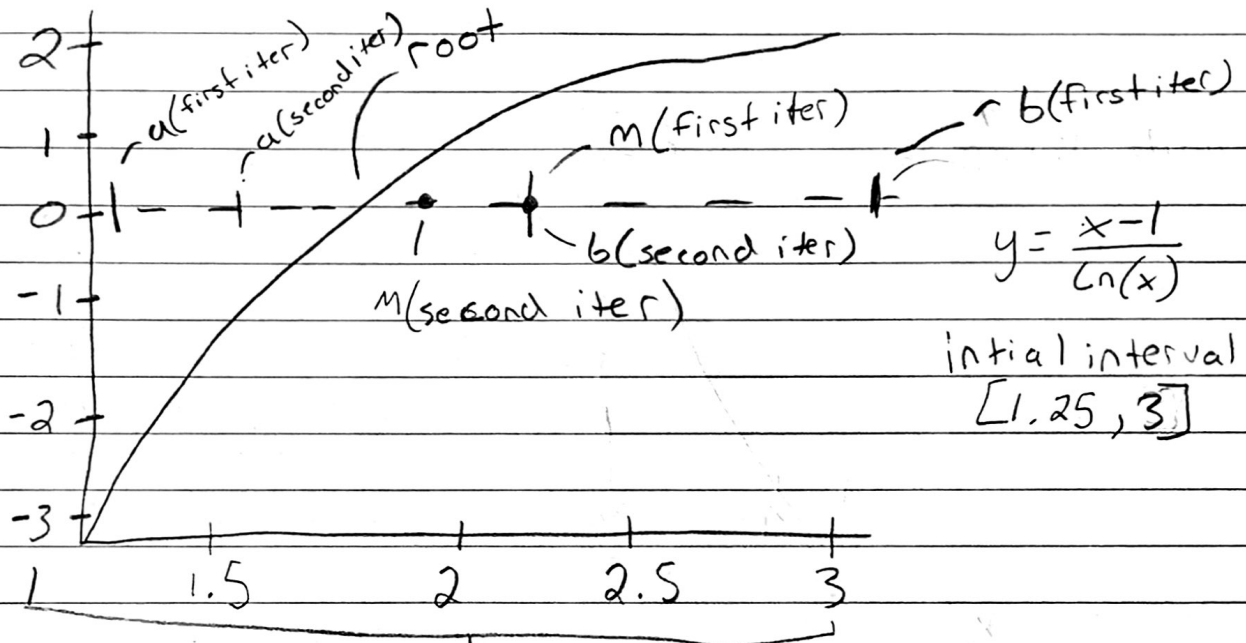


# Exam 1

1a)



First Iter

$$f(x) = \frac{x-1}{\ln(x)}$$

$$a = 1.25, b = 3$$

initial interval

$$m = \frac{a+b}{2} = 2.125$$

$$f(m) \approx 1.49$$

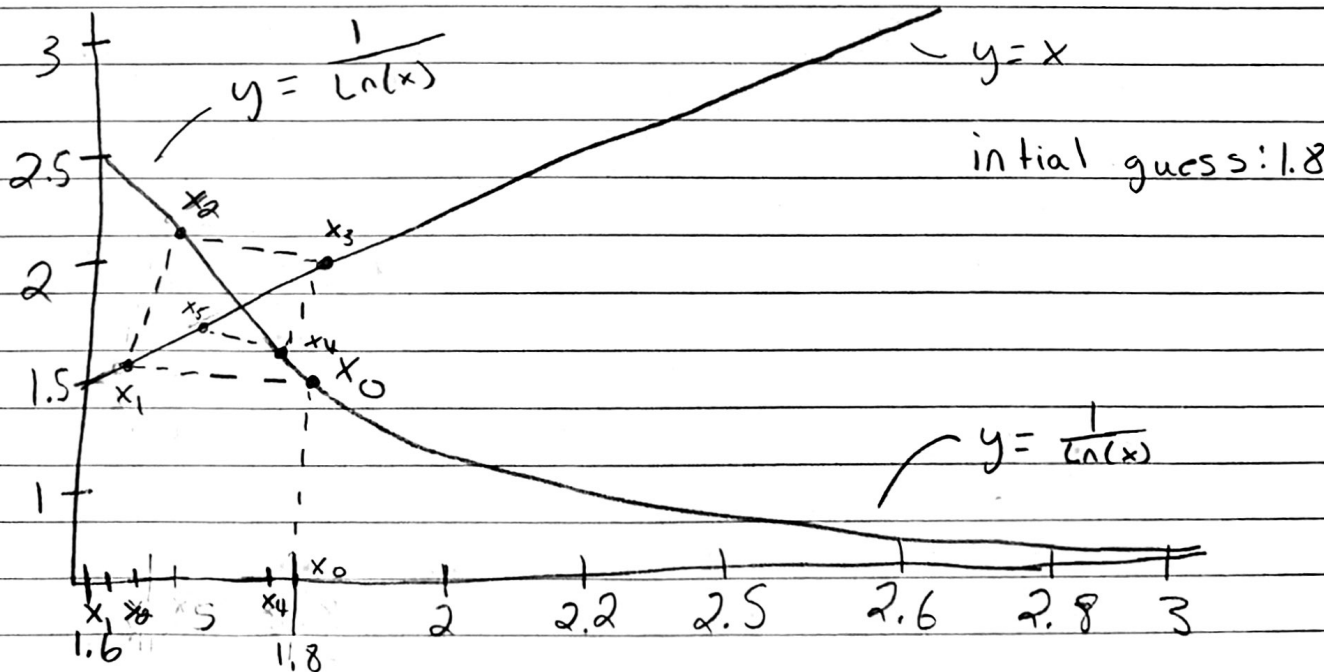
Second iter:

$$a = 1.49$$

$$m = 1.8075$$

$$b = 2.125$$

1b)



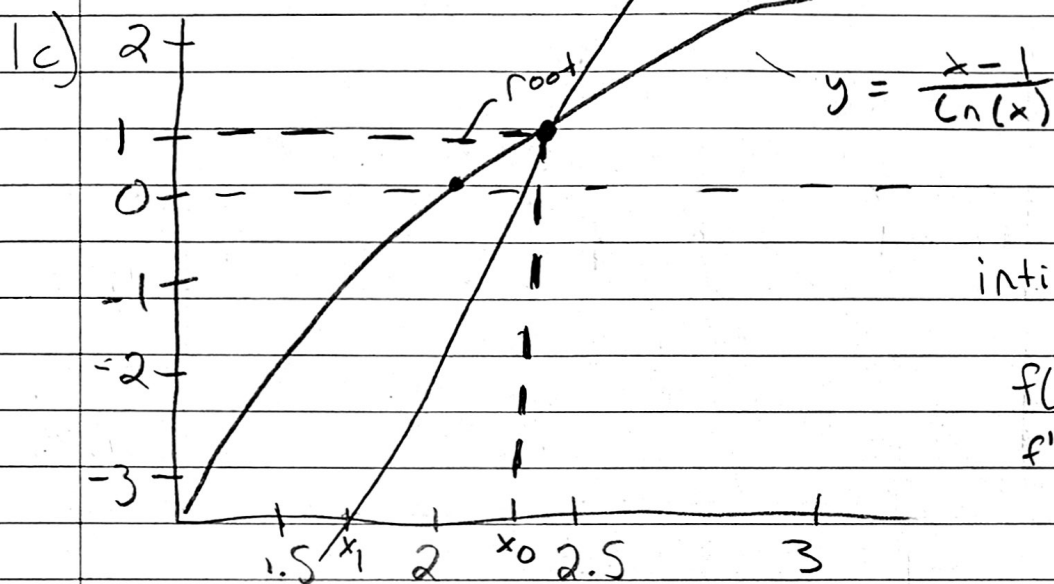
$$f(x) = \frac{1}{\ln(x)}$$

$$f'(x) = -\frac{1}{x \ln^2(x)}$$

$$x_0 = 1.8$$

$$x_1 = \frac{1}{\ln(1.8)} = 1.701$$

$$x_2 = \frac{1}{\ln(1.701)} = 1.8824$$

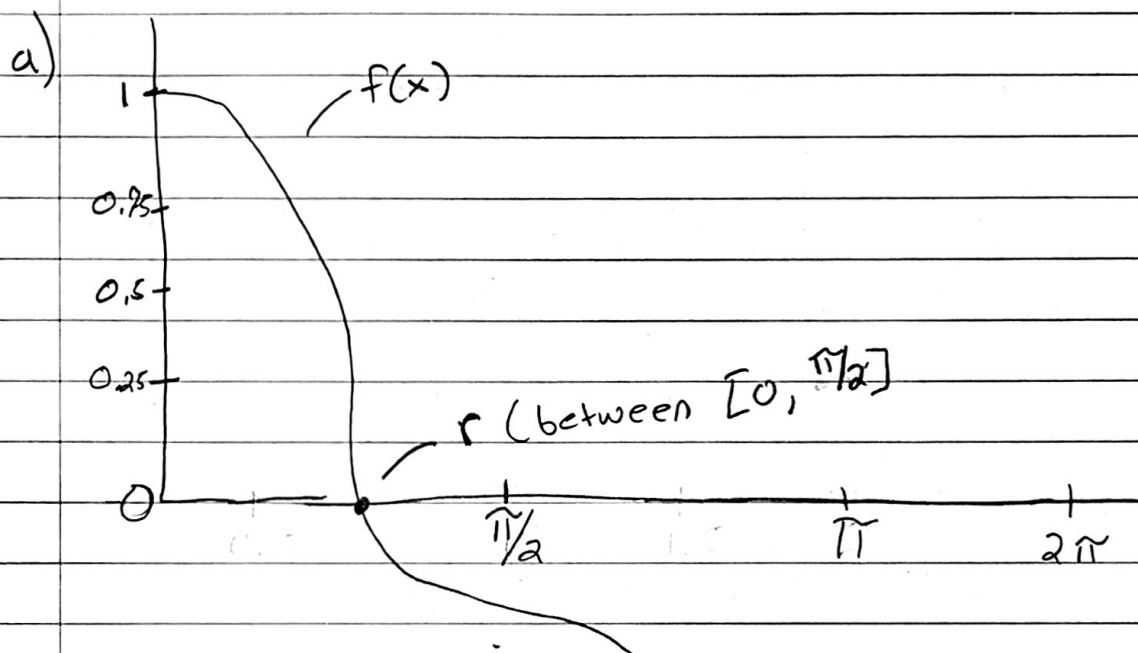


$$x_0 = 2.25$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.25 - \frac{f(2.25)}{f'(2.25)} = 1.789$$

$$2) f(x) = g(x) - x = \cos(x) - x$$

$$g(x) = \cos(x)$$



b)  $x_{k+1} = g(x_k)$

$$0 < r < \pi/2$$

$$g(x) = \cos(x)$$

$$g'(x) = -\sin(x)$$

$$0 < |-\sin(x)| < \pi/2$$

$$\text{Let } x = \pi$$

$$0 < |-\sin(x)| < \pi/2$$

$$0 < |-0.055| < \pi/2$$

$$0 < 0.055 < \pi/2 \quad \checkmark$$



converges locally at  $r$

$$3) f(x) = \sin(x) - x$$

$$a) \text{ Root } r=0 \\ \text{ Multiplicity} = 1$$

$$b) \text{ forward error} = \left| \frac{r_A - r}{r} \right| \\ \text{backward error} = |f(r_A)|$$

$$r_A = 0.001$$

$$\text{forward} = \left| \frac{0.001 - 0}{0} \right| \\ = \text{undefined}$$

$$\text{backward} = |f(0.001)| \\ = |\sin(0.001) - 0.001| \\ \approx \underline{9.825 \times 10^{-4}}$$

$$4) \frac{2}{3} = 0.\overline{66666}$$

$$a) \begin{aligned} 0.\overline{66} &= (\overline{10})_2 \\ 0 &= (0)_2 \end{aligned}$$

$$0.\overline{66} = (0.\overline{10})_2$$

$$V = (-1)^s (0.b_1 \dots b_{52}) \cdot 2^{F-1023}$$

$$\begin{aligned} \text{True exp} &= 6 & s &= 0 \\ F &= 1029 \end{aligned}$$

$$b) fl(\frac{2}{3}) = (0. \begin{array}{cccc} 1010 & 1010 & 1010 & 1010 \\ 1010 & 1010 & 1010 & 1010 \\ 1010 & 1010 & 1010 & 1010 \end{array}) \cdot 2^6$$


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$$W = (0.\overline{1010}) \cdot 2^6$$

$$5) f(x) = \ln(3-x) + x - 2$$

$$e_{14} = |x_{14} - 2| = 10^{-4}$$

$$e_{15} = |x_{15} - f| \text{ ???}$$

$$e_{k+1} \simeq M e_k^2$$

$$r = -2$$

$$M = \left| \frac{f''(r)}{2f'(r)} \right| = \left| \frac{f''(-2)}{2f'(-2)} \right| = \left| \frac{-0.04}{-0.8} \right| = 0.05$$

$$f'(x) = -\frac{1}{-x+3} + 1$$

$$f''(x) = -\frac{1}{(-x+3)^2}$$

$$e_{15} \simeq M e_{14}^2$$

$$\simeq 0.05 \cdot (10^{-4})^2$$

$$\boxed{\simeq 5 \times 10^{-9}}$$