

**1. (a)** Use the Gram-Schmidt procedure to find (by hand) a “thick”  $QR$  factorization for the matrix in the following least squares problem:

$$\begin{pmatrix} -4 & -4 \\ -2 & 7 \\ 4 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \\ 0 \end{pmatrix}.$$

Compare your answer with the factorization returned by `qr` in MATLAB. You should use `rats` on the MATLAB output. Explain any differences that you observe. Is a thick  $QR$  factorization unique?

**(b)** Use your factorization to solve (by hand) the least squares problem. Also report the length of the minimum residual, *without explicitly forming the residual*.