1. In HOMEWORK #1, we approximated $f'(x)$ for $f(x) = e^{\sin x}$ and $x = \pi$ using the one-sided stencil

$$f'(x) \simeq \frac{f(x+h) - f(x)}{h},$$

with $h = 10^{-1}, 10^{-2}, \ldots, 10^{-9}$. As observed, the error in the approximation got smaller as h decreased. However, eventually a value of h was reached, after which the error actually got worse as h was decreased even further. Assuming double precision arithmetic, estimate the *optimal value* of h for which the stencil above best approximates $f'(x)$. How does your estimate compare with the results from HOMEWORK #1? *Hint:* Assume that the computer does not perform a function evaluation $f(\xi)$ exactly, rather it computes $\hat{f}(\xi) = f(\xi) + \delta$, where δ is a number which depends on ξ and is about $\varepsilon_{\text{mach}}$ in size.

2. Use the bisection method and the MATLAB function `fzero` to compute all three real numbers x satisfying $e^{x-2} + x^3 - x$. For each of the three roots and each method, use a tolerance of 10^{-8} and list both your initial approximation (or interval in the case of bisection) and the number of iterations needed. Also print at least nine digits for each approximate root.

3. Find each fixed point and decide whether Fixed Point Iteration is locally convergent to it: **(a)** $g(x) = x^2 - \frac{3}{2}x + \frac{3}{2}$, **(b)** $g(x) = x^2 + \frac{1}{2}x - \frac{1}{2}$.

4. Assume that $\varphi(x)$ is a continuously differentiable function and that $x = \varphi(x)$ has exactly three fixed points, -3, 1, and 2. Assume that (i) $\varphi'(-3) = 2.4$ and (ii) fixed point iteration started sufficiently near the fixed point 2 converges to 2. What is $\varphi'(1)$? Justify your answer graphically.