1. For each linear system and approximate solution, (i) compute the ∞-norm condition number of the system's coefficient matrix, and (ii) find the relative forward error, relative backward error, and error magnification factor.
(a) $x_1 + 2x_2 = 3$ and $2x_1 + 4.01x_2 = 6.01$ for $\mathbf{x}_A = (-100, 52)^T$
(b) $x_1 - 2x_2 = 3$ and $3x_1 - 4x_2 = 7$ for $\mathbf{x}_A = (-2, -3)^T$

2. Let
$$A = \begin{pmatrix} 10^{-16} & 1 \\ 1 & 3 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$
Here we will see the effect of using a tiny matrix element as a pivot. **(a)** By hand solve the system $A\mathbf{x} = \mathbf{b}$ exactly, and then write your answer in a form where it is clear what good approximate values of x_1 and x_2 are. **(b)** In MATLAB enter the matrix A, and then type `cond(A)` to compute the 2-norm condition number of A. Would you describe this matrix as well conditioned or ill conditioned? **(c)** Solve this system numerically using `GE`, `LTriSol`, and `UTriSol` from the website (that is by Gaussian elimination without pivoting). Compare your numerical results with those from part **(a)**. **(d)** Repeat part **(c)**, this time using `A\b` (which does use partial pivoting) as the numerical solver.

3. For $n = 50, 100, 200, 300, 400$, do the following. Let A be the n-by-n matrix with entries $A_{ij} = 1/(|i-j|^2 + 1)$. With `repmat` in MATLAB, you can construct this matrix in two lines. Define $\mathbf{x}_{\text{exact}} = (1, 1, \ldots, 1)^T$, and then compute $\mathbf{b} = A\mathbf{x}_{\text{exact}}$. Using MATLAB's backslash, *numerically* solve the equation
$$A\mathbf{x} = \mathbf{b},$$
thereby producing a *computed* solution $\mathbf{x}_A$. In exact arithmetic $\mathbf{x}_A = \mathbf{x}_{\text{exact}}$ of course, but in IEEE double precision $\mathbf{x}_A$ will not equal $\mathbf{x}_{\text{exact}}$. The quantity $\|\mathbf{x}_A - \mathbf{x}_{\text{exact}}\|_\infty$ is the forward error. Construct a table which, for each n, collects the relative forward error, relative backward error, magnification factor, and (infinity-norm) condition number of A. Discuss your results.