$$0 \mid 76.0$$

$$1.485$$

$$20 \mid 105.7 \qquad -4.6 \times 10^{-3}$$

$$1.3 \qquad 5.35 \times 10^{-4}$$

$$40 \mid 131.7 \qquad 2.75 \times 10^{-2} \qquad -1.27 \times 10^{-5}$$

$$2.4 \qquad -4.83 \times 10^{-4} \qquad 2.12 \times 10^{-7}$$

$$60 \mid 174.4 \qquad -1.5 \times 10^{-3} \qquad 8.45 \times 10^{-6}$$

$$2.3 \qquad 1.93 \times 10^{-4}$$

$$80 \mid 226.5 \qquad 1.01 \times 10^{-2}$$

$$2.745$$

$$100 \mid 281.4$$

$$P(x) = 1.485(x) - 4.6 \times 10^{-3}(x)(x-20) + 5.35 \times 10^{-4}(x)(x-20)(x-40) - 1.27 \times 10^{-5}(x)(x-20)(x-40)(x-60) + 2.12 \times 10^{-7}(x)(x-20)(x-40)(x-60)(x-80)$$

$$P(100) \approx 4383\,52.43$$

$1) = \{(0, 76.0), (20, 105.7), (40, 131.7), (60, 179.7),$
$(80, 226.5), (100, 281.4)\}$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 20 & 400 & 8\times10^3 & 1.6\times10^5 \\ 1 & 40 & 1.6\times10^3 & 6.4\times10^4 & 2.56\times10^6 \\ 1 & 60 & 3.6\times10^3 & 2.16\times10^5 & 1.296\times10^7 \\ 1 & 80 & 6.4\times10^3 & 5.12\times10^5 & 4.096\times10^7 \\ 1 & 100 & 1\times10^4 & 1\times10^6 & 1\times10^8 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 76.0 \\ 105.7 \\ 131.7 \\ 179.7 \\ 226.5 \\ 281.4 \end{bmatrix}}_{b}$$

$Ax = b$

$A^T \cdot A \cdot x^* = A^T \cdot b$

$$\begin{bmatrix} 6 & 300 & 2.2\times10^4 & 1.8\times10^6 & 1.5664\times10^8 \\ 300 & 2.2\times10^4 & 1.8\times10^6 & 1.5664\times10^8 & 1.416\times10^{10} \\ 2.2\times10^4 & 1.8\times10^6 & 1.5664\times10^8 & 1.416\times10^{10} & 1.31296\times10^{12} \\ 1.8\times10^6 & 1.5664\times10^8 & 1.416\times10^{10} & 1.31296\times10^{12} & 1.23936\times10^{14} \\ 1.5664\times10^8 & 1.416\times10^{10} & 1.31296\times10^{12} & 1.23936\times10^{14} & 1.18523\times10^{16} \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} = \begin{bmatrix} 1001 \\ 64424 \\ 5163520 \\ 445457600 \\ 40160416000 \end{bmatrix}$$

$c_1 = 76.33328$
$c_2 = 1.53180$
$c_3 = -0.01459$
$c_4 = 0.00040$
$c_5 = -2.09797 \times 10^{-6}$

$p(x) = 76.33328 + 1.53180(x) - 0.01459x^2 + 0.004x^3 - 2.09797\times10^{-6}x^4$

$p(120) = 6527.018$

$$V = \begin{bmatrix} -4 & -4 \\ -2 & 7 \\ \boxed{14} & -5 \end{bmatrix} \qquad G_3 G_2 G_1 V = R$$

$$G_3 G_2 G_1 = Q$$

G1) $Q(-2, 4)\begin{pmatrix} -2 \\ 4 \end{pmatrix} = \dfrac{1}{\sqrt{12}}\begin{pmatrix} -2 & 4 \\ -4 & -2 \end{pmatrix}\begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 10/\sqrt{3} \\ 0 \end{pmatrix}$

$G_1 \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2/\sqrt{12} & 4/\sqrt{12} \\ 0 & -4/\sqrt{12} & -2/\sqrt{12} \end{pmatrix}\begin{pmatrix} -4 & -4 \\ -2 & 7 \\ 4 & -5 \end{pmatrix} = \begin{pmatrix} -4 & -4 \\ \boxed{10/\sqrt{3}} & -17/\sqrt{3} \\ 0 & -3\sqrt{3} \end{pmatrix}$

G2) $Q(-4, 10/\sqrt{3})\begin{pmatrix} -4 \\ 10/\sqrt{3} \end{pmatrix} = \dfrac{1}{\sqrt{148/3}}\begin{pmatrix} -4 & 10/\sqrt{3} \\ -10/\sqrt{3} & -4 \end{pmatrix}\begin{pmatrix} -4 \\ 10/\sqrt{3} \end{pmatrix} = \begin{pmatrix} 2\sqrt{37}/\sqrt{3} \\ 0 \end{pmatrix}$

$G_2 \longrightarrow \begin{pmatrix} -4/\sqrt{148/3} & 10\sqrt{3}/\sqrt{148/3} & 0 \\ -10\sqrt{3}/\sqrt{148/3} & -4/\sqrt{148/3} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -4 & -4 \\ 10\sqrt{3} & -17/\sqrt{3} \\ 0 & -3\sqrt{3} \end{pmatrix}$

$$\Downarrow$$

$$\begin{pmatrix} 2\sqrt{37}/\sqrt{3} & -61/\sqrt{111} \\ 0 & -54/\sqrt{37} \\ 0 & \boxed{-3\sqrt{3}} \end{pmatrix}$$

G3) $Q(-54/\sqrt{37}, -3\sqrt{3})\begin{pmatrix} -54/\sqrt{37} \\ -3\sqrt{3} \end{pmatrix} = \dfrac{1}{\sqrt{3360/37}}\begin{pmatrix} -54/\sqrt{37} & -3\sqrt{3} \\ 3\sqrt{3} & -54/\sqrt{37} \end{pmatrix}\begin{pmatrix} -54/\sqrt{37} \\ -3\sqrt{3} \end{pmatrix} \Downarrow$

$$\begin{pmatrix} 261\sqrt{15} \\ 4\sqrt{518} \\ 0 \end{pmatrix}$$

$G_3 \longrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & -54/\sqrt{37}/\sqrt{3360/37} & -3\sqrt{3}/\sqrt{3360/37} \\ 0 & 3\sqrt{3}/\sqrt{3360/37} & -54/54/\sqrt{3360/37} \end{pmatrix}$

$$Q = \begin{pmatrix} 2\sqrt{37}/\sqrt{3} & -61/\sqrt{111} \\ 0 & -261\sqrt{15}/4\sqrt{518} \\ 0 & 0 \end{pmatrix}$$

$$R/6_3\cdot 6_2\cdot 6_1 = \begin{pmatrix} -2\sqrt{3}/\sqrt{37} & 5/\sqrt{111} & 10/\sqrt{111} \\ -9\sqrt{15}/2\sqrt{518} & -19\sqrt{3}/2\sqrt{2590} & 109\sqrt{3}/4\sqrt{2590} \\ -3\sqrt{5}/4\sqrt{14} & 3\sqrt{7}/2\sqrt{10} & 3/2\sqrt{70} \end{pmatrix}$$

Approximate Q & R:

$$Q \approx \begin{pmatrix} 6 & -3 \\ 0 & -9 \\ 0 & 0 \end{pmatrix}$$

$$R \approx \begin{pmatrix} -6.67 & 6.67 & 3.34 \\ -3.34 & -6.67 & 6.67 \\ 6.67 & 3.34 & 6.67 \end{pmatrix}$$