

$$y = h(x_1, x_2) = x_1 \cdot x_2$$

$$\begin{aligned} y_A = h_A(x_1, x_2) &= f_1(x_1) \odot f(x_2) \\ &= x_1(1+\alpha) \odot x_2(1+\beta) \\ &= x_1 \cdot x_2(1+\alpha)(1+\beta)(1+\gamma) \\ &= \left| \frac{x_1 \cdot x_2(1+\alpha)(1+\beta)(1+\gamma) - x_1 \cdot x_2}{x_1 \cdot x_2} \right| \\ &= |(1+\alpha)(1+\beta)(1+\gamma) - 1| = O(\epsilon_{mach}) \end{aligned}$$



$$\begin{aligned} \delta x_1 &= x_1 [(1+\alpha)(1+\beta)(1+\gamma) - 1] \\ \delta x_2 &= x_2 [(1+\alpha)(1+\beta)(1+\gamma) - 1] \end{aligned}$$

$$\begin{aligned} h_A x_1 &= x_2 \cdot x_1 (1+\alpha)(1+\beta)(1+\gamma) \\ &= x_2 [x_1(1+\alpha)(1+\beta)(1+\gamma) - x_1 + x_1] \\ &= x_2 (x_1 + \delta x_1) \\ &= \underline{h(x_1 + \delta x_1)} \end{aligned}$$

$$\begin{aligned} h_A x_2 &= x_1 \cdot x_2 (1+\alpha)(1+\beta)(1+\gamma) \\ &= x_1 [x_2(1+\alpha)(1+\beta)(1+\gamma) - x_2 + x_2] \\ &= x_1 (x_2 + \delta x_2) \\ &= \underline{h(x_2 + \delta x_2)} \end{aligned}$$

$$\begin{aligned} \left| \frac{\delta x_1}{x_1} \right| &= \left| \frac{x_1 [(1+\alpha)(1+\beta)(1+\gamma) - 1]}{x_1} \right| \\ &= (1+\alpha)(1+\beta)(1+\gamma) - 1 \end{aligned}$$

$$O(\epsilon_{mach}) = (1+\alpha)(1+\beta)(1+\gamma) - 1 \quad \boxed{\checkmark}$$

$$\begin{aligned} \left| \frac{\delta x_2}{x_2} \right| &= \left| \frac{x_2 [(1+\alpha)(1+\beta)(1+\gamma) - 1]}{x_2} \right| \\ &= (1+\alpha)(1+\beta)(1+\gamma) - 1 \end{aligned}$$

$$O(\epsilon_{mach}) = (1+\alpha)(1+\beta)(1+\gamma) - 1 \quad \boxed{\checkmark}$$

$$h_A(x_1, x_2) = h(x_1 + \delta x_1, x_2 + \delta x_2)$$

$$h_A x_1 = h(x_1 + \delta x_1) \quad h_A x_2 = (x_2 + \delta x_2)$$

$$h(x_1 + \delta x_1, x_2 + \delta x_2) = h(x_1 + \delta x_1, x_2 + \delta x_2) \quad \boxed{\checkmark}$$