Please label all plots and tables and include any MATLAB scripts and functions with your solution.

1. The following *finite difference quotient*

$$\frac{f(x+h)-f(x)}{h}$$

is a one-sided formula which approximates the derivative $f'(x)$ of a differentiable function $f(x)$. Write a MATLAB function, with calling sequence (`fp` for f prime) `fp = onesidediff(f,x,h)`, which evaluate this quotient on a *vector* `x`, given a scalar h and a general function `f` passed using the `@` notation. Use this MATLAB function to approximately compute the derivative of $f(x) = \exp(\sin x)$ on $[0, 2\pi]$, using `x = linspace(0,2*pi,100)` and `h = 1e-4`. Prepare the following plot with two subplots, using the `subplot(2,1,1)` and `subplot(2,1,2)` commands. The top plot should depict the function on $[0, 2\pi]$, and the bottom plot the error (in absolute value) between the one-sided approximation and exact derivative on $[0, 2\pi]$.

2. For the same function and h in the range $[10^{-1}, 10^{-2}, \ldots, 10^{-9}]$, use your `onesidediff` to approximate $f'(\pi)$ (the exact value is obviously -1). Make a table which lists h in the first column, the one-sided approximation in the second column, and the error in the one-sided approximation in the third column. Report h in scientific notation which a minimum number of displayed digits. Report errors in absolute value using scientific notation keeping 3 digits past the decimal. Approximations should be reported in fixed-point non-scientific notation with a full field of digits (say 14 past the decimal point). Plot the set of errors versus h. What do you observe?

3. Here we approximate $f'(x)$ for a periodic function $f(x)$ defined on $[0, 2\pi]$ using a *nodal Fourier method* (please don't worry if you have never heard of such). Define a grid of x values via

$$x_k = 2\pi(k-1)/N, \qquad \text{for } k = 1, 2, \cdots, N.$$

Assume throughout that N is an even integer. Define a column vector $\boldsymbol{f}$ with components $f_k = f(x_k)$, and a *derivative matrix* $D \in \mathbb{R}^{N\times N}$ with entries

$$d_{jk} = \begin{cases} \frac{1}{2}(-1)^{j+k-2}\cot(\pi(j-k)/N) & \text{if } j \neq k \\ 0 & \text{if } j = k. \end{cases}$$

The kth component g_k of $\boldsymbol{g} = D\boldsymbol{f}$ then approximates the derivative value $f'(x_k)$. A MATLAB function `FourierDerivativeMatrix` which returns D (for N even) is posted on UNM Learn.

For $f(x) = \exp(\sin x)$ use `FourierDerivativeMatrix` to approximate $f'(\pi)$ as the component

$$\boldsymbol{g}(1 + N/2) = g_{1+N/2}.$$

That is, approximate $f'(\pi)$ as the $(1+N/2)$st component of $D\boldsymbol{f}$. For $N = 4, 8, 12, 16, 20, 24, 28, 32, 36$ compute the error (in absolute value) between the approximation and the exact answer (still -1). Plot the set of errors versus N. What do you observe?