1. Consider the data $\{(0,4),(\pi/2,-1),(\pi,-2)\}$. Construct the corresponding interpolant with the following basis functions: $\{1, \cos x, \sin x\}$. This will be a *trigonometric polynomial.* Evaluate your interpolating function at $x = \pi/4$.

2. Consider the data set $\{(-1,8),(2,-1),(3,4)\}$ **(a)** Find the polynomial which interpolates the data using the monomial, Newton, and Lagrange approaches. **(b)** Show that the three polynomial representations you found in **(a)** are all the same polynomial.

3. The following MATLAB function

```
  function c=interpnewt(x,y)
% function c=interpnewt(x,y)
% computes coefficients c of Newton interpolant through (x_k,y_k), k=1:length(x)
n=length(x);
for k=1:n-1
   y(k+1:n)=(y(k+1:n)-y(k))./(x(k+1:n)-x(k));
end
c=y;
```

uses Newton divided differences to compute the expansion coefficients for an interpolating polynomial with respect to the Newton basis. Use this function along with the routine

```
  function p = hornernewt(c,x,z)
% function p = hornernewt(c,x,z)
% Uses Horner method to evaluate in nested form a polynomial defined
% by coefficients c and shifts x. Polynomial is evaluated at z.
n = length(c); % 1 + degree of polynomial.
p = c(n);
for k = n-1:-1:1
    p = p.*(z-x(k))+c(k);
end
```

for polynomial evaluation to find and plot the polynomials which interpolate the following data sets: $\{(x_j, y_j) : j = 1, \ldots, n\}$, where $y_j = f(x_j)$ with $f(x) = 1/(x^2+1)$.

1. $x_j = -4 + 8\dfrac{j-1}{n-1}$, $n = 11$. (uniformly space points)
2. $x_j = 4\cos\dfrac{\pi(2j-1)}{2n}$, $n = 11$. (Chebyshev points)

In your `hornernewt(c,x,z)` evaluation routine `x` should be the (coarse) set of interpolation points, whereas `z` should be the (fine) set of evaluation points. Choose, say, 500 uniformly spaced points `linspace(-4,4,500)` for `z`. You will then plot `p` versus `z`. Also include on your plot (as, say, red crosses) the data points defining the interpolant. So to make the first plot, you might use the following (but make sure your plots have labels and appropriate fonts):

```
% Get the data and construct the polynomial interpolant.
n = 11; ind = transpose([1:n]); x = -4+8*(ind-1)/(n-1); y = 1./(1+x.^2);
c=interpnewt(x,y);
plot(x,y,'rx')
hold on

% Plot the interpolant.
z = transpose(linspace(-4,4,500));
p = hornernewt(c,x,z);
plot(z,p)
```

What do you observe?