

2a) Linear system:

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \quad M_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad M_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 6 & 0 & 1 \end{bmatrix} \quad M_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \leftarrow \text{Don't need this}$$

$$M_2 \cdot M_1 \cdot A \rightarrow \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

① Backward substitution (U)

$$\begin{aligned} x_3 &= 3/3 = \underline{1} \\ x_2 &= (b_2 - u_{23} \cdot x_3) / u_{22} = (1 - 0 \cdot 1) / 1 = 1/1 = \underline{1} \\ x_1 &= (b_1 - u_{12} \cdot x_2 - u_{13} \cdot x_3) / u_{11} = (0 - 1 \cdot 1 - 2 \cdot 1) / 3 \\ &= -3/3 = \underline{-1} \end{aligned}$$

$$\boxed{x_3 = 1, x_2 = 1, x_1 = -1}$$

② Forward substitution (L)

$$\begin{aligned} x_1 &= 0/1 = 0 \\ x_2 &= (b_2 - l_{21} \cdot x_1) / l_{22} = (1 - 2 \cdot 0) / 1 = \underline{1} \\ x_3 &= (b_3 - l_{31} \cdot x_1 - l_{32} \cdot x_2) / l_{33} = (3 - 1 \cdot 0 - 0 \cdot 1) / 1 \\ &= 2/1 = \underline{2} \end{aligned}$$

$$\boxed{x_1 = 0, x_2 = 1, x_3 = 2}$$