1. Consider the data set $\mathcal{D}_4 = \{(0,0), (\frac{\pi}{6}, \frac{1}{2}), (\frac{\pi}{3}, \frac{\sqrt{3}}{2}), (\frac{\pi}{2}, 1)\}$.

(a) Construct the divided–difference table associated with $\mathcal{D}_4$, and then write down the associated degree–3 Newton interpolating polynomial $p_4(x)$. [The 4 in $p_4(x)$ indicates four points, but the polynomial is cubic. Some would use the notation $p_3(x)$.] *This is a "pen-and-paper" problem, and you should use exact numbers in your calculation, not decimal approximations.*

(b) Use the error formula for polynomial interpolation to estimate the error in using $p_4(1)$ to approximate $\sin(1)$, that is find a bound for $|\sin(1) - p_4(1)|$. How does the estimate compare to the true error?

2. Addition-of-angle formulas imply the following identity: $\sin(\pi - x) = \sin x$. Indeed, $\sin(\pi - x) = \sin\pi\cos x - \cos\pi\sin x = \sin x$. Similarly $\sin(2\pi - x) = -\sin x$. **(a)** Suppose that you are tasked with the design of a `sin` (sine) key for a calculator. First, show that $[0, \pi/2]$ can be used as a fundamental domain for $\sin x$. That is, any sine value may be expressed as $\pm\sin x$ for an $x \in [0, \pi/2]$. **(b)** Suppose the key is to display six accurate digits to the right of the decimal place. Find the degree d for which an interpolating polynomial using Chebyshev points will deliver this accuracy on the interval $[0, \pi/2]$. **(c)** What would be the corresponding accuracy for this value of d had equally spaced points been used instead? *Hint:* to get an estimate for $\max_{x\in[0,\pi/2]} |\Psi(x)|$ for equally space interpolation points, plot $|\Psi(x)|$ as follows:

```
x = linspace(0,pi/2,d+1);
z = linspace(0,pi/2,1000);
Psi = ones(size(z));
for k = 1:length(x)
    Psi = Psi.*(z-x(k));
end
plot(z,abs(Psi))
```

From the graph of $|\Psi(x)|$ read off the maximum.

3. Using the basis $\mathcal{B}_2 = \{2, x-1\}$, find the line which best fits the data

$$\mathcal{D}_4 = \{(-1,-1), (0,0), (1,1), (2,8)\}$$

in the least squares sense. **Hint:** this problem is a little different than the ones considered in the notes. But you can set up a 4×2 Vandermonde system, and then construct the normal equations to solve in the least squares sense. For $\mathcal{B}_3 = \{2, x-1, x^2\}$, set up the normal equations which defines the best quadratic fit to $\mathcal{D}_4$ (no need to solve this system of normal equations).