1. Apply Newton's method to find the three roots of the function $f(x) = e^{\sin^3 x} + x^6 - 2x^4 - x^3 - 1$ on the interval $[-2, 2]$. For each root, print out the sequence of iterates, the errors e_i, and the relevant error ratio, e_{i+1}/e_i^2 or e_{i+1}/e_i, that converges to a nonzero limit. Match the limit with the expected value of $M = |f''(r)/(2f'(r))|$ or $S = (m-1)/m$, where m is the order of a multiple root. To measure errors, treat the *last* iterate as the exact root. Please make sure all tables are well-formatted with numbers reported to enough precision. See page 6 of the lecture **root3** for details.

Note: for a root of multiplicity Newton's method does not converge quadratically, rather $e_{k+1} \simeq S e_k$, with S as above. For example, consider $f(x) = (x-1)^4$. Then

$$x - f(x)/f'(x) = x - (x-1)^4/(4(x-1)^3) = x - \tfrac{1}{4}(x-1).$$

So Newton's method is $x_{k+1} = x_k - \frac{1}{4}(x_k - 1)$. This implies $x_{k+1} - 1 = \frac{3}{4}(x_k - 1)$ and so $e_{k+1} = \frac{3}{4} e_k$. Linear convergence!

2. The *ideal gas law* (Émile Clapeyron, 1834) for a gas, assumed to be at low temperature and pressure, is $PV = nRT$, where P is pressure (in `atm`), V is volumne (in `L`), T is temperature (in `K`), n is the amount of gas (in `mol`), and $R = 0.0820578$ is the molar gas constant. Van der Waal's equation

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

covers the non-ideal case, where the stated assumptions do not hold. Use the ideal gas law to compute an initial guess, followed by Newton's Method applied to the Van der Waal equation to find the volume of 1 `mol` of oxygen at 320 degrees `K` and a pressure of 15 `atm`. For oxygen $a = 1.36$ $\mathtt{L}^2\mathtt{atm/mol}^2$ and $b = 0.003183$ `L/mol`. State your initial condition and solution with six significant digits. *Coding tip:* Suppose that you have coded a function `f(V,P,T,R,n,a,b)` which depends on many variables, but wish to use it as a function of just one variable (viewing all others as fixed). This you can do with `@(V)f(V,P,T,R,n,a,b)`.