1a) $D_4 = \{(0,0), (\pi/6, 1/2), (\pi/3, \sqrt{3}/2), (\pi/2, 1)\}$

0	$\boxed{0}$
$\pi/6$	$1/2$
$\pi/3$	$\sqrt{3}/2$
$\pi/2$	1

$$\frac{1/2 - 0}{\pi/6 - 0} = \boxed{6/2\pi}$$

$$\frac{\sqrt{3}/2 - 1/2}{\pi/3 - \pi/6} = \frac{6\sqrt{3} - 6}{2\pi}$$

$$\frac{1 - \sqrt{3}/2}{\pi/2 - \pi/3} = \frac{-6\sqrt{3} + 12}{2\pi}$$

$$\frac{6\sqrt{3} - 6/2\pi - 6/2\pi}{\pi/3 - 0} = \boxed{\frac{18\sqrt{3} - 36}{2\pi^2}}$$

$$\frac{-6\sqrt{3} - 12/2\pi - 6\sqrt{3} - 6/2\pi}{\pi/2 - \pi/6} = \frac{-36\sqrt{3} - 18}{2\pi^2}$$

$\Downarrow$

$$\frac{18\sqrt{3} - 36}{2\pi^2} \qquad \frac{-36\sqrt{3} - 18}{2\pi^2}$$

$$\frac{-36\sqrt{3} - 18/2\pi^2 - 18\sqrt{3} - 36/2\pi^2}{\pi/2 - 0} = \boxed{\frac{-108\sqrt{3} + 36}{2\pi^3}}$$

$$\boxed{P_4(x) = 6/2\pi(x) + 18\sqrt{3} - 36/2\pi^2(x)(x - \pi/6) + \frac{-108\sqrt{3} + 36}{2\pi^3}(x)(x - \pi/6)(x - \pi/3)}$$

or

0	0
$\pi/6$	$1/2$
$\pi/3$	$\sqrt{3}/2$
$\pi/2$	1

$$\boxed{3/\pi} \qquad \frac{3(\sqrt{3} - 1)}{\pi} \qquad \frac{3(2 - \sqrt{3})}{\pi}$$

$$\boxed{\frac{9(\sqrt{3} - 2)}{\pi^2}} \qquad \frac{9(3 - 2\sqrt{3})}{\pi^2}$$

$$\boxed{\frac{18(5 - 3\sqrt{3})}{\pi^3}}$$

$$\boxed{P_4(x) = 3/\pi(x) + 9(\sqrt{3} - 2)/\pi^2(x)(x - \pi/6) + 18(5 - 3\sqrt{3})/\pi^3(x)(x - \pi/6)(x - \pi/6)}$$

2b) Display six accurate digits.

root = ??

$$= \left(\frac{\pi/2 - 0}{2}\right)^{30} \approx 7.12 \times 10^{-4}$$

$$= \left(\frac{\pi/2 - 0}{2}\right)^{40} \approx 6.36 \times 10^{-5}$$

$$= \left(\frac{\pi/2 - 0}{2}\right)^{48} \approx \underline{9.21 \times 10^{-6}} \checkmark$$

$a = 0$

$b = \pi/2$

degree $\boxed{d = 48}$.

$$\begin{aligned} \sin(\pi - x) &= \sin\pi\cos x - \cos\pi\sin x \\ &= 0\cdot\cos x - (-1)\cdot\sin x \\ &= \sin x \ \checkmark \end{aligned}$$

2a) $$\begin{aligned} \sin(2\pi - x) &= \sin 2\pi\cos x - \cos 2\pi\sin x \\ &= 0\cdot\cos x - 1\ \sin x \\ &= -\sin x \ \checkmark \end{aligned}$$

$$T_n(x) = \sin(n\pi - x)$$

$$\begin{aligned} T_0 &= \sin(0 - x) \\ &= \sin(-x) \\ &= -\sin(x) \end{aligned} \qquad \begin{aligned} T_1 &= \sin(\pi - x) \\ &= \sin x \end{aligned} \qquad \begin{aligned} T_2 &= \sin(2\pi - x) \\ &= -\sin x \end{aligned}$$

$$\begin{aligned} T_{n+1}(x) &= \sin((n+1)\pi - x) \\ &= \sin(n + \pi - x) \\ &= \sin n\cos\pi\cos x + \cos n\cos\pi\sin x + \sin n\sin\pi\sin x \\ &= \cos n(\sin\pi\cos x - \cos\pi\sin x) + \sin n(\sin\pi\cos x - \cos n\sin x) \\ &= \underline{T_n(x)\cdot\cos n + \sin n\cdot T_n(x)} \ \checkmark \quad \square \end{aligned}$$

$$T_n = 2^{n-1}x^n + O(x^{n-1}), \quad n > 0$$

$$x_k = \sin\left({}^{(2k-1)\pi}/_{2n}\right) \text{ for } k = 1, \dots, n$$

$$\begin{aligned} T_n(x_k) &= \sin(n\pi - {}^{k\pi}/_n) \\ &= \sin(n\ {}^{k\pi}/_n) \\ &= \sin k\pi \\ &= \underline{(-1)^k} \quad \square \end{aligned}$$