The Logistic Map and COVID-19 Dynamics

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***Abstract*—This project is intended to provide a deeper look into how utilizing a Logistic map can provide information on COVID-19 dynamics. Information theoretic measures such as Mutual Information and Transfer Entropy were used to develop the findings throughout the paper. Comparing our results found within this paper to the transition between bottom up to top down within the Walker paper helps us confirm our findings and further supports the findings of the Walker paper.**

***Index Terms*—COVID-19, top down, bottom up, Walker, shannon entropy, transfer entropy, logistic map, mutual information**

I. Introduction

The Logistic Map is a polynomial model that shows how a population will grow over time at different rates to reach its carrying capacity. To achieve this, a nonlinear difference equation uses time steps. The term ”map” is used because time stamps of the population will be mapped to the next interval of time stamps at a given value. Sensitive dependence on initial conditions is an assessment of how microscopic measurements of position and momentum can result in large errors when computing quantities over a long time interval. While sensitive dependence on initial conditions seems to have sweeping effects on chaotic simulations, in real-world applications the effects are minuscule. In our model we are examining the value of r at 3.4. Transfer entropy is the information passed between two random processes. The equation for transfer entropy is as follows:

$$T_{X\to Y}^{(K)} \sum_{n=1} P(x_{n+1}, x_n^{(K)}, y_n^{(K)}) log[\frac{p(x_{n+1}|x_n^{(K)}, y_n^{(K)})}{p(x_{n+1}|x_n^{(K)})}] \quad (1)$$

The purpose of the Walker paper is to examine the transition from bottom-up to top-down to model and replicate behavior in simulations. To achieve this, the Walker paper uses an example of a toy model to show how the reverse of information flow emerges. The Walker paper uses transfer entropy (TE) and mutual information (MI) for analysis. We evaluated the analysis of Walker's methods by inserting COVID-19 infection data into the transfer entropy equation to find mutual information.

Our motivation for using the logistic map is to examine chaotic and non chaotic systems. We then take our findings of COVID-19 infection from transfer entropy and mutual information to make our conclusions based upon the logistic map.

II. Methods and Results

A. Four Populations - Mutual Information and Transfer Entropy

Focusing our first part of the project on researching about Shannon transfer entropy (TE) and mutual information (MI) could provide some findings that allow our group to make an educated conclusion relating to the bottom-up to top-down process presented in the Walker paper. Other conclusions here could explain how logistic map could help understand both chaotic and non-chaotic systems with dependence on the numerical values used within the process.

We were tasked with providing a figure of four time series generated by the logistic map. Representation of different initial condition values and R values will be used to further our understanding of how different values for R can have dependence on the initial condition values. To generate the four time series population, we used the same reproduction fitness coefficient, but we did not use the same initial condition values. The initial condition values represent the initial population of the current time series. All four initial condition values are separated by a difference of 0.01 to have a slight, but not over-impacting change to the outcome. The fitness coefficients refer to the R values and are very important to the complexity of the population. Initial condition values are dependent on the R value and could differentiate between a non-chaotic and chaotic system. Chaotic systems are formed when the R >= 3.56995 and non-chaotic systems are generated when R <= 3.56995. To demonstrate non-chaotic behavior, the fitness coefficient that our group choose was R = 1.7 and 3.7. The non-chaotic system is presented in the upper plot of Fig. 1. All four populations start with the same R, but different initial conditions. As the simulation increases by 1 time step, the values of the non-chaotic population converge to the same populations by 10 time steps. The population throughout seem to stay attached and follow the same pattern and proves to be non-chaotic system. In the bottom plot of Fig. 1, the same four populations and initial values are used to simulate a chaotic system instead of a non-chaotic system. Throughout the first 10 time steps of the simulation, the four population seem to be following the same pattern, but after the 10 time step, all four of the systems diverge and the chaos occurs. Overall, Fig. 1 is used to demonstrate how the values for R in the logistic map can influence a system to be chaotic and non-chaotic.

Initial conditions are reliant on R to control the behavior of the system.

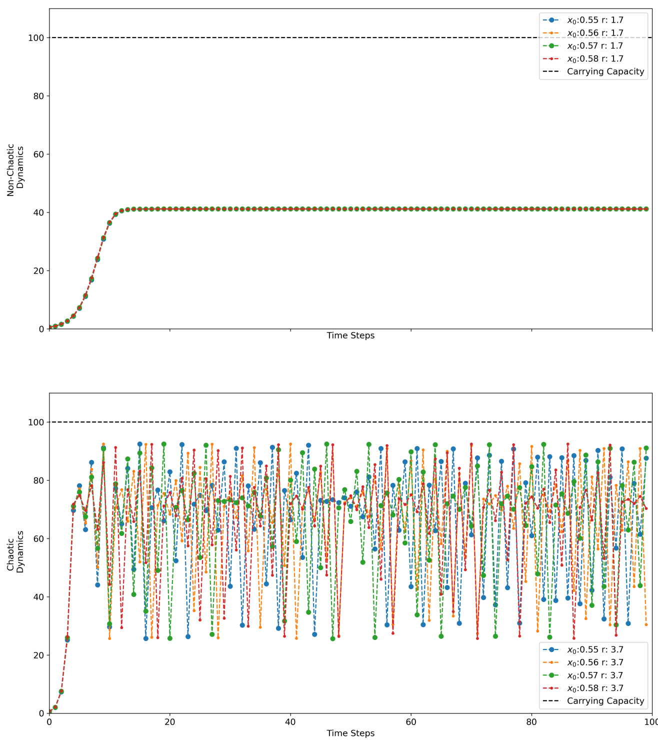

Fig. 1. Chaotic vs. Non Chaotic populations when initial condition values shift by 0.1 and R = 1.7 and 3.7.

Behavior represented in Fig. 1 is fairly conclusive but to further conclude our results, we can dive deeper into how the behavior of chaotic and non-chaotic systems vary by applying four Venn diagrams that show the data generated by our logistic map. Shannon transfer entropy refers to a non-parametric statistic measuring the amount of directed transfer of information between two random processes. This measurement could help understand the uncertainty of inheritance to a variable's possible outcome based on another. Mutual information refers to the measure of the mutual dependence between the two variables. In Fig. 2, one non-chaotic and chaotic time series population that were generated by the logistic map are chosen for comparison of entropies and mutual information measures. Entropy and mutual information values are calculated for the first 10 time steps and last 10 time steps. These time steps are appropriately chosen based on the number of time steps before the chaotic series diverges. Each entropy calculated by the population is represented by H(X) and H(Y). The distribution I(X, Y) refers to the mutual information in each time series. All measures utilized the bin method of measurement and followed the practices demonstrated in the Deterding and Wright paper. Data was separated into 10 distinct bins. By separating the data into bins, it allowed for more accurate computation of the entropies of each population. Numerically, the entropies for the time series were 3.1951 and 2.894, which led to the ability to compute the mutual information between populations and the plotting of entropies within the Venn diagram on Fig. 2. Both mutual information and entropy would not be possible with the JIDT tool [10] used within our research. Each diameter of each circle is scaled to have the diameter equal the entropy values. Overlap between the two circles in Fig. 2 demonstrate mutual information between the two populations. If not overlap appears, then there is no mutual information between populations. The non-chaotic populations seem to share most, if not all, mutual information between the two for the first 10 time steps. For the last 10 time steps of the non-chaotic system, there is no representation of entropies because the entropies of both populations in this case are equal to zero. This is due to the little to no change in the system during the last 10 time steps. The chaotic populations entropies are very interesting because there is more mutual information shared that we assumed. Both time series have similar entropies, which results in the sharing of mutual information. This information proves to be very important with how the logistic map is very pertinent to the way multiple populations behave.

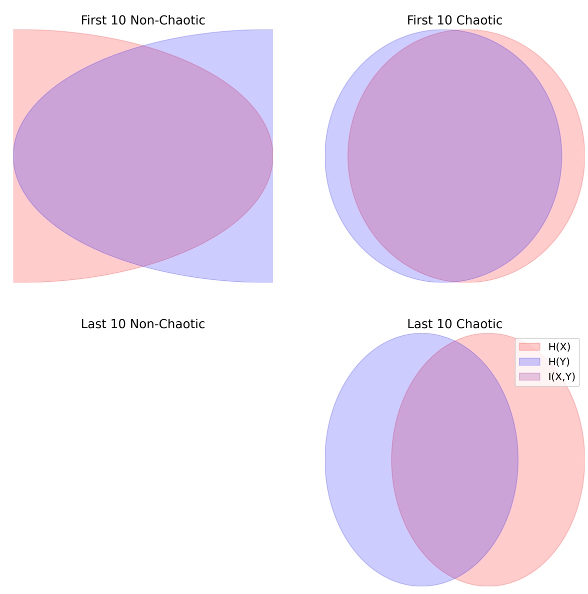

Fig. 2. Venn Diagram of Chaotic vs. Non Chaotic populations.

B. Transfer Entropy - Bottom Up to Top Down Uncertainties

In the first section, the discovery of transfer entropy and mutual information between populations proved to be very important to the insight of how the logistic map could influence behavior in systems.

The transition between bottom-up to top-down information flow was a substantial topic within the Walker paper. Transfer entropy values can be used as indicators when the transition is made. Deterding and Wright provided some insightful evidence that transfer entropy switches from a bottom-up to a top-down information flow when the value of epsilon is between 0.2 and 0.3. The value epsilon refers to the global coupling strength and it defined as the constant that determines the connection between individual behavior and the total population. The epsilon value is very important to the calculations of transfer entropy, as well as the value for the dimension of embedding which is referred to as K. The value K also defines how many time steps are used to calculate the information transferred between the variables between compared. Using both of those concepts, there can be further research done by picking three values of 0.2 <= epsilon <= 0.3 and calculating for transfer entropy. The transfer entropy will be calculated for 1 <= K <= 4. The three epsilon values that will be explored are 2.225, 2.25, and 2.275. Calculation was achieved by following the Information Transfer section in the Deterding and Wright paper. Using the equation for transfer entropy to measure in bits for system Y to system X with the dimension K and two systems X and Y as input allowed for more conclusive findings. The value of epsilon was then used to compare to the transfer entropy to find where the values of certainty and uncertainty of information sharing is throughout the two systems. In Fig. 3, a plot of the calculated transfer entropies of each epsilon value is displayed and the shaded area around the line is indicated as the 95% confidence interval for each finding. The plot shows where the uncertainty of the transfer entropy between because of the very low transfer entropies within the region of 0.2-0.3. It is very interesting how the transfer entropy for each of these parts signify some uncertainty and for seemingly no reason. One reason that we inferred is because of the transition between bottom-up to top-down information flow.

To help better understand how the value of epsilon could display uncertainty of the mutual information between populations, a comparison mutual information amount sub-population with a lag for some values of epsilon could provide more insight. A group of sub-population is created through utilizing the JIDT toolkit. These sub-populations first would have their entropy calculated, then compared to another sub-population until all sub-populations have been compared to one another. This would then give us a set of values for the mutual information between each sub-population. Mutual information values will then be averaged and plotted to show the average MI between all sub-populations. Fig. 4 shows the level of global coupling strength (epsilon) compared to the mutual information the all the sub-populations share. This figure has the same numerical values and plot as it does in the Deterding and Wright paper. Out group re-plotted with the transfer entropies we calculated and came up with the same results as they did. Sub-populations within the example here share a great amount

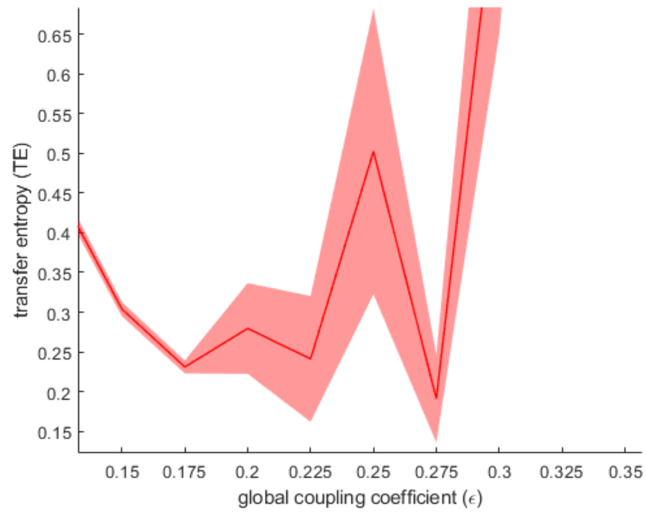

Fig. 3. Transfer Entropy calculated for Epsilon values 2.225, 2.25, and 2.275 along with a 95% Confidence intervals as the outer red.

of mutual information, except when the value of epsilon is converging to 0.2-0.3, the uncertainty appears. Along with what we concluded in Fig. 3, as the transfer entropy reaches level of uncertainty, so does the levels of mutual information among multiple sub-populations. Overall, our findings indicate that mutual exclusion and transfer entropy can help understand the shift between bottom-up to top-down information flow and how top-down causation dominates for collective states in the regimes when epsilon is between 0.2 and 0.7. This also comes with uncertainty. Nothing is ever perfect and when calculating for transfer entropy and mutual information, the uncertainty level seems to sky rocket when epsilon is between 0.2 and 0.3.

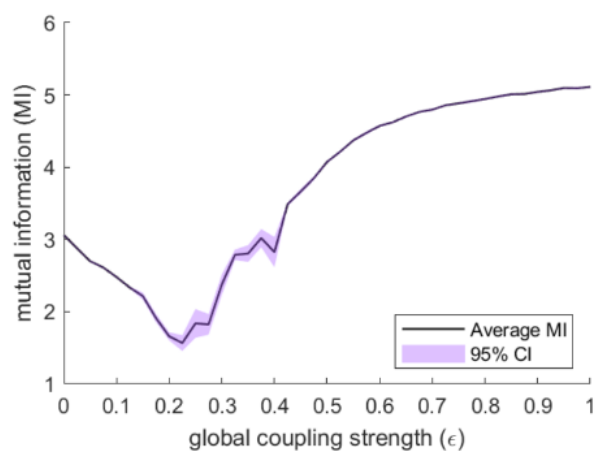

Fig. 4. Shows the mean mutual information between sub-populations and its 95% confidence interval..

C. COVID-19 Behavior

COVID-19 is a very chaotic system that has many factors that can contribute to the unpredictability of the infection rate. Numerous variables such as policies, time of year, location, ect. Using the techniques we learned about transfer entropy and mutual information, we decided to tackle two states and compare them to see if we can learn anything about how COVID-19 infections are a system of chaos or non-chaos.

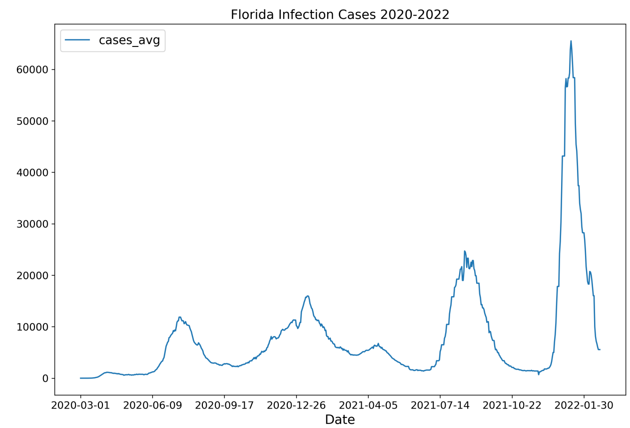

Fig. 5. COVID-19 infections throughout Florida

The two states that we choose were California and Florida. We choose these two states because the infection rates of each states are very high and somewhat similar. Since the infection rates are similar, we expect for these two systems to have a great deal of mutual information shared. The choice of Florida and California is very interesting being that Florida has very minimal policies in place to reduce spread of the virus, while California has maintained a good deal of policies throughout the entirety of the pandemic. Florida does have smaller population size compared to California. For reference, there are 10 million more people that live in California then Florida. This can be a contributing factor with how many infections are recorded. Finding the mutual information can also demonstrate if the policies in place play a big role in how the COVID-19 virus spreads.

Before we can find any significant results and make a conclusion, a proper data set must be used to help with the research. The New York Times [5] provide a significantly good data set that could act as a time-series and accounts for the total number of infections per state. Before using the data, the data was cleaned and smoothed for better results and to account for the transfer entropy and mutual exclusion equations. The main point of interest within the data set was an attribute called cases_avg, which accounts for the average number of infection cases reported each week. In Fig. 5, the average weekly infection cases are shown for the state of Florida and in Fig. 6, the average weekly infection cases are shown for the state of California.

To find the mutual information between the two states, we approached the problem just like how we did in section A of the methods and results section. Finding the entropy for each

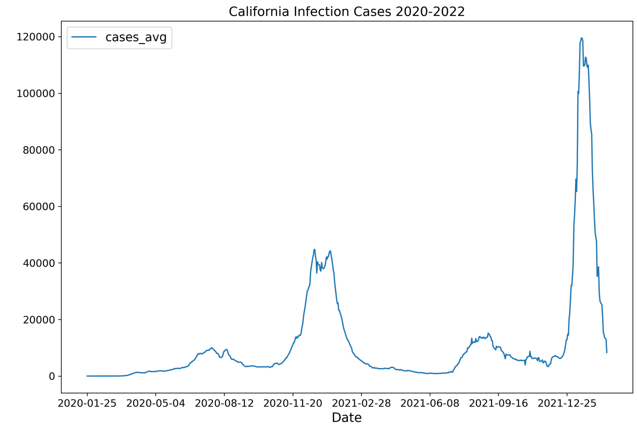

Fig. 6. COVID-19 infections throughout California

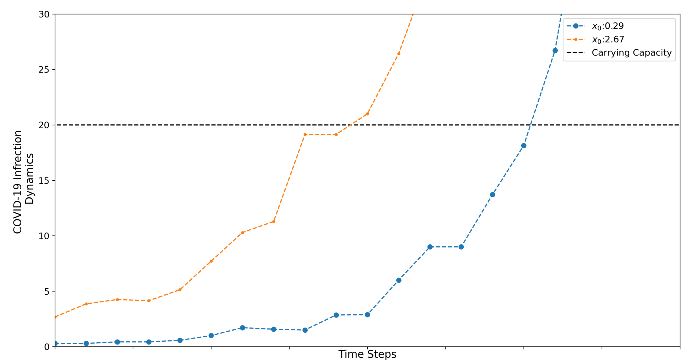

Fig. 7. COVID-19 system dynamics of both Florida and California. Florida is represented with the yellow line, and California is represented with the blue line. Shows a total of 20 time steps.

state was the first step. We found that the Shannon entropy for Florida was 0.16519544 and the entropy for California was 0.1175359. Both have very low entropies, which was not expected. Nevertheless, they were somewhat similar entropies so we made assumptions the two states having a high entropy. In Fig. 7, the entropies are distinctly different. Our assumptions that there will be a high amount of mutual information shared between the two states was questioned here because of the difference between the two states dynamics. Using the entropies calculated, we then used a Venn diagram to display any mutual information. This resulted in a large amount of mutual information being shared between the two entities. In Fig. 8, the first two Venn diagrams show the entropies of both Florida and California. A large amount of shared entropy is shared within these two states which is very interesting and provides good insight. This allows us to come to a conclusion about the two states dynamics.

California and Florida share a large amount of mutual information which results in both systems having the same

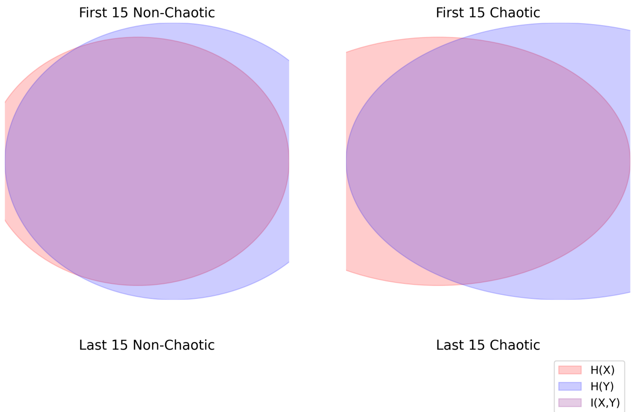

Fig. 8. COVID-19 system entropies and mutual information represented through a Venn diagram.

dynamics. COVID-19 is a chaotic system, but these two states beg to differ about how much chaos is actually involved. We conclude that COVID-19 infections are not strongly influenced by policies and location between California and Florida. Even though Florida had less policies in place to prevent infection rate compared to California, our findings show that the two systems do not have a large difference.

D. *Choose Our Own Adventure*

Unfortunately, we did not have enough time to choose our own adventure. Instead, here is a outline of what we originally wanted to do with this part of the project. Accounting for the COVID-19 behavior is difficult, due to the endless attributes that change the dynamics on how the system works. One of those factors is the policies that the United States government imposes and how that effects the infection rate of the virus, specifically the mandates for the vaccination. We were going to take two similar states with strong COVID-19 policies and compare infection rates, the dynamics behind how the complex system operated within those two states, calculate transfer entropy, and mutual exclusion values between the two. Then, we were going to aim for the opposite and take two states that have relatively relaxed or no policies in place to prevent the infection rate. This would allow us to see how chaotic systems can get and if policies have a large impact on the infection rate. Furthermore, it allows us further understand how the logistic map can make a conclusion on whether two states share mutual information revolving their policies.

III. Discussion and Conclusions

Our results were in part due to the great project that was given as an example called The Coupled Logistic Map [2]. Within our findings we can see that most, if not all, of the results were in agreement with the Coupled Logistic map and Walker paper.

The Walker paper concluded that as biological systems increase in size at a steady growth rate, the more complex that they become overtime. Our data agreeing with the Walker paper suggests that the spread of COVID-19 will increase steadily but become more complex over time. Initially with the growth of COVID-19 in the population, only a few people were infected, but the growth rate increased rapidly and spread very fast among the population.

The logistic map explains aspects of growth by mapping chaotic and no chaotic data. The spread of COVID-19 can be looked at as chaotic data since it grew rapidly with heavy reliance on initial conditions. In people these initial conditions could include where someone lived, their daily activities, aspects of their physical health, and their level of cleanliness. An insight that we gained based upon top-down and bottom-up drivers of COVID-19 dynamics is that top-down will be reliant on the location and government policies within a state. Bottom-up dynamics are reliant on herd immunity and the population of a state in a given area.

IV. Contributions

Work done throughout the project was very evenly distributed. Both students learned the topics mutually and development of the python code. Collaboration was achieved by Overleaf and Google Colab. The figures and code development were shared using cohesive communication techniques. Meiling wrote the abstract, introduction, and discussion sections. Damian reviewed the write up involving the explanation of the figures. While each team member worked on respective sections individually, at the end we both reviewed the entire document. Keeping an open line of communication allowed ease of discussion between partners to resolve any issues or problems that occurred. Together both students were able to learn a lot and contribute to the project.

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