CAS Project 1

**##** - fill in number

**Four Population Simulation**

Focusing our first part of the project on researching about Shannon transfer entropy and mutual information could provide some findings that allow our group to make an educated conclusion relating to the bottom-up to top-down process presented in the Walker paper. Other conclusions here could explain how logistic map could help understand both chaotic and non-chaotic systems with dependence on the numerical values used within the process.

We were tasked with providing a figure of four time series generated by the logistic map. Representation of different initial condition values and R values will be used to further our understanding of how different values for R can have dependence on the initial condition values. To generate the four time series population, we used the same reproduction fitness coefficient, but we did not use the same initial condition values. The initial condition values represent the initial population of the current time series. All four initial condition values are separated by a difference of 0.01 to have a slight, but not over-impactful change to the outcome. The fitness coefficients refer to the R values and are very important to the complexity of the population. Initial condition values are dependent on the R value and could differentiate between a non-chaotic and chaotic system. Chaotic systems are formed when the R >= 3.56995 and non-chaotic systems are generated when R <= 3.56995. To demonstrate non-chaotic behavior, the fitness coefficient that our group choose was R = 2.9. The non-chaotic system is presented in the upper plot of Fig. 1. All four populations start with the same R, but different initial conditions. As the simulation increases by 1 time step, the values of the non-chaotic population converge to the same populations by 30 time steps. The population throughout seem to stay attached and follow the same pattern and proves to be non-chaotic system. In the bottom plot of Fig. 1, the same four populations and initial values are used to simulate a chaotic system instead of a non-chaotic system. Throughout the first **##** time steps of the simulation, the four population seem to be following the same pattern, but after the **##** time step, all four of the systems diverge and the chaos occurs. Overall, Fig. 1 is used to demonstrate how the values for R in the logistic map can influence a system to be chaotic and non-chaotic. Initial conditions are reliant on R to control the behavior of the system.

Behavior represented in Fig. 1 is fairly conclusive but to further conclude our results, we can dive deeper into how the behavior of chaotic and non-chaotic systems vary by applying four Venn diagrams that show the data generated by our logistic map. Shannon transfer entropy refers to a non-parametric statistic measuring the amount of directed transfer of information between two random processes. This measurement could help understand the uncertainty of inheritance to a variable’s possible outcome based on another. Mutual information refers to the measure of the mutual dependence between the two variables. In Fig. 2, one non-chaotic and chaotic time series population that were generated by the logistic map are chosen for comparison of entropies and mutual information measures. Entropy and mutual information values are calculated for the first **##** time steps and last ## time steps. These time steps are appropriately chosen based on the number of time steps before the chaotic series diverges. Each entropy calculated by the population is represented by H(X) and H(Y). The distribution I(X, Y) refers to the mutual information in each time series. All measures utilized the bin method of measurement and followed the practices demonstrated in the Deterding and Wright paper. Data was separated into 10 distinct bins. By separating the data into bins, it allowed for more accurate computation of the entropies of each population. Numerically, the entropies for the time series were 3.1951 and 2.894, which led to the ability to compute the mutual information between populations and the plotting of entropies within the Venn diagram on Fig. 2. Both mutual information and entropy would not be possible with the JIDT tool [**##**] used within our research. Each diameter of each circle is scaled to have the diameter equal the entropy values. Overlap between the two circles in Fig. 2 demonstrate mutual information between the two populations. If not overlap appears, then there is no mutual information between populations. The non-chaotic populations seem to share most, if not all, mutual information between the two for the first **##** time steps. For the last **##**time steps of the non-chaotic system, there is no representation of entropies because the entropies of both populations in this case are equal to zero. This is due to the little to no change in the system during the last **##** time steps. The chaotic populations entropies are very interesting because there is more mutual information shared that we assumed. Both time series have similar entropies, which results in the sharing of mutual information. This information proves to be very important with how the logistic map is very impactful to the way multiple populations behave.

**More on Transfer Entropy and Mutual Information**

In the first section, the discovery of transfer entropy and mutual information between populations proved to be very important to the insight on how the logistic map could influence behavior in systems, and in this section, we will expand on that further.

The transition between bottom-up to top-down information flow was a substantial topic within the Walker paper. Transfer entropy values can be used as indicators when the transition is made. Deterding and Wright provided some insightful evidence that transfer entropy switches from a bottom-up to a top-down information flow when the value of epsilon is between 0.2 and 0.3. The value epsilon refers to the global coupling strength and it defined as the constant that determines the connection between individual behavior and the total population. The epsilon value is very important to the calculations of transfer entropy, as well as the value for the dimension of embedding which is referred to as K. The value K also defines how many time steps are used to calculate the information transferred between the variables between compared. Using both of those concepts, there can be further research done by picking three values of 0.2 < epsilon < 0.3 and calculating for transfer entropy. The transfer entropy will be calculated for 1 <= K <= 4. The three epsilon values that will be explored are 2.225, 2.25, and 2.275. Calculation was achieved by following the Information Transfer section in the Deterding and Wright paper. Using the equation for transfer entropy to measure in bits for system Y to system X with the dimension K and two systems X and Y as input allowed for more conclusive findings. The value of epsilon was then used to compare to the transfer entropy to find where the values of certainty and uncertainty of information sharing is throughout the two systems. In Fig. 3, a plot of the calculated transfer entropies of each epsilon value is displayed and the shaded area around the line is indicated as the 95% confidence interval for each finding. The plot shows where the uncertainty of the transfer entropy between because of the very low transfer entropies within the region of 0.2-0.3. It is very interesting how the transfer entropy for each of these parts signify some uncertainty and for seemingly no reason. One reason that we inferred is because of the transition between bottom-up to top-down information flow.

To help better understand how the value of epsilon could display uncertainty of the mutual information between populations, a comparison mutual information amount subpopulation with a lag for some values of epsilon could provide more insight.