

# Midterm Studying

1)  $(\text{nor fls}) \text{ fls} \rightarrow^* \text{tru}$

$\text{tru} = \lambda x. \lambda y. x$   
 $\text{fls} = \lambda x. \lambda y. y$

$\text{nor} \rightarrow \text{if } b \text{ then fls else (not c)}$

b	c	out
0	0	1
0	1	0
1	0	0
1	1	0

$\text{nor} = \lambda b. \lambda c. b \text{ fls (not c)}$

$= (\text{nor fls}) \text{ fls}$

$= ((\lambda b. \lambda c. b \text{ fls (not c)}) \text{ fls}) \text{ fls}$

$= (\lambda c. \text{fls fls (not c)}) \text{ fls}$

$= (\text{fls fls}) (\text{not fls})$

$= (\lambda x. \lambda y. y \text{ fls}) (\text{not fls})$

$= (\lambda y. y) (\text{not fls})$

$= \text{not fls}$

$= (\lambda b. b \text{ fls tru}) \text{ fls}$

$= (\text{fls fls}) \text{ tru}$

$= (\lambda x. \lambda y. y \text{ fls}) \text{ tru}$

$= (\lambda y. y) \text{ tru}$

$= \text{tru} \checkmark$

$\text{not}$

$= \lambda b. b \text{ fls tru}$

$= \text{if } b \text{ then fls else tru}$

$b = \text{tru} \text{ out} \rightarrow \text{tru}$

$b = \text{fls} \text{ out} \rightarrow \text{fls}$

and

→ if b then c else fls

A	B	out
0	0	0
0	1	0
1	0	0
1	1	1

and =  $\lambda b. \lambda c. b \ c \ fls$

(and fls) fls  
 = (( $\lambda b. \lambda c. b \ c \ fls$ ) fls) fls

Ex1 = ( $\lambda c. fls \ c \ fls$ ) fls  
 = (fls fls) fls  
 = ( $\lambda x. \lambda y. y \ fls$ ) fls  
 = fls

Ex2 = (and tru) tru  
 = ( $\lambda b. \lambda c. b \ c \ fls$ ) tru) tru  
 = ( $\lambda c. tru \ c \ fls$ ) tru  
 = (tru tru) fls  
 = ( $\lambda x. \lambda y. x \ tru$ ) fls  
 = ( $\lambda y. tru$ ) fls  
 = tru

2)  $x \oplus \text{or}$   $\rightarrow$  if b then (if c then tru else fls) else (if c then fls else tru)

b	c	out
0	0	1
0	1	0
1	0	0
1	1	1

$\lambda b. \lambda c. b (c \text{ tru fls}) (c \text{ fls tru})$

$= (x \text{ nor fls}) \text{ fls}$   
 $= ((\lambda b. \lambda c. b (c \text{ tru fls}) (c \text{ fls tru})) \text{ fls}) \text{ fls}$   
 $= (\lambda c. \text{fls} (c \text{ tru fls}) (c \text{ fls tru})) \text{ fls}$   
 $= \text{fls} (\text{fls tru fls}) (\text{fls fls tru})$   
 $= (\lambda x. \lambda y. y) (\text{fls tru fls}) (\text{fls fls tru})$   
 $= (\lambda y. y) (\text{fls fls tru})$   
 $= (\text{fls fls}) \text{ tru}$   
 $= (\lambda x. \lambda y. y \text{ fls}) \text{ tru}$   
 $= \lambda y. y \text{ tru}$   
 $= \text{tru} \checkmark$

3)  $\text{imply}$   $\rightarrow$  if b then (if c then tru) else tru else fls

b	c	out
0	0	1
0	1	1
1	0	0
1	1	1

$\lambda b. \lambda c. b (c \text{ tru fls}) \text{ tru}$

$= (\text{imply tru}) \text{ tru}$   
 $= ((\lambda b. \lambda c. b (c \text{ tru fls}) \text{ fls}) \text{ tru}) \text{ tru}$   
 $= (\lambda c. \text{tru} (c \text{ tru fls}) \text{ fls}) \text{ tru}$   
 $= \text{tru} (\text{fls tru fls}) \text{ fls}$   
 $= (\lambda x. \lambda y. x) (\text{fls tru fls}) \text{ tru}$   
 $= (\lambda y. \text{fls tru fls}) \text{ tru}$   
 $= \text{fls tru fls}$

$$\begin{aligned}
 &= (\lambda x. \lambda y. y \text{ tru}) \text{ fls} \\
 &= (\lambda y. y) \text{ fls} \\
 &= \text{fls} \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 4) \quad c_0 &= \lambda s. \lambda z. z \\
 c_1 &= \lambda s. \lambda z. s \ z \\
 c_2 &= \lambda s. \lambda z. s (s \ z) \\
 c_3 &= \lambda s. \lambda z. s (s (s \ z)) \quad \checkmark
 \end{aligned}$$

5) pred

$$\begin{aligned}
 \text{pair} &= \lambda x. \lambda y. \lambda z. z \times y \\
 \text{fst} &= \lambda p. p \text{ tru} \\
 \text{snd} &= \lambda p. p \text{ fls}
 \end{aligned}$$

$$\begin{aligned}
 \text{pred} &= \lambda n. \text{fst} (\wedge \text{ ss } z z) \\
 \text{where } \text{ss} &= \text{pair } c_0 \ c_0 \\
 z z &= \lambda p. \text{pair} (\text{snd } p) (\text{succ} (\text{snd } p))
 \end{aligned}$$

$$\text{pred } c_2 \xrightarrow{*}_B c_1$$

6) times

$$\begin{aligned}
 \text{succ} &= \lambda n. \lambda s. \lambda z. s (\wedge s z) \\
 \text{add} &= \lambda i. \lambda j. i \text{ succ } j
 \end{aligned}$$

$$\text{times} = \lambda i. \lambda j. i (\text{add } j) \ c_0$$

7)

$$\begin{array}{c}
 \frac{}{0:\text{Nat}} \quad \frac{}{0:\text{Nat}} \\
 \frac{}{\text{succ } 0:\text{Nat}} \quad \frac{}{0:\text{Nat}} \quad \frac{}{\text{succ } 0:\text{Nat}} \\
 \frac{}{\text{if succ } 0 \text{ then } 0 \text{ else succ } 0: ?} \\
 \frac{}{\text{pred (if succ } 0 \text{ then } 0 \text{ else succ } 0): ?} \\
 \frac{}{\text{isZero (pred (if succ } 0 \text{ then } 0 \text{ else succ } 0): ?}}
 \end{array}$$

Wrong!

This is not well-typed

8)

$$\begin{array}{c}
 \frac{}{0:\text{Nat}} \quad \frac{}{\text{false}:\text{Bool}} \quad \frac{}{\text{true}:\text{Bool}} \\
 \frac{}{\text{isZero } 0:\text{Bool}} \quad \frac{}{\text{succ } 0:\text{Nat}} \quad \frac{}{\text{pred } 0:\text{Nat}} \\
 \frac{}{\text{if isZero } 0 \text{ then false else true}:\text{Bool}} \quad \frac{}{\text{succ } 0:\text{Nat}} \quad \frac{}{\text{pred } 0:\text{Nat}} \\
 \frac{}{\text{if (if isZero } 0 \text{ then false else true) then succ } 0 \text{ else pred } 0:\text{Nat}}
 \end{array}$$

This is well-typed

9)

Step 1

$$\begin{array}{c}
 \frac{}{\lambda f. \lambda g. f (\lambda y. y y) (\lambda x. x) \rightarrow \lambda f. \lambda g. f (\lambda y. y y)} \\
 \frac{}{(\lambda f. \lambda g. f (\lambda y. y y) (\lambda x. x)) (\lambda y. y y)} \\
 \rightarrow \lambda f. \lambda g. f (\lambda y. y y) (\lambda x. x) (\lambda y. y y)
 \end{array}$$

Step 2

$$\begin{array}{c}
 \frac{}{\lambda f. \lambda g. f (\lambda y. y y) (\lambda x. x) \rightarrow \lambda f. \lambda g. f (\lambda y. y y)} \\
 \frac{}{(\lambda f. \lambda g. f (\lambda y. y y) (\lambda x. x)) (\lambda y. y y)} \\
 \rightarrow (\lambda f. \lambda g. f (\lambda y. y y) (\lambda x. x)) (\lambda y. y y)
 \end{array}$$

step 3  $(E\text{-app})$   $(E\text{-?})$

$$\frac{\lambda f. \lambda y. f (\lambda y. y y) \rightarrow \text{Value!}}{(\lambda f. \lambda y. f (\lambda y. y y)) (\lambda y. y y) \rightarrow ?}$$

Reduction/Evaluation

10)  $(\lambda x. x) (\lambda z. (\lambda z. x) z) (\lambda x. x)$

step 1:  $\rightarrow (\lambda x. x) (\lambda z. (\lambda z. x) z)$

step 2:  $\rightarrow (\lambda z. \lambda z. x) z$

$\rightarrow \text{Value!}$

11)  $(\lambda x. \lambda z. x) (\lambda f. \lambda y. f y) (\lambda g. g)$

step 1  $\rightarrow (\lambda x. \lambda z. x) (\lambda f. \lambda y. f y)$

step 2  $\rightarrow (\lambda x. \lambda z. x)$

step 3  $\rightarrow \text{DONE value!}$