$$\text{Prove } \underbrace{\text{map } f \text{ (map } f\ l) = \text{map } (\backslash x \to f(f x))\ l}_{P(l)}$$

for all $l :: [a]$ and for all $f :: a \to a$

① Base case: Prove $P([\,])$

i.e. $\text{map} f\ (\text{map} f\ [\,]) = \text{map}\ (\backslash x \to f(f x))\ [\,]$

LHS $\Rightarrow$ map f (map f [ ])

By definition of map

$\to$ [ ] ✓

RHS $\Rightarrow$ map $(\backslash x \to f(f x))$ [ ]

By definition of map

$\to$ [ ] ✓

LHS = RHS

[ ] = [ ] ✓

(2) Inductive case: Prove $P(xs) \Rightarrow P(x:xs)$
for $l = (x:xs)$ for some $(x:xs)$

Assume IH: $P(xs)$
i.e. map f (map f xs) = map (\x -> f (f x)) xs

implies => map f (map f (x:xs)) = map (\x -> f (f x)) (x:xs)

RHS = map f (map f (x:xs))
By definition of map
-> map f (f x : map f xs)
By definition of map
-> (f (f x)) : map f (map f xs)
By IH
-> (f (f x)) : map (\x -> f (f x)) xs
By definition of map
-> map (\x -> f (f x)) (x:xs)
✓ DONE!
-> LHS = RHS

map f (map f (x:xs)) = map (\x -> f (f x)) (x:xs)