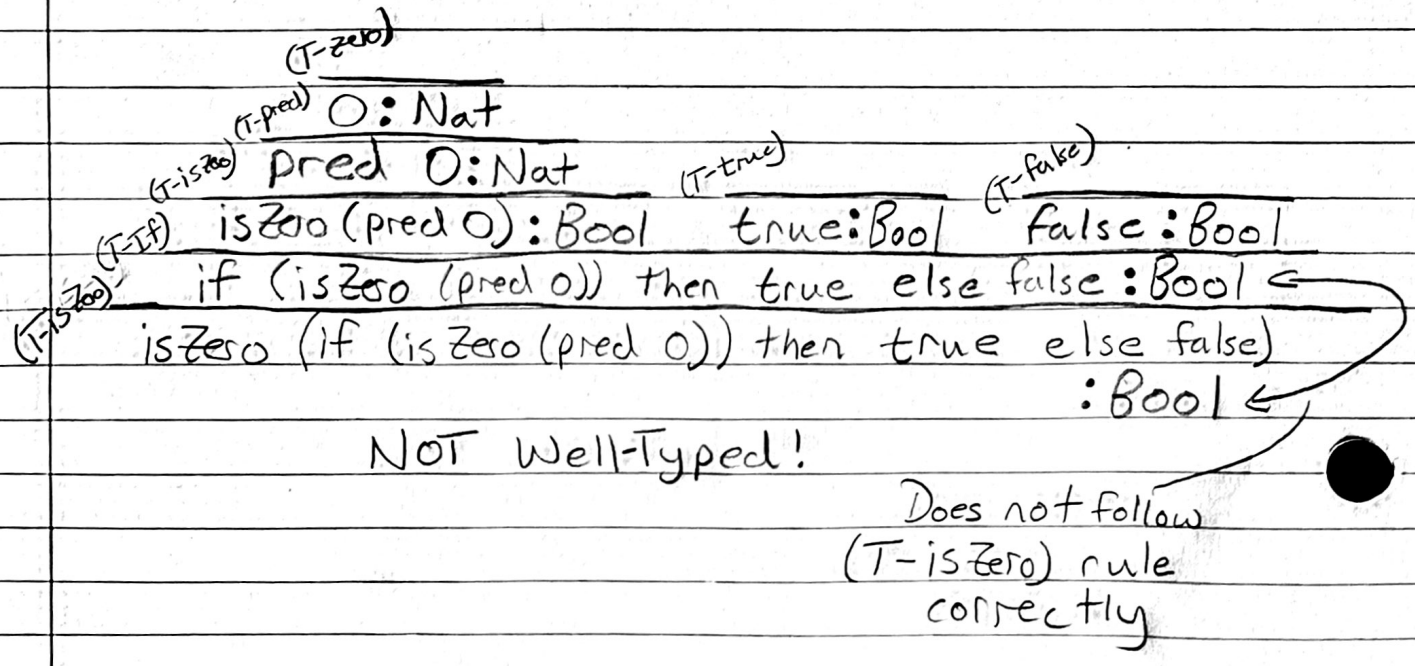
4.1 (a)

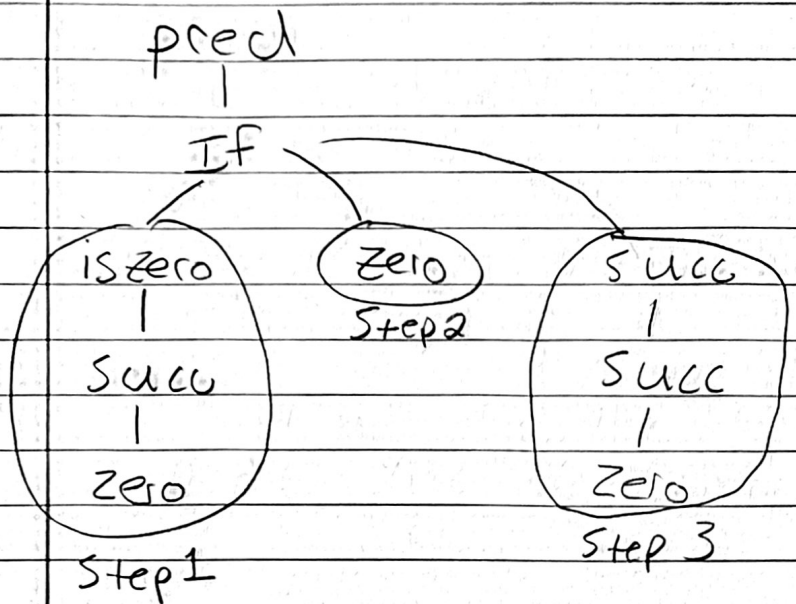
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{0 : \text{Nat}}\,(\text{T-Zero})}{\text{succ}\,0 : \text{Nat}}\,(\text{T-Succ})}{\text{iszero}\,(\text{succ}\,0) : \text{Bool}}\,(\text{T-IsZero}) \quad \dfrac{}{0 : \text{Nat}}\,(\text{T-Zero}) \quad \dfrac{\dfrac{\dfrac{}{0 : \text{Nat}}\,(\text{T-Zero})}{\text{succ}\,0 : \text{Nat}}\,(\text{T-Succ})}{\text{succ}\,(\text{succ}\,0) : \text{Nat}}\,(\text{T-Succ})}{\text{if}\,(\text{iszero}\,(\text{succ}\,0))\,\text{then}\,0\,\text{else}\,(\text{succ}\,(\text{succ}\,0)) : \text{Nat}}\,(\text{T-If})}{\text{pred}\,(\text{if}\,(\text{iszero}\,(\text{succ}\,0))\,\text{then}\,0\,\text{else}\,(\text{succ}\,(\text{succ}\,0))) : \text{Nat}}\,(\text{T-Pred})$$

Well-Typed!

4.1 (b)


(T-zero)
0 : Nat
(T-pred)
pred 0 : Nat
(T-iszero)
iszero (pred 0) : Bool
(T-true)
true : Bool
(T-false)
false : Bool
(T-If)
if (iszero (pred 0)) then true else false : Bool
(T-iszero)
iszero (if (iszero (pred 0)) then true else false)
: Bool
NOT Well-Typed!
Does not follow (T-iszero) rule correctly

4.2(a)



Step 1 : Find t'

$$\frac{\dfrac{\dfrac{}{\text{iszero}(\text{succ } 0) \to \text{false}}\,(E\text{-IsZeroSucc})}{\text{if } (\text{iszero}(\text{succ } 0)) \text{ then } 0 \text{ else } (\text{succ}(\text{succ } 0)) \to \text{if false then } 0 \text{ else } (\text{succ}(\text{succ } 0))}\,(E\text{-If})}{\text{pred } (\text{if } (\text{iszero}(\text{succ } 0)) \text{ then } 0 \text{ else } (\text{succ}(\text{succ } 0))) \to \text{pred } (\text{if false then } 0 \text{ else } (\text{succ}(\text{succ } 0))) \Rightarrow t'}\,(E\text{-pred})$$

Step 2 : Find t''

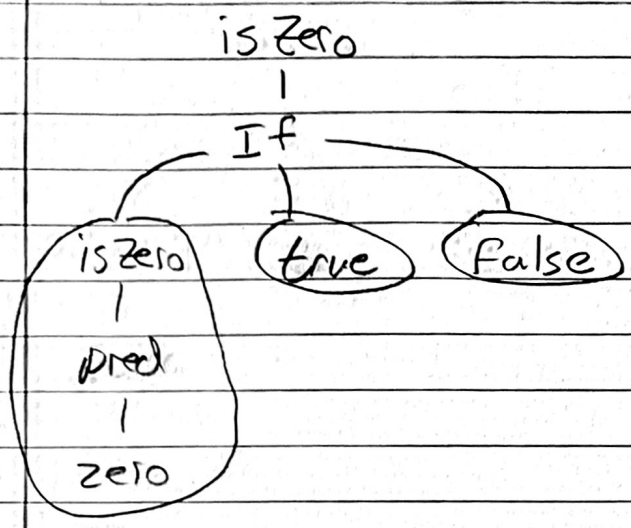
$$\frac{\dfrac{}{\text{if false then } 0 \text{ else } (\text{succ}(\text{succ } 0)) \to (\text{succ}(\text{succ } 0))}\,(E\text{-IfFalse})}{\text{pred } (\text{if false then } 0 \text{ else } (\text{succ}(\text{succ } 0))) \to \text{pred } (\text{succ}(\text{succ } 0)) \Rightarrow t''}\,(E\text{-pred})$$

Step 3 : Find t'''

$$\text{(T-pred)}\ \frac{succ(succ\ 0) \rightarrow\ ?}{pred\ (succ\ (succ\ 0)) \rightarrow\ ?}$$

t'' cannot be reduced to a value so this is STUCK!

4.2 (b)



Step 1: Find t'

$$\frac{\dfrac{\dfrac{}{\text{pred } 0 \to 0}\,(E\text{-pred zero})}{\text{isZero}(\text{pred } 0) \to \text{true}}\,(E\text{-isZero zero})}{\text{if (isZero (pred 0)) then true else false} \to \text{if true then true else false}}\,(E\text{-if})$$

$$\frac{\text{isZero (if (isZero (pred 0)) then true else false} \to \text{isZero (if true then true else false)} \Rightarrow t'}{}\,(E\text{-isZero})$$

Step 2: Find t''

$$\frac{\dfrac{}{\text{if true then true else false} \to \text{true}}\,(E\text{-if true})}{\text{isZero (if true then true else false)} \to \text{isZero (true)}}\,(E\text{-isZero})$$

Step 3: Find t'''

$$\frac{}{\text{isZero (true)} \to ?}$$

↙

t'' does not reduce to anything because true is not a 0 or numeric value so this is STUCK!