$$\text{Prove } \underbrace{((\text{map } f) \cdot (\text{Map } g))\ l = \text{Map } (f \cdot g)\ l}_{P(l)}$$

$l :: [a]$, $g :: a \to b$ and $f :: b \to c$

① Base case: Prove $P([\ ])$

i.e. $((\text{Map} f) \cdot (\text{map } g))\ [\ ] = \text{Map } (f \cdot g)\ [\ ]$

LHS $\Rightarrow ((\text{map } f) \cdot (\text{map } g))\ [\ ]$

By definition of $\cdot$ (function composite operator)

$\to (\text{Map } f\ (\text{map } g\ [\ ]))$

By definition of Map

$\to [\ ]$ ✓

RHS $\Rightarrow \text{Map } (f \cdot g)\ [\ ]$

By definition of map

$\to [\ ]$ ✓

LHS = RHS

$[\ ] = [\ ]$ ✓

② Inductive case: Prove $P(xs) \Rightarrow P(x:xs)$
for $l = (x:xs)$ for some $(x:xs)$

Assume IH: $P(xs)$

i.e. $((map\ f) \cdot (map\ g))\ xs = map\ (f \cdot g)\ xs$

implies $\Rightarrow ((map\ f) \cdot (map\ g))\ (x:xs) = map\ (f \cdot g)\ (x:xs)$

RHS $= map\ (f \cdot g)\ (x:xs)$

By definition of $\cdot$ (function composite operator)

$\rightarrow (map\ f\ (map\ g\ (x:xs)))$

By definition of map

$\rightarrow map\ f\ (g\ x : map\ g\ xs)$

By definition of map

$\rightarrow (f(g\ x)) : map\ f\ (map\ g\ xs)$

By definition of $\cdot$ (function composite)

$\rightarrow (f \cdot g)\ x : ((map\ f) \cdot (map\ g))\ xs$

Stop here, can't proceed further

$\rightarrow$ Do LHS.

LHS $= ((map\ f) \cdot (map\ g))\ (x:xs)$

By definition of map

$\rightarrow ((f \cdot g)\ x)) : map\ (f \cdot g)\ xs$

By definition of $\cdot$ (function composite)

$\rightarrow (f \cdot g)\ x : ((map\ f) \cdot (map\ g))\ xs$

✓ DONE!

$\rightarrow$ LHS = RHS

✓ $(f \cdot g)\ x : ((map\ f) \cdot (map\ g)\ xs = (f \cdot g)\ x : ((map\ f) \cdot (map\ g)\ xs$