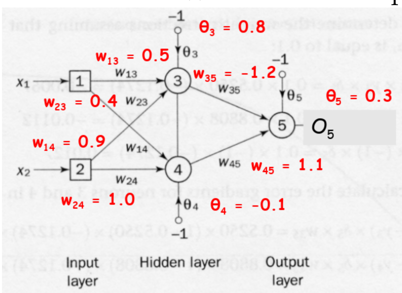
COMP 6721 Applied Artificial Intelligence (Fall 2023)

Worksheet #6: Introduction to Deep Learning

Matrix Notation. Last time, we labeled each weight in a neural network individually ($w_{14}, w_{45}, \ldots$), which is not practical for larger networks. From now on, we'll use *matrix notation*:[1] We can encode the sample input x as a matrix of size 1×2, the weights W as a matrix of size 2×2 and the bias b as a matrix of size 1×2. We can then compute the net activation as the dot product and add the bias: $\text{net} = x \cdot W + b$:



$$\text{net}_h = \begin{bmatrix} 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0.5 & 0.9 \\ 0.4 & 1.0 \end{bmatrix} + \begin{bmatrix} -0.8 & 0.1 \end{bmatrix} = \begin{bmatrix} \ldots & \ldots \end{bmatrix}$$

(Next, you'd have to compute the activation function to get the output: $h = \text{sigmoid}(\text{net}_h)$, as before; and finally do the same calculation for the output layer.)

Autoencoder. Assume that the 3×3 matrix below represents a gray scale image:

$X =$

0.2	0.3	0.2
0.4	0.1	0.3
0.1	0.9	0.5

Your job is to use X to train an *autoencoder*. So, the input to our network is going to be X. But what is the expected output (labels)? ....

Define the size of the input vector: .... and output vector: .... of your autoencoder.

How many hidden neurons H would you use? (Assuming we use a single hidden layer): ....
(Hint: there is no single correct answer, but you can define a sensible range.)

How many *weights* are there to train in total? (For H hidden neurons): ....

Autoencoder Activation. Assume that we use an autoencoder with the following hyperparameters: the activation function is *sigmoid* and the hidden layer has a size of 5. Perform a single forward pass through the autoencoder. You can assume an input value of 1 for the biases, and all the weights (including the biases) are initialized to 0.5. Note, our input vector corresponding to the image above is

$$x = \begin{bmatrix} 0.2 & 0.3 & 0.2 & 0.4 & 0.1 & 0.3 & 0.1 & 0.9 & 0.5 \end{bmatrix}$$

1. First, compute the pre-activation function (the *net*), by multiplying the input and weights, plus the bias b. Note that W is a matrix of size 9×5:

$$\text{net}_h = x \cdot W_{ih} + b_{ih} \tag{1}$$

the result is a matrix of size 1×5: $\text{net}_h = [$ .... $]$

[1] See https://medium.com/from-the-scratch/deep-learning-deep-guide-for-all-your-matrix-dimensions-and-calculations-415012de1568 for a review of matrix calculations for neural networks

2. Now, to compute the result for h, the sigmoid function $S(x) = \frac{1}{1+e^{-x}}$ is applied to the pre-activation (net) result (eq. 1):

$$h = S(\text{net}_h) \tag{2}$$

   where h is again a matrix of size 1×5: $h = [$ .... $]$

3. To compute the output o, the result of the hidden layer (eq. 2) should be multiplied with the weights of the output layer, W_{ho}, then we apply sigmoid again:

$$o = S(h \cdot W_{ho} + b_{ho}) \tag{3}$$

   where o is now a matrix of size 1×9: $o = [$ .... $]$

CNN Activation Map. Assume the following matrix that represents an image. This image will be fed to a convolutional neural network (CNN).

1	1	2	2	2	0	0
2	0	1	1	2	1	2
0	1	0	0	1	1	2
0	2	1	2	0	2	2
1	2	0	0	1	0	1
0	0	0	0	1	2	1
2	0	0	0	2	1	1

Assume that we use the following convolution filter (kernel) with a stride of 2 (no padding):

0	1	1
0	1	0
0	-1	-1

1. What will be the size of the activation map? ....
2. What will be the resulting activation map?

Pooling Layer. What will be the output of the pooling layer with a size of 2×2 and a stride of 1, on the activation map of the question above, if we use the following strategies:

1. Average pooling:

2. Max pooling: