

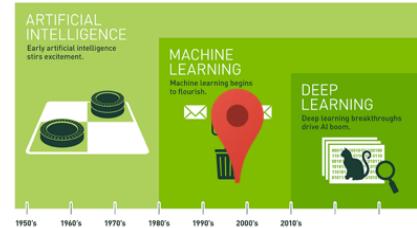
Artificial Intelligence: Naive Bayes Classification

Remember this slide...

History of AI

- 1980s-2010
- The rise of Machine Learning
 - More powerful CPUs -> usable implementation of neural networks
 - Big data -> Huge data sets are available
 - document repositories for NLP (e.g. emails)
 - billions of images for image retrieval
 - billions of genomic sequences, ...

😊 Rules are now learned automatically !

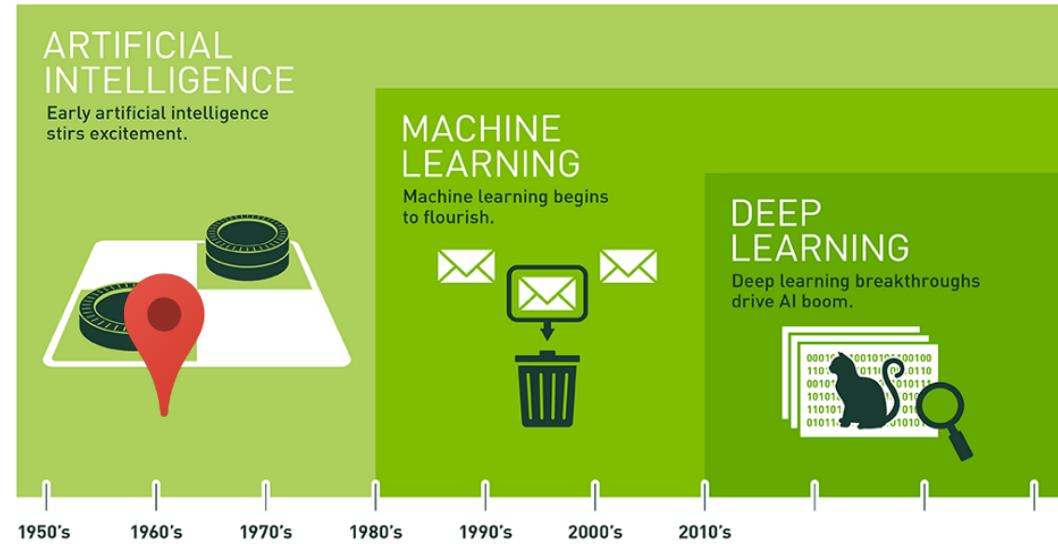


2011: Watson wins at Jeopardy!

Today

YOU ARE HERE!

1. Introduction to ML
2. Naïve Bayes Classifier
3. Evaluation



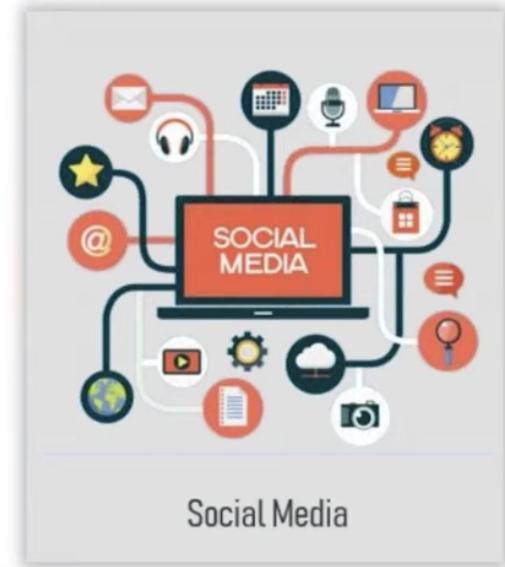
Why Machine Learning?



Cloud Data



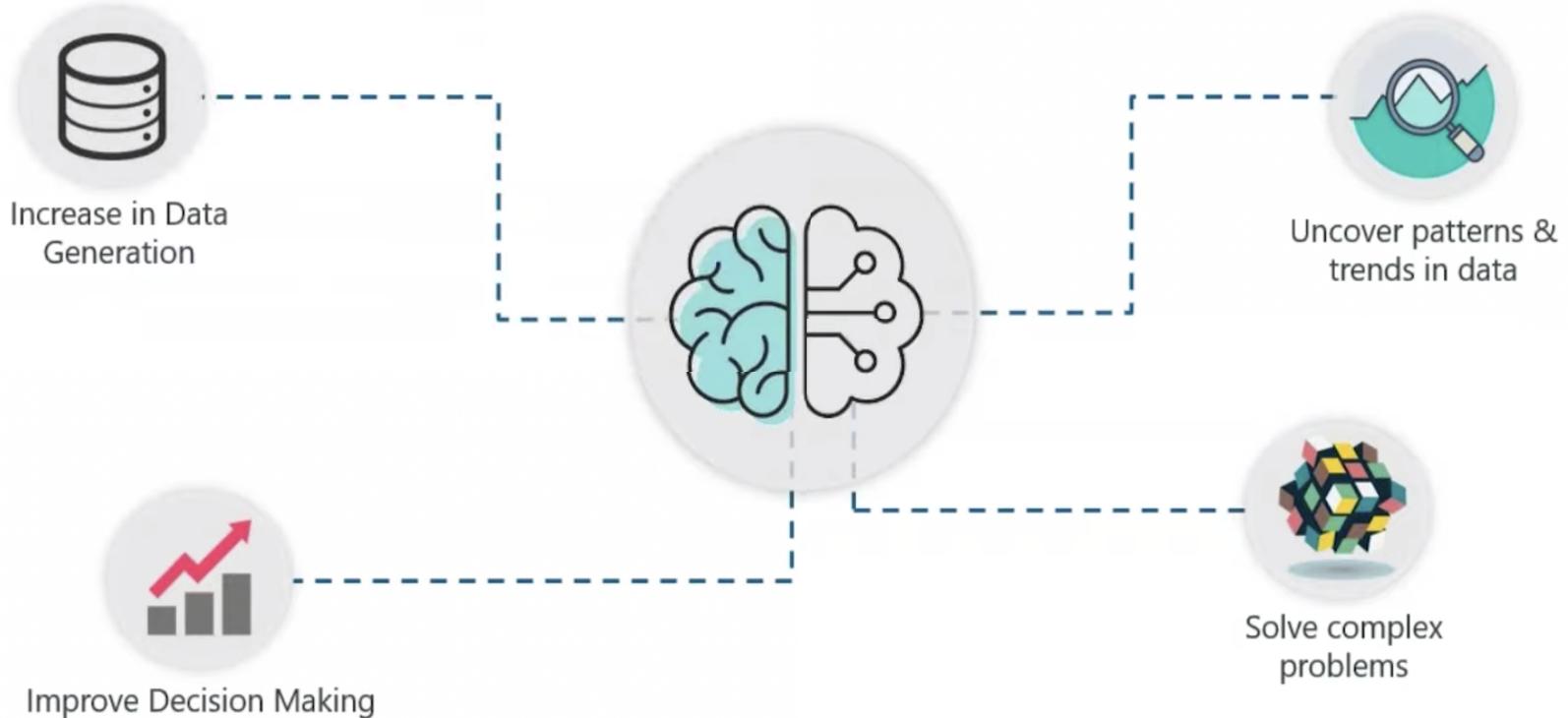
Internet Of Things



Social Media

- Estimated 2.5 quintillion bytes (2.5 exabytes) of data are created every single day
- ...and still growing

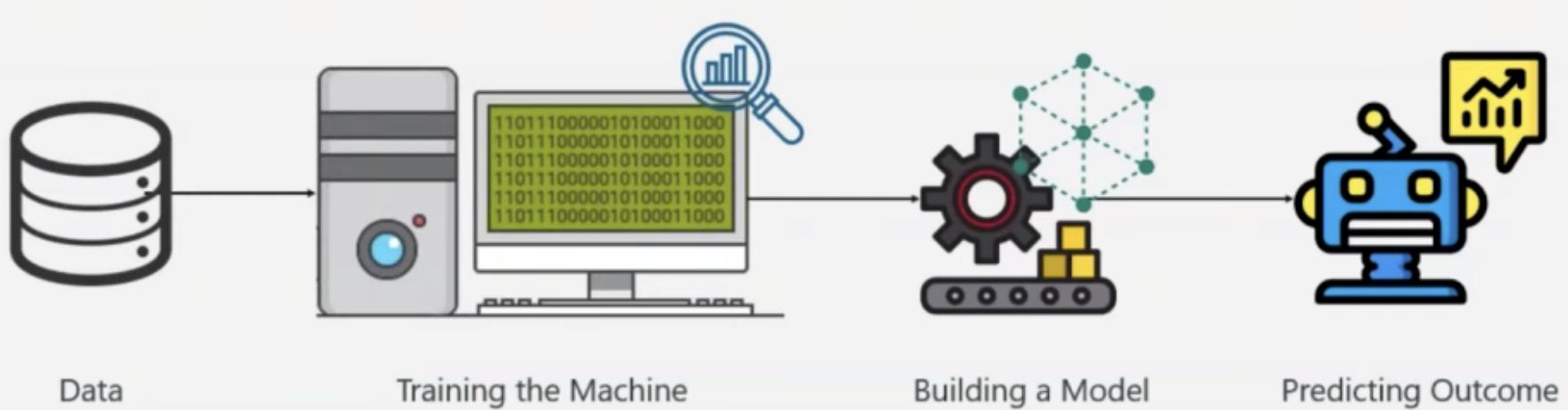
Why Machine Learning?



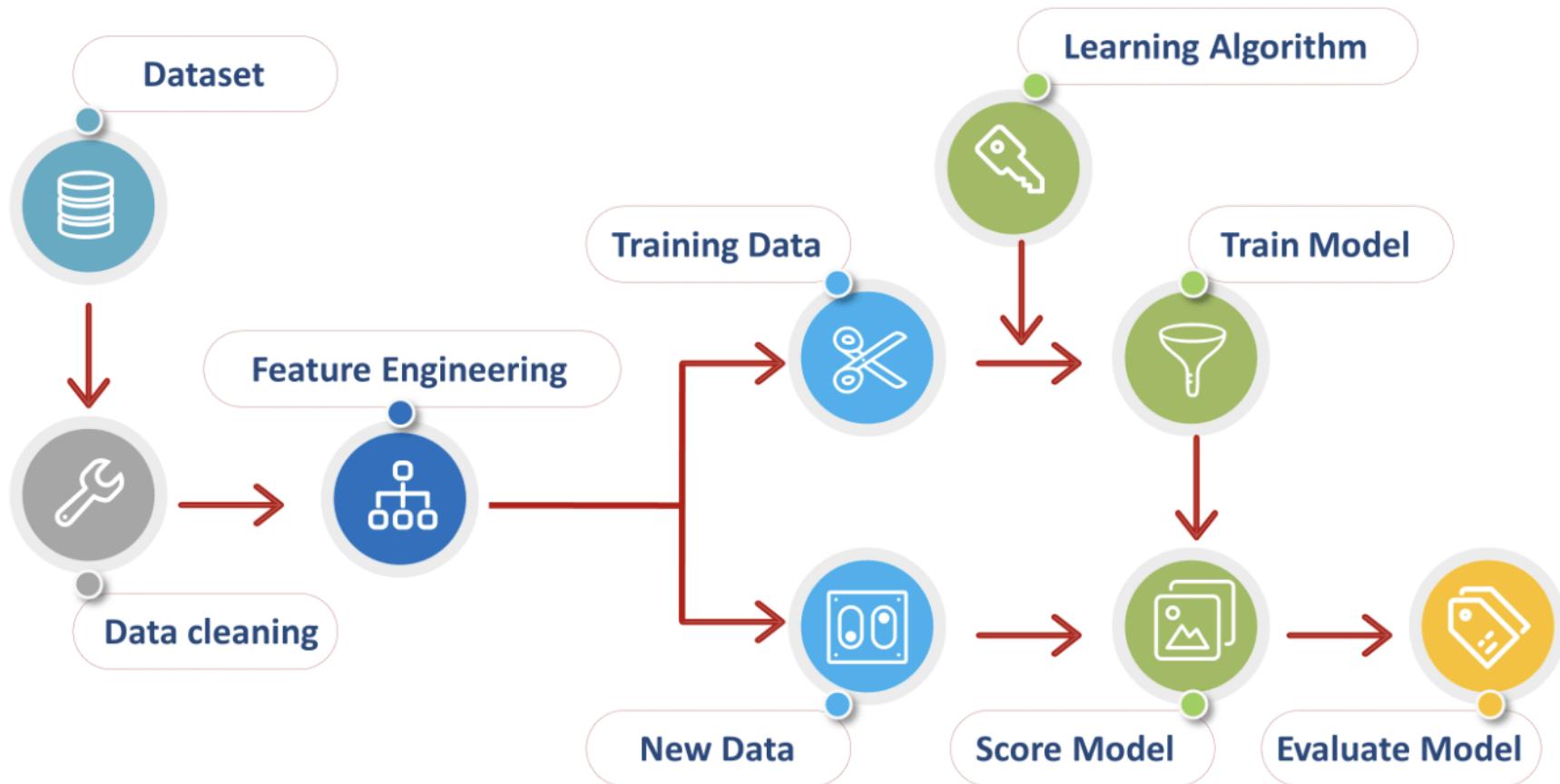
Machine Learning History

In 1959, Arthur Samuel first proposed the concept Machine Learning:

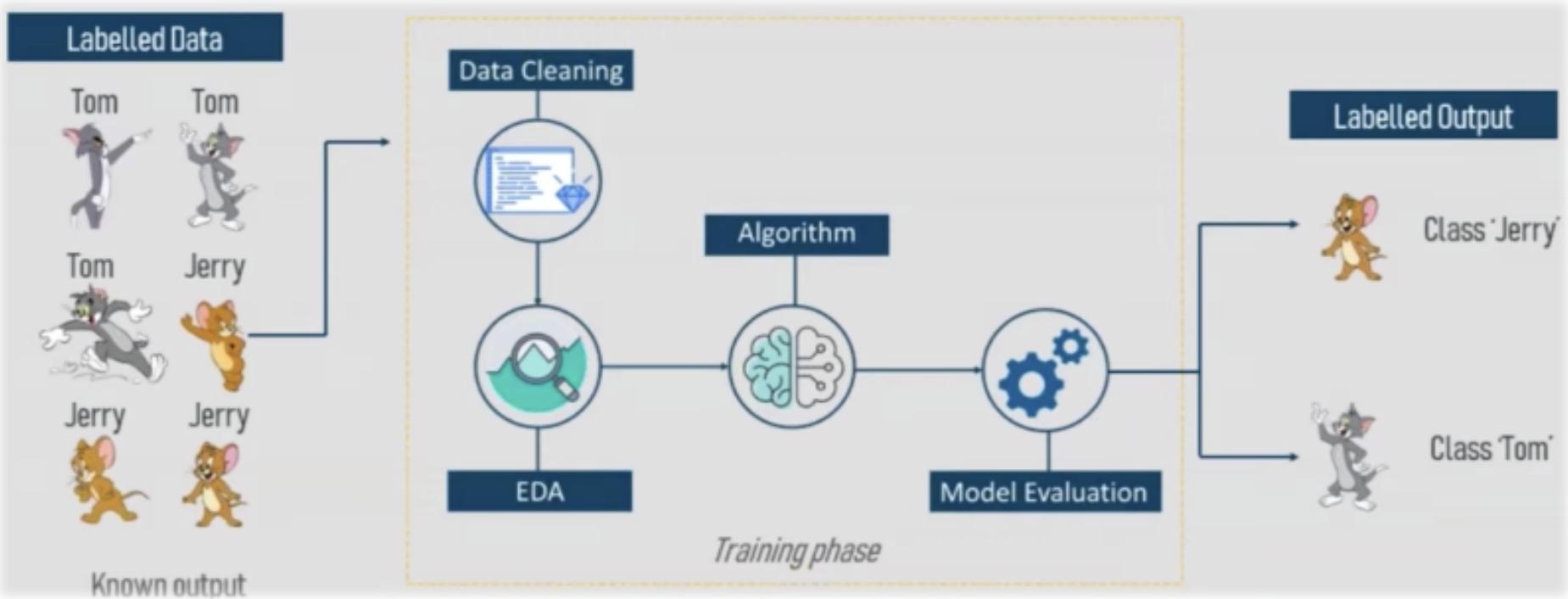
"A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P if its performance at tasks in T, as measured by P, improves with experience E."



Machine Learning Process

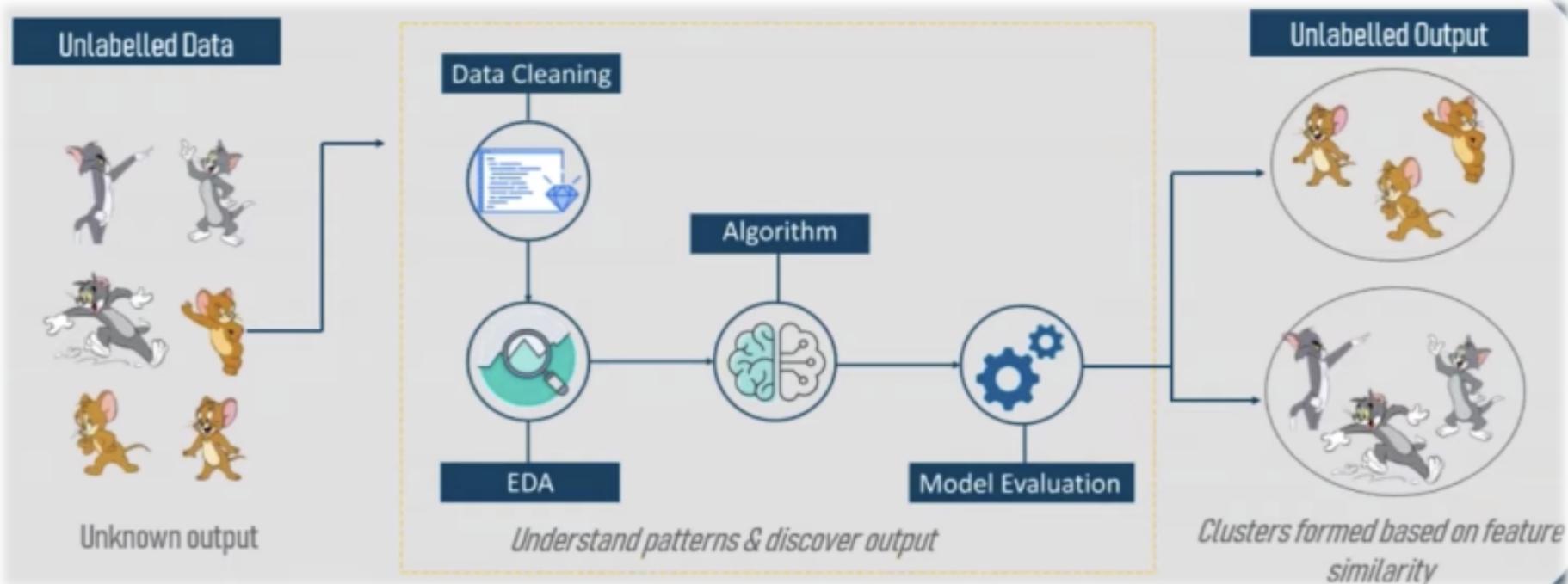


Supervised Learning



(EDA: Exploratory Data Analysis)

Unsupervised Learning

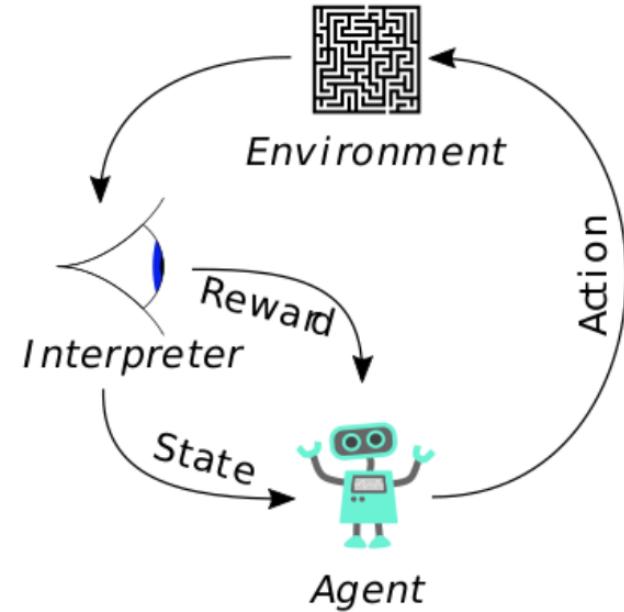
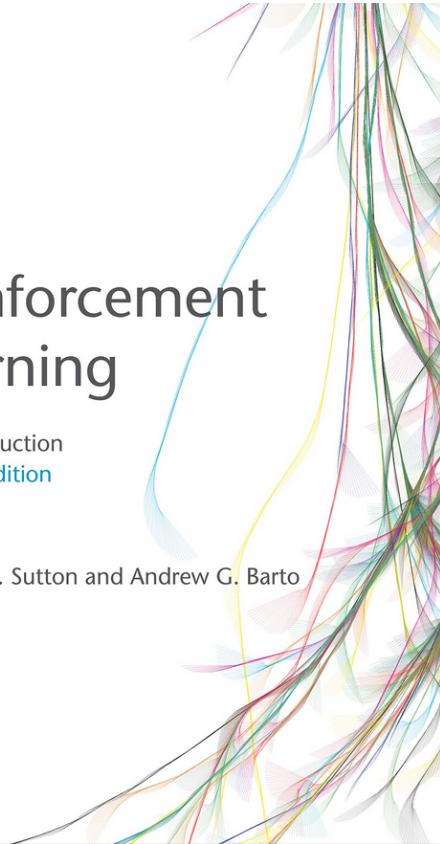


Reinforcement Learning

Reinforcement Learning

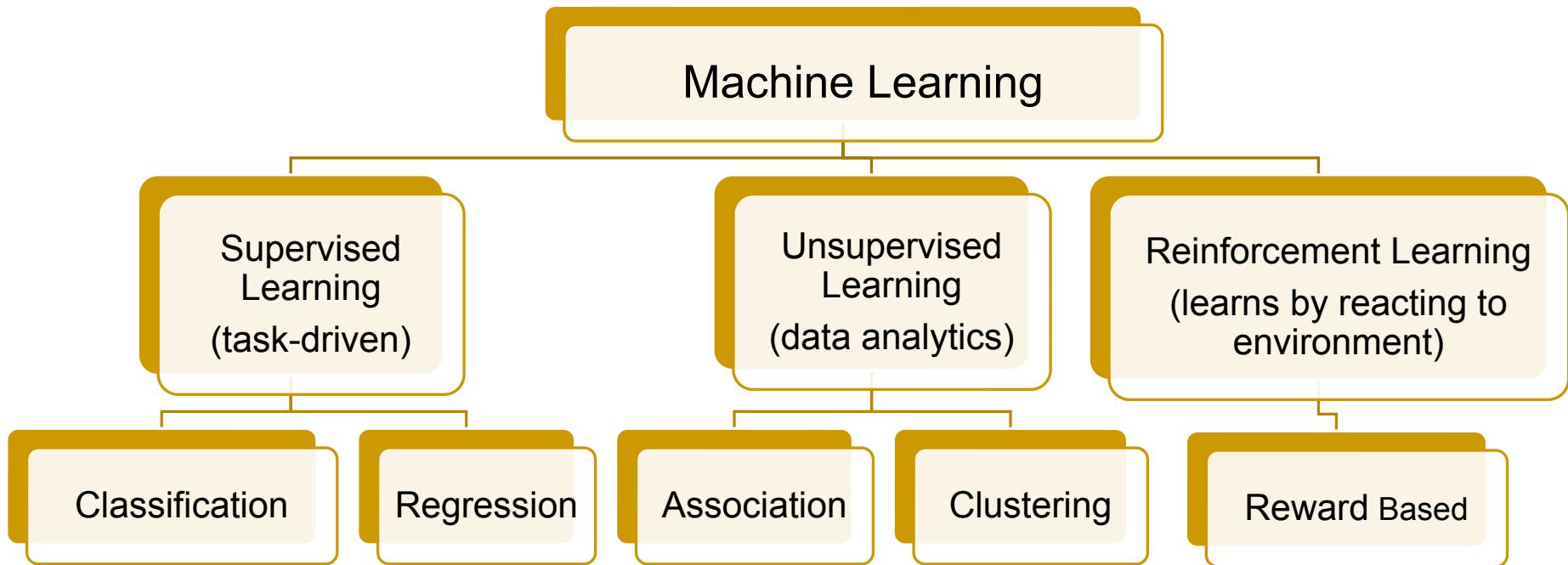
An Introduction
second edition

Richard S. Sutton and Andrew G. Barto



The typical RL scenario: an agent takes actions in an environment, which is interpreted into a reward and a representation of the state, which are fed back into the agent.

Types of ML Algorithms



ML outside of AI

ML is widely used in Data Mining

- a.k.a. Knowledge Discovery in Databases (KDD)
- e.g. Clustering, Anomaly Detection, Association Rule Mining
- Example: predict if a customer is likely to purchase certain goods according to history of shopping activities.



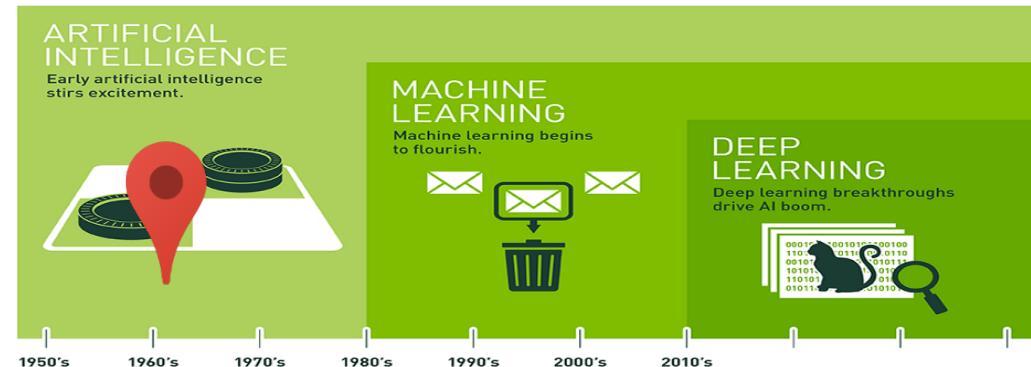
Types of Machine Learning

	Supervised Learning	Unsupervised Learning	Reinforcement Learning
Definition	The machine learns by using labelled data	The machine is trained on unlabeled data without any guidance	An agent interacts with its environment by producing actions & discovers errors and rewards
Types of problems	Regression & Classification	Association & Clustering	Reward based
Type of data	Labelled data	Unlabelled data	No pre-defined data
Training	External supervision	No supervision	No supervision
Approach	Map labelled input to known output	Understand patterns and discover output	Follow trial and error method
Popular Algorithms	Linear Regression, Logistic Regression, KNN, etc	K-means, C-means, etc	Q-learning, etc

Today

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2. Naïve Bayes Classifier
3. Evaluation

YOU ARE HERE!



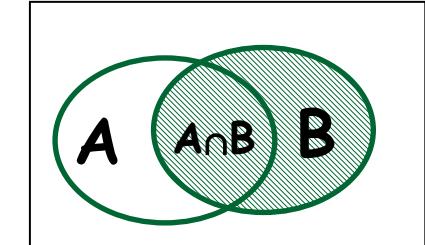
Motivation

- How do we represent and reason about non-factual knowledge?
 - It *might* rain tonight
 - If you have red spots on your face, you *might* have the measles
 - This e-mail is *most likely* spam
 - I can't read this character, but it *looks* like a "B"
 - These 2 pictures are *very likely* of the same person
 - ...

Remember...

- P is a probability function:

- $0 \leq P(A) \leq 1$
 - $P(A) = 0 \Rightarrow$ the event A will never take place
 - $P(A) = 1 \Rightarrow$ the event A must take place
 - $\sum_i P(A_i) = 1 \Rightarrow$ one of the events A_i will take place
 - $P(A) + P(\sim A) = 1$



- Joint probability

- intersection $A_1 \cap \dots \cap A_n$ is an event that takes place if **all** the events A_1, \dots, A_n take place
 - denoted $P(A \cap B)$ or $P(A, B)$

- Sum Rule

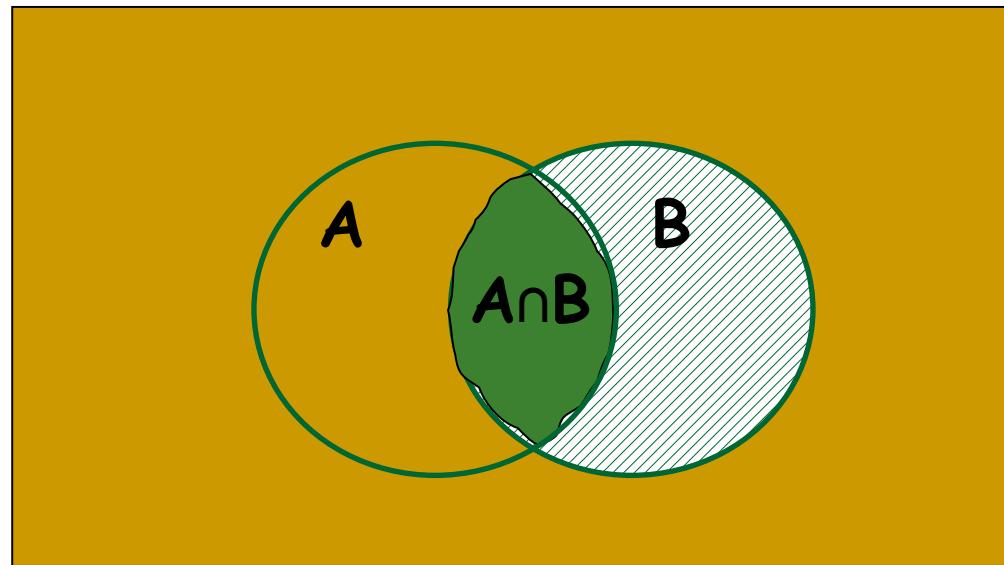
- union $A_1 \cup \dots \cup A_n$ is an event that takes place if **at least one** of the events A_1, \dots, A_n takes place
 - denoted $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional Probability

- Prior (or unconditional) probability
 - Probability of an event before any evidence is obtained
 - $P(A) = 0.1$ $P(\text{rain today}) = 0.1$
 - i.e. your belief about A given that you have no evidence
- Posterior (or conditional) probability
 - Probability of an event given that you know that B is true ($B = \text{some evidence}$)
 - $P(A|B) = 0.8$ $P(\text{rain today} | \text{cloudy}) = 0.8$
 - i.e. your belief about A given that you know B

Conditional Probability (con't)

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A, B)}{P(B)}$$



Example



- Rolling two dice (together):

2	3	4	5	6	7	8	9	10	11	12
1/36	1/18	1/12	1/9	5/36	1/6	5/36	1/9	1/12	1/18	1/36

- Rolling two dice one after the other, first dice rolled 1:

2	3	4	5	6	7	8	9	10	11	12
1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0

Chain Rule

- With 2 events, the probability that A and B occur is:

$$P(A|B) = \frac{P(A,B)}{P(B)} \text{ so } P(A,B) = P(A|B) \times P(B)$$

- With 3 events, the probability that A, B and C occur is:
 - The probability that A occurs
 - Times, the probability that B occurs, assuming that A occurred
 - Times, the probability that C occurs, assuming that A and B have occurred
- With n events, we can generalize to the Chain rule:

$$\begin{aligned} & P(A_1, A_2, A_3, A_4, \dots, A_n) \\ &= P(\cap A_i) \\ &= P(A_1) \times P(A_2 | A_1) \times P(A_3 | A_1, A_2) \times \dots \times P(A_n | A_1, A_2, A_3, \dots, A_{n-1}) \end{aligned}$$

So what?

- we can do probabilistic inference
 - i.e. infer new knowledge from observed evidence

Example 1

- Joint probability distribution:

$P(\text{Toothache} \cap \text{Cavity})$

evidence

hypothesis

	Toothache	\sim Toothache
Cavity	0.04	0.06
\sim Cavity	0.01	0.89

$$P(H | E) = \frac{P(H \cap E)}{P(E)}$$

→ Worksheet #3 (“Joint Probabilities”)

Getting the Probabilities

- in most applications, you just count from a set of observations

$$P(A) = \frac{\text{count_of_}A}{\text{count_of_all_events}}$$

$$P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{\text{count_of_}A\text{_and_}B\text{_together}}{\text{count_of_all_}B}$$

Combining Evidence

- Assume now 2 pieces of evidence:
- Suppose, we know that
 - $P(\text{Cavity} \mid \text{Toothache}) = 0.12$
 - $P(\text{Cavity} \mid \text{Young}) = 0.18$
- A patient complains about Toothache and is Young...
 - what is $P(\text{Cavity} \mid \text{Toothache} \cap \text{Young})$?

Combining Evidence

	Toothache		\sim Toothache	
	Young	\sim Young	Young	\sim Young
Cavity	0.108	0.012	0.072	0.008
\sim Cavity	0.016	0.064	0.144	0.576

$P(\text{Toothache} \cap \text{Cavity} \cap \text{Young})$

- But how do we get the data ?
- In reality, we may have dozens, hundreds of variables
- We cannot have a table with the probability of all possible combinations of variables
 - Ex. with 16 binary variables, we would need 2^{16} entries

Independent Events

- In real life:
 - some variables are independent...
 - ex: living in Montreal & tossing a coin
 - $P(\text{Montreal, head}) = P(\text{Montreal}) * P(\text{head})$
 - probability of 2 heads in a row:
 - $P(\text{head, head}) = 1/2 * 1/2 = 1/4$
 - some variables are not independent...
 - ex: living in Montreal & wearing snow boots
 - $P(\text{Montreal, snow boots}) \neq P(\text{Montreal}) * P(\text{snow boots})$

Independent Events

- Two events A and B are independent:
 - if the occurrence of one of them does not influence the occurrence of the other
 - i.e. A is independent of B if $P(A) = P(A|B)$
- If A and B are independent, then:
 - $P(A,B) = P(A|B) \times P(B)$ (by chain rule)
 $= P(A) \times P(B)$ (by independence)
- To make things work in real applications, we often assume that events are independent
 - $P(A,B) = P(A) \times P(B)$

Conditional Independent Events

- Two events A and B are conditionally independent given C:
 - Given that C is true, then any evidence about B cannot change our belief about A
 - $P(A, B \mid C) = P(A \mid C) \times P(B \mid C)$.

Bayes' Theorem

- given: $P(A|B) = \frac{P(A,B)}{P(B)}$ so $P(A,B) = P(A|B) \times P(B)$
 $P(B|A) = \frac{P(A,B)}{P(A)}$ so $P(A,B) = P(B|A) \times P(A)$
- then: $P(A|B) \times P(B) = P(B|A) \times P(A)$
- and: $P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$

→ Worksheet #3 (“Bayes’ Theorem”)

So?

- We typically want to know: $P(\text{Hypothesis} \mid \text{Evidence})$
 - $P(\text{Disease} \mid \text{Symptoms}) \dots P(\text{meningitis} \mid \text{red spots})$
 - $P(\text{Cause} \mid \text{Side Effect}) \dots P(\text{misaligned brakes} \mid \text{squeaky wheels})$
- But $P(\text{Hypothesis} \mid \text{Evidence})$ is hard to gather
 - *ex: out of all people who have red spots... how many have meningitis?*
- However $P(\text{Evidence} \mid \text{Hypothesis})$ is easier to gather
 - *ex: out of all people who have the meningitis ... how many have red spots?*
- So

$$P(\text{Hypothesis} \mid \text{Evidence}) = \frac{P(\text{Evidence} \mid \text{Hypothesis}) \times P(\text{Hypothesis})}{P(\text{Evidence})}$$

Example 2

Assume we only have 1 hypothesis

Assume:

- $P(\text{spots=yes} \mid \text{meningitis=yes}) = 0.4$
- $P(\text{meningitis=yes}) = 0.00003$
- $P(\text{spots=yes}) = 0.05$

$$P(\text{meningitis=yes} \mid \text{spots=yes})$$

$$= \frac{P(\text{spots=yes} \mid \text{meningitis=yes}) \times P(\text{meningitis=yes})}{P(\text{spots=yes})}$$

$$= \frac{0.4 \times 0.00003}{0.05} = 0.00024$$

→ If you have spots... you are more likely to have meningitis than if we don't know about you having spots

Example 3

- Predict the weather tomorrow based on tonight's sunset...
- Assume we have 3 hypotheses...
 - H_1 : weather will be *nice* $P(H_1) = 0.2$
 - H_2 : weather will be *bad* $P(H_2) = 0.5$
 - H_3 : weather will be *mixed* $P(H_3) = 0.3$
- And 1 piece of evidence with 3 possible values
 - E_1 : today, there's a *beautiful sunset*
 - E_2 : today, there's a *average sunset*
 - E_3 : today, there's *no sunset*

$P(E_2 | H_1)$

$P(E_x H_i)$	E_1	E_2	E_3
H_1	0.7	0.2	0.1
H_2	0.3	0.3	0.4
H_3	0.4	0.4	0.2

Example 3

- Observation: average sunset (E_2)
- Question: how will be the weather tomorrow?
 - $P(H_i | E_2)$?
 - predict the weather that maximizes the probability
 - select H_i such that $P(H_i | E_2)$ is the greatest

$$P(H_i | E_2) = \frac{P(H_i) \times P(E_2 | H_i)}{P(E_2)}$$

$$\begin{aligned}P(E_2) &= P(H_1) \times P(E_2 | H_1) + P(H_2) \times P(E_2 | H_2) + P(H_3) \times P(E_2 | H_3) \\&=.2 \times .2 + .5 \times .3 + .3 \times .4 = .04 + .15 + .12 = 0.31\end{aligned}$$

→ Worksheet #3 (“AI Weather Prediction”)

Bayes' Reasoning

- Out of n hypothesis...
 - we want to find the most probable H_i given the evidence E
- So we choose the H_i with the largest $P(H_i|E)$

$$H_{NB} = \operatorname{argmax}_{H_i} P(H_i|E) = \operatorname{argmax}_{H_i} \frac{P(H_i) \times P(E|H_i)}{P(E)}$$

- But... $P(E)$
 - is the same for all possible H_i (and is hard to gather anyways)
 - so we can drop it
- So Bayesian reasoning:

$$H_{NB} = \operatorname{argmax}_{H_i} \frac{P(H_i) \times P(E|H_i)}{P(E)} = \operatorname{argmax}_{H_i} P(H_i) \times P(E|H_i)$$

Representing the Evidence

- The evidence is typically represented by many attributes/features
 - beautiful sunset? clouds? temperature? summer?, ...
- so often represented as a feature/attribute vector

	evidence				hypothesis
	sunset a ₁	clouds a ₂	temp a _{3.}	summer a ₄	
e1	beautiful	no	high	yes	Nice

- $e1 = \langle a_1, \dots, a_n \rangle$
- $e1 = \langle \text{sunset:beautiful, clouds:no, temp:high, summer:yes} \rangle$

Combining Evidence

toothache	young	cavity
yes	yes	?

$$P(\text{Cavity} = \text{yes} | \text{Toothache} = \text{yes} \cap \text{Young} = \text{yes}) = ?$$

with Bayes Rule :

$$= \frac{P(\text{Toothache} = \text{yes} \cap \text{Young} = \text{yes} | \text{Cavity} = \text{yes}) \times P(\text{Cavity} = \text{yes})}{P(\text{Toothache} = \text{yes} \cap \text{Young} = \text{yes})}$$

with independence assumption :

$$= \frac{P(\text{Toothache} = \text{yes} \cap \text{Young} = \text{yes} | \text{Cavity} = \text{yes}) \times P(\text{Cavity} = \text{yes})}{P(\text{Toothache} = \text{yes}) \times P(\text{Young} = \text{yes})}$$

with conditional independence assumption :

$$= \frac{P(\text{Toothache} = \text{yes} | \text{Cavity} = \text{yes}) \times P(\text{Young} = \text{yes} | \text{Cavity} = \text{yes}) \times P(\text{Cavity} = \text{yes})}{P(\text{Toothache} = \text{yes}) \times P(\text{Young} = \text{yes})}$$

Now we have decomposed the joint probability distribution into much smaller pieces...

Combining Evidence

toothache	young	cavity
yes	yes	yes? or no?

But since we only care about ranking the hypothesis...

? > $P(\text{Cavity} = \text{yes} | \text{Toothache} = \text{yes} \cap \text{Young} = \text{yes})$

$P(\text{Cavity} = \text{no} | \text{Toothache} = \text{yes} \cap \text{Young} = \text{yes})$

$$\frac{P(\text{Cavity} = \text{yes}) \times P(\text{Toothache} = \text{yes} | \text{Cavity} = \text{yes}) \times P(\text{Young} = \text{yes} | \text{Cavity} = \text{yes})}{P(\text{Toothache} = \text{yes}) \times P(\text{Young} = \text{yes})}$$

$$\frac{P(\text{Cavity} = \text{no}) \times P(\text{Toothache} = \text{yes} | \text{Cavity} = \text{no}) \times P(\text{Young} = \text{yes} | \text{Cavity} = \text{no})}{P(\text{Toothache} = \text{yes}) \times P(\text{Young} = \text{yes})}$$

? > $P(\text{Cavity} = \text{yes}) \times P(\text{Toothache} = \text{yes} | \text{Cavity} = \text{yes}) \times P(\text{Young} = \text{yes} | \text{Cavity} = \text{yes})$

$P(\text{Cavity} = \text{no}) \times P(\text{Toothache} = \text{yes} | \text{Cavity} = \text{no}) \times P(\text{Young} = \text{yes} | \text{Cavity} = \text{no})$

$$H_{NB} = \underset{H_i}{\operatorname{argmax}} \frac{P(H_i) \times P(E|H_i)}{P(E)} = \underset{H_i}{\operatorname{argmax}} P(H_i) \times P(E|H_i) = \underset{H_i}{\operatorname{argmax}} P(H_i) \times P(< a_1, a_2, a_3, \dots, a_n > | H_i) = \underset{H_i}{\operatorname{argmax}} P(H_i) \times \prod_{j=1}^n P(a_j | H_i)$$

Example 4

evidence

Day	Outlook	Temperature	Humidity	Wind	Play Tennis
Day1	Sunny	Hot	High	Weak	No
Day2	Sunny	Hot	High	Strong	No
Day3	Overcast	Hot	High	Weak	Yes
Day4	Rain	Mild	High	Weak	Yes
Day5	Rain	Cool	Normal	Weak	Yes
Day6	Rain	Cool	Normal	Strong	No
Day7	Overcast	Cool	Normal	Strong	Yes
Day8	Sunny	Mild	High	Weak	No
Day9	Sunny	Cool	Normal	Weak	Yes
Day10	Rain	Mild	Normal	Weak	Yes
Day11	Sunny	Mild	Normal	Strong	Yes
Day12	Overcast	Mild	High	Strong	Yes
Day13	Overcast	Hot	Normal	Weak	Yes
Day14	Rain	Mild	High	Strong	No

Example 4

- Goal: Given a new instance $X = \langle a_1, \dots, a_n \rangle$, classify as Yes/No

$$H_{NB} = \underset{H_i}{\operatorname{argmax}} \frac{P(H_i) \times P(E|H_i)}{P(E)} = \underset{H_i}{\operatorname{argmax}} P(H_i) \times P(E|H_i) = \underset{H_i}{\operatorname{argmax}} P(H_i) \times P(\langle a_1, a_2, a_3, \dots, a_n \rangle | H_i) = \underset{H_i}{\operatorname{argmax}} P(H_i) \times \prod_{j=1}^n P(a_j | H_i)$$

- Naïve Bayes: Assumes that the attributes/features are conditionally independent

Example 4

- Goal: Given a new instance $X = \langle a_1, \dots, a_n \rangle$, classify as Yes/No

$$H_{NB} = \underset{H_i}{\operatorname{argmax}} P(H_i) \times \prod_{j=1}^n P(a_j | H_i)$$

1st estimate the probabilities from the training examples:

- a) For each hypothesis H_i
estimate $P(H_i)$
- b) For each attribute value a_j of each instance (evidence)
estimate $P(a_j | H_i)$

Example 4

1. TRAIN:

- compute the probabilities from the training set

$$\left. \begin{array}{l} P(\text{PlayTennis} = \text{yes}) = 9/14 = 0.64 \\ P(\text{PlayTennis} = \text{no}) = 5/14 = 0.36 \end{array} \right\} \text{prior probabilities } P(H_i)$$

$$P(\text{Out} = \text{sunny} | \text{PlayTennis} = \text{yes}) = 2/9 = 0.22$$

$$P(\text{Out} = \text{sunny} | \text{PlayTennis} = \text{no}) = 3/5 = 0.60$$

$$P(\text{Out} = \text{rain} | \text{PlayTennis} = \text{yes}) = 3/9 = 0.33$$

$$P(\text{Out} = \text{rain} | \text{PlayTennis} = \text{no}) = 2/5 = 0.4$$

...

$$P(\text{Wind} = \text{strong} | \text{PlayTennis} = \text{yes}) = 3/9 = 0.33$$

$$P(\text{Wind} = \text{strong} | \text{PlayTennis} = \text{no}) = 3/5 = 0.60$$

conditional probabilities
 $P(a_j | H_i)$

Example 4

2. TEST:

classify the new case: $X = (\text{Outlook: Sunny}, \text{Temp: Cool}, \text{Hum: High}, \text{Wind: Strong})$

$$H_{NB} = \underset{H_i \in \{\text{yes, no}\}}{\operatorname{argmax}} P(H_i) \times P(X | H_i)$$

$$= \underset{H_i \in \{\text{yes, no}\}}{\operatorname{argmax}} P(H_i) \times \prod_j P(a_j | H_i)$$

$$= \underset{H_i \in \{\text{yes, no}\}}{\operatorname{argmax}} P(H_i) \times P(\text{Outlook} = \text{sunny} | H_i) \times P(\text{Temp} = \text{cool} | H_i) \\ \times P(\text{Humidity} = \text{high} | H_i) \times P(\text{Wind} = \text{strong} | H_i)$$

1) $P(\text{PlayTennis} = \text{yes})$

$$\times P(\text{Outlook} = \text{sunny} | \text{PlayTennis} = \text{yes}) \times P(\text{Temp} = \text{cool} | \text{PlayTennis} = \text{yes}) \times P(\text{Hum} = \text{high} | \text{PlayTennis} = \text{yes}) \times P(\text{Wind} = \text{strong} | \text{PlayTennis} = \text{yes}) \\ = 0.0053$$

2) $P(\text{PlayTennis} = \text{no})$

$$\times P(\text{Outlook} = \text{sunny} | \text{PlayTennis} = \text{no}) \times P(\text{Temp} = \text{cool} | \text{PlayTennis} = \text{no}) \times P(\text{Hum} = \text{high} | \text{PlayTennis} = \text{no}) \times P(\text{Wind} = \text{strong} | \text{PlayTennis} = \text{no}) \\ = 0.0206$$

\Rightarrow answer : $\text{PlayTennis}(X) = \text{no}$

Application of Bayesian Reasoning

- Categorization: $P(\text{Category} \mid \text{Features of Object})$
 - Diagnostic systems: $P(\text{Disease} \mid \text{Symptoms})$
 - Text classification: $P(\text{sports_news} \mid \text{text})$
 - Character recognition: $P(\text{character} \mid \text{bitmap})$
 - Speech recognition: $P(\text{words} \mid \text{acoustic signal})$
 - Image processing: $P(\text{face_person} \mid \text{image features})$
 - Spam filter: $P(\text{spam_message} \mid \text{words in e-mail})$
 - ...

Naive Bayes Classifier

- A simple probabilistic classifier based on Bayes' theorem
 - with strong (naive) independence assumption
 - i.e. the features/attributes are conditionally independent
- The assumption of conditional independence, often does not hold...
- But Naïve Bayes works very well in many applications anyways!
 - ex: Medical Diagnosis
 - ex: Text Categorization (spam filtering)

Ex. Application: Spam Filtering

- Task: classify e-mails (documents) into a pre-defined class
 - ex: spam / ham
 - ex: sports, recreation, politics, war, economy, ...
 - ex: customer email → order, complaint, support request, ...
- Given
 - N sets of training texts (1 set for each class)
 - Each set is already tagged by the class name



Strictly speaking, what we will see is called a Multinomial Naïve Bayes classifier, because we will count the number of words, as opposed to just using binary values for the presence/absence of words...

e-mail Representation

- each e-mail is represented by a vector of feature/value:
 - feature = actual words in the e-mail
 - value = number of times that word appears in the e-mail
- each e-mail in the training set is tagged with the correct category.

data instance	features / evidence / X						$f(X)$
	offer	money	viagra	laptop	exam	study	
email 1	3	2	5	1	0	1	SPAM
email 2	1	1	0	5	4	3	HAM
email 3	0	3	2	1	0	1	SPAM
...							

- task: correctly tag a new e-mail

	offer	money	viagra	laptop	exam	study	category
new email	2	1	0	1	1	2	?

Naïve Bayes Algorithm

```
// 1. training
for all classes ci // ex. ham or spam
    for all words wj in the vocabulary
        compute P(wj | ci) =  $\frac{\text{count}(w_j, c_i)}{\sum_j \text{count}(w_j, c_i)}$ 
for all classes ci
    compute P(ci) =  $\frac{\text{count}(\text{documents in } c_i)}{\text{count}(\text{all documents})}$ 

// 2. testing a new document D
for all classes ci // ex. ham or spam
    score(ci) = P(ci)
    for all words wj in the D
        score(ci) = score(ci) x P(wj | ci)

choose c* = with the greatest score(ci)
```

	w ₁	w ₂	w ₃	w ₄	w ₅	w ₆
c ₁ : SPAM	p(w ₁ c ₁)	p(w ₂ c ₁)	p(w ₃ c ₁)	p(w ₄ c ₁)	p(w ₅ c ₁)	p(w ₆ c ₁)
c ₂ : HAM	p(w ₁ c ₂)	p(w ₂ c ₂)	p(w ₃ c ₂)	p(w ₄ c ₂)	p(w ₅ c ₂)	p(w ₆ c ₂)

Example

■ Dataset

□ c1: SPAM

doc1: "cheap meds for sale"

doc2: "click here for the best meds"

doc3: "book your trip"



Spam

□ c2: HAM

doc4: "cheap book sale, not meds"

doc5: "here is the book for you"



Ham

■ Question:

□ doc6: "the cheap book"

□ should it be classified as HAM or SPAM?



?

→ Worksheet #3 ("Email Spam Detection")

Be Careful: Smooth Probabilities

- normally: $P(w_i | c_j) = \frac{\text{(frequency of } w_i \text{ in } c_j)}{\text{total number of words in } c_j}$
- what if we have a $P(w_i | c_j) = 0 \dots ?$
 - ex. the word "dumbo" never appeared in the class SPAM?
 - then $P(\text{"dumbo"} | \text{SPAM}) = 0$
- so if a text contains the word "dumbo", the class SPAM is completely ruled out !
- to solve this: we assume that every word always appears at least once (or a smaller value)
 - ex: add-1 smoothing:

$$P(w_i | c_j) = \frac{\text{(frequency of } w_i \text{ in } c_j) + 1}{\text{total number of words in } c_j + \text{size of vocabulary}}$$

Be Careful: Use Logs

- if we really do the product of probabilities...
 - $\operatorname{argmax}_{c_j} P(c_j) \prod P(w_i | c_j)$
 - we soon have numerical underflow...
 - ex: $0.01 \times 0.02 \times 0.05 \times \dots$
- so instead, we add the log of the probs
 - $\operatorname{argmax}_{c_j} \log(P(c_j)) + \sum \log(P(w_i | c))$
 - ex: $\log(0.01) + \log(0.02) + \log(0.05) + \dots$

Example COOKING SPORTS

Dataset

c1: COOKING	c2: SPORTS
<p>doc₁: ... stove... kitchen... the... heat doc₂: ... kitchen... pasta... stove... ... doc₁₀₀₀₀₀: ... stove...heat... ball...</p>	<p>doc₁: ... ball... heat... doc₂: ... the... referee... player... ... doc₇₅₀₀₀: goal... injury ...</p>

Assume:

- ◻ |V| = 100 vocabulary = {ball, heat, kitchen, referee, stove, the, ... }
- ◻ 500,000 words in Cooking
- ◻ 300,000 words in Sports
- ◻ 100,000 docs in Cooking
- ◻ 75,000 docs in Sports

Example

- Training - Unsmoothed / Smoothed probs:

- | | |
|--|---|
| <ul style="list-style-type: none">□ $P(\text{ball} \text{COOKING}) =$□ $P(\text{heat} \text{COOKING}) =$□ $P(\text{kitchen} \text{COOKING}) =$□ $P(\text{referee} \text{COOKING}) =$□ $P(\text{stove} \text{COOKING}) =$□ $P(\text{the} \text{COOKING}) =$□ ...□ $P(\text{COOKING}) =$ | <ul style="list-style-type: none">$P(\text{ball} \text{SPORTS}) =$$P(\text{heat} \text{SPORTS}) =$$P(\text{kitchen} \text{SPORTS}) =$$P(\text{referee} \text{SPORTS}) =$$P(\text{stove} \text{SPORTS}) =$$P(\text{the} \text{SPORTS}) =$$P(\text{SPORTS}) =$ |
|--|---|

- Testing: "the referee hit the blue bird"

- $\text{Score}(\text{COOKING}) = \log() + \log(P(\text{the}|\text{COOKING})) + \log(P(\text{referee}|\text{COOKING})) + \log(P(\text{hit}|\text{COOKING})) + \log(P(\text{the}|\text{COOKING}))$
 - $\text{Score}(\text{SPORTS}) = \log() + \log(P(\text{the}|\text{SPORTS})) + \log(P(\text{referee}|\text{SPORTS})) + \log(P(\text{hit}|\text{SPORTS})) + \log(P(\text{the}|\text{SPORTS}))$

Example

■ Training - Unsmoothed / Smoothed probs:

- | | | | |
|--|-----------------|---|-----------------|
| □ $P(\text{ball} \text{COOKING}) = \frac{10,000}{500,000}$ | $\frac{??}{??}$ | $P(\text{ball} \text{SPORTS}) = \frac{10,000}{300,000}$ | $\frac{??}{??}$ |
| □ $P(\text{heat} \text{COOKING}) = \frac{255}{500,000}$ | $\frac{??}{??}$ | $P(\text{heat} \text{SPORTS}) = \frac{1,8000}{300,000}$ | $\frac{??}{??}$ |
| □ $P(\text{kitchen} \text{COOKING}) = \frac{2,600}{500,000}$ | $\frac{??}{??}$ | $P(\text{kitchen} \text{SPORTS}) = \frac{0}{300,000}$ | $\frac{??}{??}$ |
| □ $P(\text{referee} \text{COOKING}) = \frac{0}{500,000}$ | $\frac{??}{??}$ | $P(\text{referee} \text{SPORTS}) = \frac{1,500}{300,000}$ | $\frac{??}{??}$ |
| □ $P(\text{stove} \text{COOKING}) = \frac{3,600}{500,000}$ | $\frac{??}{??}$ | $P(\text{stove} \text{SPORTS}) = \frac{4}{300,000}$ | $\frac{??}{??}$ |
| □ $P(\text{the} \text{COOKING}) = \frac{400,000}{500,000}$ | $\frac{??}{??}$ | $P(\text{the} \text{SPORTS}) = \frac{19,000}{300,000}$ | $\frac{??}{??}$ |
| □ ... | | | |
| □ $P(\text{COOKING}) = \frac{100,000}{175,000}$ | | $P(\text{SPORTS}) = \frac{75,000}{175,000}$ | |

■ Testing: "the referee hit the blue bird"

- $\text{Score}(\text{COOKING}) = \log\left(\frac{100,000}{175,000}\right) + \log(P(\text{the}|\text{COOKING})) + \log(P(\text{referee}|\text{COOKING})) + \log(P(\text{hit}|\text{COOKING})) + \log(P(\text{the}|\text{COOKING}))$
- $\text{Score}(\text{SPORTS}) = \log\left(\frac{75,000}{175,000}\right) + \log(P(\text{the}|\text{SPORTS})) + \log(P(\text{referee}|\text{SPORTS})) + \log(P(\text{hit}|\text{SPORTS})) + \log(P(\text{the}|\text{SPORTS}))$

Another Application: Postal Code Recognition

BAM BAM
42 T-REX RD.
PANGAEA, RB 48016

FRED FLINSTONE
69 OLD SCHOOL AVE
BEDROCK, OLDEN-TOWN
77005

Digit Recognition

- MNIST dataset
- data set contains handwritten digits from the American Census Bureau employees and American high school students
- 28×28 grayscale images
- training set: 60,000 examples
- test set: 10,000 examples.
- Features: each pixel is used as a feature so:
 - there are $28 \times 28 = 784$ features
 - each feature = 256 greyscale value
- Task: classify new digits into one of the 10 classes



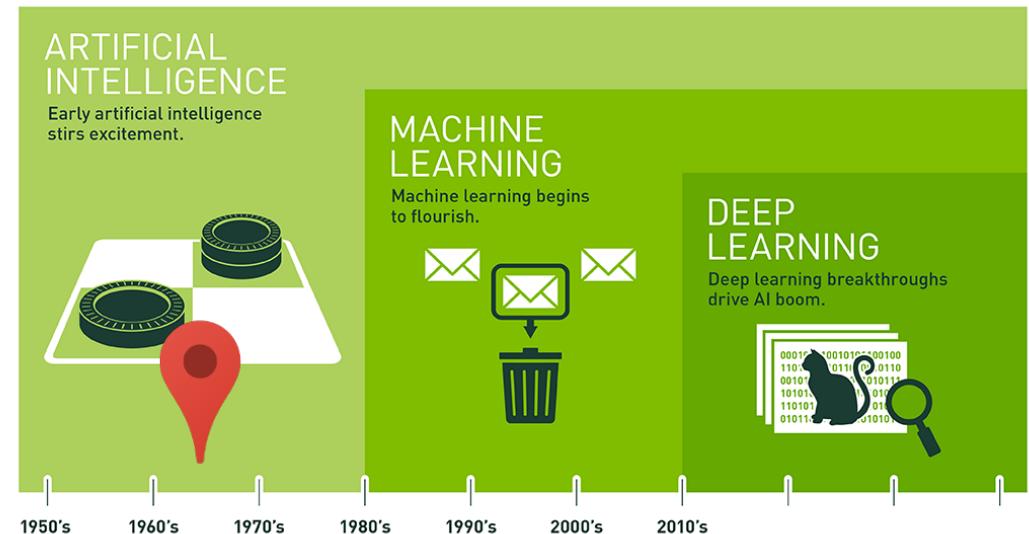
Comments on Naïve Bayes Classification

- Makes a strong assumption of conditional independence
 - that is often incorrect
 - ex: the word *ambulance* is not conditionally independent of the word *accident* given the class SPORTS
- BUT:
 - surprisingly very effective on real-world tasks
 - basis of many spam filters
 - fast, simple
 - gives confidence in its class predictions (i.e., the scores)
- Fast, easy to apply
 - often used as a *baseline* algorithm before trying other methods

Today

1. Introduction to ML
2. Naïve Bayes
3. Evaluation

YOU ARE HERE!



Evaluation of Learning Model

- How do you know if what you learned is correct?
- You run your classifier on a data set of **unseen** examples (that you did not use for training) for which you know the correct classification
- Split data set into 3 sub-sets
 1. Actual **training** set (~80%)
 2. **Validation** set (~20%)
 3. **Test** set

The diagram illustrates the splitting of a data set into three subsets. A large bracket on the right side groups the first two items (Training and Validation) and is labeled with a tilde followed by 80%, representing approximately 80% of the data. Another bracket below the third item (Test) is labeled with a tilde followed by 20%, representing approximately 20% of the data.

~80%

~20%

Standard Methodology

1. Collect a large set of examples (all with correct classifications)
2. Divide collection into training, validation and test set

Loop:

3. Apply learning algorithm to training set to learn the parameters
4. Measure performance with the validation set, and adjust hyper-parameters* to improve performance
5. Measure performance with the test set

■ DO NOT LOOK AT THE TEST SET until step 5.

Parameters:

basic values learned by the ML model. eg.

- for NB: prior & conditional probabilities
- for DTs: features to split
- for ANNs: weights

Hyper-parameters: parameters

used to set up the ML model. eg.

- for NB: value of delta for smoothing,
- for DTs: pruning level
- for ANNs: nb of hidden layers, nb of nodes per layer...

Metrics

- Accuracy
 - % of instances of the test set the algorithm correctly classifies
 - when all classes are equally important and represented
- Recall, Precision & F-measure
 - when one class is more important than the others

Accuracy

- % of instances of the test set the algorithm correctly classifies
- when all classes are equally important and represented
- problem:
 - when one class C is more important than the others
 - eg. when data set is unbalanced

	<i>Target</i>	<i>system 1</i>
	X1 ✓	X1 ✗
	X2 ✓	X2 ✗
	X3 ✓	X3 ✗
	X4 ✓	X4 ✗
	X5 ✓	X5 ✗
	X6 ✗	X6 ✗
	X7 ✗	X7 ✗

	X500 ✗	X500 ✗

$$\begin{aligned} \text{Accuracy} &= 495/500 \\ &= 99\% \end{aligned}$$

Recall, Precision, Accuracy

- Recall: How many % of instances of C were found correctly?
- Precision: Of the detected instances of C , how many % were correct?
- Accuracy: How many % were correct overall (both C and not C)?
- See confusion matrix:

Model says...	In reality, the instance is...	
	in class C	Is not in class C
instance is in class C	True Positive (TP)	False Positive (FP)
instance is NOT in class C	False Negative (FN)	True Negative (TN)

$$\text{Precision} = \frac{\text{TP}}{\text{TP} + \text{FP}}$$
$$\text{Recall} = \frac{\text{TP}}{\text{TP} + \text{FN}}$$
$$\text{Accuracy} = \frac{\text{TP} + \text{TN}}{\text{TP} + \text{TN} + \text{FP} + \text{FN}}$$

Example

	<i>Target</i>	<i>system 1</i>	<i>system 2</i>	<i>system 3</i>
	X1 ✓	X1 ✗	X1 ✓	X1 ✓
	X2 ✓	X2 ✗	X2 ✗	X2 ✓
	X3 ✓	X3 ✗	X3 ✓	X3 ✓
	X4 ✓	X4 ✗	X4 ✓	X4 ✓
	X5 ✓	X5 ✗	X5 ✗	X5 ✓
	X6 ✗	X6 ✗	X6 ✗	X6 ✓
	X7 ✗	X7 ✗	X7 ✗	X7 ✓
	... ✗ ✗	... ✗
	... ✗ ✗	... ✗
	X500 ✗	X500 ✗	X500 ✗	X500 ✗

→ Worksheet #3 (“Machine Learning System Evaluation”)

Error Analysis

- Where did the learner go wrong ?
- Use a confusion matrix / contingency table

correct class (that should have been assigned)	classes assigned by the learner							Total
	C1	C2	C3	C4	C5	C6	...	
C1	94	3	0	0	3	0		100
C2	0	93	3	4	0	0		100
C3	0	1	94	2	1	2		100
C4	0	1	3	94	2	0		100
C5	0	0	3	2	92	3		100
C6	0	0	5	0	10	85		100
...								

Today

1. Introduction to ML
2. Naïve Bayes Classifier
3. Evaluation

