19 Antiderivatives

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December 27 2023

1 First Part: Antiderivatives Part 1

Goal: Understand the concept of antiderivative and solve simple problems involving anti-erivatives.

Motivation: Understand meaningfully how antiderivatives solve the problem of determining a function with a known/prescribed instantaneous rate of change.

Definition: A function $F(x)$ is called an *antiderivative* of $f(x)$ on an interval I if $F'(x) = f(x)$ for all x in I.

$$f(x) = 32x$$

$$F(x) = 16x^2 + 100$$
$$F'(x) = f(x)$$

Theorem: If F is an antiderivative of f on I, then any antiderivative of f is of the form $F + C$, where C is a constant.

$$F(x) = 16x^2 + C$$

Where C is any constant, $F'(x)$ will have the same derivative $f(x) = 32x$. It is clear that two functions that differ by a constant have the same derivative since it will be zero. Therefore, the **theorem tells us** that any two functions with the same derivative differ by a constant.

$$F(x) = 16x^2 + C$$

All functions whose derivative is $32x$ are in the family of $F(x) = 16x^2 + C$. Take for example:

Find the general antiderivative of:

$$\frac{dy}{dx} = 6x^6 - 7x^2 - 3$$

As you can see, the derivative has been taken for y already based on $\frac{dy}{dx}$. We also know that:

$$F'(x) = 6x^6 = f(x)$$

But what is $F(x)$?

Damiam's Theorem to find out: to find the general antiderivative of a function:

$$\frac{dF}{dx} = ax^n + b$$

$$F(x) = \frac{a}{n+1}x^{n+1} + bx + C$$

Also, this is not part of the theorem but if you happen to have $F(n) = n_2$, you will need to solve for C after applying **Damiam's Theorem** to find $F(x)$.

Take the following **Geometric Formulas** into account:

$$f(x) = \sin x \to F(x) = -\cos x - C$$

$$f(x) = \cos x \to F(x) = \sin x + C$$

$$f(x) = \sec^2 x \to F(x) = \tan x + C$$

$$f(x) = \ln x \to F(x) = \frac{1}{x} + C$$

$$f(x) = e^x \to F(x) = e^x + C$$

$$f(x) = \sin 2x \to F(x) = \sin^2 x + C$$

$$f(x) = ag(x) + bh(x) \to F(x) = dG(x) + bH(x) + C$$

$$f(x) = \cos \frac{x}{n} \to F(x) = n \sin \frac{x}{n}$$

$$f(x) = \sqrt{n} \to F(x) = \sqrt{nx}$$

2 Second Part: Antiderivatives Part 2

Take the following **Formulas of Antiderivatives** into account:

$$f(x) = \frac{1}{x} \to F(x) = \ln|x| + C$$

$$f(x) = \sec x \tan x \to F(x) = \sec x + C$$

$$f(x) = \frac{1}{\sqrt{1-x^2}} \to F(x) = \sin^{-1} x + C = \arcsin x + C$$

$$f(x) = -\frac{1}{\sqrt{1-x^2}} \to F(x) = \cos^{-1} x + C = \arccos x + C$$

$$f(x) = \frac{1}{1+x^2} \to F(x) = \tan^{-1} x + C = \arctan x + C$$

There will be times where one will need to **apply antiderivative several times** E.g. Find f(x) given:

Find $f''(x)$ by taking antiderivative of $f'''(x)$

$$f'''(x) = e^x; f''(0) = 4; f'(0) = 4$$

$$f'''(x) = e^x$$

$$f''(x) = e^x + C$$

Solve for C using given $f''(0) = 4$

$$4 = e^0 + C$$

$$3 = C$$

$$f''(x) = e^x + 3$$

Find $f'(x)$ by taking antiderivative of $f''(x)$

$$f''(x) = e^x + 3$$

$$f'(x) = e^x + 3x + C$$

Solve for C using given $f'(0) = 4$

$$4 = e^0 + 3(0) + C$$

$$3 = C$$

$$f'(x) = e^x + 3x + 3$$

Find $f(x)$ by taking antiderivative of $f'(x)$

$$f'(x) = e^x + 3$$

$$f(x) = e^x + \frac{3}{2}x^2 + 3x + C$$

Therefore solving for $f(x)$.

$$\int f(x)dx$$

Indefinite integral: $\int f(x)dx$ is the notation for the general antiderivative of the function f with respect to x.

$$\int \cos x dx = \sin x + C$$

It isn's always the sharpest eyes that see things first. -Plato, The Republic