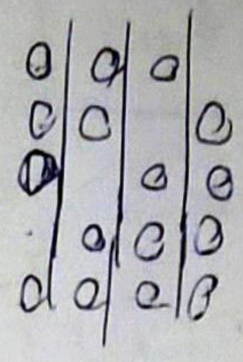
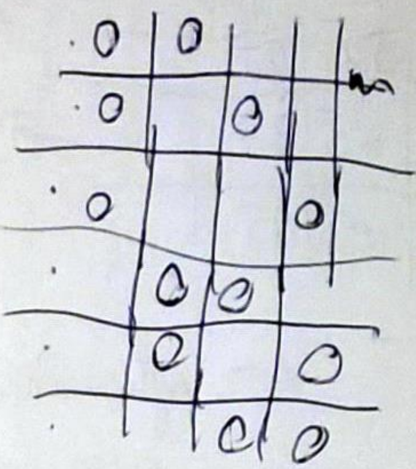
RPIS, ćw 1

1.2

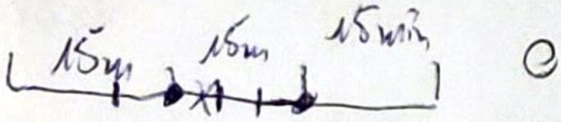
20 osób

$$\binom{1}{20}\binom{1}{20}\binom{1}{20}$$





1.3



1.9

A - wyp. karty $P(A) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{32}$

B - para liczb ... $P(B) = \frac{7}{32}$

C - więcej niż ... $P(C) = \frac{5}{32}$

$$P(A|C) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{32}}{\frac{7}{32}} = \frac{1}{32} \cdot \frac{32}{7} = \frac{32}{224}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{1}{32}}{\frac{1}{32}} = \frac{1}{32} \cdot \frac{32}{1} = \underline{1}$$

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{32}}{\frac{5}{32}} = \frac{1}{32} \cdot \frac{32}{5} = \frac{1}{5}$$

$P(C|A) = \underline{1}$

$$P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{\frac{1}{32}}{\frac{5}{32}} = \frac{1}{5}$$

$P(C|B) = \underline{1}$

RPIS ćw2

$\binom{10}{4} = \frac{10!}{4!\,6!} = \frac{7\cdot 8\cdot 9\cdot 10}{24} = 210$

1.11

$Z_1 \quad Z_2 \quad Z_3$

	%	Wym.	
Z_1	50	95	
Z_2	35	80	
Z_3	15	85	

$210 \cdot \frac{1}{16}\left(\frac{1}{2}\right)^6 = 13,125 \cdot \frac{1}{64} = \frac{13,125}{64}$

$$\begin{array}{r} 0,1275 \\ 0,2800 \\ +\ 0,4750 \\ \hline 0,8825 \end{array}$$

A – wylosowanie zespołu odpowiadającego wymaganiom

$\cancel{P(Z_1|A_1) = \frac{P(Z_1 \cap A_1)}{P(A_1)} = \frac{95\%}{}}$

$P(A|Z_1) = 95\%$

$P(A|Z_2) = 80\%$

$P(A|Z_3) = 85\%$

$P(A) = 85\% \cdot 50\% + 80\% \cdot 35\% + 85\% \cdot 15\%$

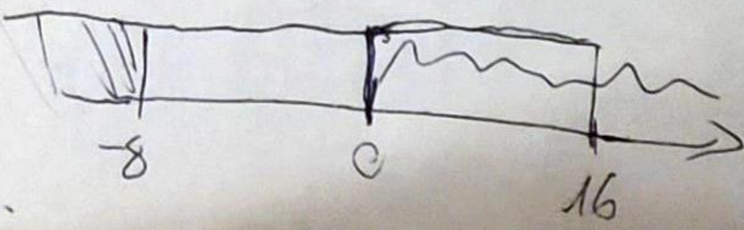
$0,475 + 0,28 + 0,1275 =$

$120 \cdot \frac{1}{8} \cdot \frac{1}{2}^7 = 120 \cdot \frac{1}{8} \cdot \frac{1}{128} = \frac{15}{128}$

b) $Z_1 \quad 5\% \cdot 95\%$

$Z_2 \quad 20\% \cdot 80$

$Z_3 \quad 15\%$



$P(A') = 0,1175$

$P(A'|Z_1) = \frac{5\%}{0,1175} = 0,425\ldots$

$\frac{10!}{1!(10-1)!} = \frac{10!}{9!} = \frac{9!\cdot 10}{9!} = 10$

$\binom{10}{2} = \frac{10!}{2!(10-2)!} = \frac{8!\cdot 9\cdot 10}{2!\,8!} = \frac{90}{2} = 45$

$10 \cdot \frac{1}{2}\left(\frac{1}{2}\right)^9 = 5 \cdot \frac{1}{1024} = \frac{5}{1024}$

$45 \cdot \frac{1}{4}(1-p)^{10-2} = 11,25 \cdot \frac{1}{512} = \frac{11,25}{512}$

$\binom{10}{3} = \frac{10!}{3!\,7!} = \frac{8\cdot 9\cdot 10}{6} = 120$

RPIS cw3

1.13

I	II	III	IV
k	k	k	k
k	k	k	
	k	k	k
k	k		
	k	k	
		k	k
k		k	
		k	
k	k	k	
k	k	k	

Z 300 zdających

Z_1 200 mat-fiz $0{,}9$

Z_2 75 ogól. $0{,}25$

Z_3 25 human $0{,}1$

A - zaliczenie zdania egzaminu

~~$P(A) = P(A|Z_1) + P(A|Z_2) + P(A|Z_3)$~~

$P(A|Z_1) = 0{,}9$

$P(A|Z_2) = 0{,}25$

$P(A|Z_3) = 0{,}1$

$$P(A) = P(A|Z_1)\cdot P(Z_1) + P(A|Z_2)\cdot P(Z_2) + P(A|Z_3)\cdot P(Z_3) =$$

$$= \frac{0{,}9\cdot 200 + 0{,}25\cdot 75 + 0{,}1\cdot 25}{300} = \frac{180 + 18{,}75 + 2{,}5}{300} =$$

$$= \frac{201{,}25}{300} \approx \frac{2}{3}$$

	$\frac{1}{3}$			$\frac{2}{3}$
I	V	X	X	X
II	X	V	X	X
III	X	X	V	X
IV	X	X	X	V

RPIS ćw 4 | 1.25

Z_k – ~~prawdop.~~ zdarz. zniszczenia celu i k strzałów; T – zdarz. trafi w cel

$$P(Z_k) = 1 - q^k$$

$$P(T) = q$$

$$P(Z_n | T) = \sum_{i=1}^{n} P(Z_n | T) P(Z_n)$$

$q = 0,5$

$$P(Z_1|T) = P(Z_1|T)(1 - q^4) = \frac{P(Z_1 \cap T)}{}$$

Bayes

$$P(Z_k | T) = \frac{P(T|Z_k) P(Z_k)}{\sum_{i=1}^{k} P(T|Z_i) P(Z_i)}$$

$$P(Z_k | T) = \frac{P(T|Z_k) P(Z_k)}{\sum_{n=1} P(T|Z_n) P(Z_n)}$$

CW RPIS 5

~~1.32~~

$$\begin{array}{ll} AAAA & 0.3 \\ BBBB & 0.4 \\ CCCC & 0.3 \end{array}$$

h - funkcja

ξ - zmienna losowa

1.37

A - p

B - q

C - r

$|p+q+r| = \underline{1}$

~~2.1~~ ~~$f(x) = \begin{cases} \frac{a}{x^2} & |x| \geq 1 \\ 0 & \text{dla } |x| < 1 \end{cases}$~~

~~2.9 jest OK~~

2.9 $F_\xi(t) = 1 - e^{-t/8}$ dla $t > 0$

$P(\xi \in (-8, 16)) = F_\xi(16) - F_\xi(-8) = F(16) - 0 =$

$= 1 - e^{-16/8} = 1 - e^{-2} = 1 - \frac{1}{e^2}$

2.10 $\boxed{\begin{array}{ccc} 5 & 3 & 2 \\ B & C & Z \end{array}}$ $n = 4$ losowanie niezależne cw RPIS 6

Zadanie

~~Wyznaczyć c dla której f dana wzorem~~

~~$f(x) = cx(1-x)^2 I_{(0,1)}(x)$~~

~~jest gęst. zm. losowej~~

2.9

$$F_\xi(t) = 1 - e^{-t/8} \text{ dla } t > 0$$

$$P(\xi \in (-8, 16)) = P(-8 < \xi < 0) = \cancel{F(16) - F(-8)}$$

$$= \int_0^{16} (1 - e^{-x/8})\,dx = \left[x - 8e^{-x/8}\right]_0^{16} =$$

$$= 16 - 8e^{-16/8} - 0 - 8e^{0/8} = 16 - \frac{8}{e^2} - 1 =$$

$$= -\frac{8}{e^2} + 15$$

Egzamin 2 z 1 r

		Y: -1	1
X	-5	0,2	0,1
	1	0,3	0,4

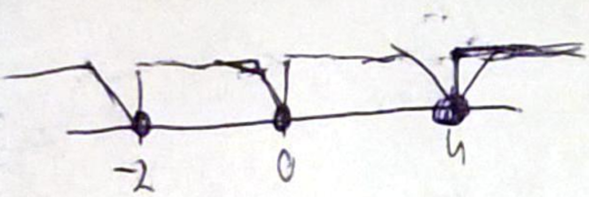
$E(-4X - 2Y^2)$ oraz $E(XY)$

$$0 + 0 + \frac{1}{3} + \frac{4}{3} - \frac{16}{24} + 0 = \frac{5}{3} - \frac{16}{24} = \frac{40}{24} - \frac{16}{24} = \frac{24}{24} = 1$$

ćw RPIS 7

Zmienna los X ma rozkład $f(x)\begin{cases} \frac{x}{6}+\frac{1}{3} & \text{gdy } -2\le x<0 \\ \frac{1}{3}-\frac{x}{12} & \text{gdy } 0\le x<4 \\ 0 & \text{poza tym} \end{cases}$

wyznaczyć a) Dystrybuantę zmiennej losowej X



1. $t\in(-\infty,-2)$ $F(t)=0$

2. $t\in\langle -2,0)$, $F(t)=\int_{-2}^{t}\left(\frac{x}{6}+\frac{1}{3}\right)dx=\left[\frac{x^2}{12}+\frac{x}{3}\right]_{-2}^{t}=$

$=\frac{t^2}{12}+\frac{t}{3}-\frac{1}{3}+\frac{2}{3}=\frac{t^2}{12}+\frac{t}{3}+\frac{1}{3}$

3. $t\in\langle 0,4)$, $F(t)=\int_{0}^{t}\left(\frac{1}{3}-\frac{x}{12}\right)dx=\left[\frac{x}{3}-\frac{x^2}{24}\right]_{0}^{t}=$

$=\frac{t}{3}-\frac{t^2}{24}-0-0=\frac{t}{3}-\frac{t^2}{24}$

4. $t\in\langle 4,+\infty)$ $F(t)=0$

$t(-\infty,+\infty)$ $0+\frac{t^2}{12}+\frac{t}{3}+\frac{1}{3}+\frac{t}{3}-\frac{t^2}{24}+0=$

$$F(t)\begin{cases} 0 \\ \frac{t^2}{12}+\frac{t}{3}+\frac{1}{3} \\ \frac{t}{3}-\frac{t^2}{24} \\ 0 \end{cases}$$

~~RPIS 8~~ b) $P(X \in \langle 1,3 \rangle) = F(3) - F(1) =$

$$\cancel{= -\frac{3^2}{24} + \frac{3}{3} - \frac{1^2}{24} - \frac{1}{3} = \frac{9}{24} + 1 + \frac{1}{24} - \frac{1}{3} =}$$

$$\cancel{= \frac{1}{3} + 1 - \frac{1}{3} - 1 = \frac{10}{24} + \frac{2}{3} = \frac{10+16}{24} = \frac{26}{24}}$$

$$= \frac{-3^2}{24} + 1 + \frac{1^2}{24} - \frac{1}{3} = -\frac{8}{24} + \frac{2}{3} = -\frac{1}{3} + \frac{2}{3} = \frac{1}{3}$$

c) Prawdopodobieństwo zdarzenia, że w 4 niezal. dośw. co najmniej dwa razy zmienna losowa X przyjmie wartość z przedziału $\langle 1,3 \rangle$

$$P(X \in \langle 1,3 \rangle) = \frac{1}{3}$$

$$P(S_4 \geq 2) = 1 - P(S_4 < 2) = 1 - P(S_4 = 0) - P(S_4 = 1) = 1 - \left(\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\right) -$$

$$- \frac{1}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3} = \cancel{1 - \frac{16}{81} - \frac{8}{81}} = 1 - \quad - \frac{4}{81}$$

$27 \cdot 3$

$60 + 24$ 5.3 $D(0.5)$

$$\eta_n = \sum_{i=1}^{n} \xi_i \qquad \text{Oblicz: } P(\eta_{10} = 4 \text{ i } \eta_n \leq 6 \text{ dla } n = 1,2,\ldots 9)$$

$P(\eta_{10} = 4) \cup P(\eta_n \leq 6$

2 4 8 16 32 64 128 256 512 1024 2048 4096

RPiS 9

2.2

$$f(x) = \begin{cases} 0 & , x<0 \\ axe^{-x} & x \geq 0 \end{cases}$$

$$\cancel{\int_0^\infty axe^{-x} dx} = \quad \cancel{axe}^{-x}$$

$$\int_0^\infty axe^{-x} dx = \left[-ae^{-x}(x+1)\right]_0^\infty =$$

$$= \lim_{0 \to \infty} -a^{-x}(x+1) = \left[\cancel{\quad} - \frac{1}{a^x}(x+1) = \right.$$

$$= -\frac{x}{a^x} - \frac{1}{a^x} = \infty \Big] = \infty$$

∞, 0, 0

2.5

$$f(x) = \begin{cases} \frac{1}{4|ax|^3}, & dla\ |x| \geq 1 \\ 0 & , dla\ |x| \end{cases}$$

2.1

$$f(x) \begin{cases} \frac{a}{x^4} & x \geq 1 \\ 0 & x < 1 \end{cases} \qquad f(x) > 0 \Rightarrow a > 0$$

$$\int_0^\infty \frac{a}{x^4} dx = \left[\cancel{\frac{4a}{4x^5}} - \frac{a}{3x} \right]^\infty$$

$$\int_{-\infty}^\infty f(x) dx = \int_{-1}^{1} 0 dx + \int_1^\infty \frac{a}{x^4} dx = a\int_0^\infty \frac{1}{x^4} dx = a \left[\frac{1}{4x^3 dx} \right]_0^\infty \neq$$

$$\cancel{= a + \lim_{x \to \infty} \frac{1}{4x^3} = a + 0}$$

$$\int_1^\infty \frac{a}{x^4} dx + \int_{-\infty}^{-1} \frac{a}{x^4} dx = a\left[\frac{1}{4x^3}\right]_1^\infty + a\left[\frac{1}{4x^3}\right]_{-\infty}^{1} =$$

$$= -a\frac{1}{4 \cdot 1^3} + a\frac{1}{4 \cdot -1^3} - 0 = \frac{a}{4} + \frac{a}{-4}$$