

(Towards) A Unified Topological Kashiwara-Vergne Theory

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Contents

Introduction	3
Acknowledgements	4
1 Formality	5
1.1 Filtrations	5
1.2 Associated Graded Functor	5
1.3 Finite Type Invariants and Chord Diagrams	5
2 Lie Theory and the Kashiwara-Vergne Problem	6
3 Existing Topological Interpretations of the Kashiwara Vergne Equations	7
3.1 Lie Theory	7
3.2 Goldman-Turaev	7
3.3 Welded Foams	7
4 Topological Approaches	8
5 Emergent Tangles: Lifting Goldman-Turaev to 3 dimensions	9
6 Emergent w-foams: Lifting goldman-Turaev to 4 dimensions	10
7 Virtual Knot Tabulation?	11
A appendixname	12

Introduction

This is the introduction.

Acknowledgements

These are the acknowledgements.

Chapter 1

Formality

Filtered structures, associated graded structures, formality and how this leads to connections between Vassiliev Invariants and quantum algebra via a general application of Von Dyck's Theorem. Can include intro to PaT Vassiliev filtration, chord diagrams, Drinfeld Associators on a story sort of level.

1.1 Filtrations

1.2 Associated Graded Functor

This is a reference to [[Ada94](#)].

1.3 Finite Type Invariants and Chord Diagrams

Chapter 2

Lie Theory and the Kashiwara-Vergne Problem

Base this on Alekseev-Torossian or even WKOII.

Chapter 3

Existing Topological Interpretations of the Kashiwara Vergne Equations

3.1 Lie Theory

3.2 Goldman-Turaev

3.3 Welded Foams

Chapter 4

Topological Approaches

Welded foams (WKOII) vs Goldman-Turaev (AKKN) - pointing out the differences.
Would be good to try writing here.

Chapter 5

Emergent Tangles: Lifting Goldman-Turaev to 3 dimensions

Zsuzsi et al paper in the works

Chapter 6

Emergent w-foams: Lifting goldman-Turaev to 4 dimensions

Needs to be done mathematically

Chapter 7

Virtual Knot Tabulation?

Appendix A

appendixname

This is the first appendix. The subject will be bialgebras.

References

- [Ada94] C.C. Adams. *The Knot Book*. W.H. Freeman, 1994. ISBN: 9780821886137. URL: https://www.math.cuhk.edu.hk/course_builder/1920/math4900e/Adams--The%20Knot%20Book.pdf.