

(Towards) A Unified Topological Kashiwara-Vergne Theory

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An essay submitted in partial fulfilment of
the requirements for the degree of
Master of Philosophy (Science)

Pure Mathematics
University of Sydney



February 24, 2025

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Acknowledgements

These are the acknowledgements.

Introduction

The idea of Finite Type invariants

NOTE: Here I want to motivate the study of Finite-type invariants in general, using analogies from discriminant theory.

Various geometric or topological objects can be studied by considering not only those objects, but also their singular versions, and using this ‘discriminant set’ to define and compute invariants. Take for example the following (simple) analogy:

EXAMPLE 1 (Finite type invariants of Quadratic Equations). aksldjkhalsjd aksjd lkajs dlkasj lkda j dklasj dlkajslkdjkashg fkjfs h kjshks aksldjkhalsjd aksjd lkajs dlkasj lkda j dklasj dlkajslkdjkashg fkjfs h kjshks aksldjkhalsjd aksjd lkajs dlkasj lkda j dklasj dlkajslkdjkashg fkjfs h kjshks aksldjkhalsjd aksjd lkajs dlkasj lkda j dklasj dlkajslkdjkashg fkjfs h kjshks aksldjkhalsjd aksjd lkajs dlkasj lkda j dklasj dlkajslkdjkashg fkjfs h kjshks

Countless other geometric and topological objects and their invariants can be described this way, for example, the inertial index of a quadratic form, the number of roots of cubic equations, etc. [Sos23].

NOTE: Then I want to talk about the Dror Bar-Natan multiavriable calculus analogy a bit here.

Chapter 1

Formality

Filtered structures, associated graded structures, formality and how this leads to connections between Vassiliev Invariants and quantum algebra via a general application of Von Dyck's Theorem. Can include intro to PaT Vassiliev filtration, chord diagrams, Drinfeld Associators on a story sort of level.

1.1 Filtrations

1.2 Associated Graded Functor

This is a reference to [[Ada94](#)].

1.3 Finite Type Invariants and Chord Diagrams

Chapter 2

Lie Theory and the Kashiwara-Vergne Problem

Base this on Alekseev-Torossian or even WKOII.

Chapter 3

Existing Topological Interpretations of the Kashiwara Vergne Equations

3.1 Lie Theory

3.2 Goldman-Turaev

3.3 Welded Foams

Chapter 4

Topological Approaches

Welded foams (WKOII) vs Goldman-Turaev (AKKN) - pointing out the differences.
Would be good to try writing here.

Chapter 5

Emergent Tangles: Lifting Goldman-Turaev to 3 dimensions

Zsuzsi et al paper in the works

Chapter 6

Emergent w-foams: Lifting goldman-Turaev to 4 dimensions

Needs to be done mathematically

Chapter 7

Virtual Knot Tabulation?

Appendix A

appendixname

This is the first appendix. The subject will be bialgebras.

References

- [Ada94] C.C. Adams. *The Knot Book*. W.H. Freeman, 1994. ISBN: 9780821886137. URL: https://www.math.cuhk.edu.hk/course_builder/1920/math4900e/Adams--The%20Knot%20Book.pdf.
- [Sos23] A.B. Sossinsky. *Knots, Links and Their Invariants: An Elementary Course in Contemporary Knot Theory*. Student Mathematical Library. American Mathematical Society, 2023. ISBN: 9781470471514.