

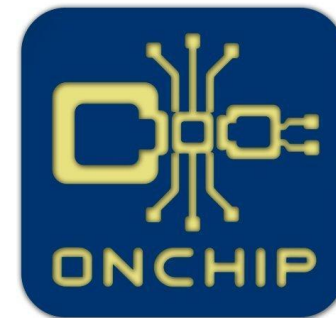
# Lecture 04: Small Signal Modeling

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# Outline

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□ Introduction

□ Signal Definitions

□ Small-Signal Modeling

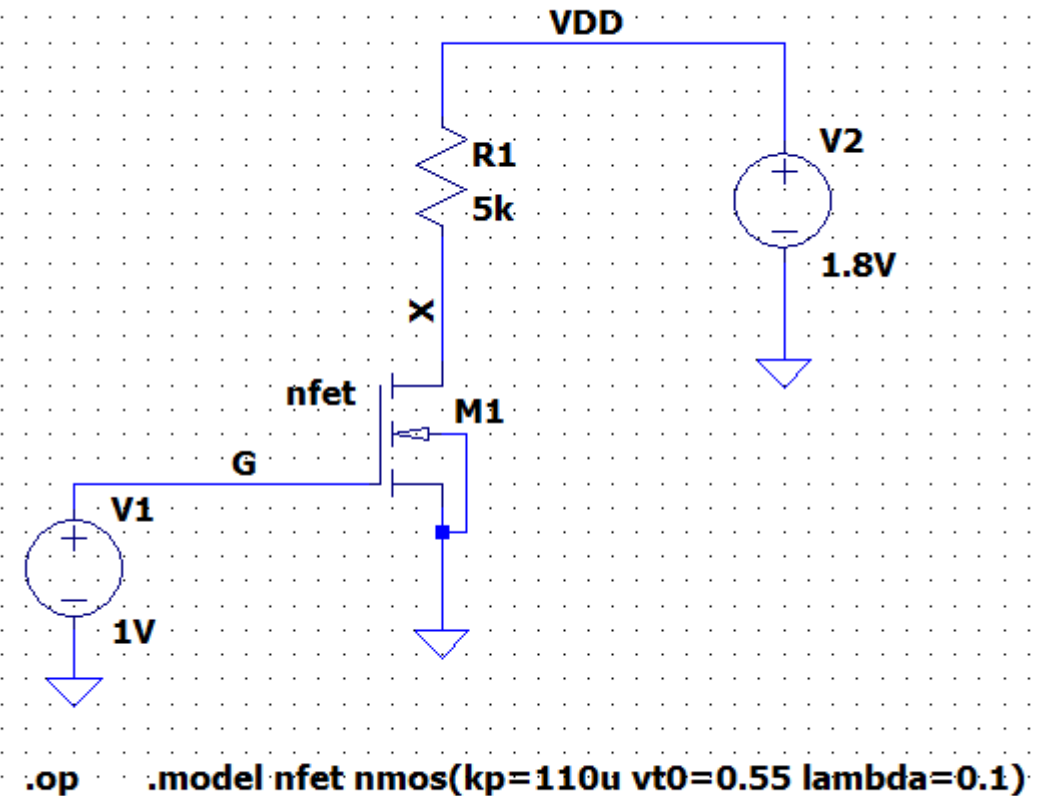
□ Conclusions

# Introduction

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \cdot (1 + \lambda V_{DS})$$

$$I_D = (185.6 \mu) (1 + 0.1(1.8 - 5000 I_D))$$

$$\underline{I_D = 200.4 \mu A}$$



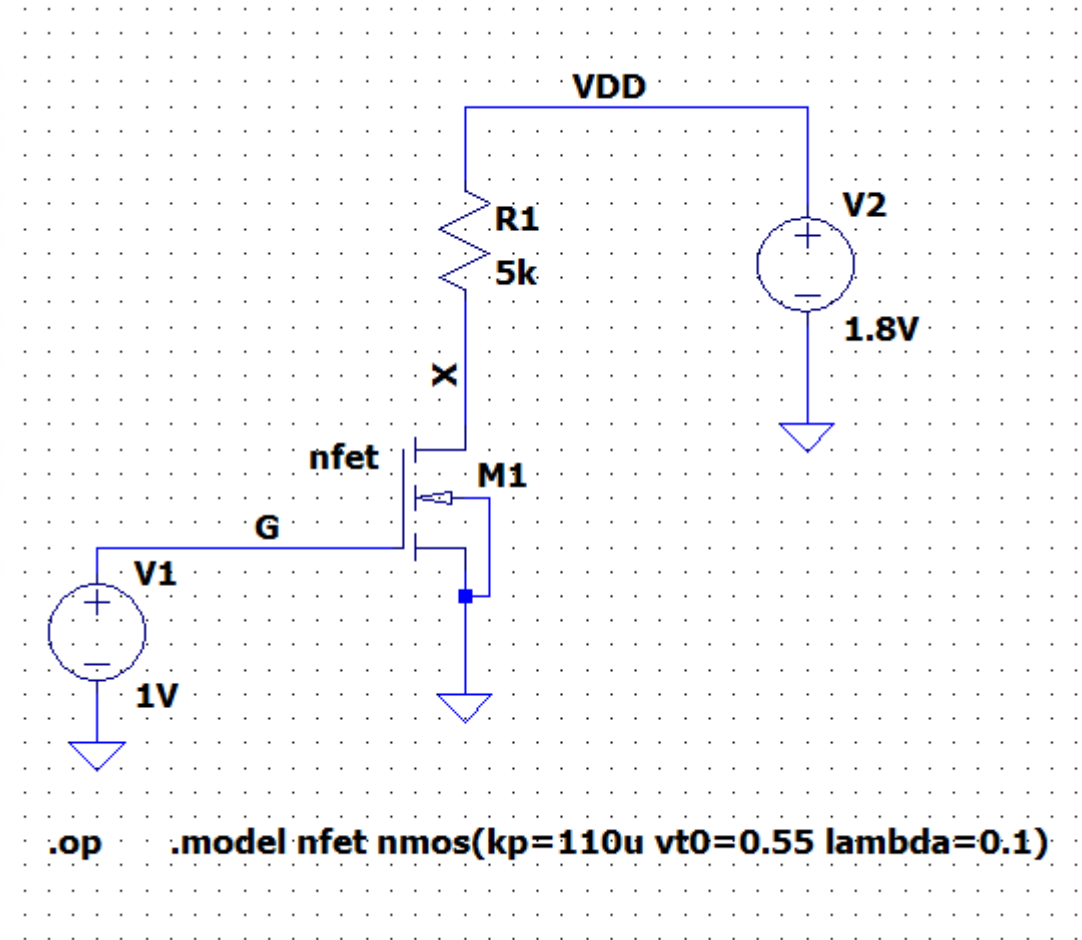
# Introduction – Changing Values

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \cdot (1 + \lambda V_{DS})$$

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$$V_{GS2} = V_{GS} + \Delta V_{GS} \quad ; \quad \Delta V_{GS} = 10 \text{ mV}$$



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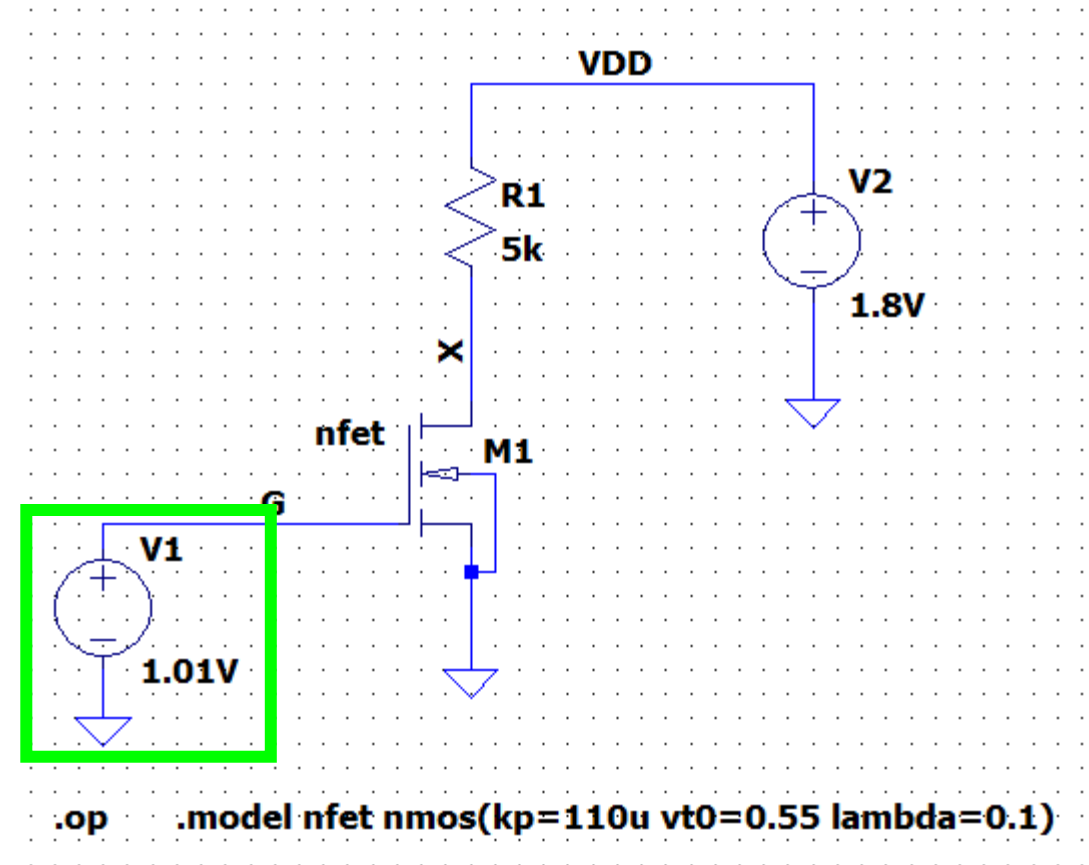
$$I_D = (185.6 \mu) (1 + 0.1 (1.8 - 5000 I_D))$$

$$\underline{I_D = 200.4 \mu A}$$

$$V_{GS2} = V_{GS} + \Delta V_{GS} \quad ; \quad \Delta V_{GS} = 10 mV$$

$$V_{GS2} = 1.01 V \rightarrow \text{we repeat the same process.}$$

$$I_D = (193.97 \mu) (1 + 0.1 (1.8 - 5000 I_D))$$



# Introduction – Changing Values

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \cdot (1 + \lambda V_{DS})$$

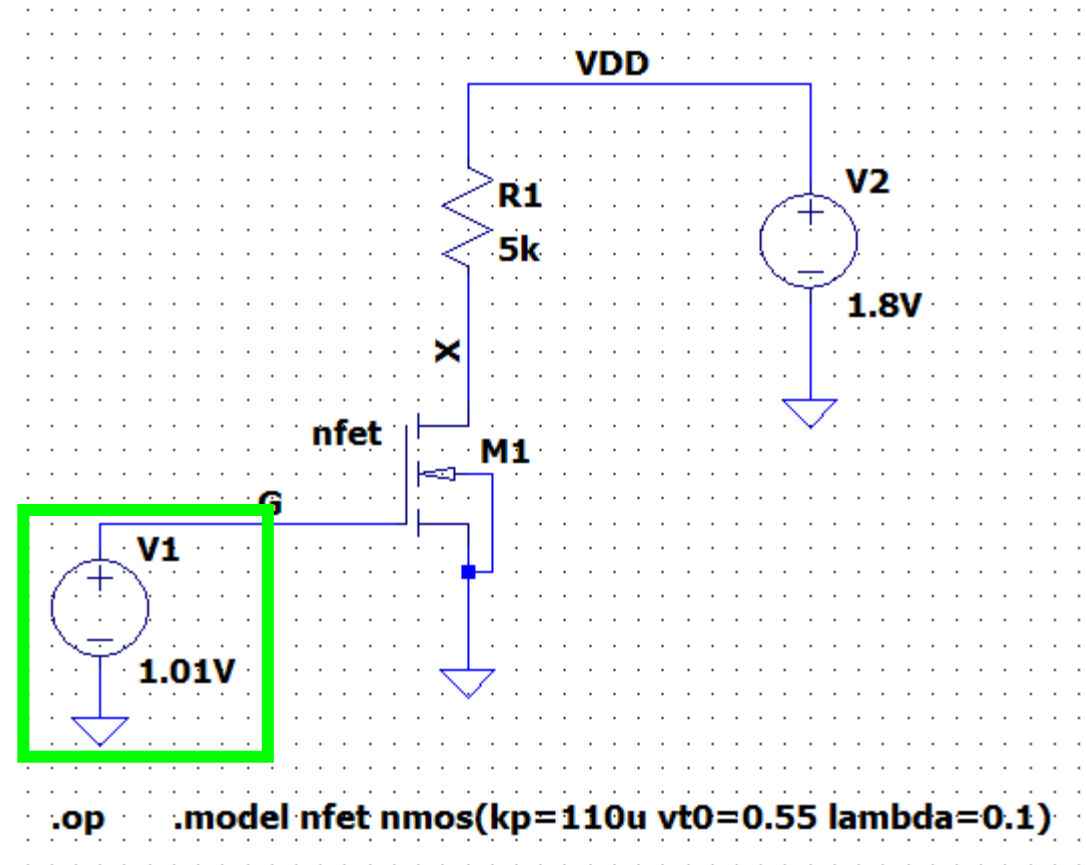
$$I_D = (185.6 \mu) (1 + 0.1(1.8 - 5000 I_D))$$

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$$V_{GS2} = V_{GS} + \Delta V_{GS} \quad ; \quad \Delta V_{GS} = 10 mV$$

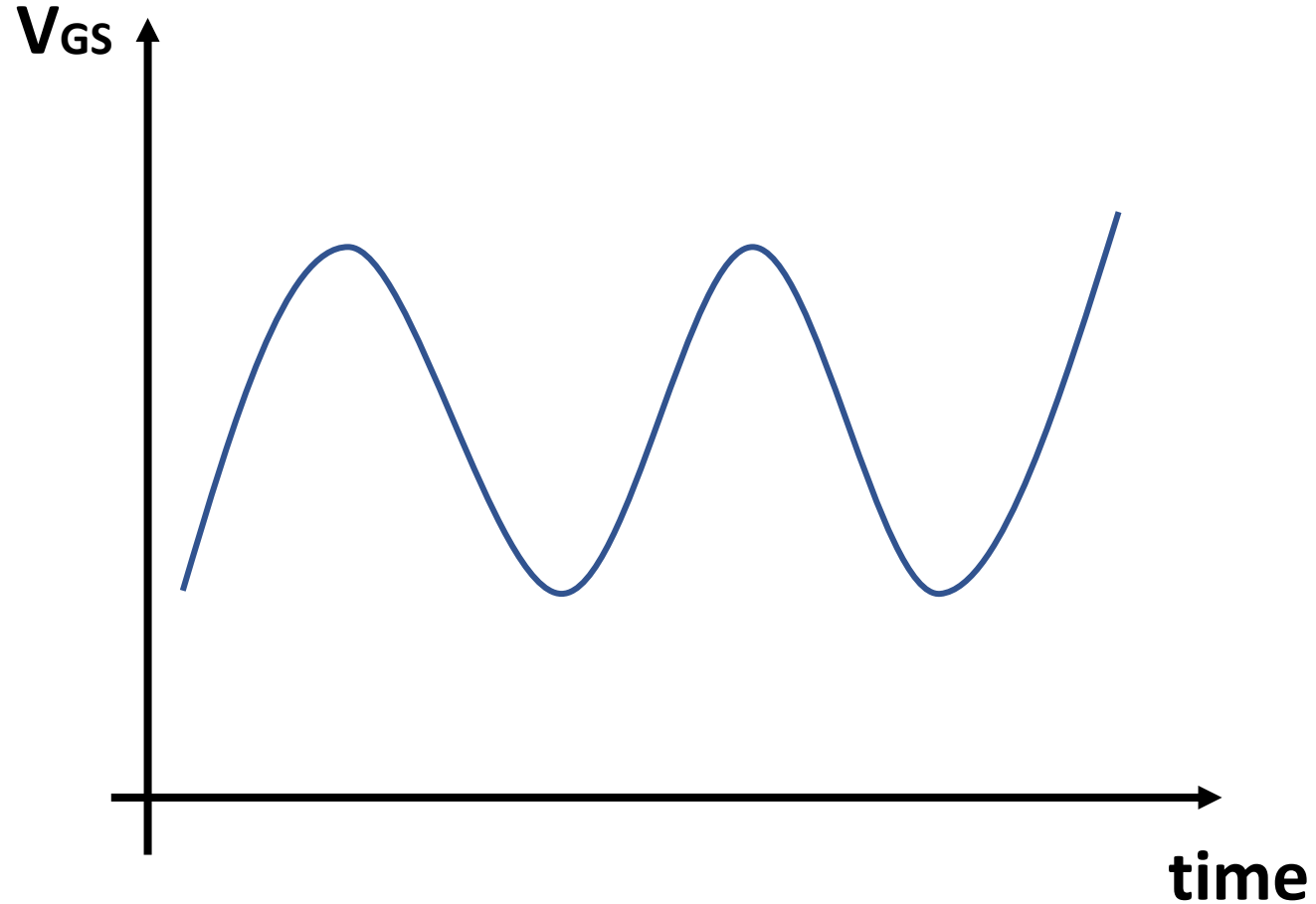
$$I_{D2} = 208.65 \mu A \rightarrow I_{D2} = I_D + \Delta I_D$$

$$\underline{\Delta I_D = 8.25 \mu A}$$



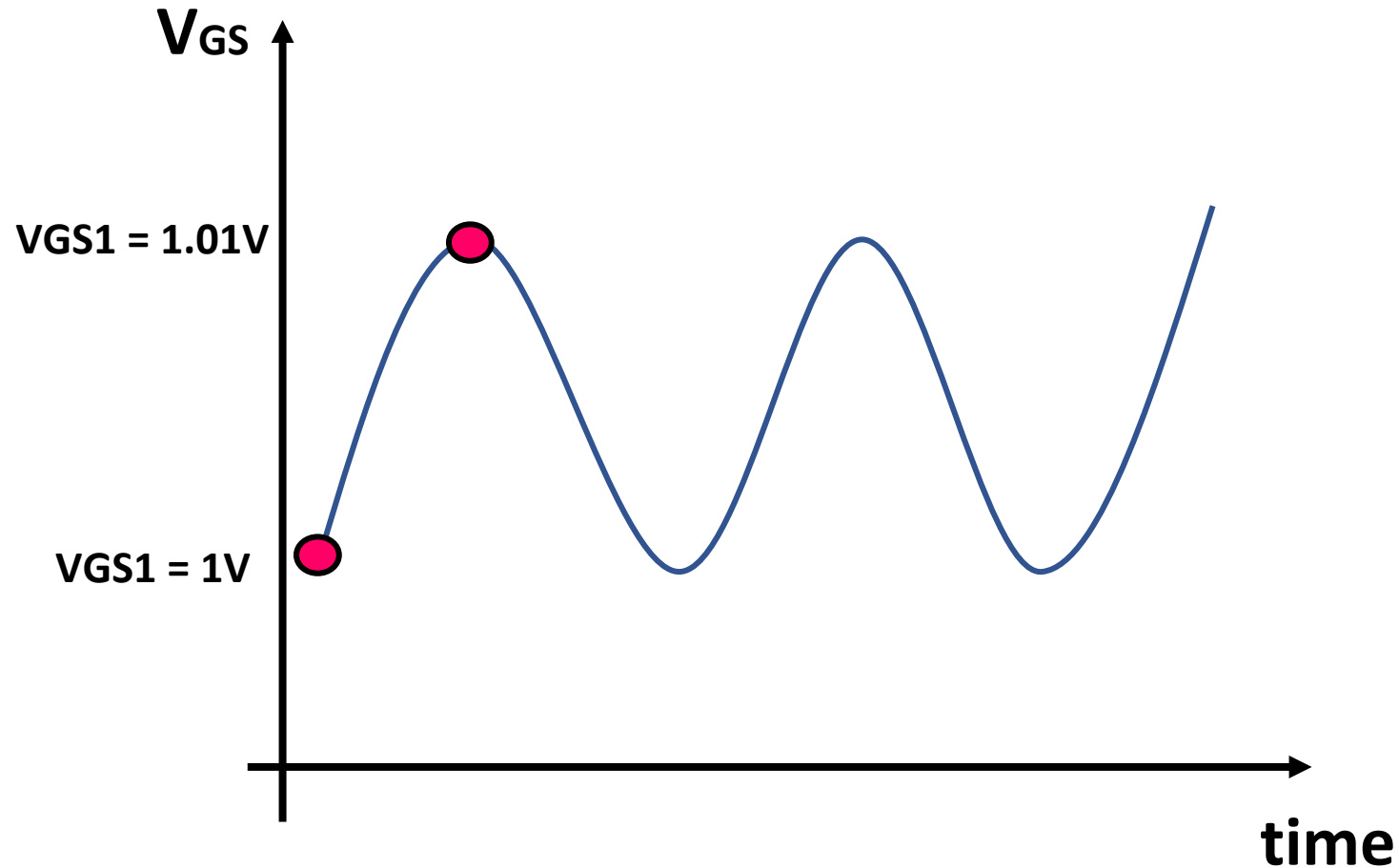
# Signals

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**What if we put a  
signal?**

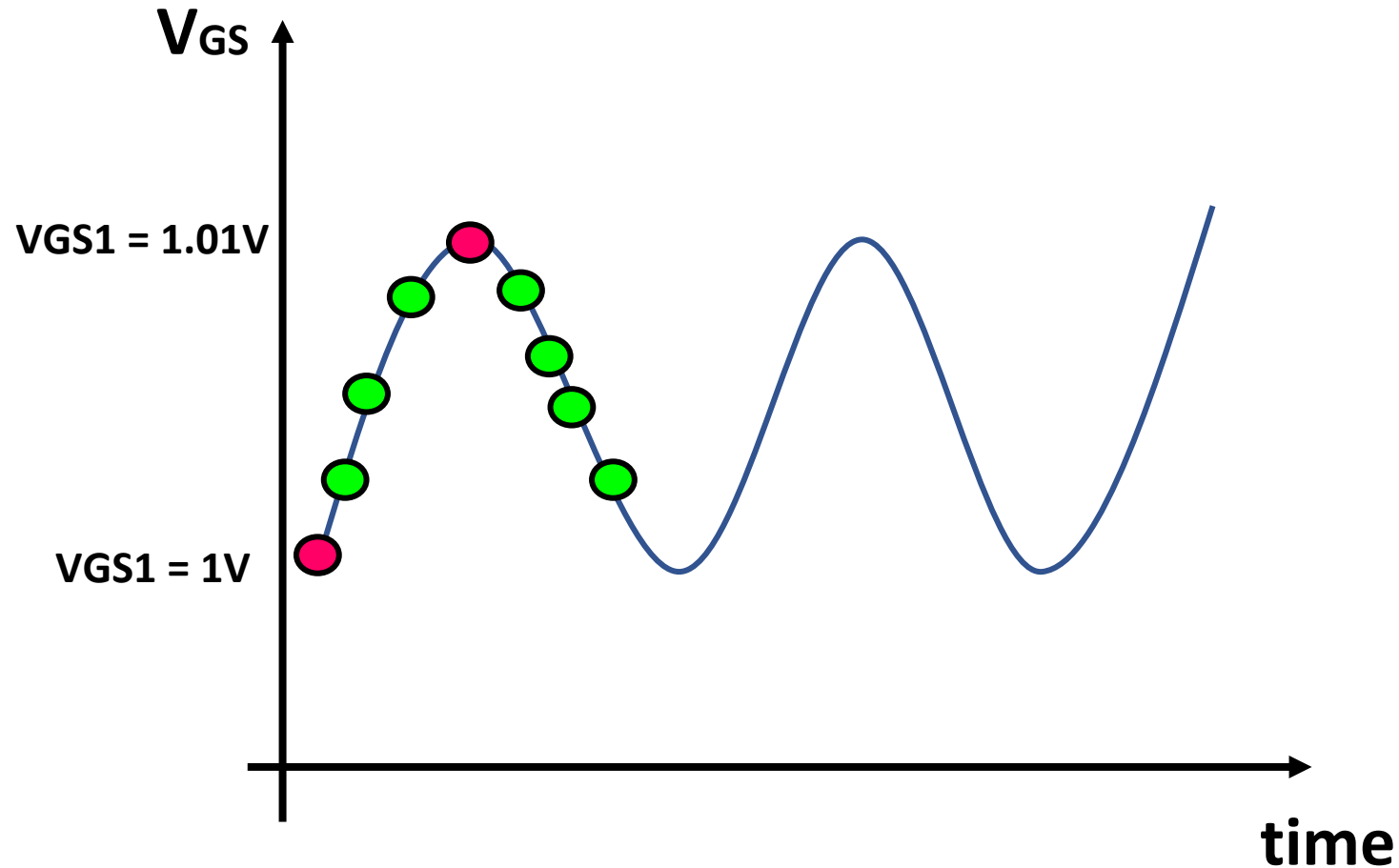
# Signals



**Think of a signal  
between the two  
previous values**



# Signals



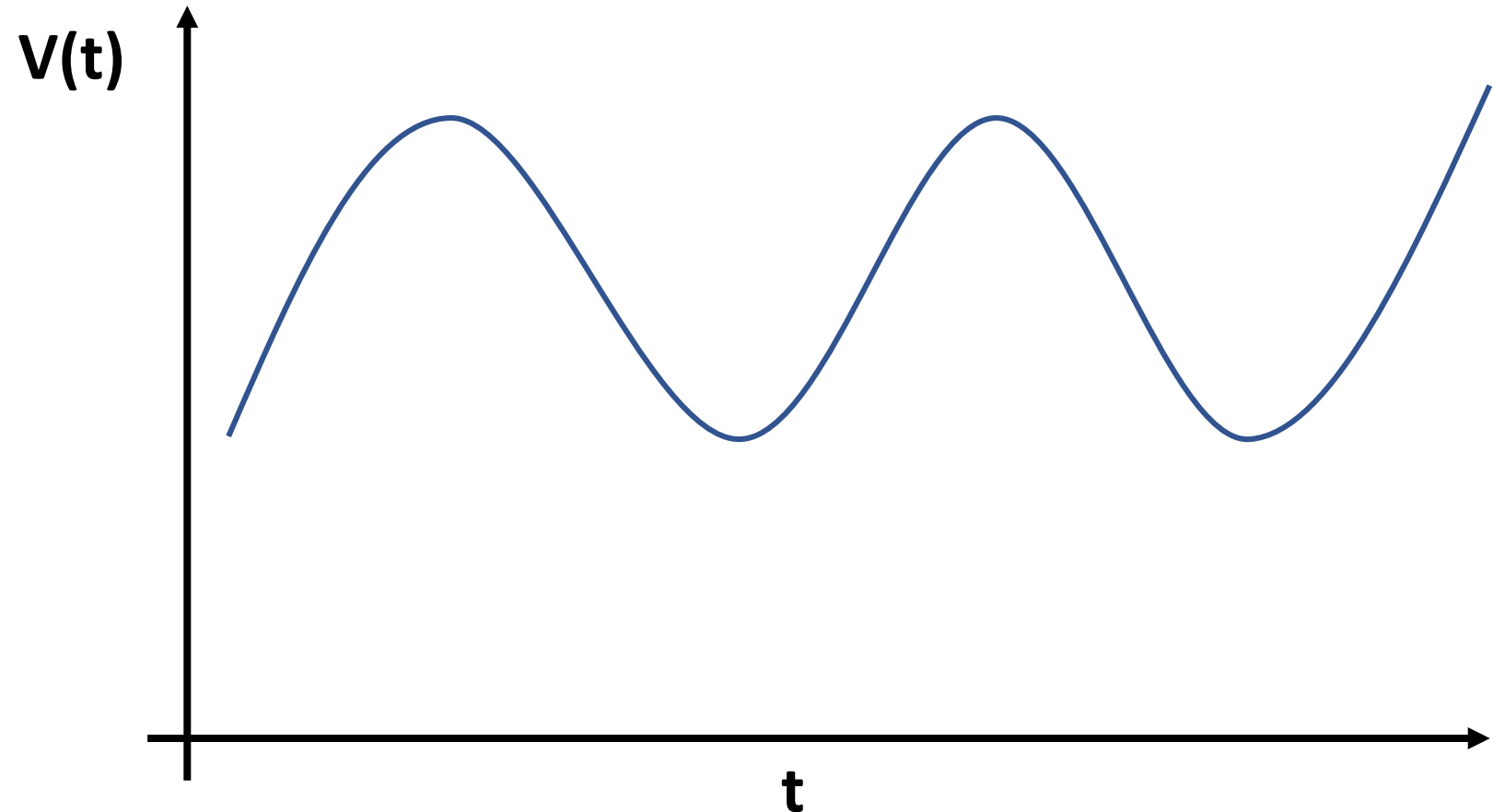
We need to solve  
for each point

**Lots of  
calculations!**

# Signals: Definitions

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- ☐ DC Level
- ☐ Amplitude
- ☐ Peak Value
- ☐ Peak-peak Value
- ☐ Something else?



# Signals: Definitions

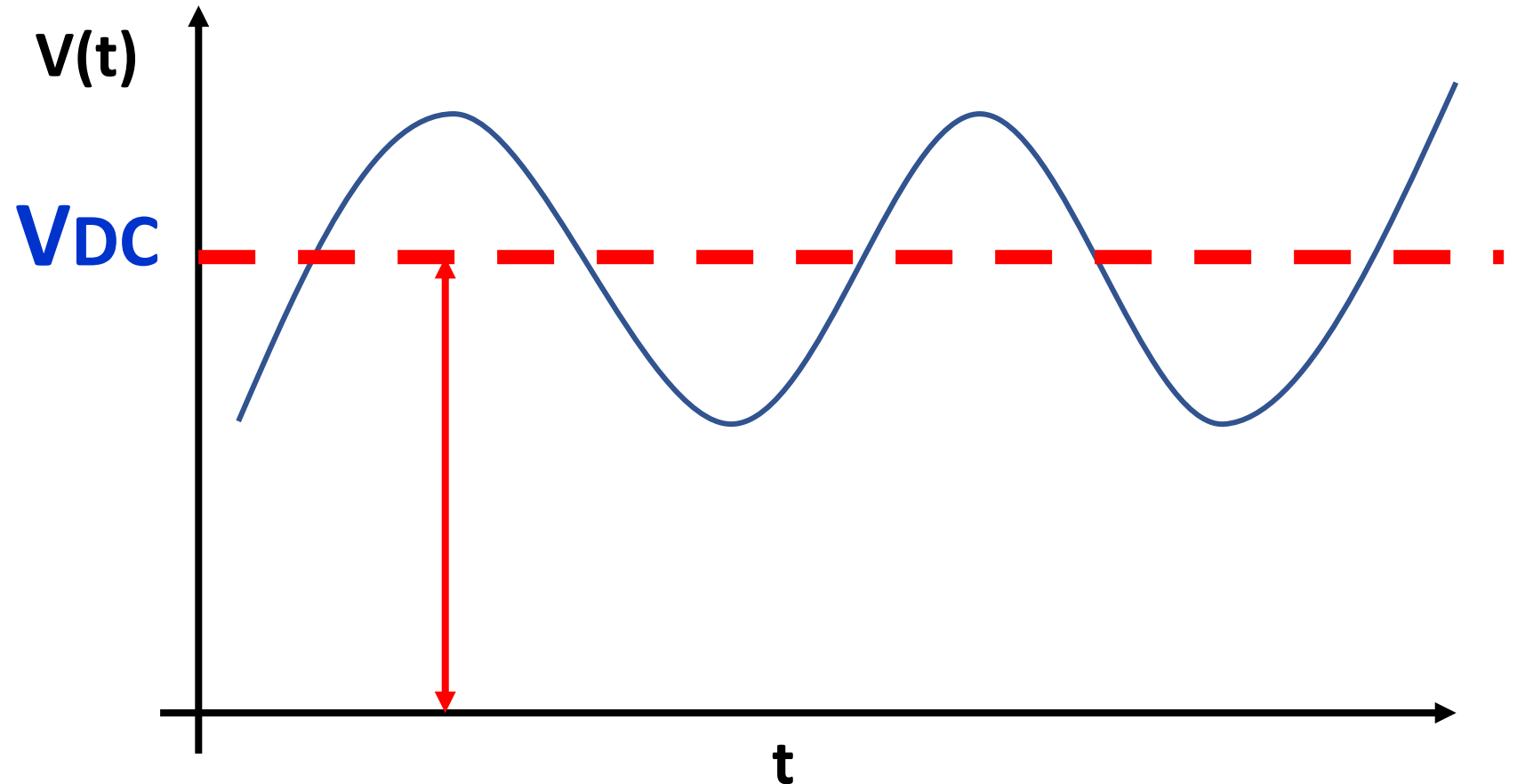
☒ DC Level

☐ Amplitude

☐ Peak Value

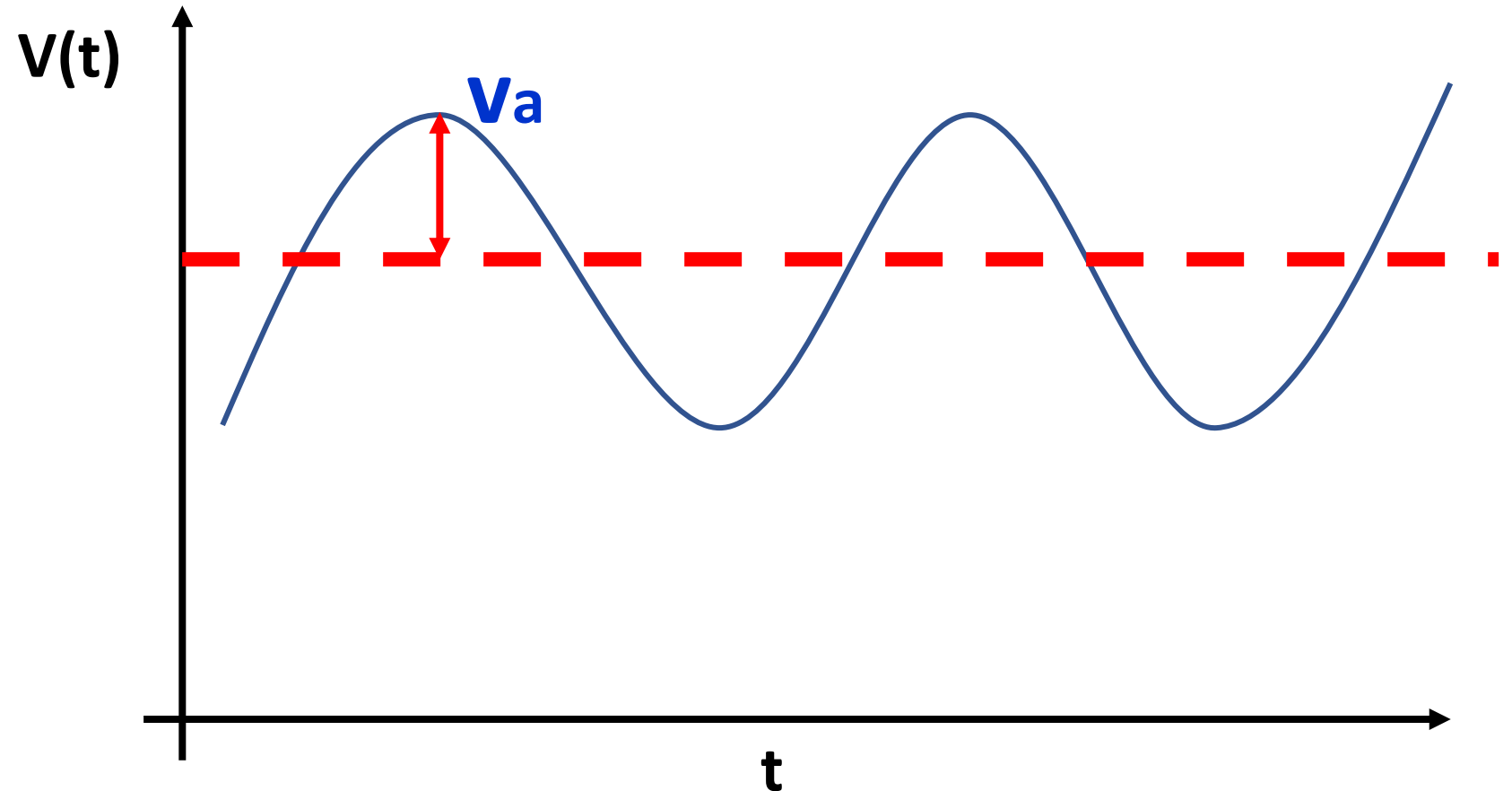
☐ Peak-peak Value

☐ Something else?



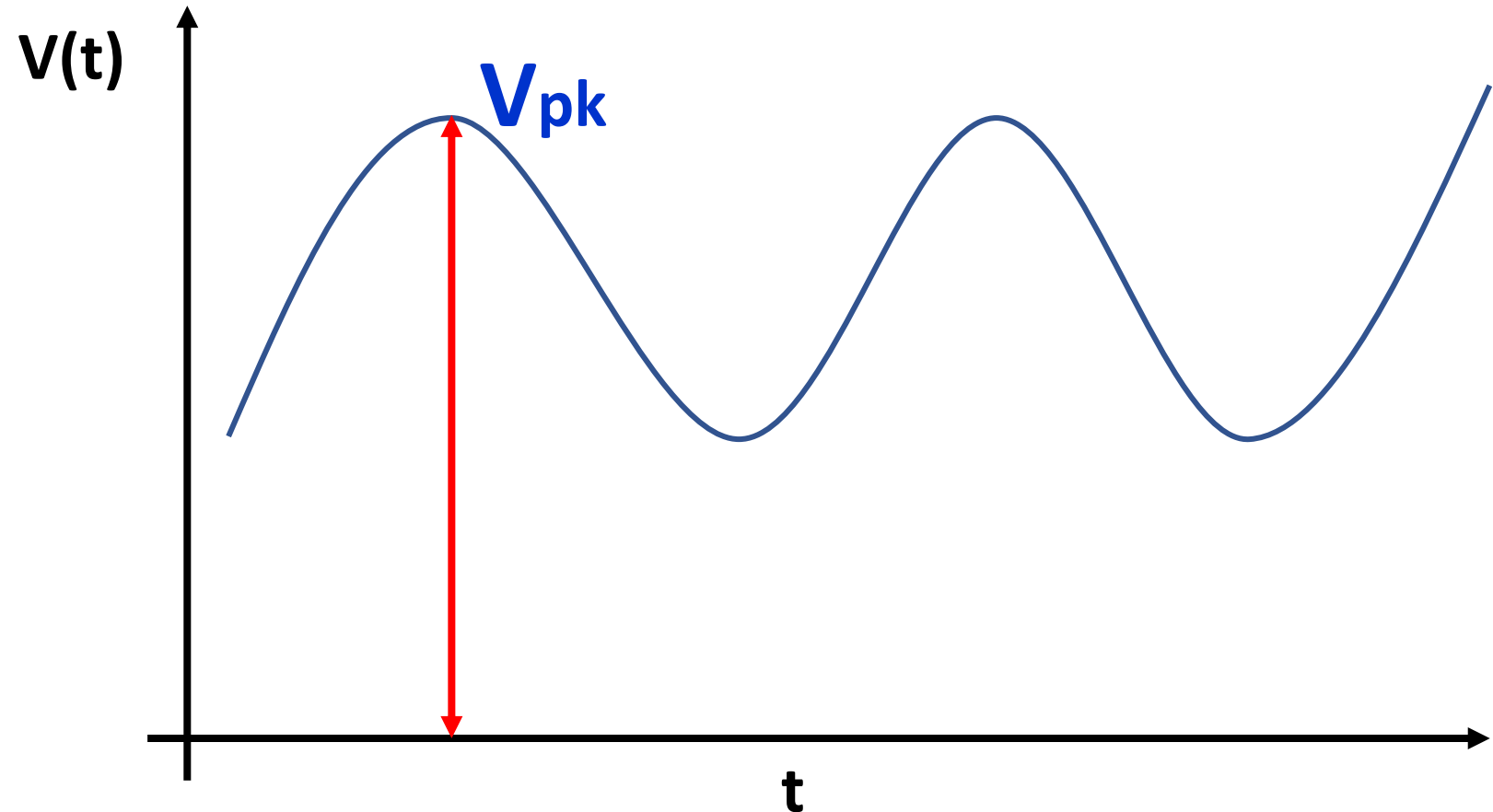
# Signals: Definitions

- ☐ DC Level
- ☒ **Amplitude\***
- ☐ Peak Value
- ☐ Peak-peak Value
- ☐ Something else?



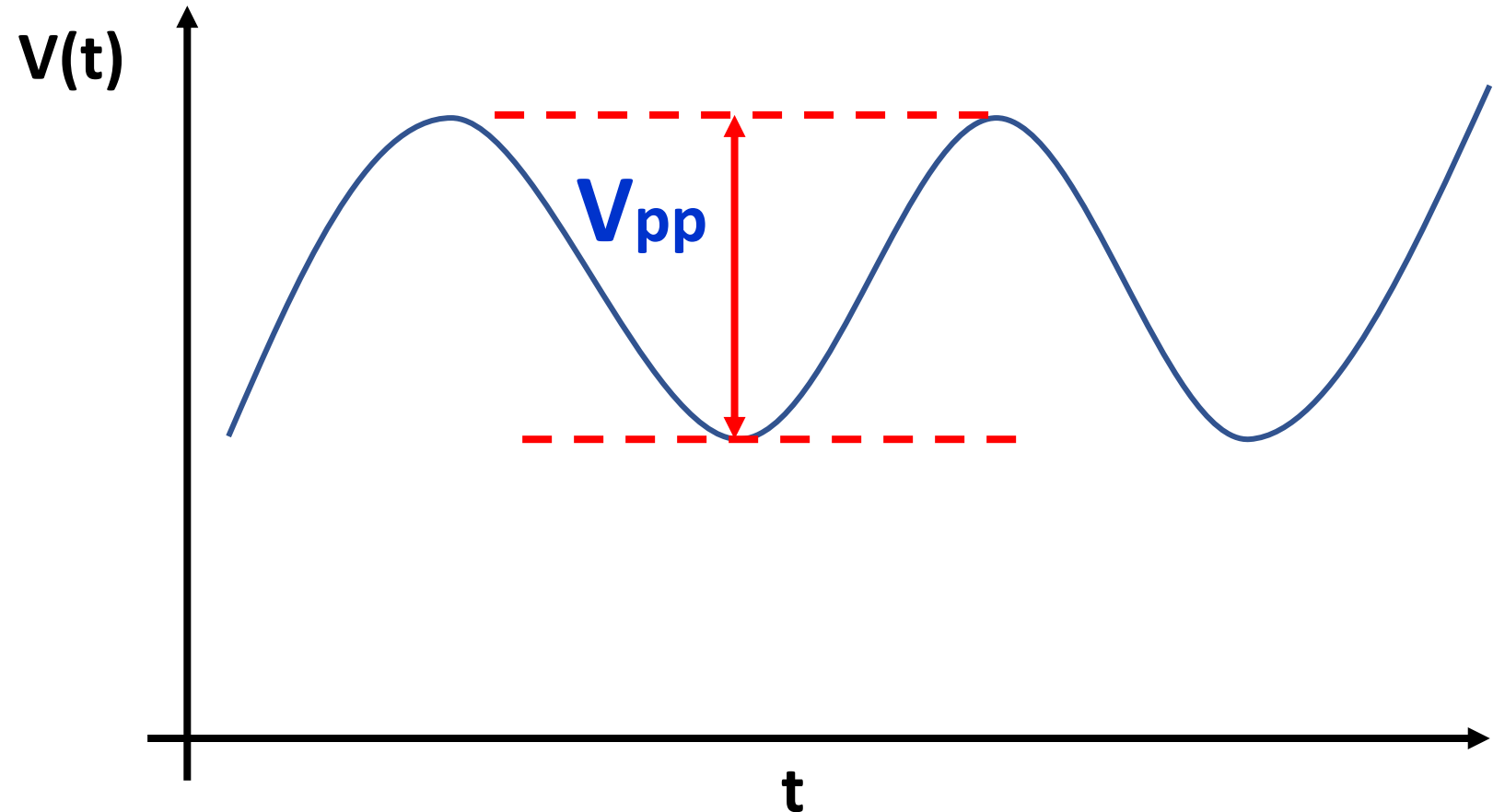
# Signals: Definitions

- ☐ DC Level
- ☐ Amplitude
- ☒ **Peak Value**
- ☐ Peak-peak Value
- ☐ Something else?



# Signals: Definitions

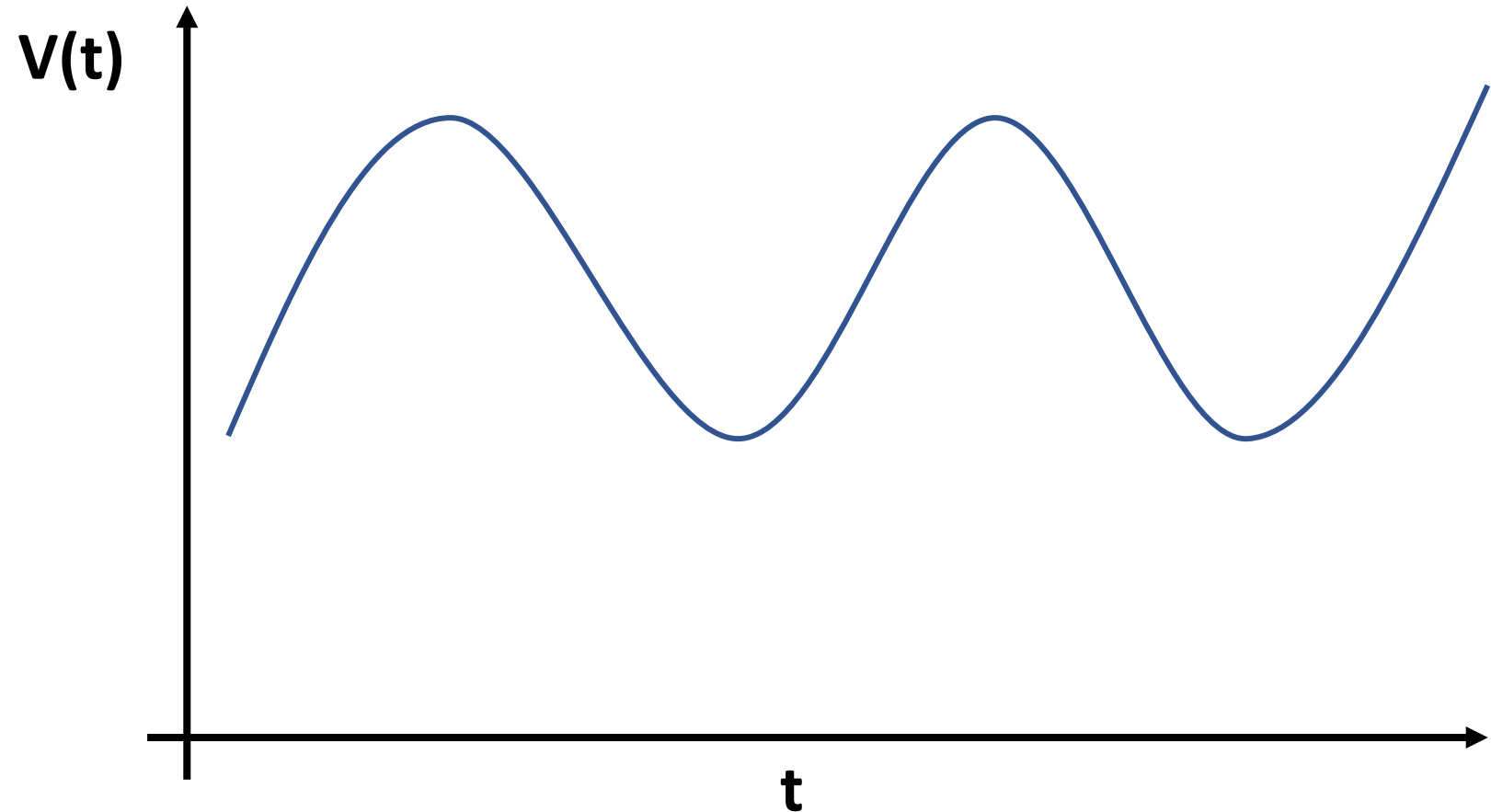
- ☐ DC Level
- ☐ Amplitude
- ☐ Peak Value
- ☒ **Peak-peak Value**
- ☐ Something else?



# Signals: Definitions

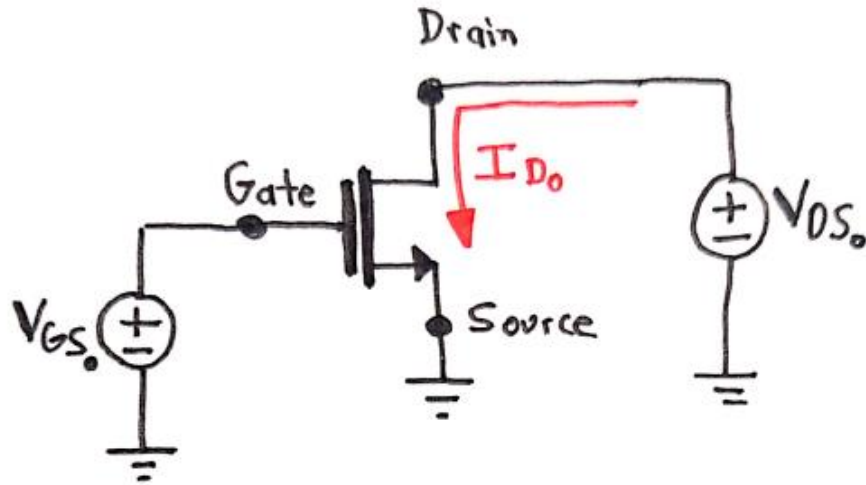
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- ☐ DC Level
- ☐ Amplitude
- ☐ Peak Value
- ☐ Peak-peak Value
- ☒ Something else?

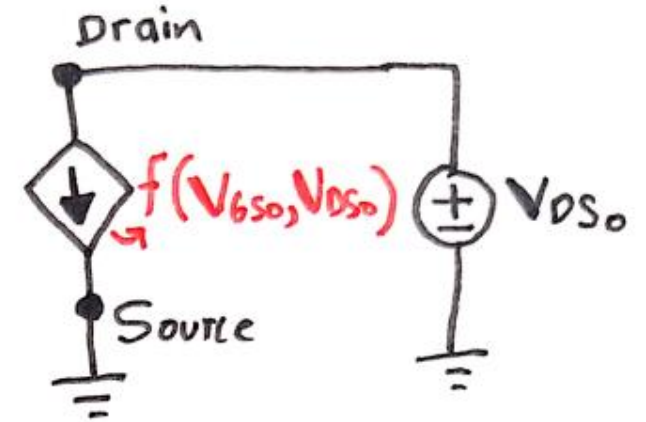
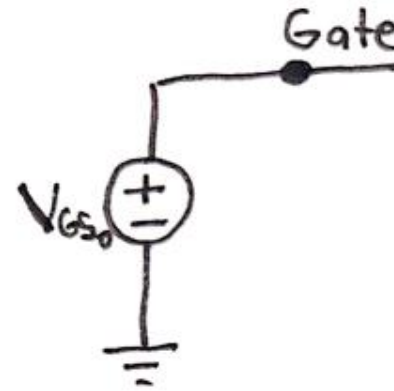


# How to simplify the analysis?

## Bias Point



$$I_{D0} = f(V_{GS0}, V_{DS0})$$



$f(V_{GS0}, V_{DS0}) \rightarrow$  Bias point



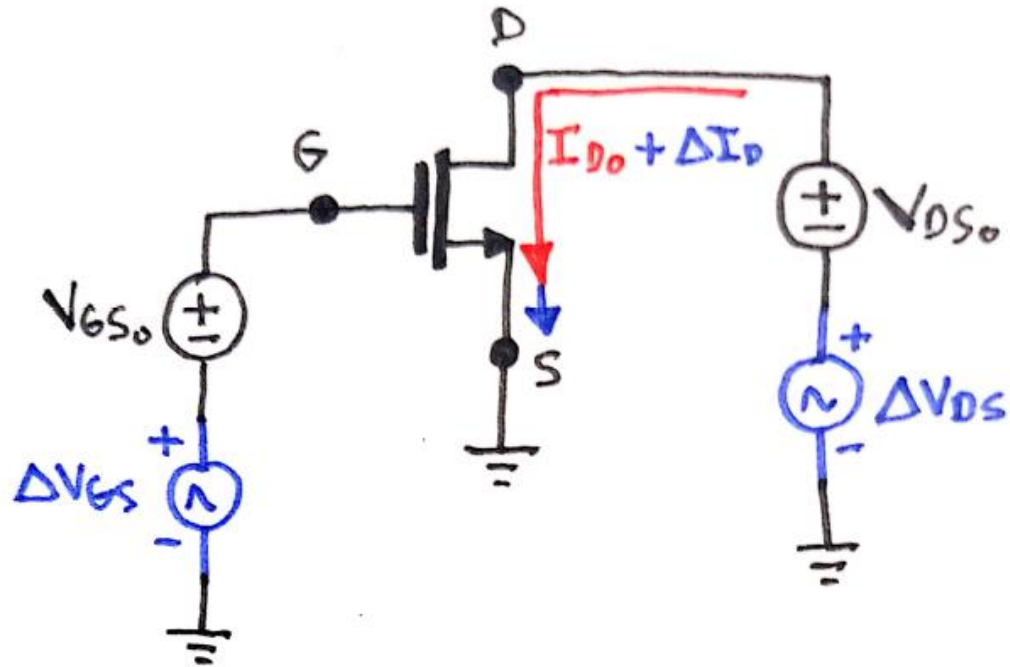
# Bias Point

$$f(V_{GS0}, V_{DS0}) =$$

$V_{GS0} \leq V_{TH}$	$V_{GS0} > V_{TH}$
$\sim 0.$	<ul style="list-style-type: none"> <li>• <math>V_{DS0} &lt; V_{GS0} - V_{TH}</math></li> </ul> $K' \left[ (V_{GS0} - V_{TH}) V_{DS0} - \frac{1}{2} V_{DS0}^2 \right]$ <p>TRIODE</p>
	<ul style="list-style-type: none"> <li>• <math>V_{DS0} \geq V_{GS0} - V_{TH}</math></li> </ul> $\frac{1}{2} K' (V_{GS0} - V_{TH})^2$ <p>or</p> $\frac{1}{2} K' (V_{GS0} - V_{TH})^2 (1 + \lambda V_{DS0})$ <p>SATURATION <span style="margin-left: 50px;">↪ C.L.M</span></p>

$$K' = \mu_n C_{ox} \frac{W}{L}$$

# Changes around Bias



$$I_{D0} + \Delta I_D = f(V_{GS0} + \Delta V_{GS}, V_{DS0} + \Delta V_{DS})$$

Diagram illustrating the relationship between the drain current and the gate-source and drain-source voltages, showing the bias point and small signal variations.

- $I_{D0} + \Delta I_D$  is the total drain current, consisting of the DC bias current  $I_{D0}$  and the AC signal current  $\Delta I_D$ .
- $V_{GS0} + \Delta V_{GS}$  is the total gate-source voltage, consisting of the DC bias voltage  $V_{GS0}$  and the AC signal voltage  $\Delta V_{GS}$ .
- $V_{DS0} + \Delta V_{DS}$  is the total drain-source voltage, consisting of the DC bias voltage  $V_{DS0}$  and the AC signal voltage  $\Delta V_{DS}$ .

Annotations:

- $I_{D0}$  is labeled as the **Bias point**.
- $\Delta I_D$  is labeled as a **Small signal** or **"small change"**.
- $\Delta V_{GS}$  is labeled as a **Small signal**.
- $\Delta V_{DS}$  is labeled as a **Small signal**.

# Changes around Bias

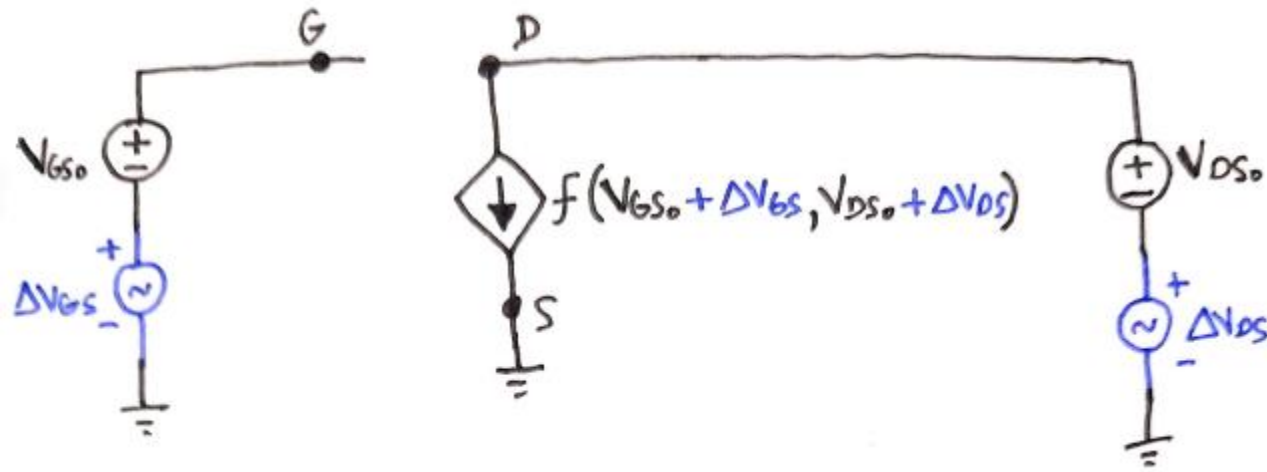
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$$I_{D0} + \Delta I_D = f(V_{GS0}, V_{DS0}) + \frac{\partial f}{\partial V_{GS}} \cdot \Delta V_{GS} + \frac{\partial f}{\partial V_{DS}} \cdot \Delta V_{DS}$$

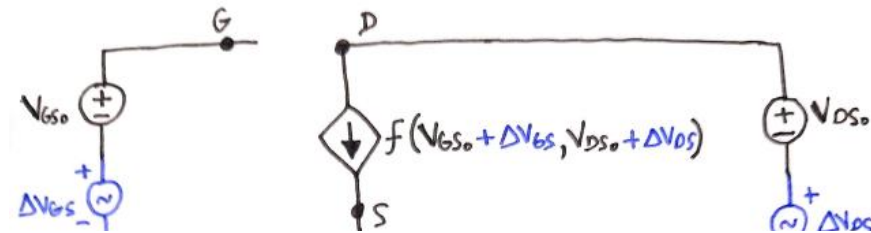
$$I_{D0} = f(V_{GS0}, V_{DS0}) \longrightarrow \text{Bias point}$$

$$\Delta I_D = \frac{\partial f}{\partial V_{GS}} \cdot \Delta V_{GS} + \frac{\partial f}{\partial V_{DS}} \cdot \Delta V_{DS} \longrightarrow \text{small signal components}$$

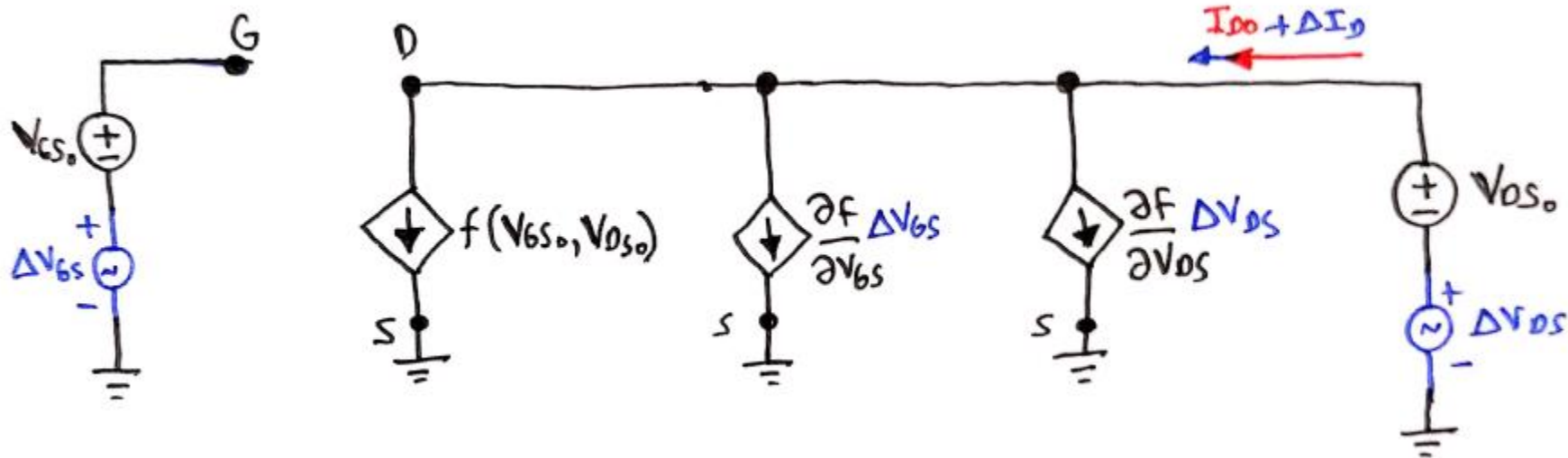
# Circuit meaning



# Circuit meaning: going to ss-model



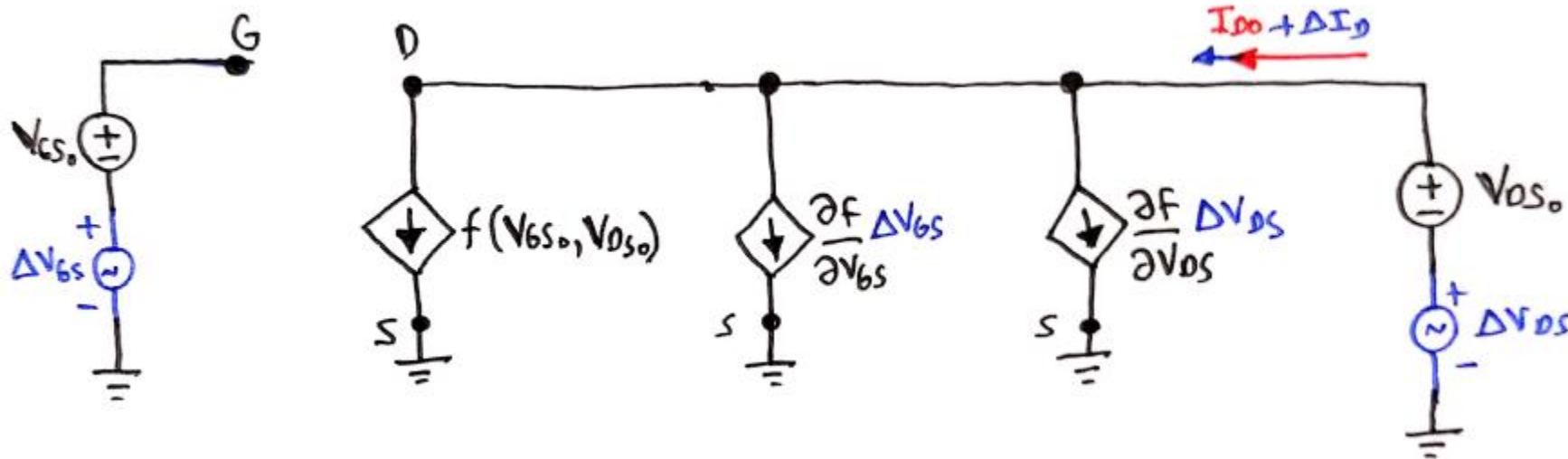
$$I_{D0} + \Delta I_D = f(V_{GS0}, V_{DS0}) + \frac{\partial f}{\partial V_{GS}} \cdot \Delta V_{GS} + \frac{\partial f}{\partial V_{DS}} \cdot \Delta V_{DS}$$



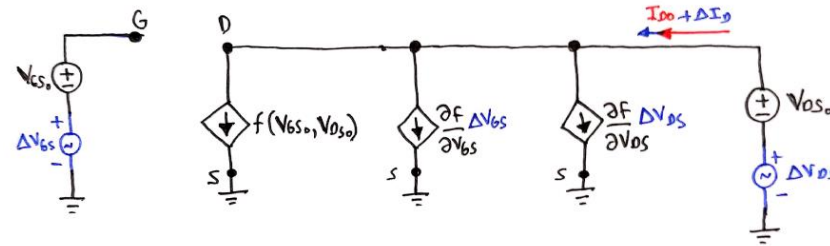
# Circuit meaning going to ss-model

## Current Law at D node

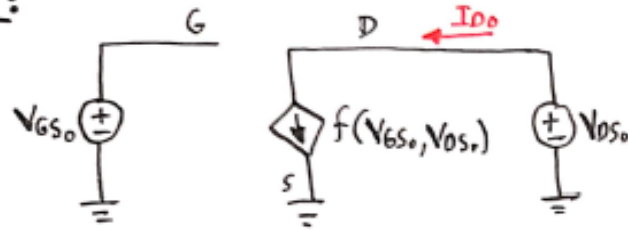
$$I_{D0} + \Delta I_D = f(V_{GS0}, V_{DS0}) + \frac{\partial f}{\partial V_{GS}} \Delta V_{GS} + \frac{\partial f}{\partial V_{DS}} \Delta V_{DS}$$



# Circuit meaning going to ss-model

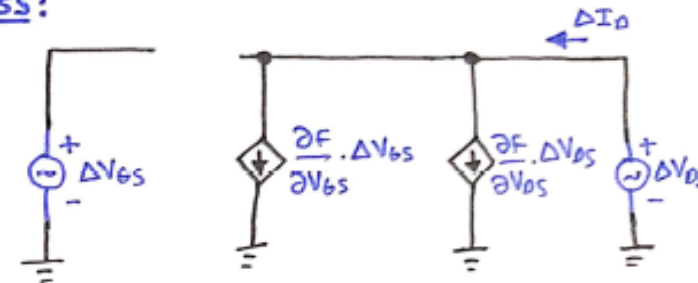


DC:



Bias Point  
"DC Analysis"  
" $\Delta_s$ " = 0.

SS:



$$\Delta V_{GS} \rightarrow V_{gs}$$

$$\Delta V_{DS} \rightarrow V_{ds}$$

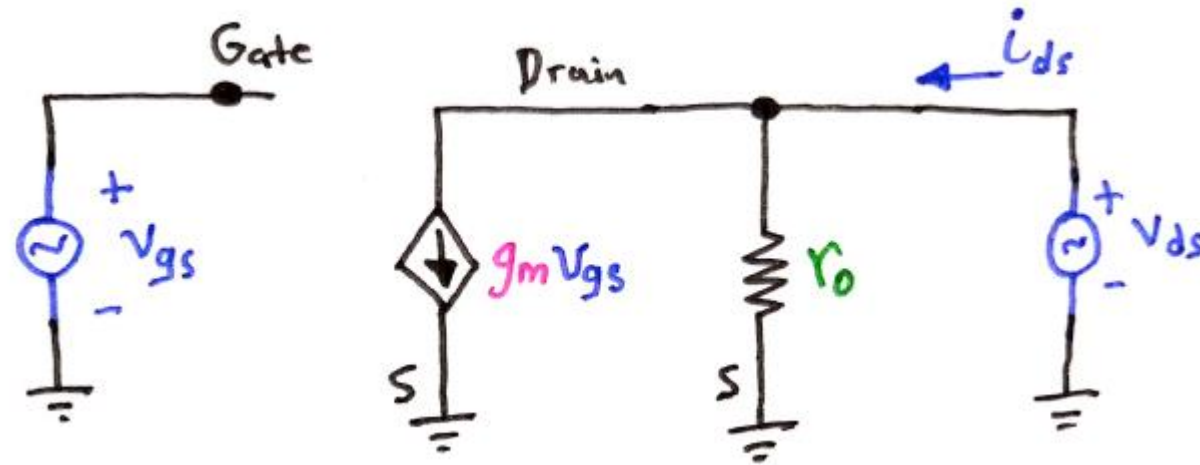
$$\Delta I_D \rightarrow i_{ds}$$

$$\frac{\partial f}{\partial V_{GS}} = \frac{\partial I_D}{\partial V_{GS}} \rightarrow g_m \rightarrow \text{"Transconductance"}$$

$$\frac{\partial f}{\partial V_{DS}} = \frac{\partial I_D}{\partial V_{DS}} \rightarrow \frac{1}{r_o} \rightarrow \text{"Output resistance"}$$

# Small Signal Model (ss-model)

## Low Frequency Small-Signal Model





# Going to the details

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$$I_{DS} = f(V_{GS}, V_{DS}, V_{BS})$$

$$\Delta I_{DS} = \frac{\partial I_{DS}}{\partial V_{GS}} \Delta V_{GS} + \frac{\partial I_{DS}}{\partial V_{DS}} \Delta V_{DS} + \frac{\partial I_{DS}}{\partial V_{BS}} \Delta V_{BS}$$

# Going to the details

---

$$I_{DS} = f(V_{GS}, V_{DS}, V_{BS})$$

$$\Delta I_{DS} = \frac{\partial I_{DS}}{\partial V_{GS}} \Delta V_{GS} + \frac{\partial I_{DS}}{\partial V_{DS}} \Delta V_{DS} + \frac{\partial I_{DS}}{\partial V_{BS}} \Delta V_{BS}$$

# Going to the details

---

$$I_{DS} = f(V_{GS}, V_{DS}, V_{BS})$$

$$i_{ds} = \frac{\partial I_{DS}}{\partial V_{GS}} v_{gs} + \frac{\partial I_{DS}}{\partial V_{DS}} v_{ds} + \frac{\partial I_{DS}}{\partial V_{BS}} v_{bs}$$

# How to simplify the analysis?

---

$$i_{ds} = \frac{\partial I_{DS}}{\partial V_{GS}} v_{gs} + \frac{\partial I_{DS}}{\partial V_{DS}} v_{ds} + \frac{\partial I_{DS}}{\partial V_{BS}} v_{bs}$$

# Transconductance

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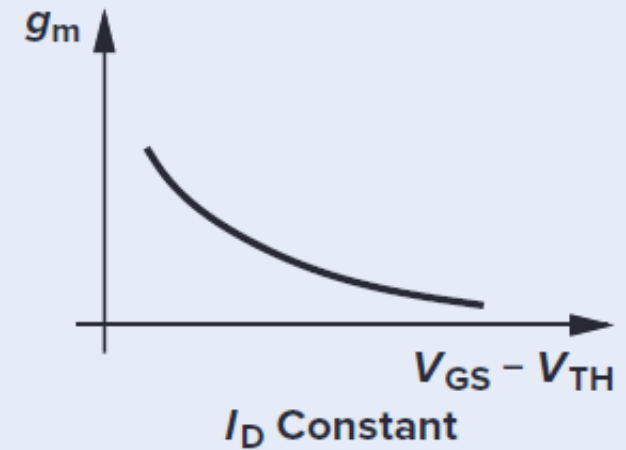
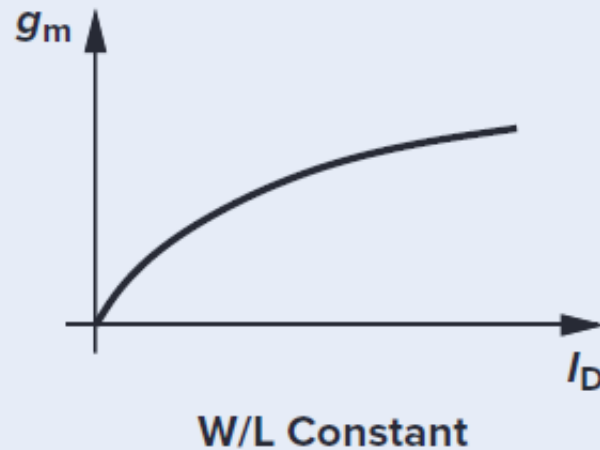
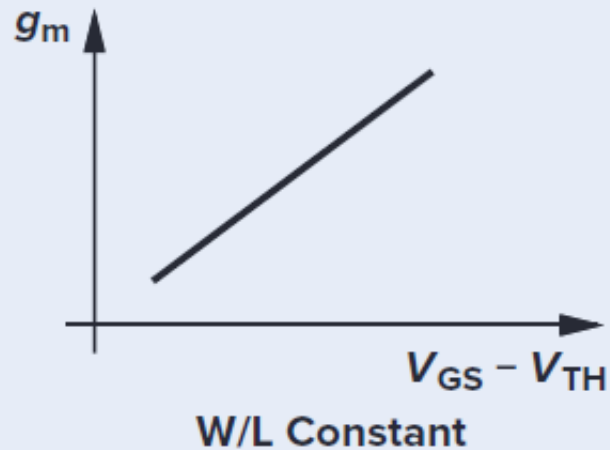
$$i_{ds} = \frac{\partial I_{DS}}{\partial V_{GS}} v_{gs} + \frac{\partial I_{DS}}{\partial V_{DS}} v_{ds} + \frac{\partial I_{DS}}{\partial V_{BS}} v_{bs}$$

$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}}$$

# How to simplify the analysis?

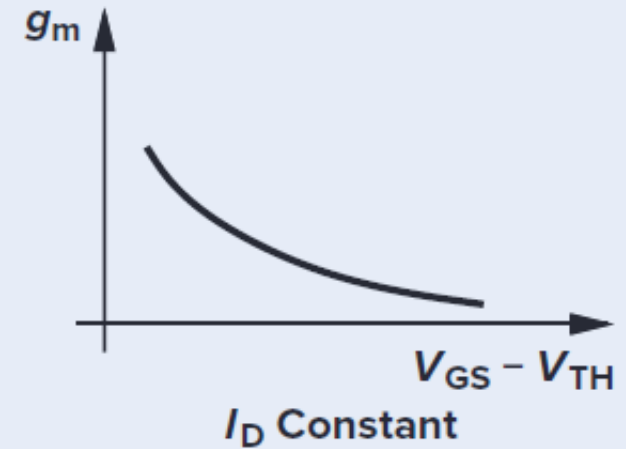
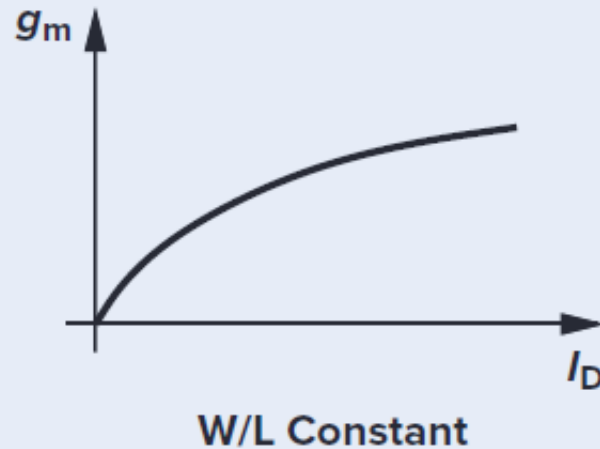
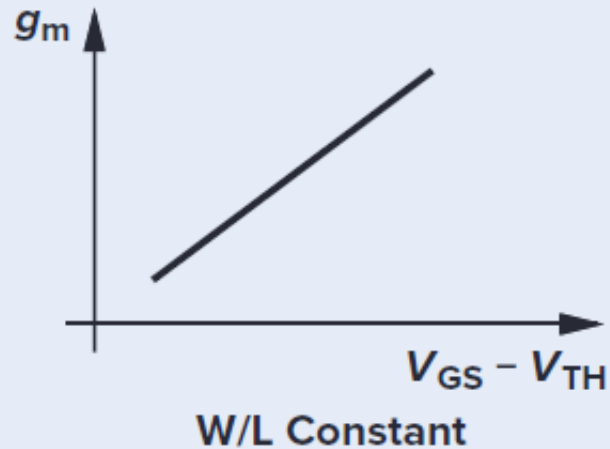
$$g_m = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V_{DS} \text{ const.}}$$
$$= \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$
$$= \frac{2I_D}{V_{GS} - V_{TH}}$$



# How to simplify the analysis?

$$\begin{aligned} g_m &= \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})(1 + \lambda V_{DS}) \\ &= \sqrt{2\mu_n C_{ox} (W/L) I_D (1 + \lambda V_{DS})} \\ &= \frac{2I_D}{V_{GS} - V_{TH}} \end{aligned}$$



# How to simplify the analysis?

---

$$i_{ds} = \frac{\partial I_{DS}}{\partial V_{GS}} v_{gs} + \frac{\partial I_{DS}}{\partial V_{DS}} v_{ds} + \frac{\partial I_{DS}}{\partial V_{BS}} v_{bs}$$

$$r_o = \left[ \frac{\partial I_{DS}}{\partial V_{DS}} \right]^{-1}$$



# MOSFET – Initial Thoughts

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$$\begin{aligned} r_O &= \frac{\partial V_{DS}}{\partial I_D} \\ &= \frac{1}{\partial I_D / \partial V_{DS}} \\ &= \frac{1}{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \cdot \lambda} \end{aligned} \quad \begin{aligned} &\approx \frac{1 + \lambda V_{DS}}{\lambda I_D} \\ &\approx \frac{1}{\lambda I_D} \end{aligned}$$

# How to simplify the analysis?

---

$$i_{ds} = \frac{\partial I_{DS}}{\partial V_{GS}} v_{gs} + \frac{\partial I_{DS}}{\partial V_{DS}} v_{ds} + \frac{\partial I_{DS}}{\partial V_{BS}} v_{bs}$$

$$g_{mb} = \frac{\partial I_{DS}}{\partial V_{BS}}$$

# MOSFET – Initial Thoughts

$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}}$$

$$= \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) \left( -\frac{\partial V_{TH}}{\partial V_{BS}} \right)$$

$$\frac{\partial V_{TH}}{\partial V_{BS}} = -\frac{\partial V_{TH}}{\partial V_{SB}}$$

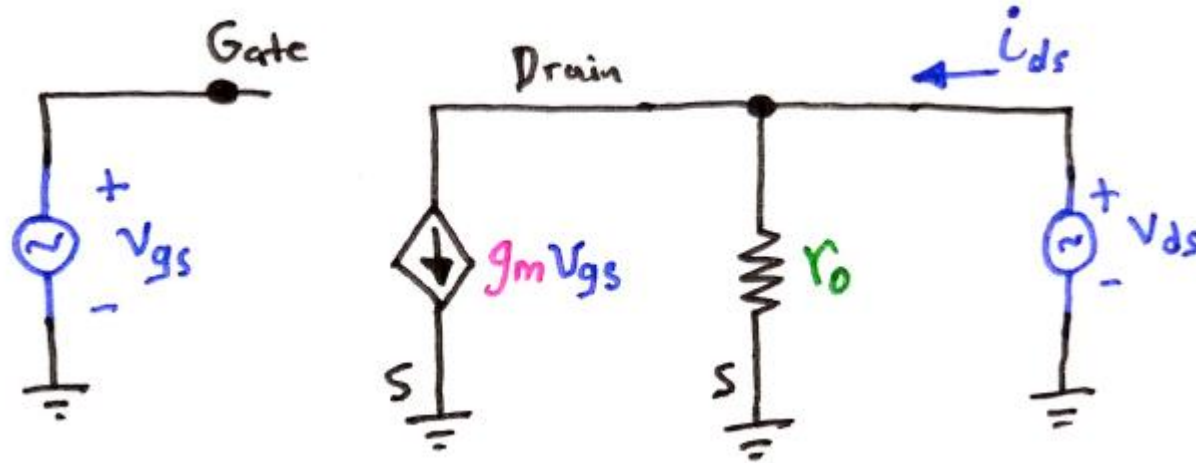
$$= -\frac{\gamma}{2} (2\Phi_F + V_{SB})^{-1/2}$$

$$g_{mb} = g_m \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}$$

$$= \eta g_m$$

# Small-Signal Model Building

Something is missing → ?



$$i_{ds} = g_m v_{gs} + \frac{1}{r_o} v_{ds} + g_{mb} v_{bs}$$

# PMOS ss-model

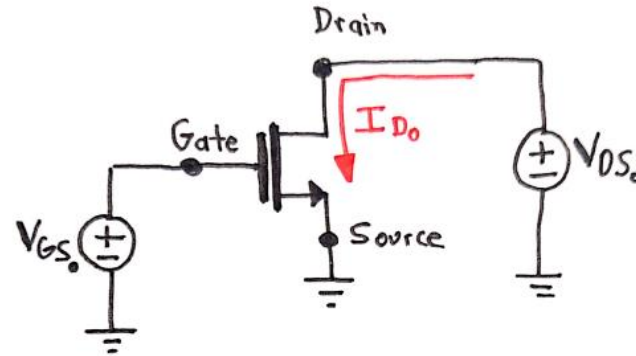
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$$i_{ds} = g_m v_{gs} + \frac{1}{r_o} v_{ds} + g_{mb} v_{bs}$$

**For a PMOS device it is easy to demonstrate that the ss-model is EXACTLY the same as in NMOS case IF and ONLY IF the same terminal labels and voltage references are used.**

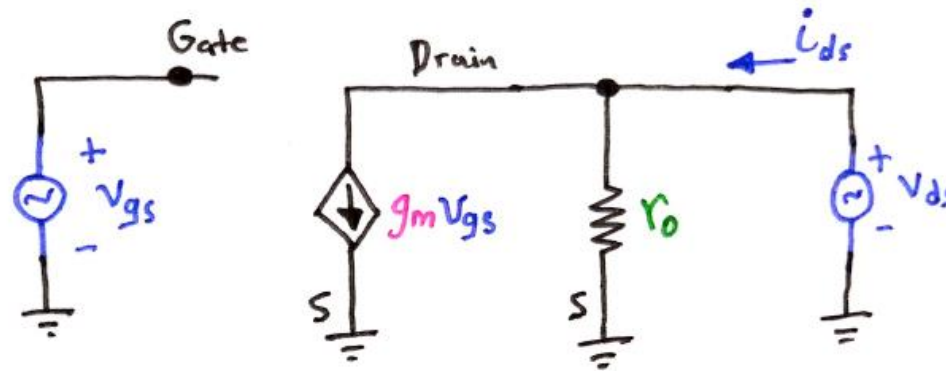
# How to solve problems?

## ➤ DC analysis



$$I_{D_0} = f(V_{GS_0}, V_{DS_0})$$

## ➤ ss Analysis



# Let's solve our initial problem again!

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# Conclusions

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- Note that the small signal model works on “Deltas” instead of the total voltage (or current) value.
- Small-signal model does not tell us anything about ABSOLUTE values!
- But it allow us to calculate the variations in a very useful way!
- We transform the MOSFET into a linear circuit!
- SS analysis is just one half of the work

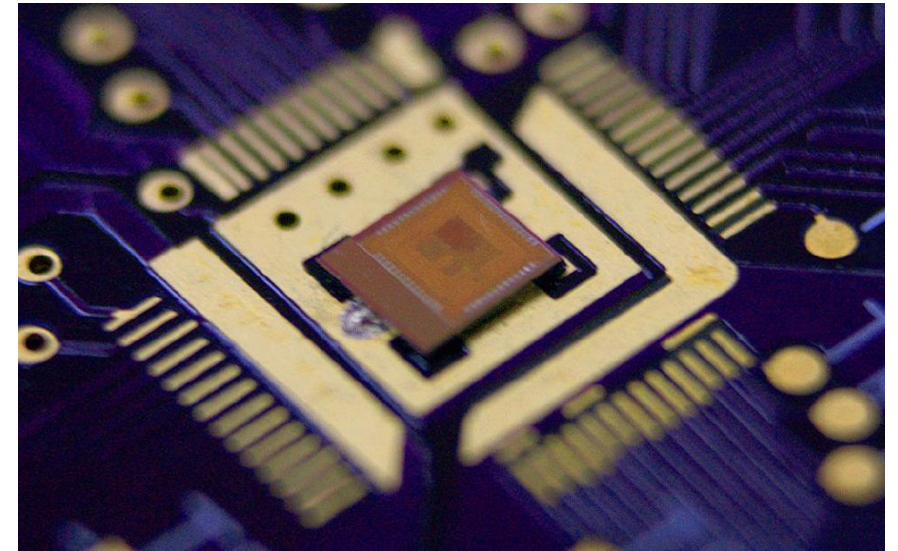
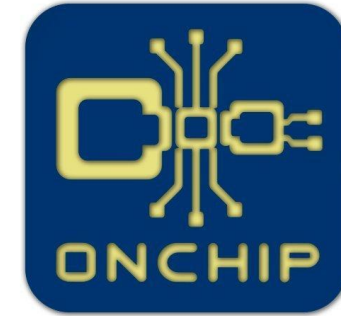


# Thanks

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