Lecture 04: Small Signal Modeling

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Outline

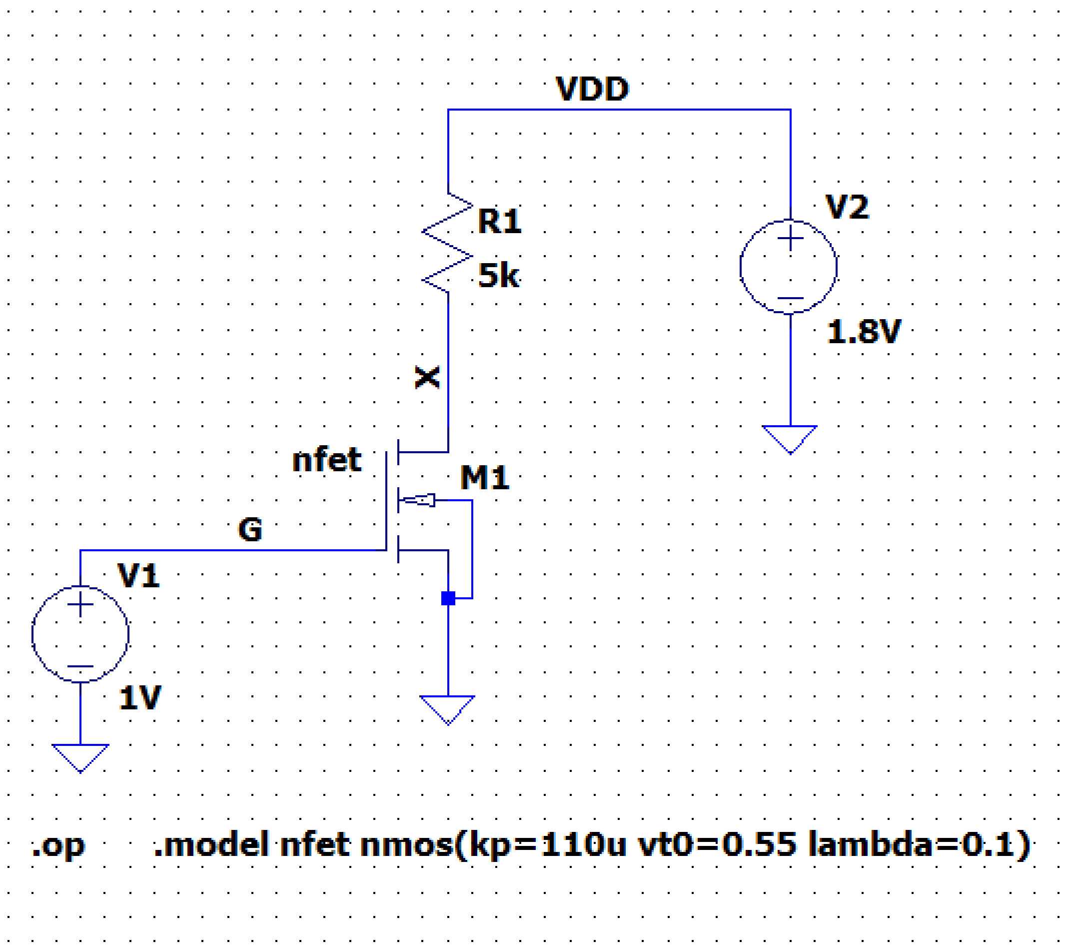
- ❑Introduction
- ❑Signal Definitions
- ❑Small-Signal Modeling
- ❑Conclusions

Introduction

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2\cdot(1+\lambda V_{DS})$$

$$I_D = (185.6\,\mu)(1+0.1(1.8-5000\,I_D))$$

$$\underline{I_D = 200.4\ \mu A}$$



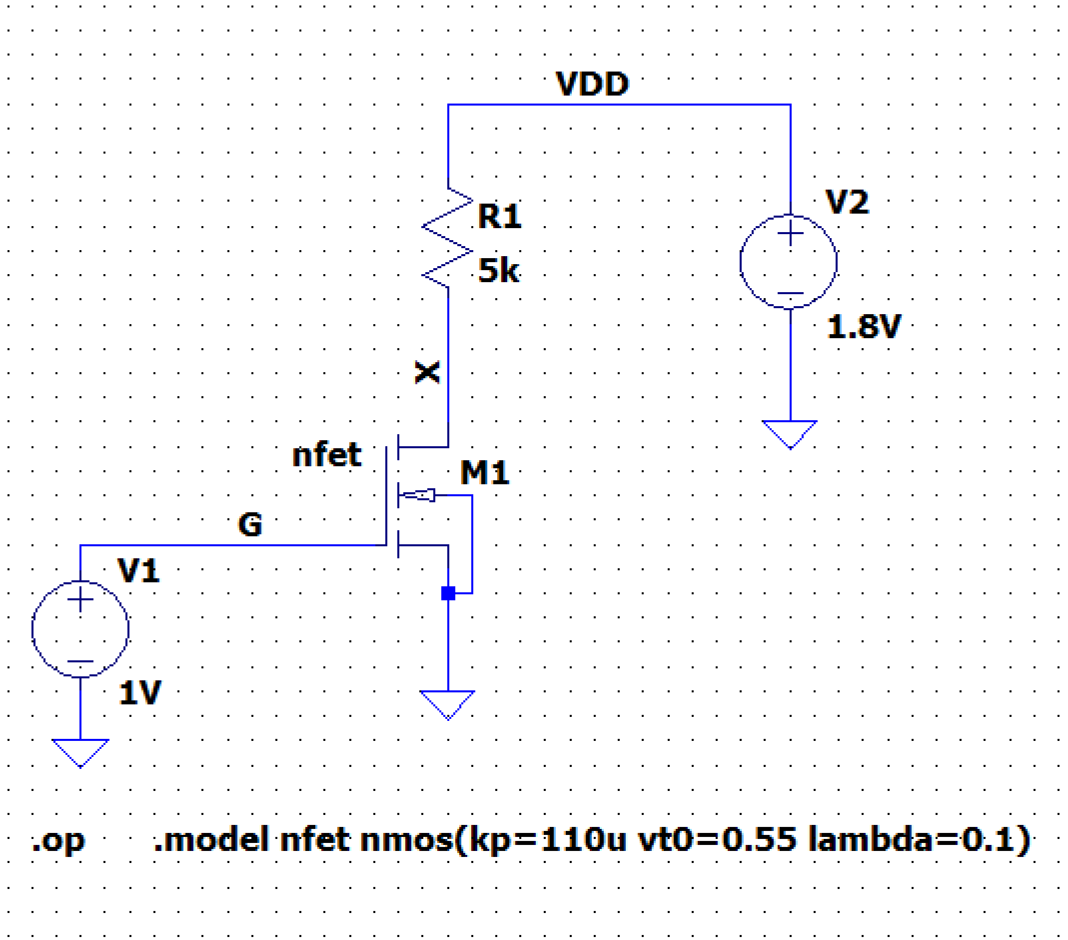
Introduction – Changing Values

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2\cdot(1+\lambda V_{DS})$$

$$I_D = (185.6\,\mu)(1+0.1(1.8-5000\,I_D))$$

$$\underline{I_D = 200.4\ \mu A}$$

$$V_{GS2} = V_{GS} + \Delta V_{GS} \quad ; \quad \Delta V_{GS} = 10mV$$



Introduction – Changing Values

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2\cdot(1+\lambda V_{DS})$$

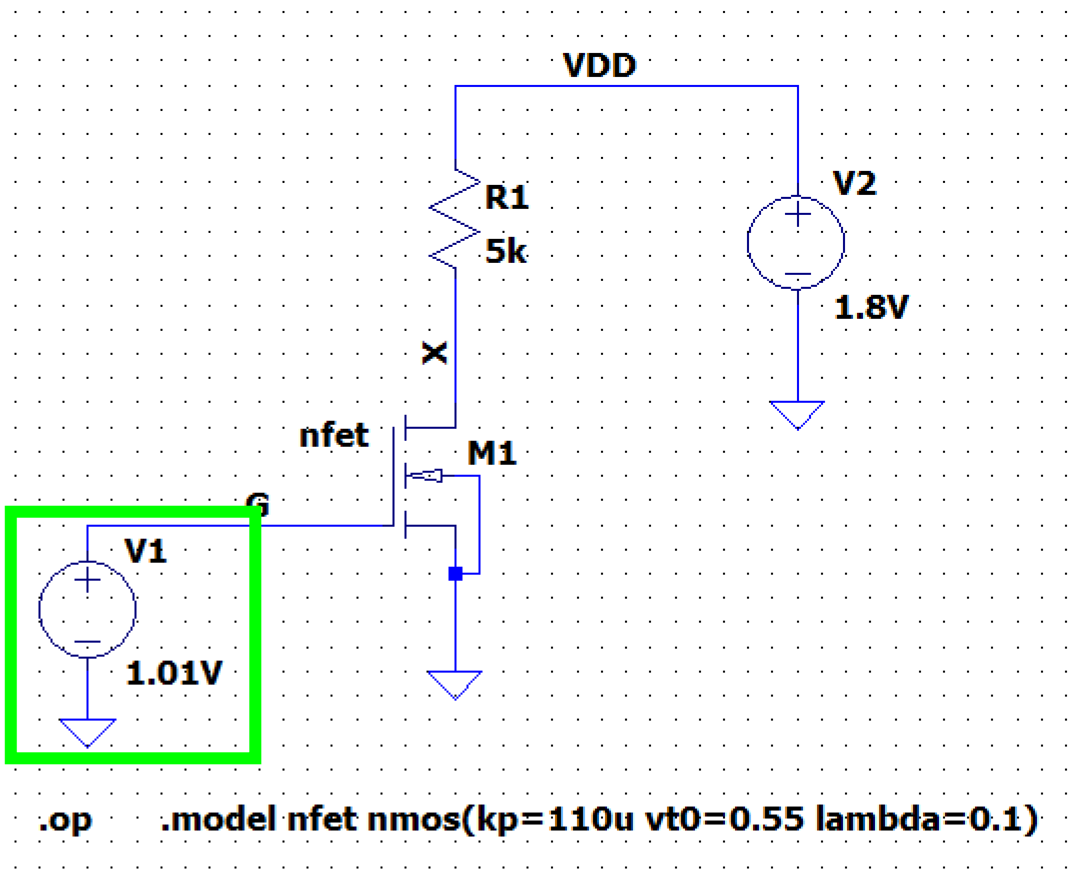
$$I_D = (185.6\mu)(1+0.1(1.8-5000\, I_D))$$

$$\underline{I_D = 200.4\ \mu A}$$

$$V_{GS2} = V_{GS} + \Delta V_{GS} \quad ; \quad \Delta V_{GS} = 10mV$$

$V_{GS2} = 1.01\ V$ → we repeat the same process.

$$I_{D2} = (193.97\mu)(1+0.1(1.8-5000\, I_D))$$



Introduction – Changing Values

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2 \cdot (1+\lambda V_{DS})$$

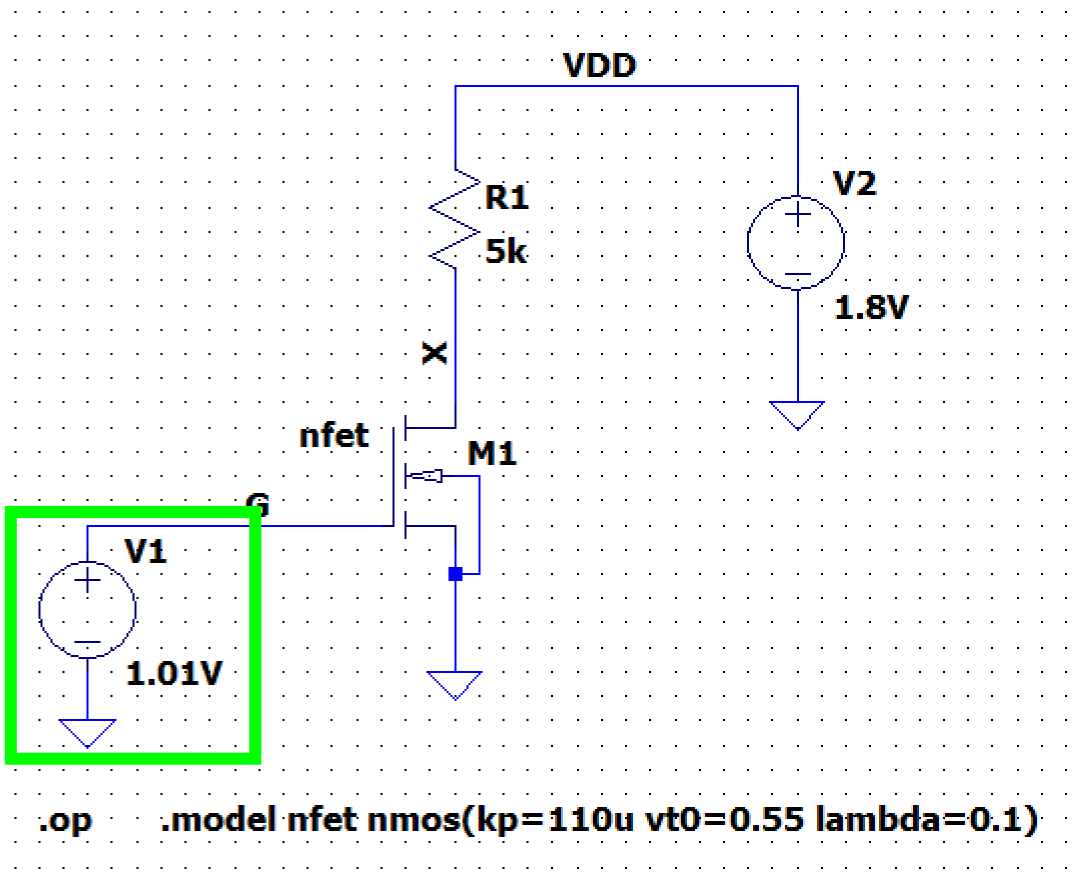
$$I_D = (185.6\,\mu)(1+0.1(1.8-5000\,I_D))$$

$$\underline{I_D = 200.4\ \mu A}$$

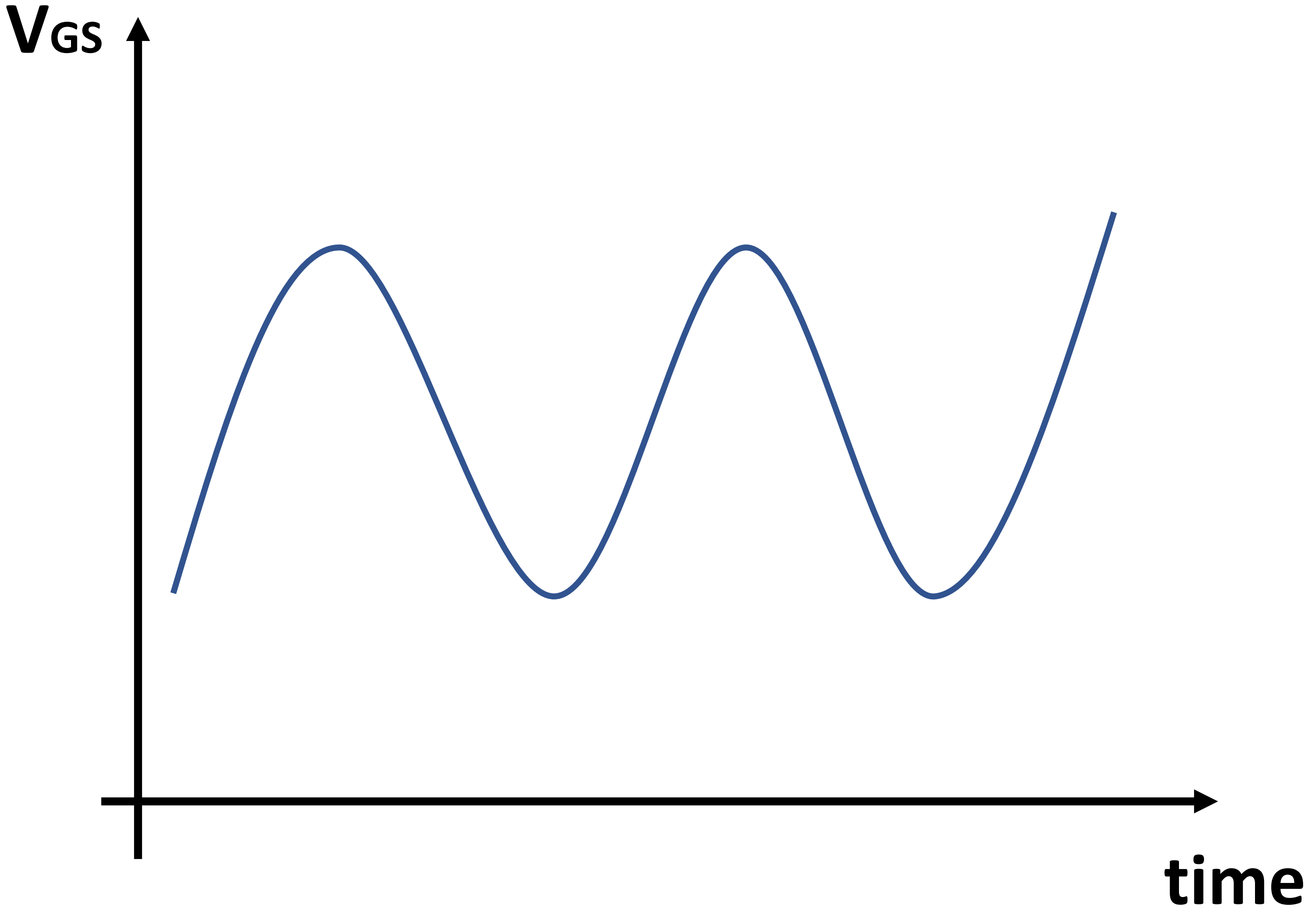
$$V_{GS2} = V_{GS} + \Delta V_{GS} \quad ; \quad \Delta V_{GS} = 10mV$$

$$I_{D2} = 208.65\ \mu A \rightarrow I_{D2} = I_D + \Delta I_D$$

$$\underline{\Delta I_D = 8.25\ \mu A}$$

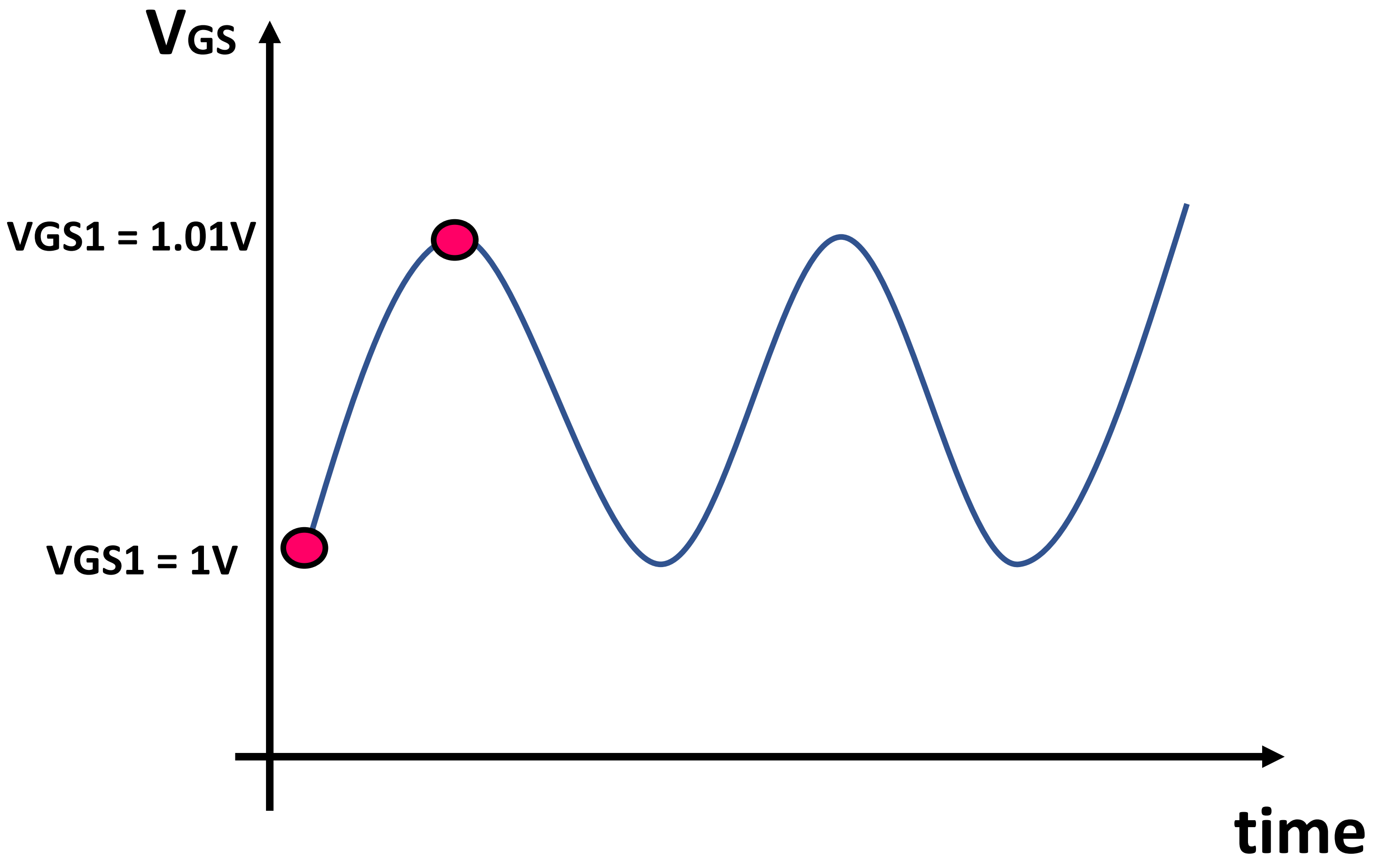


Signals



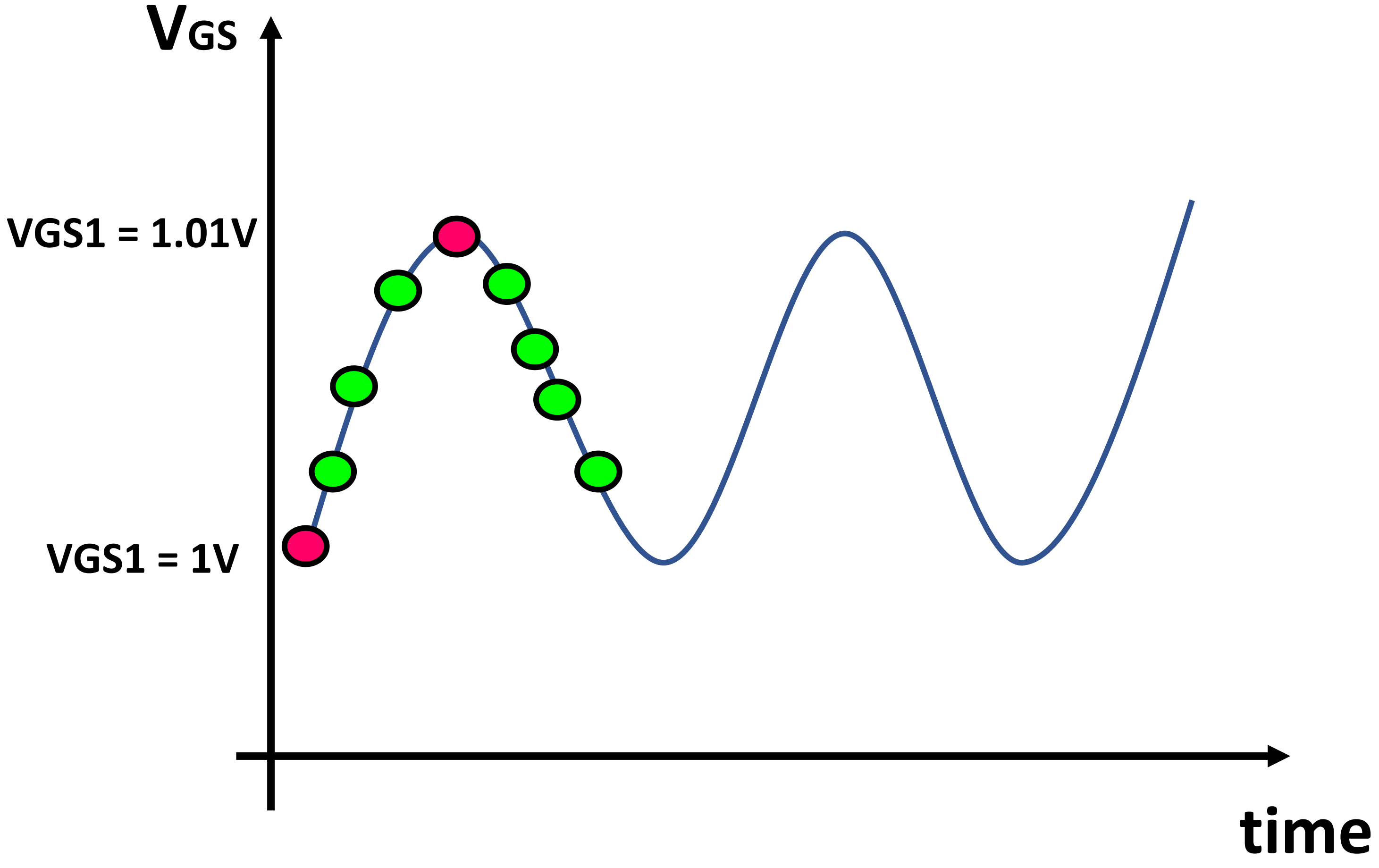
What if we put a signal?

Signals



Think of a signal between the two previous values

Signals

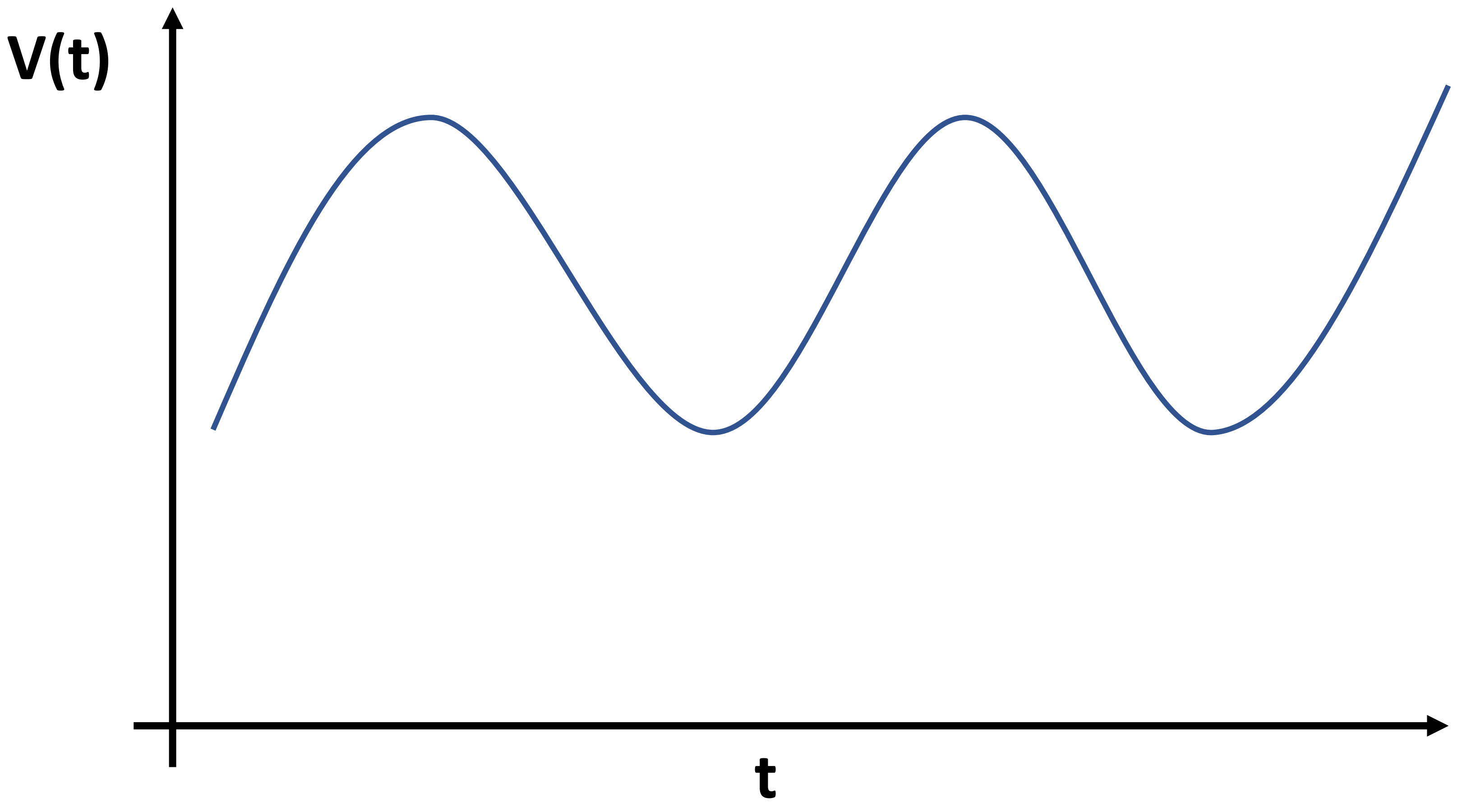


We need to solve for each point

Lots of calculations!

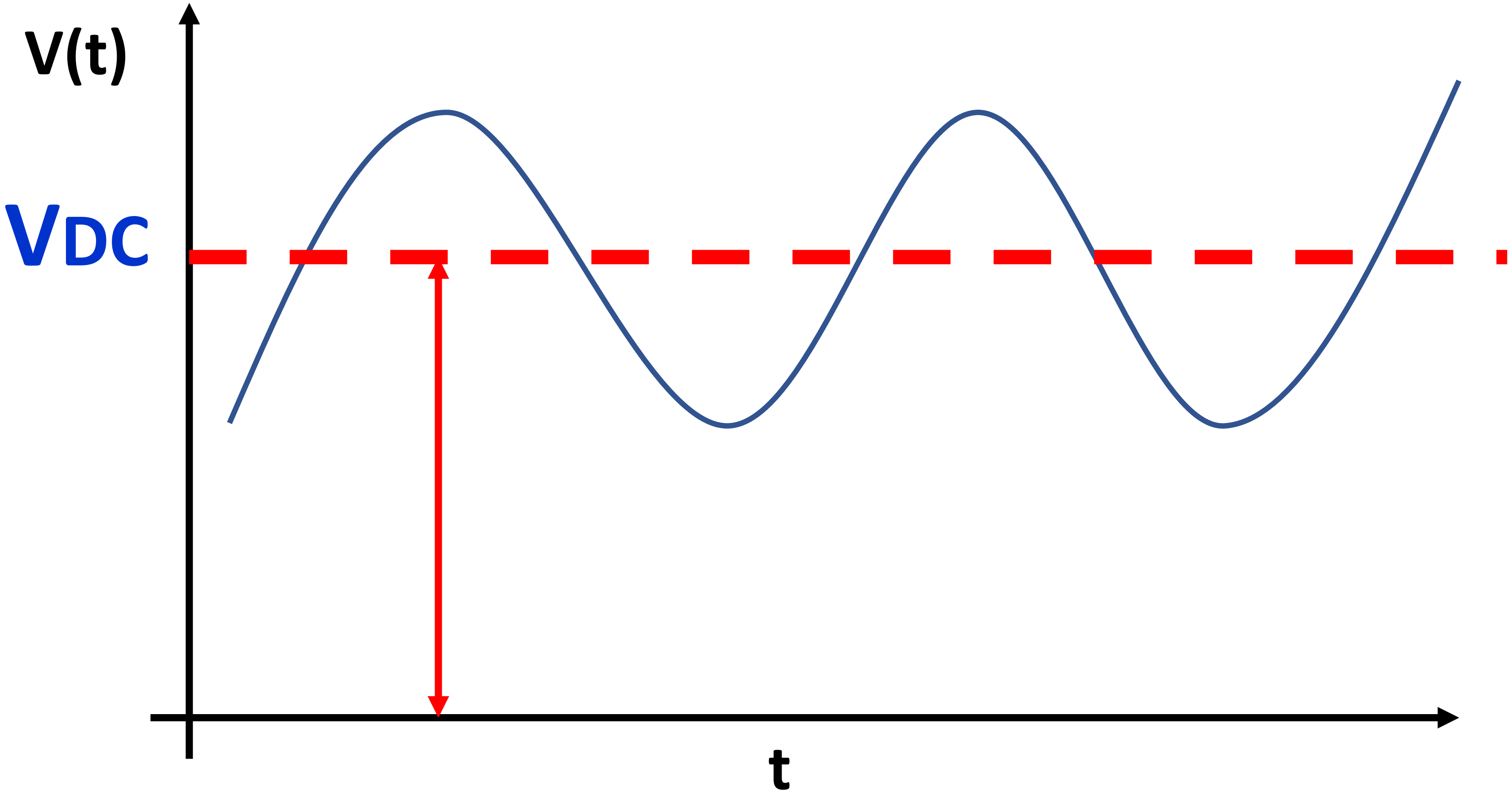
Signals: Definitions

- ❑DC Level
- ❑Amplitude
- ❑Peak Value
- ❑Peak-peak Value
- ❑Something else?



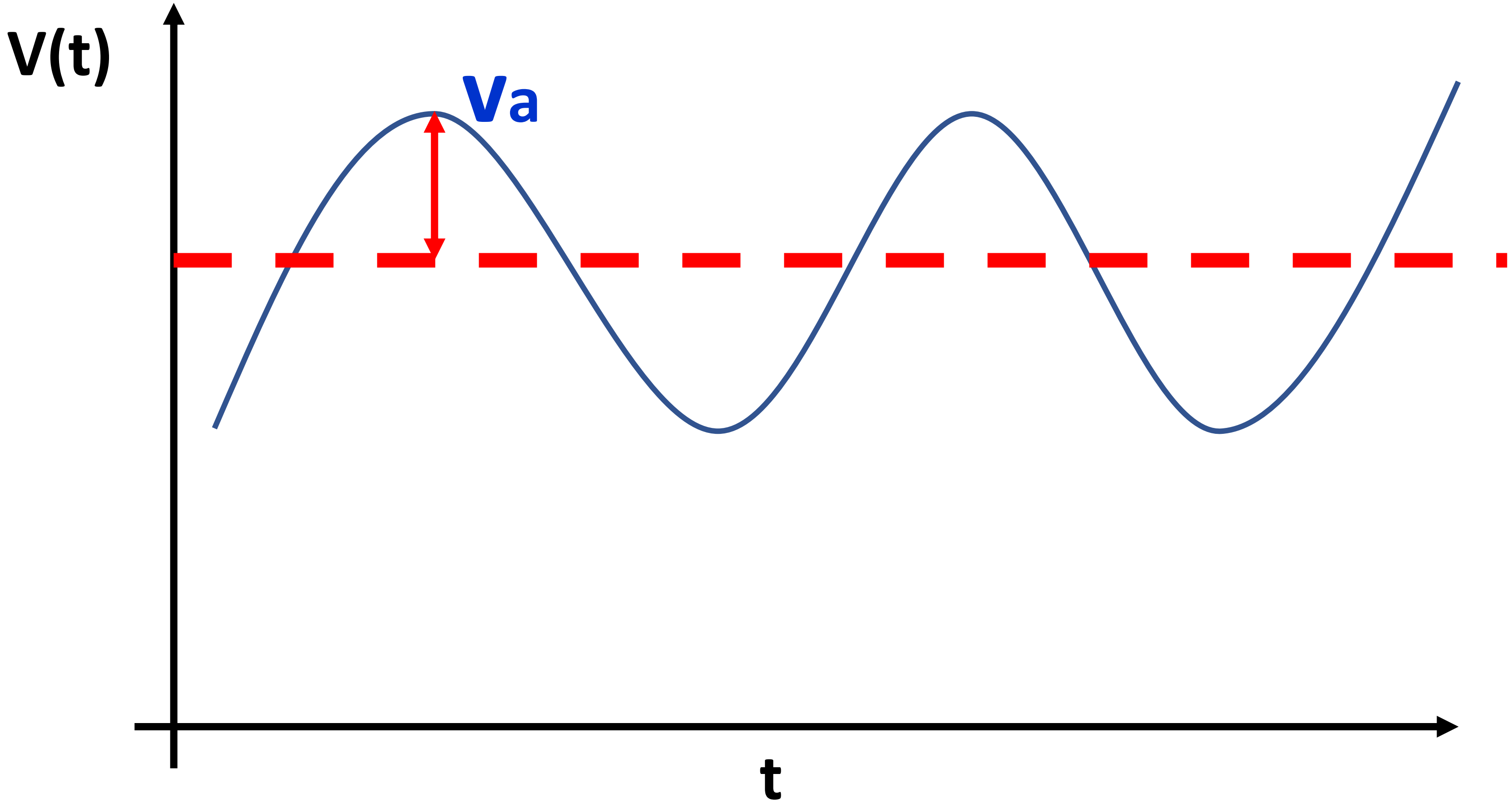
Signals: Definitions

- ❑ **DC Level**
- ❑ Amplitude
- ❑ Peak Value
- ❑ Peak-peak Value
- ❑ Something else?



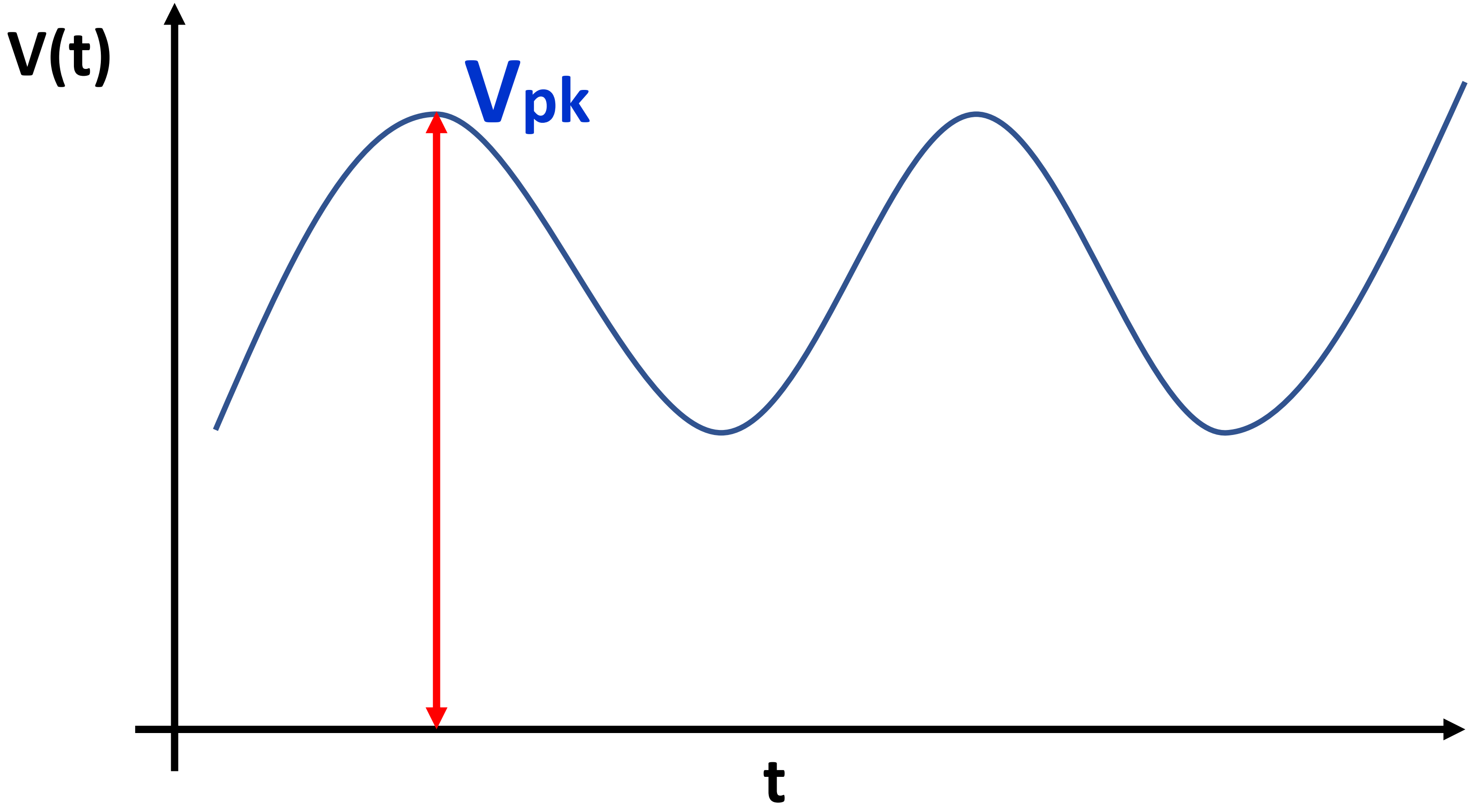
Signals: Definitions

- ❑DC Level
- ❑**Amplitude***
- ❑Peak Value
- ❑Peak-peak Value
- ❑Something else?



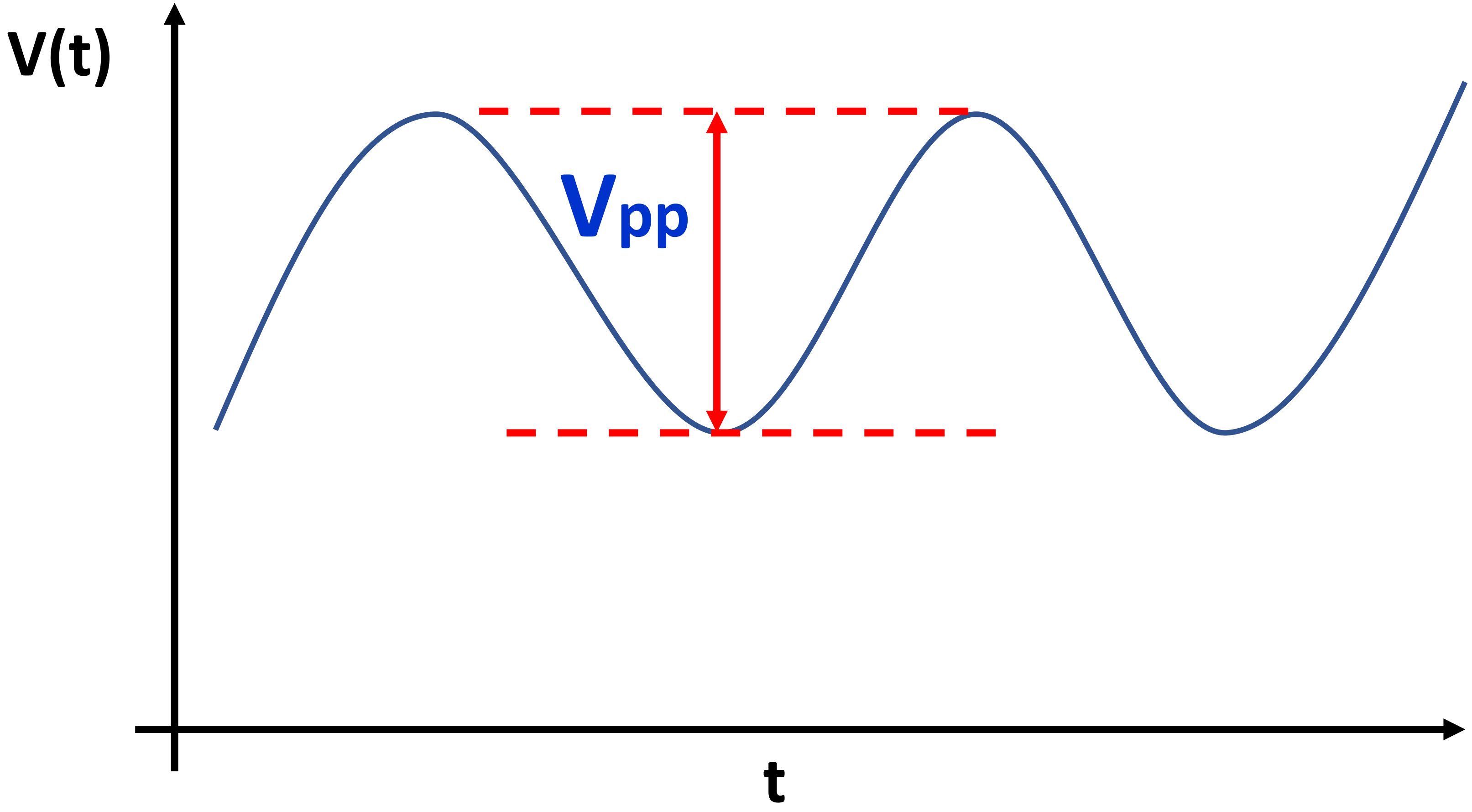
Signals: Definitions

- ❑DC Level
- ❑Amplitude
- ❑**Peak Value**
- ❑Peak-peak Value
- ❑Something else?



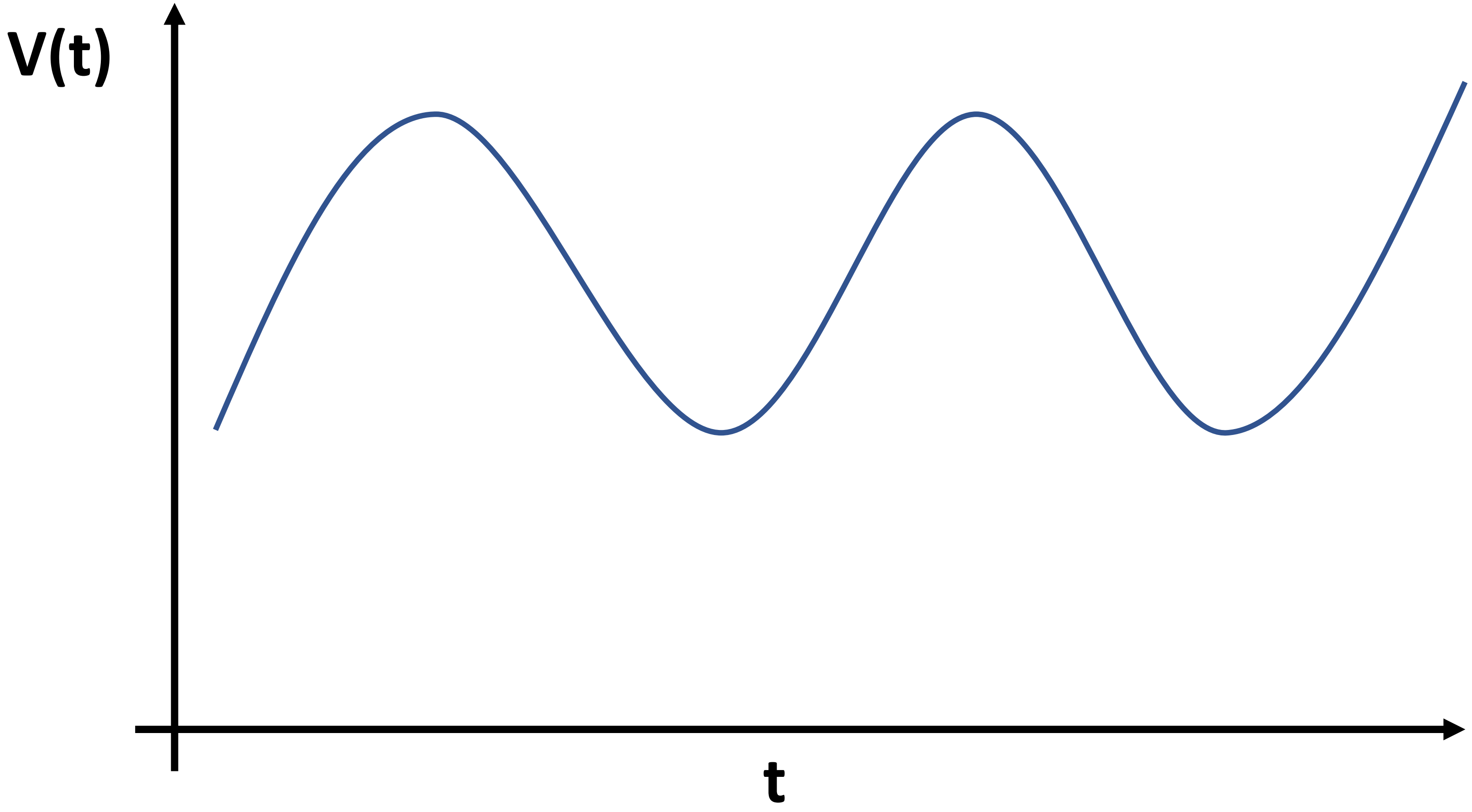
Signals: Definitions

- ❑DC Level
- ❑Amplitude
- ❑Peak Value
- **❑Peak-peak Value**
- ❑Something else?



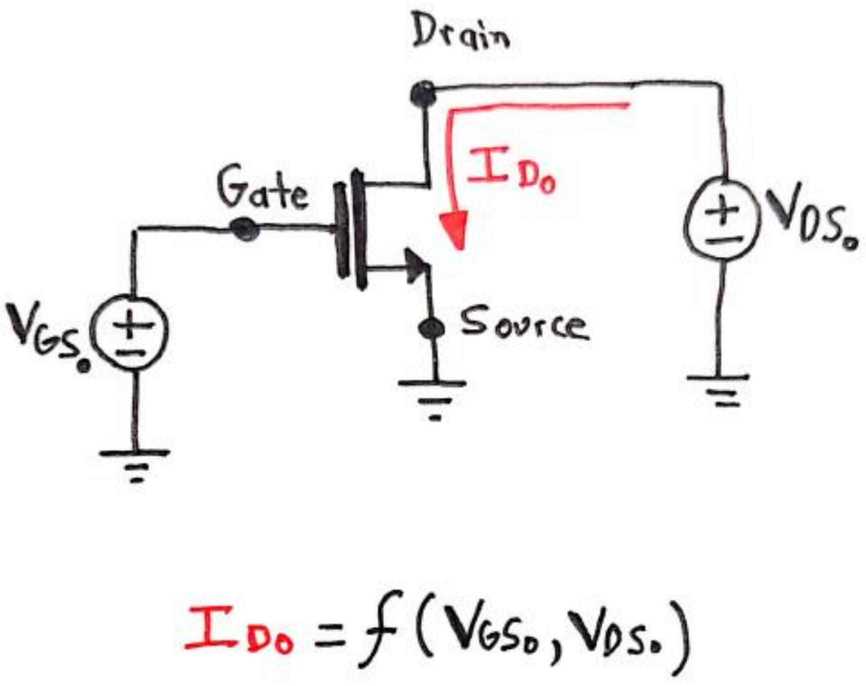
Signals: Definitions

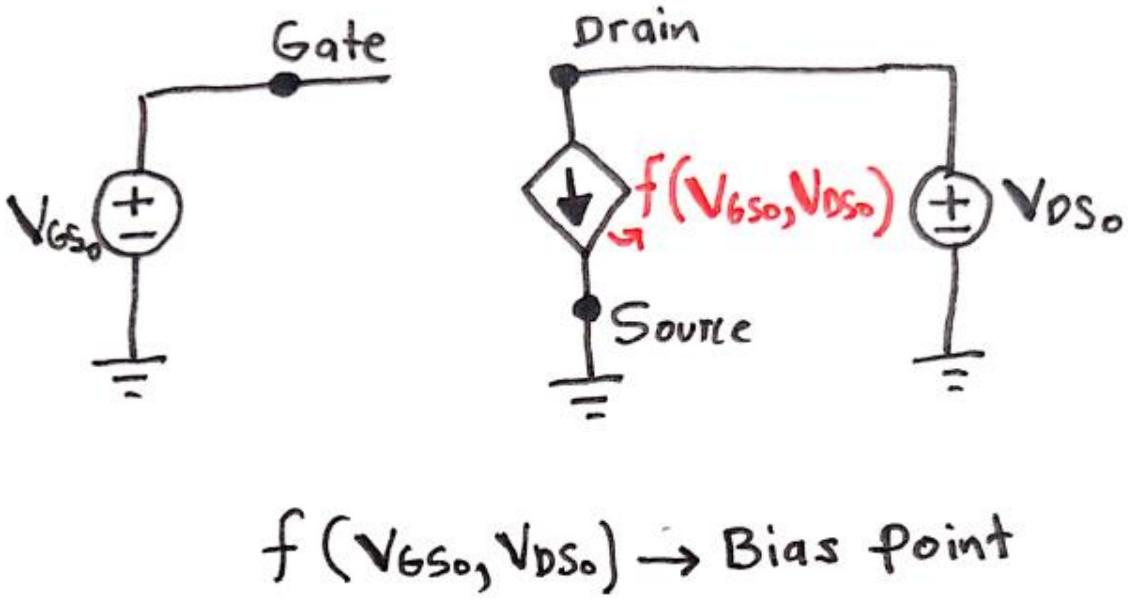
- ❑DC Level
- ❑Amplitude
- ❑Peak Value
- ❑Peak-peak Value
- ❑**Something else?**



How to simplify the analysis?

Bias Point





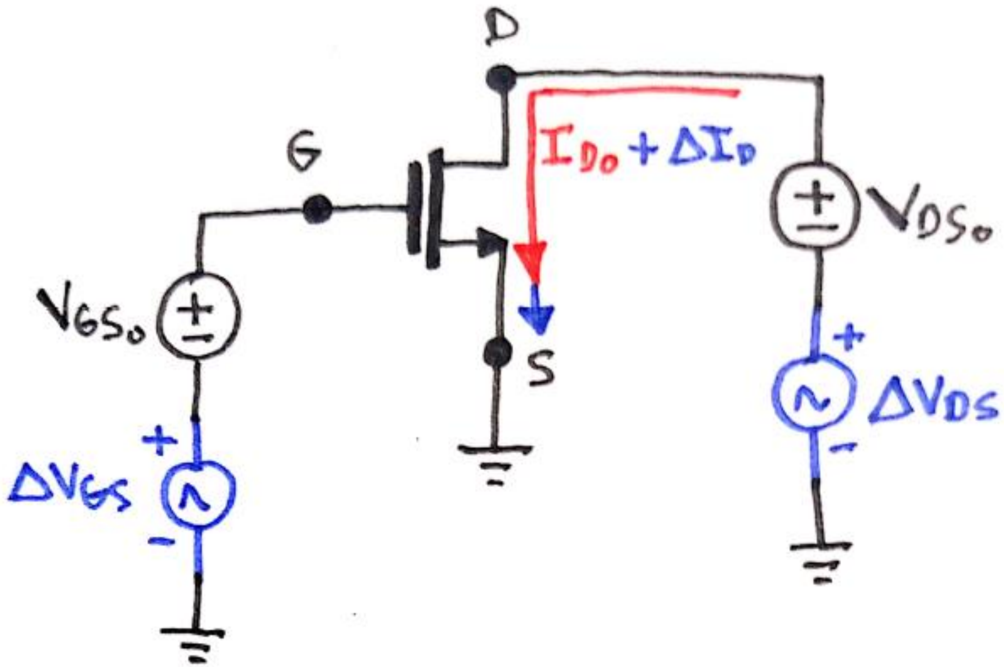
Bias Point

$$f(V_{GSo}, V_{DSo}) =$$

$V_{GSo} \leq V_{TH}$	$V_{GSo} > V_{TH}$
	• $V_{DSo} < V_{GSo} - V_{TH}$ $K'\left[(V_{GSo} - V_{TH})V_{DSo} - \frac{1}{2}V_{DSo}^2\right]$ TRIODE
$\sim 0.$	• $V_{DSo} \geq V_{GSo} - V_{TH}$ $\frac{1}{2}K'(V_{GSo} - V_{TH})^2$ or $\frac{1}{2}K'(V_{GSo} - V_{TH})^2(1 + \lambda V_{DSo})$ SATURATION ↳ C.L.M

$$\textbf{K'} = \mu_n C_{ox} \frac{W}{L}$$

Changes around Bias



$$I_{D0} + \Delta I_D = f\left(V_{GS0} + \Delta V_{GS},\ V_{DS0} + \Delta V_{DS}\right)$$

I_{D0}: Bias point

ΔI_D: Small signal "small change"

ΔV_{GS}: small signal

ΔV_{DS}: small signal

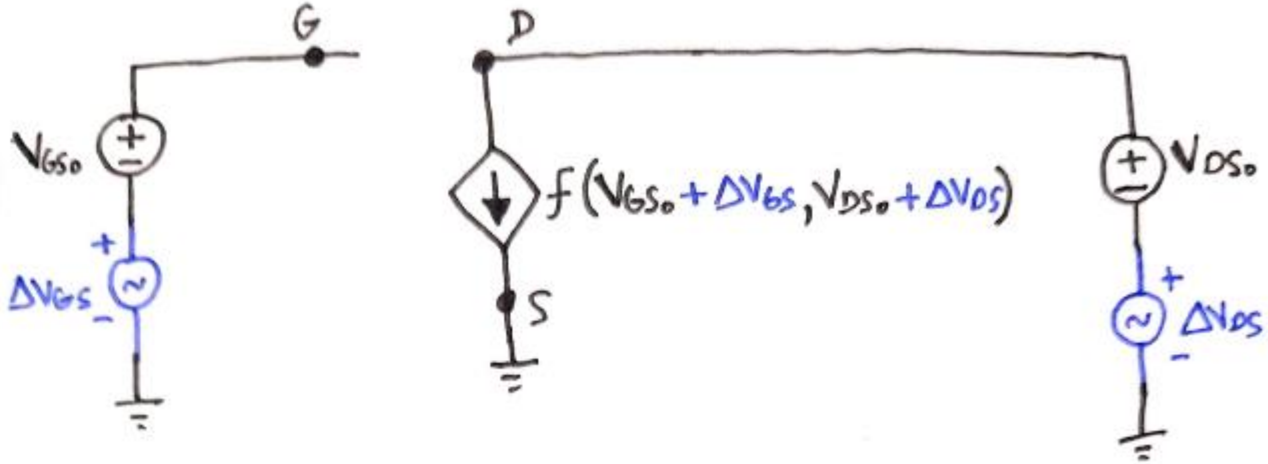
Changes around Bias

$$I_{D0} + \Delta I_D = f(V_{GS0}, V_{DS0}) + \frac{\partial f}{\partial V_{GS}} \cdot \Delta V_{GS} + \frac{\partial f}{\partial V_{DS}} \cdot \Delta V_{DS}$$

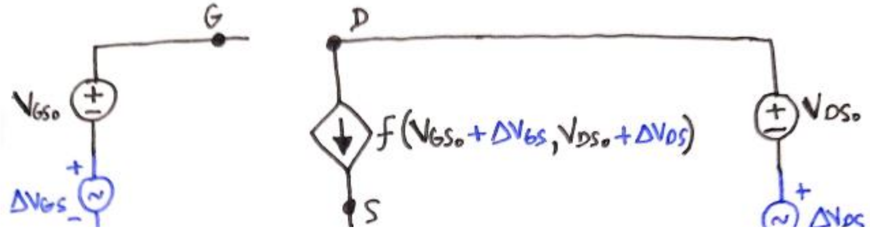
$$I_{D0} = f(V_{GS0}, V_{DS0}) \longrightarrow \text{Bias point}$$

$$\Delta I_D = \frac{\partial f}{\partial V_{GS}} \cdot \Delta V_{GS} + \frac{\partial f}{\partial V_{DS}} \cdot \Delta V_{DS} \longrightarrow \text{small signal components}$$

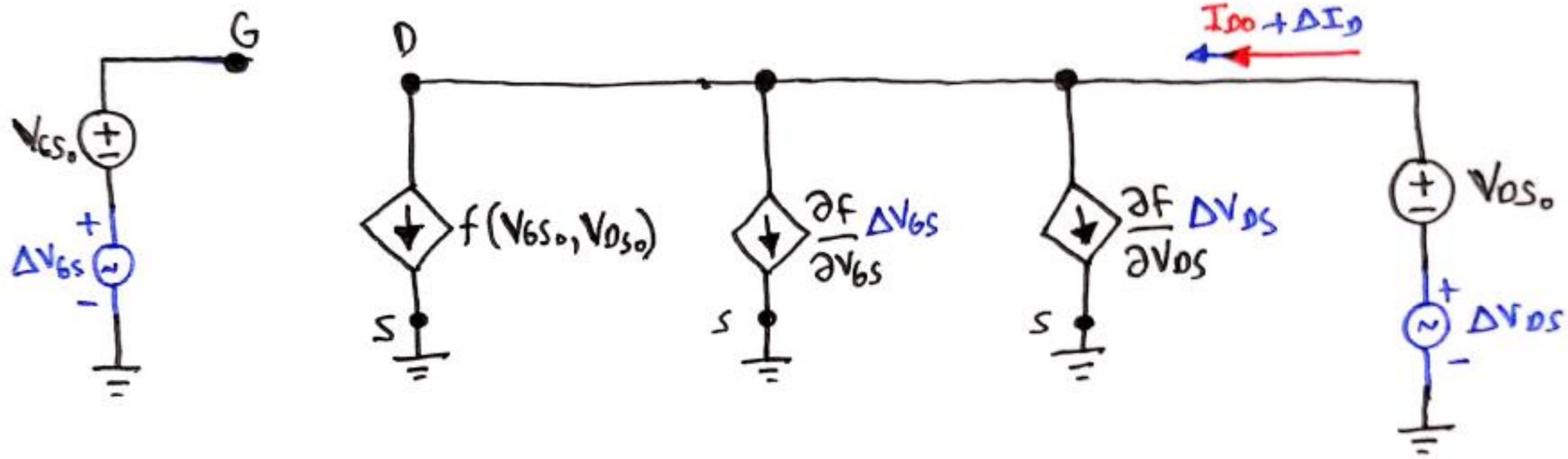
Circuit meaning



Circuit meaning: going to ss-model



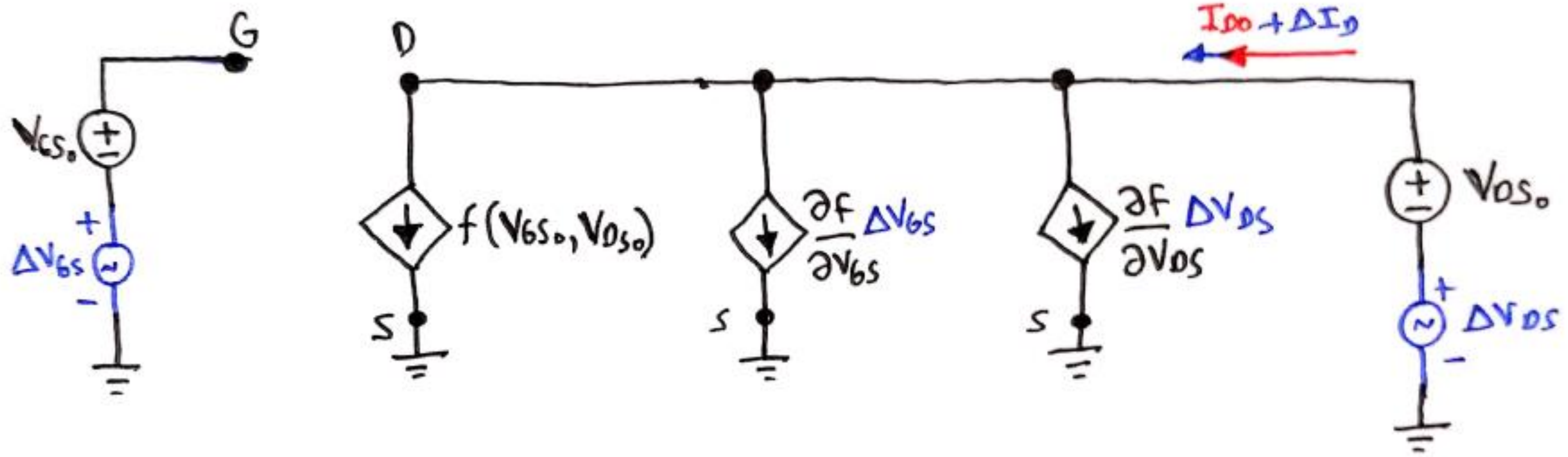
$$I_{D0} + \Delta I_D = f(V_{GS_0}, V_{DS_0}) + \frac{\partial f}{\partial V_{GS}} \cdot \Delta V_{GS} + \frac{\partial f}{\partial V_{DS}} \cdot \Delta V_{DS}$$



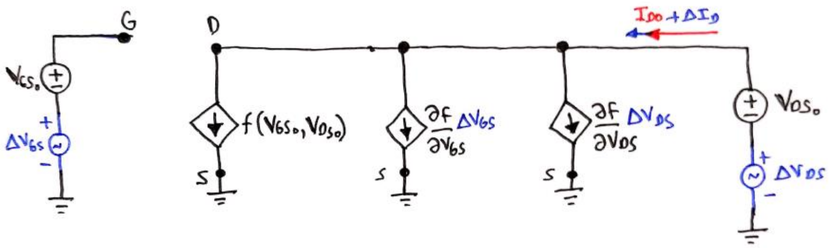
Circuit meaning going to ss-model

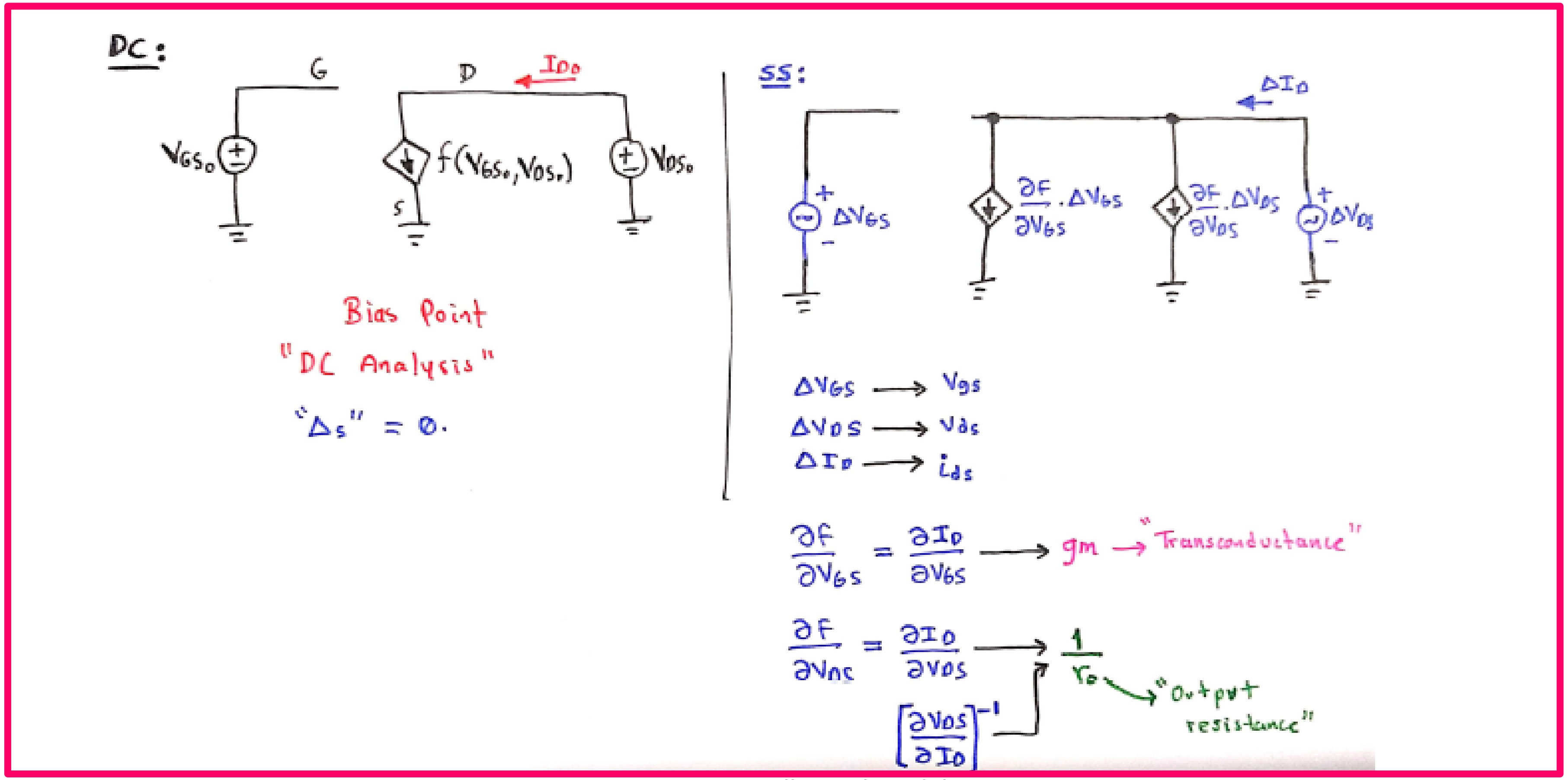
Current Law at D node

$$I_{D0} + \Delta I_D = f(V_{GS0}, V_{DS0}) + \frac{\partial f}{\partial V_{GS}} \cdot \Delta V_{GS} + \frac{\partial f}{\partial V_{DS}} \cdot \Delta V_{DS}$$



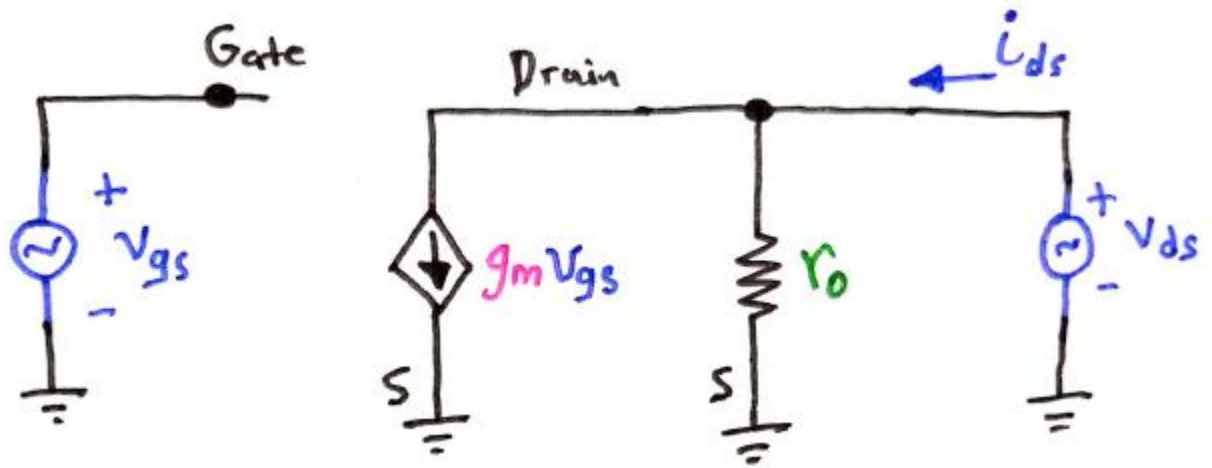
Circuit meaning going to ss-model





Small Signal Model (ss-model)

Low Frequency Small-Signal Model



Going to the details

$$I_{DS} = f(V_{GS}, V_{DS}, V_{BS})$$

$$\Delta I_{DS} = \frac{\partial I_{DS}}{\partial V_{GS}} \Delta V_{GS} + \frac{\partial I_{DS}}{\partial V_{DS}} \Delta V_{DS} + \frac{\partial I_{DS}}{\partial V_{BS}} \Delta V_{BS}$$

Going to the details

$$I_{DS} = f(V_{GS}, V_{DS}, V_{BS})$$

$$\Delta I_{DS} = \frac{\partial I_{DS}}{\partial V_{GS}} \Delta V_{GS} + \frac{\partial I_{DS}}{\partial V_{DS}} \Delta V_{DS} + \frac{\partial I_{DS}}{\partial V_{BS}} \Delta V_{BS}$$

Going to the details

$$I_{DS} = f(V_{GS}, V_{DS}, V_{BS})$$

$$i_{ds} = \frac{\partial I_{DS}}{\partial V_{GS}} v_{gs} + \frac{\partial I_{DS}}{\partial V_{DS}} v_{ds} + \frac{\partial I_{DS}}{\partial V_{BS}} v_{bs}$$

How to simplify the analysis?

$$i_{ds} = \frac{\partial I_{DS}}{\partial V_{GS}} v_{gs} + \frac{\partial I_{DS}}{\partial V_{DS}} v_{ds} + \frac{\partial I_{DS}}{\partial V_{BS}} v_{bs}$$

Transconductance

$$i_{ds} = \frac{\partial I_{DS}}{\partial V_{GS}} v_{gs} + \frac{\partial I_{DS}}{\partial V_{DS}} v_{ds} + \frac{\partial I_{DS}}{\partial V_{BS}} v_{bs}$$

$$\boxed{g_m = \frac{\partial I_{DS}}{\partial V_{GS}}}$$

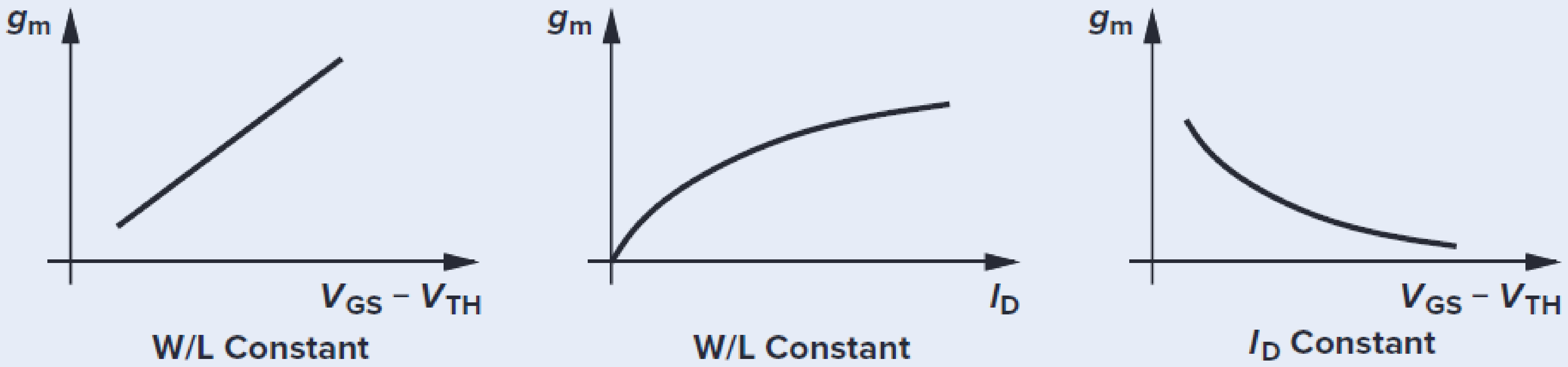
How to simplify the analysis?

$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{VDS\ \text{const.}}$$

$$= \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})$$

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}$$

$$= \frac{2I_D}{V_{GS} - V_{TH}}$$

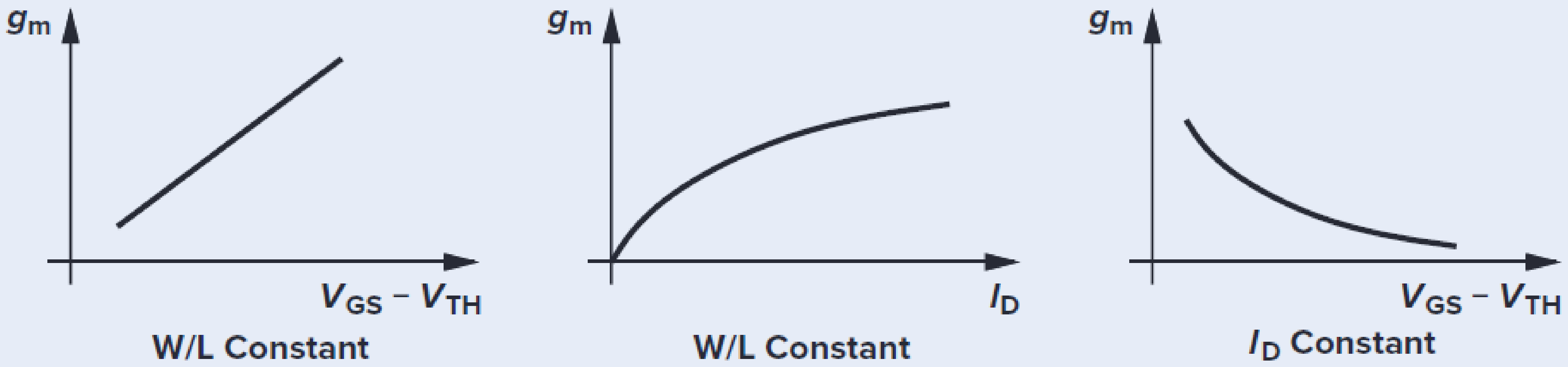


How to simplify the analysis?

$$g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})(1 + \lambda V_{DS})$$

$$= \sqrt{2\mu_n C_{ox}(W/L) I_D (1 + \lambda V_{DS})}$$

$$= \frac{2I_D}{V_{GS} - V_{TH}}$$



How to simplify the analysis?

$$i_{ds} = \frac{\partial I_{DS}}{\partial V_{GS}} v_{gs} + \frac{\partial I_{DS}}{\partial V_{DS}} v_{ds} + \frac{\partial I_{DS}}{\partial V_{BS}} v_{bs}$$

$$\boxed{r_o = \left[\frac{\partial I_{DS}}{\partial V_{DS}}\right]^{-1}}$$

MOSFET – Initial Thoughts

$$r_O = \frac{\partial V_{DS}}{\partial I_D}$$

$$= \frac{1}{\partial I_D / \partial V_{DS}}$$

$$= \frac{1}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2 \cdot \lambda}$$

$$\approx \frac{1 + \lambda V_{DS}}{\lambda I_D}$$

$$\approx \frac{1}{\lambda I_D}$$

How to simplify the analysis?

$$i_{ds} = \frac{\partial I_{DS}}{\partial V_{GS}} v_{gs} + \frac{\partial I_{DS}}{\partial V_{DS}} v_{ds} + \frac{\partial I_{DS}}{\partial V_{BS}} v_{bs}$$

$$\boxed{g_{mb} = \frac{\partial I_{DS}}{\partial V_{BS}}}$$

MOSFET – Initial Thoughts

$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}}$$

$$= \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) \left(- \frac{\partial V_{TH}}{\partial V_{BS}} \right)$$

$$\frac{\partial V_{TH}}{\partial V_{BS}} = - \frac{\partial V_{TH}}{\partial V_{SB}}$$

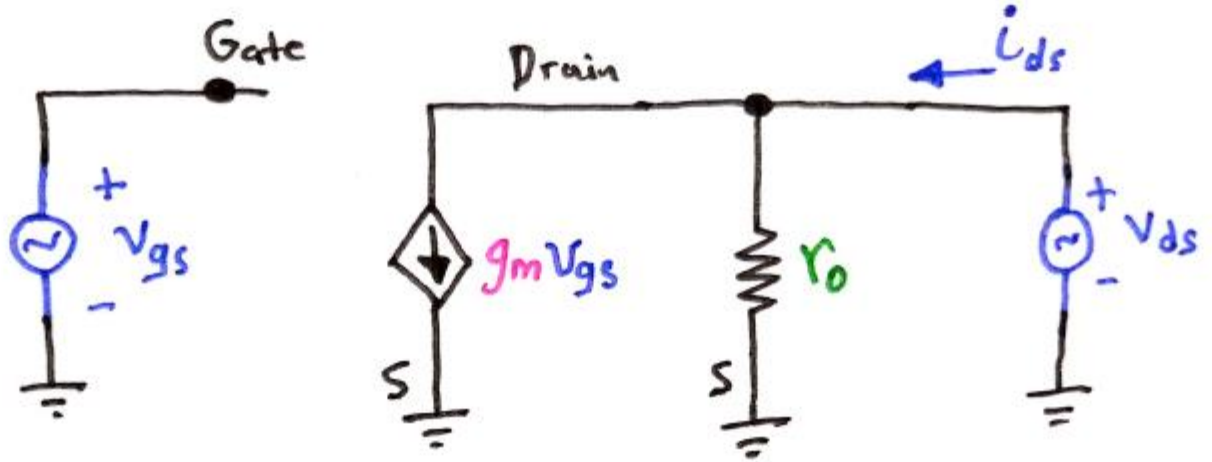
$$= - \frac{\gamma}{2} (2\Phi_F + V_{SB})^{-1/2}$$

$$g_{mb} = g_m \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}$$

$$= \eta g_m$$

Small-Signal Model Building

Something is missing → ?



$$i_{ds} = g_m v_{gs} + \frac{1}{r_o} v_{ds} + g_{mb} v_{bs}$$

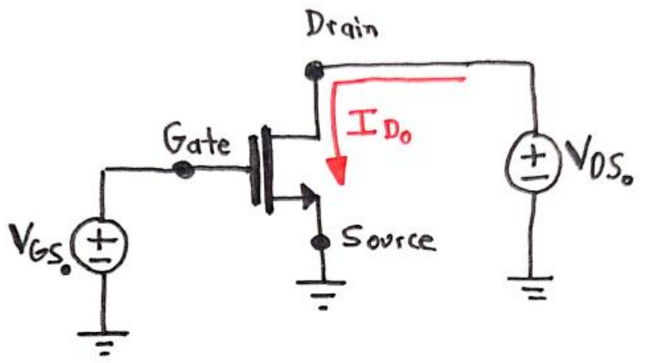
PMOS ss-model

$$i_{ds} = g_m v_{gs} + \frac{1}{r_o} v_{ds} + g_{mb} v_{bs}$$

For a PMOS device it is easy to demonstrate that the ss-model is EXACTLY the same as in NMOS case IF and ONLY IF the same terminal labels and voltage references are used.

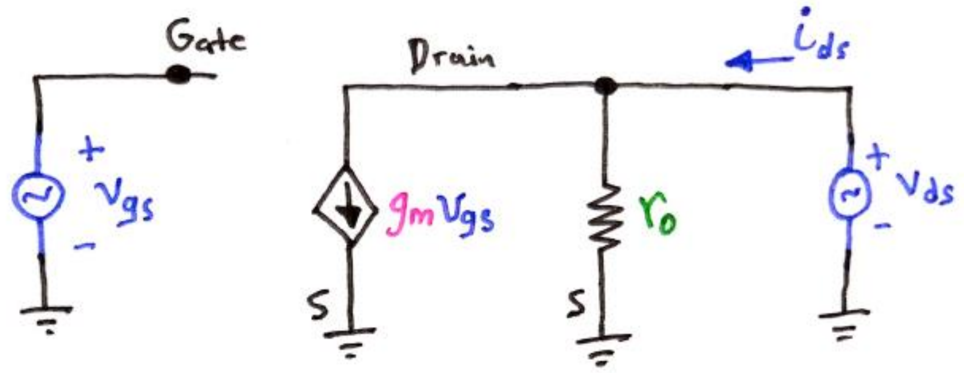
How to solve problems?

- DC analysis



$$I_{D_0} = f(V_{GS_0}, V_{DS_0})$$

- ss Analysis



Let’s solve our initial problem again!

Conclusions

- Note that the small signal model works on “Deltas” instead of the total voltage (or current) value.
- Small-signal model does not tell us anything about ABSOLUTE values!
- But it allow us to calculate the variations in a very useful way!
- We transform the MOSFET into a linear circuit!
- SS analysis is just one half of the work

Thanks

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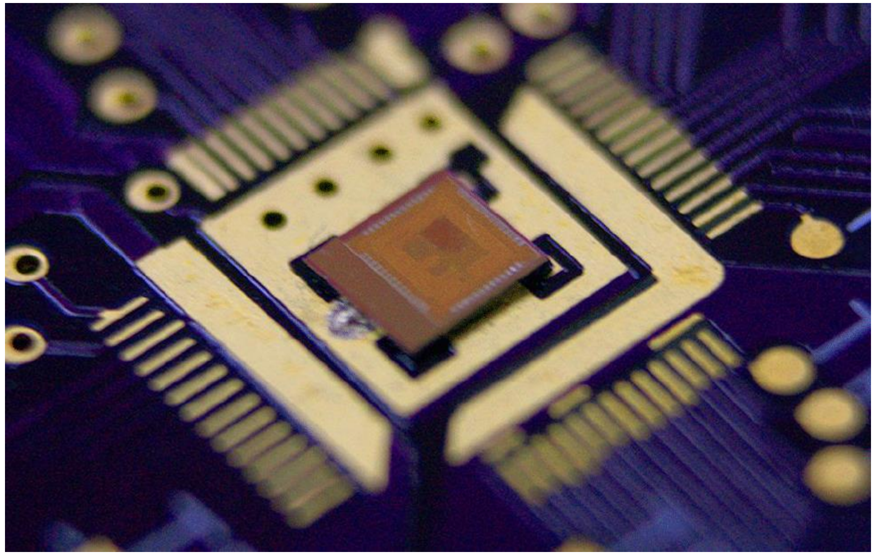

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