PASIVE FILTERS OF FIRST AND SECOND ORDER WITH RLC ELEMENTS

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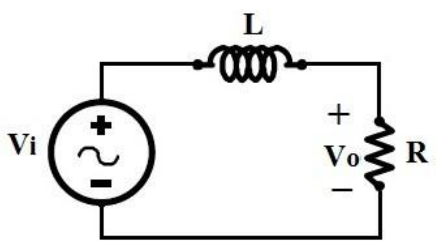
I. LOW-PASS FILTER

A. RL CIRCUIT



- Transfer function

$$H(s) = \frac{\frac{R}{L}}{S + \frac{R}{L}}$$

- Cut-off frequency

$$\omega_c = \frac{R}{L}$$

- Maximum Amplitude if $j\omega = 0$

$$|H(j\omega)|_{max} = 1$$

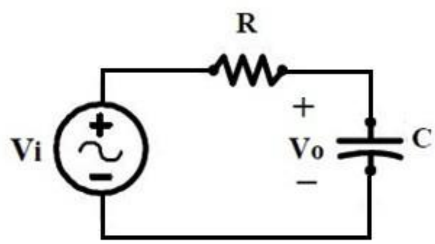
- Angle

$$\theta(j\omega) = -tan^{-1}\left(\frac{\omega L}{R}\right)$$

- Amplitude of $H(j\omega)$

$$|H(j\omega)| = \frac{\frac{R}{L}}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}}$$

B. RC CIRCUIT



- Transfer function.

$$H(s) = \frac{\frac{1}{RC}}{S + \frac{1}{RC}}$$

- Cut-off frequency

$$\omega_c = \frac{1}{RC}$$

- Maximum Amplitude if $j\omega$=0

$$|H(j\omega)|_{max} = 1$$

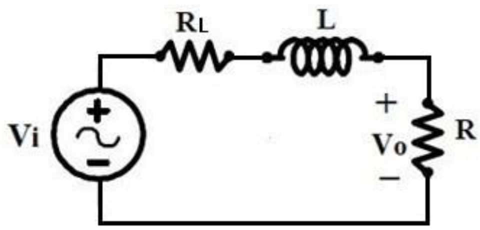
- Angle

$$\theta(j\omega) = -tan^{-1}(\omega RC)$$

- Amplitude of $H(j\omega)$

$$|H(j\omega)| = \frac{\frac{1}{RC}}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}}$$

C.RL CIRCUIT AND R_{load} SERIES



- Transfer function

$$H(s) = \frac{\frac{R}{L}}{S + \frac{R + R_L}{L}}$$

- Cut-off frequency

$$\omega_c = \frac{R_e}{L}$$

- Maximum Amplitude if $j\omega$=0

$$|H(j\omega)|_{max} = \frac{R}{R + R_L}$$

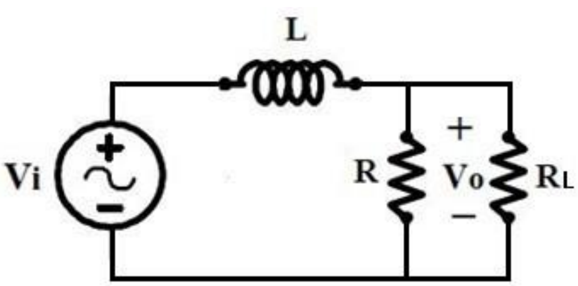
If $j\omega = \infty$

$$H(j\omega) = 0$$

- Amplitude of $H(j\omega)$

$$|H(j\omega)| = \frac{\frac{R}{L}}{\sqrt{\omega^2 + \left(\frac{R + R_L}{L}\right)^2}}$$

D. RL CIRCUIT AND R_{load} PARALLEL



- Transfer function
-

$$H(s) = \frac{\left(\frac{R_L}{R + R_L}\right)}{S + \left(\frac{R}{\mathrm{L}}\right)}$$

- Cut-off frequency

$$\omega_c = \frac{R_e}{L}$$

Where

$$R_e = \frac{R * R_L}{R + R_L}$$

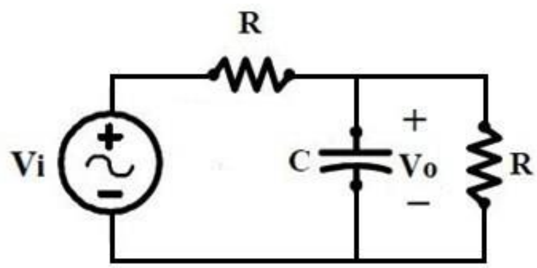
- Maximum Amplitude if $j\omega$ = 0

$$|H(j\omega)|_{max} = 1$$

if $j\omega = \infty$

$$H(j\omega) = 0$$

E. RC CIRCUIT AND R_{load} PARALLEL



- Transfer function

$$H(s) = \frac{\frac{1}{RC}}{S + \left(\frac{1}{RC}\right)\left(\frac{R + R_C}{R_C}\right)}$$

- Cut-off frequency

$$\omega_c = \left(\frac{1}{RC}\right)\left(\frac{R + R_C}{R_C}\right)$$

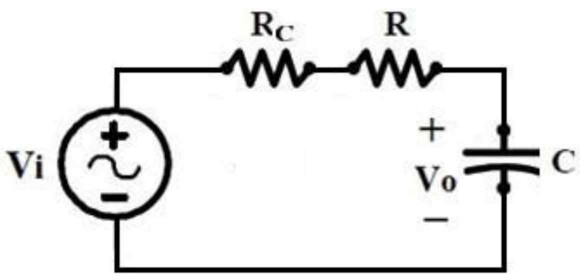
- Maximum Amplitude if $j\omega$=0

$$|H(j\omega)|_{max} = \frac{R_C}{R + R_C}$$

- Amplitude of $H(j\omega)$

$$|H(j\omega)| = \frac{\frac{1}{RC}}{\sqrt{\omega^2 + \left(\frac{1}{RC}\left(\frac{R + R_C}{R_C}\right)\right)^2}}$$

F. RC CIRCUIT AND R_{load} SERIES



- Transfer function

$$H(s) = \frac{\frac{1}{C(R + R_C)}}{S + \frac{1}{C(R + R_C)}}$$

- Cut-off frequency

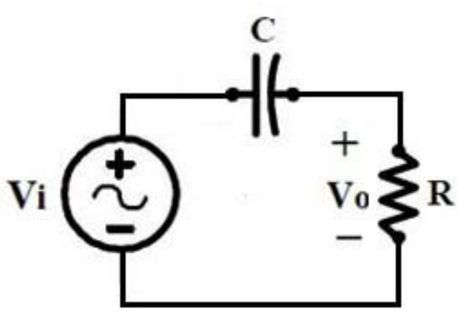
$$\omega_c = \frac{1}{C(R + R_C)}$$

- Maximum Amplitude if $j\omega$=0

$$|H(j\omega)|_{max} = 1$$

II. HIGH PASS FILTER

A. RC CIRCUIT



- Transfer function

$$H(s) = \frac{S}{S + \frac{1}{RC}}$$

- Cut-off frequency

$$\omega_c = \frac{1}{RC}$$

- Maximum Amplitude if $j\omega = \infty$

$$|H(j\omega)|_{max} = 1$$

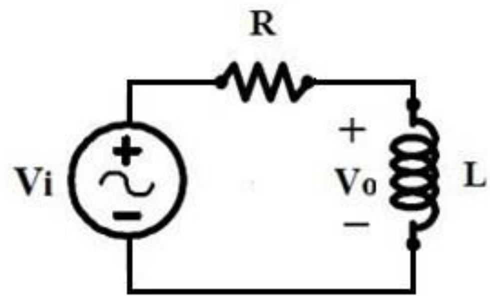
- Angle

$$\theta(j\omega) = \frac{\pi}{2} - tan^{-1}(\omega RC)$$

- Amplitude of $H(j\omega)$

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}}$$

B. RL CIRCUIT



- Transfer function

$$H(s) = \frac{S}{S + \frac{R}{L}}$$

- Cut-off frequency

$$\omega_c = \frac{R}{L}$$

- Maximum Amplitude if $j\omega = \infty$

$$|H(j\omega)|_{max} = 1$$

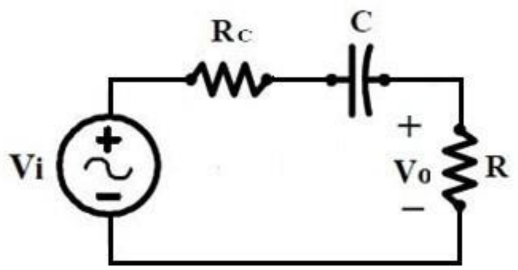
- Angle

$$\theta(j\omega) = \frac{\pi}{2} - tan^{-1}\left(\frac{\omega L}{R}\right)$$

- Amplitude of $H(j\omega)$

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}}$$

C. RC CIRCUIT AND R_{load} SERIES



- Transfer function

$$H(s) = \frac{S * \frac{R}{R + R_C}}{S + \frac{1}{C(R + R_C)}}$$

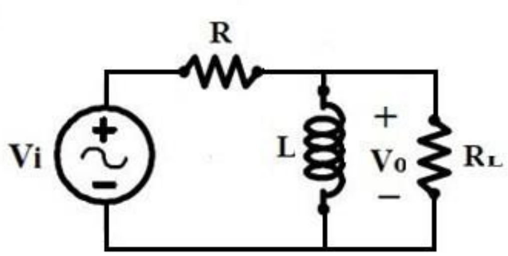
- Cut-off frequency

$$\omega_c = \frac{1}{C(R + R_C)}$$

- Maximum Amplitude if $j\omega = \infty$

$$|H(j\omega)|_{max} = \frac{R}{R + R_C}$$

D. RL CIRCUIT AND R_{load} PARALLEL



- Transfer function

$$H(s) = \frac{S * \left(\frac{R_L}{R + R_L}\right)}{S + \left(\frac{R}{L}\right)\left(\frac{R_L}{R + R_L}\right)}$$

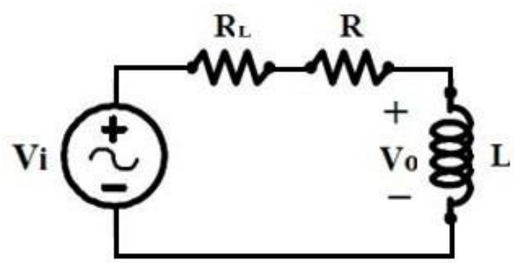
- Cut-off frequency

$$\omega_c = \left(\frac{R}{L}\right)\left(\frac{R_L}{R + R_L}\right)$$

- Maximum Amplitude if $j\omega = \infty$

$$|H(j\omega)|_{max} = \frac{R_L}{R + R_L}$$

E. RL CIRCUIT AND R_{load} SERIES



- Transfer function

$$H(s) = \frac{S}{S + \frac{R + R_L}{L}}$$

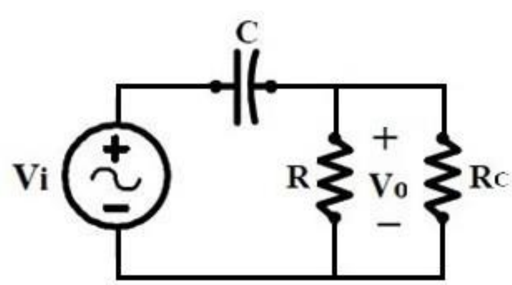
- Cut-off frequency

$$\omega_c = \frac{R + R_L}{L}$$

- Maximum Amplitude if $j\omega = \infty$

$$|H(j\omega)|_{max} = 1$$

F. RC CIRCUIT AND R_{load} PARALLEL



- Transfer function

$$H(s) = \frac{S}{S + \left(\frac{1}{RC}\right)\left(\frac{R + R_C}{R_C}\right)}$$

- Cut-off frequency

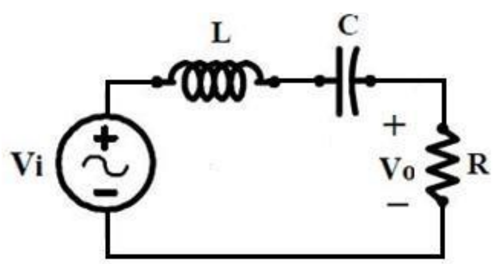
$$\omega_c = \left(\frac{1}{RC}\right)\left(\frac{R + R_C}{R_C}\right)$$

- Maximum Amplitude if $j\omega = \infty$

$$|H(j\omega)|_{max} = 1$$

III. BAND PASS FILTER

A. SERIES RLC CIRCUIT



- Transfer function

$$H(s) = \frac{S\left(\frac{R}{L}\right)}{S^2 + S\left(\frac{R}{L}\right) + \frac{1}{RC}}$$

- Maximum Amplitude if $\omega = \omega_0$

$$|H(j\omega)|_{max} = 1$$

Where ω_0 is defined as

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

- Angle

$$\theta(j\omega) = \frac{\pi}{2} - tan^{-1}\left(\frac{\frac{\omega R}{\mathrm{L}}}{\frac{1}{LC} - \omega^2}\right)$$

- Amplitude of $H(j\omega)$

$$|H(j\omega)| = \frac{\frac{\omega R}{\mathrm{L}}}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{\omega R}{\mathrm{L}}\right)^2}}$$

$$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}}$$

$$\omega_{c1} \text{ ó } \omega_{c2} = \pm\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)^2}$$

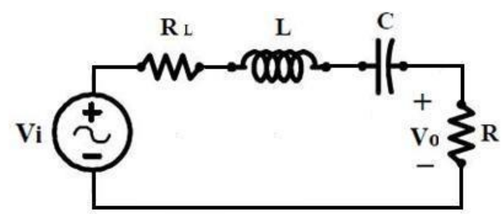
$$\beta = \frac{R}{L} \text{ ó } \beta = \omega_{c1} - \omega_{c2}$$

- Quality factor (Q)

$$Q = \sqrt{\frac{L}{CR^2}}$$

$$\omega_{c1} \text{ ó } \omega_{c2} = \omega_0\left(\pm\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right)$$

B. RLC CIRCUIT AND R_{load} SERIES



- Transfer function

$$H(s) = \frac{S\left(\frac{R}{L}\right)}{S^2 + S\left(\frac{R + R_L}{L}\right) + \frac{1}{RC}}$$

- Maximum Amplitude if $\omega = \omega_0$

$$|H(j\omega)|_{max} = \frac{R}{R + R_L}$$

- Where ω_0 is defined as

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

- Amplitude of $H(j\omega)$

$$|H(j\omega)| = \frac{\frac{\omega R}{\mathrm{L}}}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{\omega(R + R_L)}{\mathrm{L}}\right)^2}}$$

$$\beta_u = \frac{R + R_L}{L};\ with\ load$$
$$also\ \beta = \omega_{c1} - \omega_{c2}$$

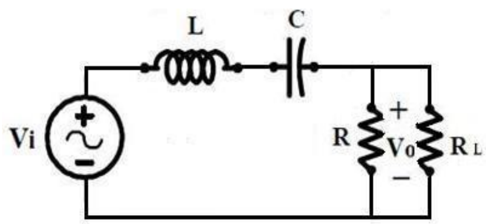
$$\beta = \frac{R_e}{L}\ ;\ R_e = R + R_L$$

- Quality factor (Q)

$$Q_u = \frac{{\omega_0}^2}{R + R_L}$$

$$\omega_{c1} \text{ ó } \omega_{c2} = \pm \frac{R + R_L}{2L} + \sqrt{{\omega_0}^2 + \left(\frac{R + R_L}{2L}\right)^2}$$

C. RLC CIRCUIT AND R_{load} PARALLEL



- Transfer function

$$H(s) = \frac{S\left(\frac{R}{L}\right)\left(\frac{R_L}{R + R_L}\right)}{S^2 + S\left(\frac{R}{L}\right)\left(\frac{R + R_L}{L}\right) + \frac{1}{LC}}$$

- Maximum Amplitude if $\omega = \omega_0$

$$|H(j\omega)|_{max} = 1$$

- Where ω_0 is defined as

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

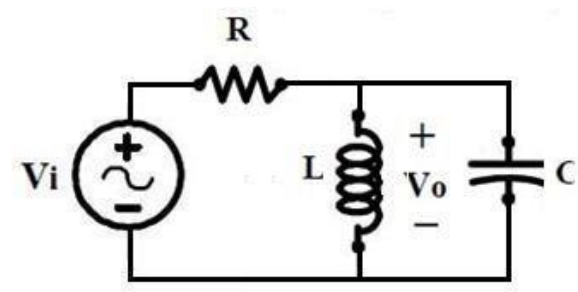
$$\beta_u = k\beta \,; with\ load$$

where $\beta = \omega_{c1} - \omega_{c2}$

$$\beta = \frac{R}{L} \,; k = \frac{\mathrm{R}}{R + R_L}$$

- $\omega_{c1} \text{ ó } \omega_{c2} = \pm \frac{\beta_u}{2} + \sqrt{{\omega_0}^2 + \left(\frac{\beta_u}{2}\right)^2}$

D. RLC CIRCUIT PARALLEL



- Transfer function

$$H(s) = \frac{S\left(\frac{1}{RC}\right)}{S^2 + S\left(\frac{1}{RC}\right) + \frac{1}{LC}}$$

- Maximum Amplitude if $\omega = \omega_0$

$$|H(j\omega)|_{max} = 1$$

- Where ω_0 is defined as

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

- Angle

$$\theta(j\omega) = \frac{\pi}{2} - tan^{-1}\left(\frac{\frac{\omega R}{\mathrm{L}}}{\frac{1}{LC} - \omega^2}\right)$$

- Amplitude of $H(j\omega)$

$$|H(j\omega)| = \frac{\omega\left(\frac{1}{\mathrm{RC}}\right)}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{\omega}{\mathrm{RC}}\right)^2}}$$

$\beta = \frac{1}{\mathrm{RC}}$

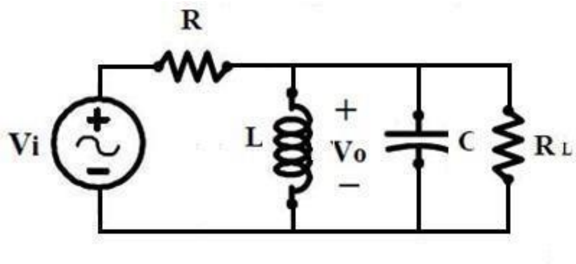
$also\ \ \beta = \omega_{c1} - \omega_{c2}$

- Quality factor (Q)

$$Q = \sqrt{\frac{R^2 C}{L}}$$

$$\omega_{c1} \text{ ó } \omega_{c2} = \pm \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \left(\frac{1}{LC}\right)^2}$$

E. RLC CIRCUIT AND R_{load} PARALLEL



- Transfer function

$$H(s) = \frac{S\left(\frac{1}{RC}\right)\left(\frac{R + R_L}{R}\right)\left(\frac{R}{R + R_L}\right)}{S^2 + S\left(\frac{1}{RC}\right)\left(\frac{R + R_L}{R_L}\right) + \frac{1}{LC}}$$

- Maximum Amplitude if $\omega = \omega_0$

$$|H(j\omega)|_{max} = 1$$

Where ω_0 is defined as

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\beta_u = \left(\frac{1}{RC}\right)\left(\frac{R + R_L}{R_L}\right) \text{; with load}$$

also $\beta = \omega_{c1} - \omega_{c2}$

$$\beta = \frac{1}{RC} \to \beta_u = \left(1 + \frac{R}{R_L}\right)\beta$$

- Rewriting $H(s = j\omega)$

$$H(s = j\omega) = \frac{\omega\left(\frac{R}{L}\right)}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{\omega(R + R_L)}{L}\right)^2}}$$

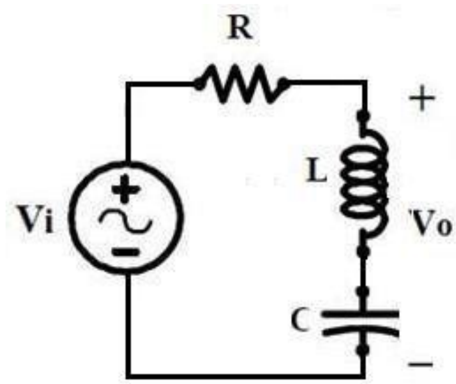
- Quality factor (Q)

$$Q = \frac{\omega_o}{\beta}$$

$$\omega_{c1} \text{ ó } \omega_{c2} = \pm\frac{\beta}{2} + \sqrt{{\omega_0}^2 + \left(\frac{\beta}{2}\right)^2}$$

IV. BAND REJECT FILTER

A. RLC SERIES CIRCUIT



- Transfer function

$$H(s) = \frac{S^2 + \left(\frac{1}{LC}\right)}{S^2 + S\left(\frac{R}{L}\right) + \frac{1}{LC}}$$

- Where ω_0 is defined as

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

- Angle

$$\theta(j\omega) = -tan^{-1}\left(\frac{\frac{\omega R}{L}}{\frac{1}{LC} - \omega^2}\right)$$

- Amplitude of $H(j\omega)$

$$|H(j\omega)| = \frac{\left|\frac{1}{LC} - \omega^2\right|}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{\omega R}{L}\right)^2}}$$

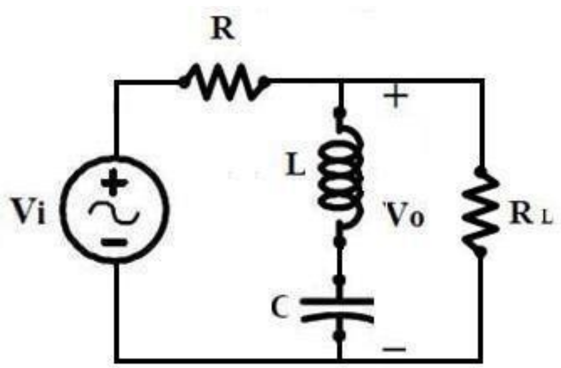
$$\beta = \frac{R}{L} \text{ ó } \beta = \omega_{c1} - \omega_{c2}$$

$$\omega_{c1} \text{ ó } \omega_{c2} == \pm\frac{\beta}{2} + \sqrt{{\omega_0}^2 + \left(\frac{\beta}{2}\right)^2}$$

- Quality factor (Q)

$$Q = \sqrt{\frac{L}{R^2 C}}$$

B. LC SERIES. R_{load} PARALLEL



- Transfer function

$$H(s) = \frac{\left(S^2 + \frac{1}{LC}\right)\left(\frac{R_L}{R + R_L}\right)}{S^2 + S\left(\frac{R}{L}\right)\left(\frac{R_L}{R + R_L}\right) + \frac{1}{LC}}$$

- Maximum Amplitude if $\omega = 0$ ó $\omega = \infty$

$$|H(j\omega)|_{max} = k$$

- Where ω_0 is defined as

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

So $k = \frac{R_L}{R + R_L}$

- $\beta = \frac{R_e}{L}$ where $R_e = \frac{RR_L}{R + R_L}$

- Rewriting $H(s)$

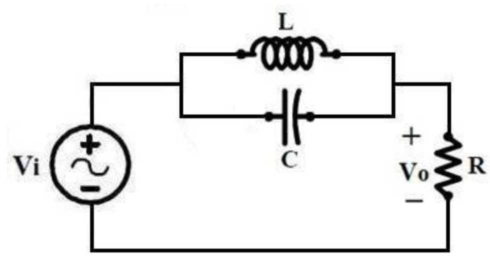
$$H(s) = \frac{k(s^2 + \omega^2)}{s^2 + \beta_u s + {\omega_0}^2}$$

- ω_{c1} ó $\omega_{c2} == \pm\frac{\beta}{2} + \sqrt{{\omega_0}^2 + \left(\frac{\beta}{2}\right)^2}$

- Quality factor (Q)

$$Q = \left(\frac{R + R_L}{R_L}\right)\left(\frac{\omega_0\,\mathrm{L}}{R}\right)$$

C. RLC CIRCUIT. LC PARALLEL



- Transfer function

$$H(s) = \frac{S^2 + \frac{1}{LC}}{S^2 + S\left(\frac{1}{RC}\right) + \frac{1}{LC}}$$

- Maximum Amplitude if $\omega = 0$ ó $\omega = \infty$

$$|H(j\omega)|_{max} = 1$$

- Where Where ω_0 is defined as

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\beta = \frac{1}{RC}$$

- Rewriting $H(s)$

$$H(s) = \frac{s^2 + \omega^2}{s^2 + \beta s + {\omega_0}^2}$$

- Amplitude of $H(j\omega)$

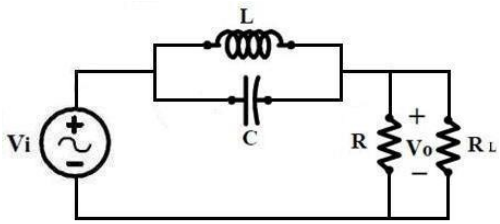
$$|H(j\omega)| = \frac{|{\omega_0}^2 - \omega^2|}{\sqrt{({\omega_0}^2 - \omega^2)^2 + (\omega\beta)^2}}$$

$$\omega_{c1} \text{ ó } \omega_{c2} == \pm\frac{\beta}{2} + \sqrt{{\omega_0}^2 + \left(\frac{\beta}{2}\right)^2}$$

- Quality factor (Q)

$$Q_u = \omega_0\,RC$$

D. RLC CIRCUIT. LC PARALLEL AND R_{load} PARALLEL



- Transfer function

$$H(s) = \frac{S^2 + \frac{1}{LC}}{S^2 + S\left(\frac{1}{RC}\right)\left(\frac{R + R_L}{R}\right) + \frac{1}{LC}}$$

- Maximum Amplitude if $\omega = 0$ ó $\omega = \infty$

$$|H(j\omega)|_{max} = 1$$

- Where ω_0 is defined as

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\beta = \frac{\omega_0}{R_e\, C}$$

$$R_e = \frac{RR_L}{R + R_L}$$

- Rewriting $H(s)$

$$H(s) = \frac{s^2 + \omega^2}{s^2 + \beta s + {\omega_0}^2}$$

- Amplitude of $H(j\omega)$

$$|H(j\omega)| = \frac{|{\omega_0}^2 - \omega^2|}{\sqrt{({\omega_0}^2 - \omega^2)^2 + (\omega\beta)^2}}$$

$$\omega_{c1} \text{ ó } \omega_{c2} == \pm\frac{\beta}{2} + \sqrt{{\omega_0}^2 + \left(\frac{\beta}{2}\right)^2}$$

- Quality factor (Q)

$$Q = \omega_0\, R_e C$$