Relativistic Quantum Field Theory

Physics 7651

Homework 3

Due: In class on Wednesday, Sept. 21.

1. **The dilatation current and scale invariance**

   A class of interesting theories are invariant under the scaling of all lengths,

   $$x^\mu \to (x')^\mu = \lambda x^\mu \qquad \text{and} \qquad \phi(x) \to \phi'(x) = \lambda^{-D}\phi(\lambda^{-1}x),$$

   where D is called the **scaling dimension** of the field. Consider the action for a real scalar field given by

   $$S = \int d^4x \, \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - g\phi^p.$$

   (a) Find the scaling dimension D such that the derivative terms remain invariant.

   (b) For what values of m and p are the scale transformation a symmetry of the theory?

   (c) How do these conclusions change for a scalar field living in an $(n+1)$-dimensional spacetime instead of a $(3+1)$-dimensional spacetime?

   (d) In $(3+1)$-dimensions, use Noerther's theorem to construct the conserved current D^μ associated to scaling invariance.

2. **Problem 2**

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