Relativistic Quantum Field Theory

Physics 7651, Fall 2011

Final Exam

Please read the following instructions carefully

1. You have 24 hours to complete this exam.
2. Upon completion, please staple all pages together, place the exam back in its envelope, and return it directly to Flip in PSB 432.
3. The *only* references allowed are Peskin & Schröeder, the lecture notes by Preskill and Coleman, your homework, homework solutions, and integral tables like Gradshteyn & Ryzhik. (Official solutions may have minor errors—you are responsible for correcting them if you use these results!) If you use one of these sources, please provide a reference.
4. You may *not* consult outside references 'for your homework' (e.g. Homework 13) while you are working on this exam.
5. You may *not* collaborate on this exam, not even with Google / Wikipedia / Siri or any other imaginary friends that you may have made while taking this course.
6. **Write clearly and box your final answers**. We cannot give partial credit if we don't know where you went wrong. Trivial algebraic/arithmetic slips will be penalized much less than fundamental misunderstandings.
7. There will be loop integrals on this exam which you will be asked to calculate using a hard cutoff. You may solve these by hand, appealing to an integral table, or using computer software. You should provide an appropriate reference for any result.
8. If you require clarification, contact Flip at `pt267@cornell.edu`.

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I have read and understand these rules to this exam and agree to abide by them.

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1. **Pion decay** [50 points]

   The dominant decay mode of the π^+ (pion) is into a μ^+ (antimuon) and ν_μ (muon neutrino). Assume that the only interactions of these particles is described by

   $$\mathcal{L}_{\text{Int}} = (\partial_\lambda \pi)\bar{\nu}\gamma^\lambda (f + g\gamma_5)\mu + \text{ complex conjugate.}$$

   The muon and neutrino are Dirac fermions, the couplings f and g are complex numbers.

   (a) Compute the decay width $\Gamma(\pi^+ \to \bar{\mu} + \nu_\mu)$ to second order in the coupling constants (the lowest non-vanishing order) in terms of the coupling constants, the pion mass m_π, and the muon mass m_μ. Assume the neutrino mass is zero. Assume the validity of the naïve Feynman rules for derivative interactions as explained in the homework.

   (b) The neutrinos emitted in this process are experimentally observed to always have helicity $-1/2$. What does this imply for the ratio f/g?

   (c) A much less frequent decay mode of the π^+ is into a positron and an electron neutrino. Assume that this is caused by an interaction of the same form as above with the following replacements:

   - Electron and electron neutrino fields replace the muon and muon neutrino
   - Couplings f' and g' replace f and g

   Electron neutrinos, like muon neutrinos, are massless and always have helicity $-1/2$. Use the following data to compute $|f'/f|$ to within 10% accuracy:

   $$\begin{aligned} m_\pi &= 140 \text{ MeV} \\ m_\mu &= 106 \text{ MeV} \\ m_e &= 0.51 \text{ MeV} \\ \frac{\Gamma(\pi^+ \to \bar{e}\nu_e)}{\Gamma(\pi^+ \to \bar{\mu}\nu_\mu)} &= 1.23 \times 10^{-4}. \end{aligned}$$

2. **Renormalization of Yukawa theory** [50 points]

   In Homework 13 you renormalized the two-point function in pseudoscalar Yukawa theory. In this problem we'll finish the job.

   $$\mathcal{L} = \bar{\Psi}(i\gamma^\mu - M)\partial_\mu \Psi + \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 - iy\varphi\bar{\Psi}\gamma^5\Psi + \frac{\lambda}{4!}\varphi^4.$$

   We shall use renormalization conditions that fix y and λ when all external momenta vanish. (This will simplify your loop calculations.) Determine the counter-terms for the interaction terms, δ_y and δ_λ, at one-loop order.

All done? *Please* make sure your answers are written legibly with answers boxed. It has been a pleasure having you part of the course. Best wishes with your remaining exams and have a good winter break.

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