Relativistic Quantum Field Theory

Physics 7651

Homework 2.

Due: In on Wednesday, Sept. 14 to Flip Tanedo's mailbox

Reminder: make-up class on Friday, Sept. 9 8:3-10 am in Rock 105. No class on Wednseday, Sept. 14

1. **Explicit expression of correlator**

   We have used the correlator

   $$\langle 0|\Phi(x)\Phi(y)|0\rangle = \Delta_+(x-y) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} e^{-ik\cdot(x-y)}$$

   many times in class when discussing causality. Find an explicit expression for $\Delta_+(r)$ in terms of Bessel functions for space-like x with $x^2 = -r^2 < 0$.

2. **Time ordered product as Green's function**

   The time-ordered product of two fields $A(x)$ and $B(x)$ is defined by

   $$T\left[A(x)B(y)\right] = \begin{cases} A(x)B(y) & \text{for } x^0 > y^0 \\ B(y)A(x) & \text{for } x^0 < y^0 \end{cases}$$

   Using only the field equation and the equal time commutation relations, show that for a free scalar field Φ with mass m,

   $$(\Box_x + m^2)\langle 0|T[\Phi(x)\Phi(y)|0\rangle = c\delta^4(x-y),$$

   and find the proportionality constant c.

3. **Feynman propagator**

   Show that

   $$\langle 0|T[\Phi(x)\Phi(y)]|0\rangle = \lim_{\epsilon\to 0^+} \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{-c}{k^2 - m^2 + i\epsilon}.$$

   The limit symbol indicates that ϵ goes to zero through positive values. If the ϵ were not present the integral would be ill-defined, because it would have poles in the domain of integration. Hint: Do the k^0 integration and compare with the expression for the left-hand side obtained by inserting a complete set of intermediate states. Here you can use anything we know about the field Φ.