Relativistic Quantum Field Theory

Physics 7651

Homework 6

Due: In class on Wednesday, October 12

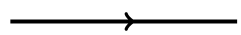
1. **Feynman rules for a theory with complex scalars** [5 points]

   In class we derived the Feynman rules for a real scalar field. We will now derive the rules for a theory with both complex fields ψ and real fields ϕ and Lagrangian

   $$\mathcal{L} = \frac{1}{2}\left(\partial\phi\right)^2 + \frac{1}{2}m^2\phi^2 + |\partial\psi|^2 - M^2|\psi|^2 - g\phi\psi^\dagger\psi$$

   Draw real scalar propagators as dashed lines and complex scalar propagators as solid lines with arrows pointing along the direction of U(1) charge flow:

   ϕ propagator

   $\psi/\psi^\dagger$ propagator

   (a) Determine all the Feynman rules for this theory.

   (b) Draw and calculate the leading order diagram(s) in g for $\psi\psi^\dagger \to \phi\phi$. Can you draw a diagram for $\psi\psi \to \phi\phi$?

   (c) Draw and calculate the leading order diagram(s) in g for $\psi\phi \to \psi\phi$. Can you draw a diagram for $\psi\phi \to \psi^\dagger\phi$?

   (d) Suppose I told you that I can draw a diagram for

   $$p\psi\,\bar{p}\psi^\dagger\,r\phi \to q\psi\,\bar{q}\psi^\dagger\,s\phi,$$

   where the coefficients are just the number of initial and final state particles. What relation must hold between the coefficients? Motivate this by appealing to a symmetry principle.

2. **A toy model of meson interactions** [5 points]

   The most common decay mode of the K_S neutral kaon ($m_{K_S} = 498$ MeV) is into two charged pions ($m_{\pi^\pm} = 145$ MeV) with a rate of $\Gamma = 0.776 \times 10^{10}$ sec^{-1}. Model this system in terms of the theory in Problem 1 with ϕ representing K_S, ψ representing π^+ and $\psi^\dagger$ representing π^-. Compute the dimensionless quantity g/m_{K_S} and comment on why we call this is a "weak interaction." *Hint:* $\hbar = 6.58 \times 10^{-22}$ MeV sec.

3. **Models of $2 \to 4$ at a real scalar collider** [10 points]

Consider the usual ϕ^4 theory of a real scalar field,

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4.$$

Suppose we have a ϕ collider and we'd like to study this theory by colliding pairs of ϕ.

(a) Draw the leading order diagrams for $2 \to 4$ scattering. Label the external states with distinct momenta.

(b) Calculate the amplitude for this process.

(c) Now consider a slightly different theory with two real fields, ϕ and φ,

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}M^2\varphi^2 + \frac{\lambda}{3!}\phi^3\varphi.$$

Determine the amplitude for $2\phi \to 4\phi$ in this modified theory.

(d) Suppose the mass of the φ is much heavier than the energies of our ϕ collider. That is, consider the limit where M is much larger than the external momenta. Write the $2\phi \to 4\phi$ amplitude in this limit.

(e) When the φ field is too heavy to produce directly, physicists must make sense of their data by writing 'effective theories' of only the low energy (ϕ) fields. Suppose physicists at our real scalar collider observe $2\phi \to 4\phi$ scattering *without* observing $2\phi \to 2\phi$ scattering. This disfavors the ϕ^4 model so that one might consider a different effective model of ϕ fields such as,

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{g}{6!}\phi^6.$$

What is the dimension of the coupling g? Suppose the actual theory of nature is that in part (d). What is the relation between the 'effective coupling' parameter g and the parameters of the actual theory?

4. **A theory with a derivative interaction** [10 points]

Work out the Feynman rules for the following theory of a real scalar field ϕ,

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{4}(\phi\partial\phi)^2.$$

What is the dimension of the coupling λ? What is the amplitude for elastic $2 \to 2$ scattering to first order in λ?

Hint, Oct 9. Even though the action is Lorentz invariant, there are subtleties regarding Lorentz covariance of the interaction Hamiltonian. (See Preskill 4.33 for a discussion.) For the purposes of this problem, assume that $H_{\text{int}} = -\mathcal{L}_{\text{int}}$. Ignore terms coming from ∂_0 acting on time ordering symbols (alternately, assume the four-point term contracts with external states). Thanks to Michael for pointing this out.