ITCT Lecture 9.3:

Predictive Coding II

A good reference taken from Internet!
(Prof. Yao Wang)

1

Predictive Coding

- Prediction
- Prediction in Images
- Principle of Differential Pulse Code Modulation (DPCM)
- DPCM and entropy-constrained scalar quantization
- DPCM and transmission errors
- Adaptive intra-interframe DPCM
- Conditional Replenishment

2

Prediction

Prediction is difficult – especially for the future.
Mark Twain

- Prediction: *Statistical estimation procedure where future random variables are estimated/predicted from past and present observable random variables.*

- Prediction from previous samples: $\hat{S}_0 = f(S_1, S_2, ..., S_N) = f(\mathbf{S})$
- Optimization criterion

$$E = \{(S_0 - \hat{S}_0)^2\} = E\{[S_0 - f(S_1, S_2, ..., S_N)]^2\} \rightarrow \min$$

- Optimum predictor:

$$\hat{S}_0 = E\{S_0 \mid (S_1, S_2, ... S_N)\}$$

3

Structure

- The optimum predictor $\hat{S}_0 = E\{S_0 \mid (S_1, S_2, ... S_N)\}$ can be stored in a table (Pixels: 8 bit ➔ size 2^{8N})
- Optimal linear prediction (zero mean, Gaussian RVs)

$$\hat{S}_0 = a_1 S_1 + a_2 S_2 + ... + a_N S_N = \mathbf{a}^t \mathbf{S}$$

- Optimization criterion

$$E\{(S_0 - \hat{S}_0)^2\} = E\{(S_0 - \mathbf{a}^t \mathbf{S})^2\}$$

- Optimum linear predictor is solution of

$$\mathbf{a}^t \mathbf{R}_S = E\{S_0 \mathbf{S}^t\}$$

- In case $\mathbf{R}_S = E\{\mathbf{S}\mathbf{S}^t\}$ is invertible

$$\mathbf{a} = \mathbf{R}_S^{-1} E\{S_0 \mathbf{S}\}$$

4

Prediction in Images: Intra-frame Prediction

- Past and present observable random variables are prior scanned pixels within that image
- When scanning from upper left corner to lower right corner:

B	C	D
A	X	

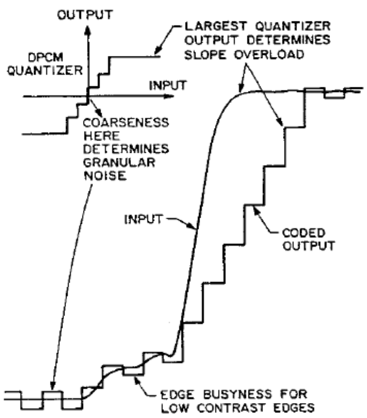
- 1-D Horizontal prediction: A only
- 1-D Vertical prediction: C only
- Improvements for 2-D approaches (requires line store)

$$\hat{s}(x,y) = \underbrace{\sum_{p=-P_1}^{P_2} \sum_{q=0}^{Q}}_{(p,q)\neq(0,0)} a(p,q)\cdot s(x-p,y-q)$$

5

Signal distortions due to intraframe DPCM coding

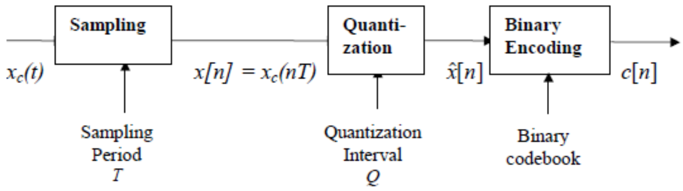
- Granular noise: random noise in flat areas of the picture
- Edge busyness: jittery appearance of edges (for video)
- Slope overload: blur of high-contrast edges, Moire patterns in periodic structures.



Bernd Girod: EE368b Image and Video Compression DPCM no. 5

6

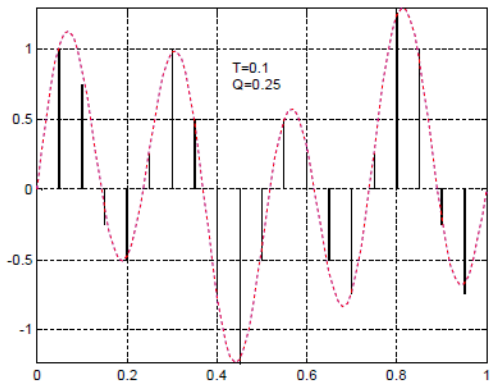
Three Processes in A/D Conversion



- Sampling: take samples at time nT
  - T: sampling period;
  - f_s = 1/T: sampling frequency
- Quantization: map amplitude values into a set of discrete values kQ
  - Q: quantization interval or stepsize
- Binary Encoding
  - Convert each quantized value into a binary codeword

7

Analog to Digital Conversion



A2D_plot.m

8

How to determine T and Q?

- T (or f_s) depends on the signal frequency range
  - A fast varying signal should be sampled more frequently!
  - Theoretically governed by the Nyquist sampling theorem
    - $f_s > 2 f_m$ (f_m is the maximum signal frequency)
    - For speech: $f_s >= 8$ KHz; For music: $f_s >= 44$ KHz;
- Q depends on the dynamic range of the signal amplitude and perceptual sensitivity
  - Q and the signal range D determine bits/sample R
    - $2^R = D/Q$
    - For speech: $R = 8$ bits; For music: $R = 16$ bits;
- One can trade off T (or f_s) and Q (or R)
  - lower R -> higher f_s; higher R -> lower f_s
- We considered sampling in last lecture, we discuss quantization in this lecture

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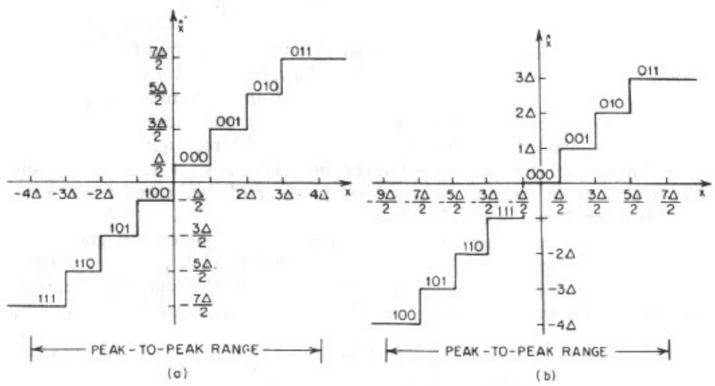
9

Signal range is symmetric



L = even, Mid-Riser

$Q_i(f) = floor(\frac{f}{Q}),\quad Q(f) = Q_i(f) * Q + \frac{Q}{2}$

L = odd, Mid-Tread

$Q_i(f) = round(\frac{f}{Q}),\quad Q(f) = Q_i(f) * Q$

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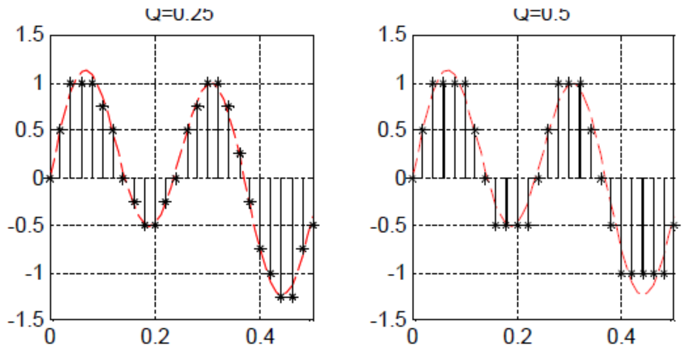
10

Effect of Quantization Stepsize



demo_sampling_quant.m

 EE3414:Quantization

11

Non-Uniform Quantization

- Problems with uniform quantization
  - Only optimal for uniformly distributed signal
  - Real audio signals (speech and music) are more concentrated near zeros
  - Human ear is more sensitive to quantization errors at small values
- Solution
  - Using non-uniform quantization
    - quantization interval is smaller near zero

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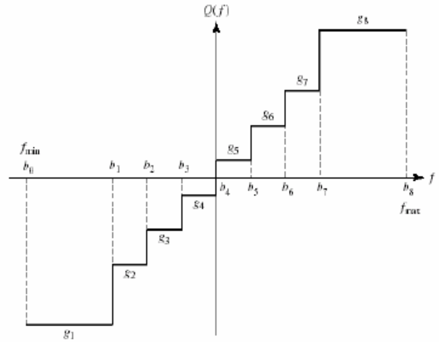
12

Function Representation



$$Q(f) = g_l, \text{ if } f \in B_l$$

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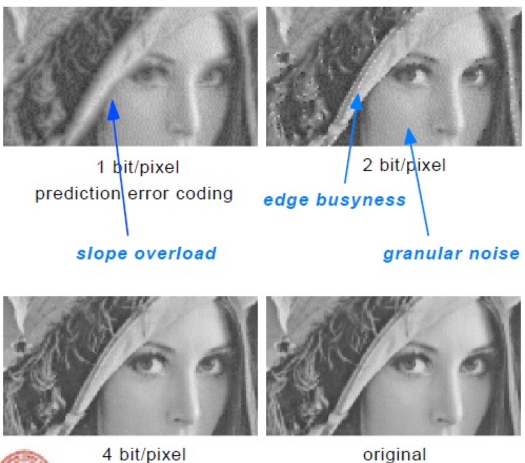
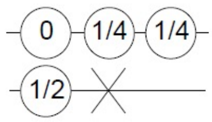
13

Example of intraframe DPCM coding



- Linear predictor:



- Lloyd-Max quantizers
- Fixed-length coding

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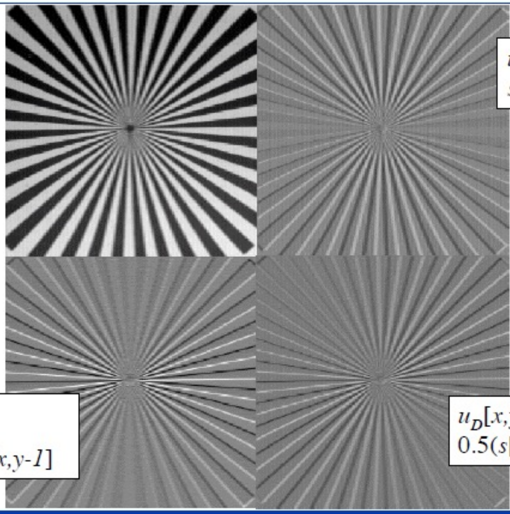
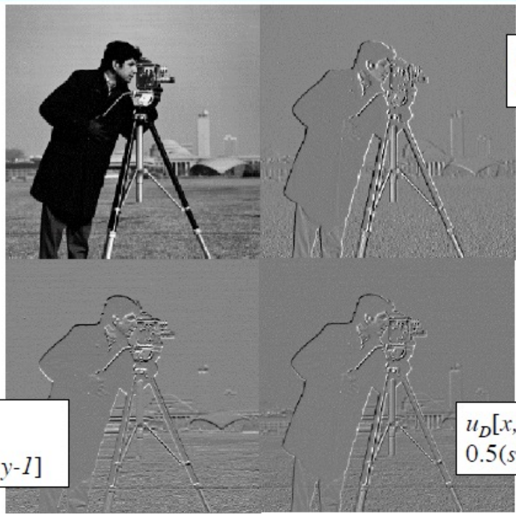
14

Prediction Example: Test Pattern



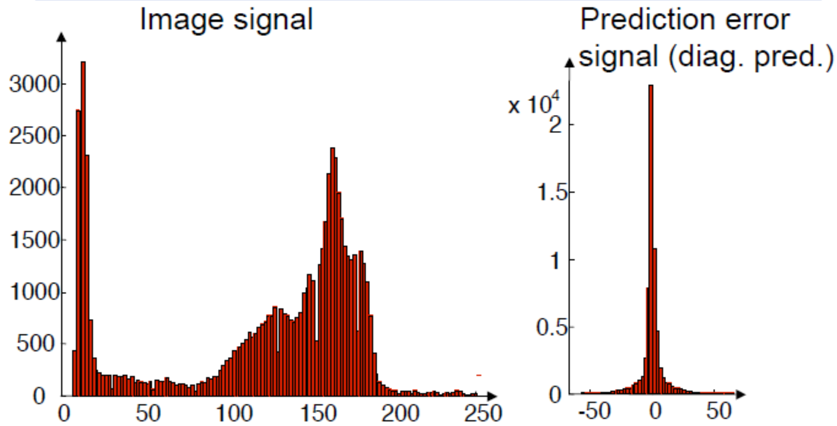
15

Prediction Example: Cameraman



16

Change of Histograms: Cameraman

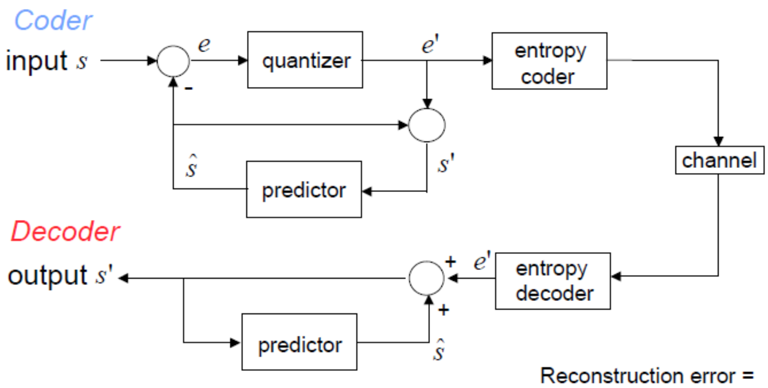


Can we use prediction for compression ?

Yes, if we reproduce the prediction signal at the decoder

17

Differential Pulse Code Modulation



Prediction error

$\boxed{e = s - \hat{s}}$

Reconstruction

$\boxed{s' = e' + \hat{s}}$

Reconstruction error = quantization error

$\boxed{s' - s = e' - e = q}$

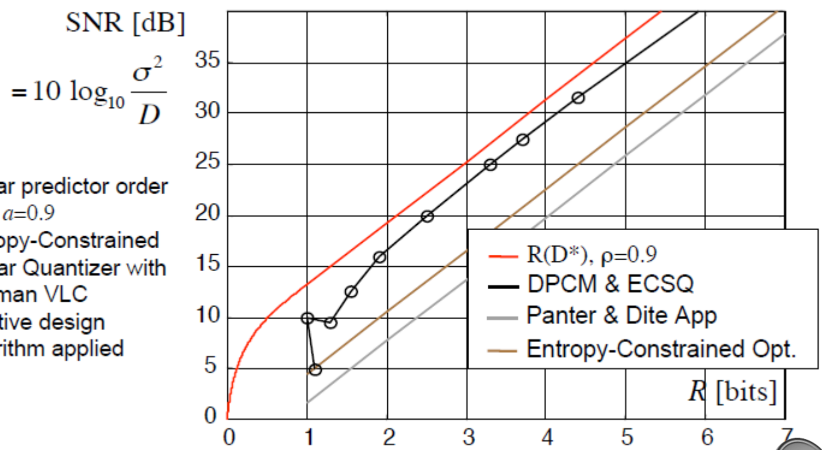
18

DPCM and Quantization

- Prediction is based on quantized samples
- Stability problems for large quantization errors
- Prediction shapes error signal (typical pdfs: Laplacian, generalized Gaussian)
- Simple and efficient: combine with entropy-constrained scalar quantization
- Higher gains: Combine with block entropy coding
- Use a switched predictor
  - Forward adaptation (side information)
  - Backward adaptation (error resilience, accuracy)
- DPCM can also be conducted for vectors
  - Predict vectors (with side information)
  - Quantize prediction error vectors

19

Comparison for Gauss-Markov Source: $\rho=0.9$



- Linear predictor order $N=1$, $a=0.9$
- Entropy-Constrained Scalar Quantizer with Huffman VLC
- Iterative design algorithm applied

20

DPCM with Entropy-Constrained Scalar Quantization

Example: Lena, 8 b/p

K=511, H=4.79 b/p K=15, H=1.98 b/p K=3, H=0.88 b/p

K...number of reconstruction levels, H...entropy

from: O

21

Transmission Errors in a DPCM System

- For a linear DPCM decoder, the transmission error response is superimposed to the reconstructed signal S'
- For a stable DPCM decoder, the transmission error response decays
- Finite word-length effects in the decoder can lead to residual errors that do not decay (e.g., limit cycles)

from: Gi

22

Transmission Errors in a DPCM System II

Example: Lena, $3\ b/p$ (fixed code word length)

Error rate $p=10^{-3}$.
1D pred., hor. $a_H=0.95$ | 1D pred., ver. $a_V=0.95$ | 2D pred.*, $a_H=a_V=0.5$

from: Ohm

23

Inter-frame Coding of Video Signals

- Inter-frame coding exploits:
  - Similarity of temporally successive pictures
  - Temporal properties of human vision
- Important inter-frame coding methods:
  - Adaptive intra/inter-frame coding
  - Conditional replenishment
  - Motion-compensating prediction (in Hybrid Video Coding)
  - Motion-compensating interpolation

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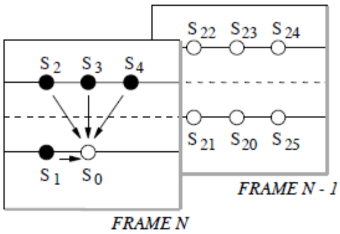
24

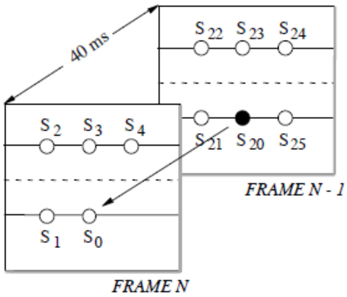
Principle of Adaptive Intra/Inter-Frame DPCM

Predictor is switched between two states:
for moving or changed areas.

Intra-frame prediction
for moving or changed areas.

Inter-frame prediction (previous frame
prediction) for still areas of the picture.





$$\hat{S} = a_1 \cdot S_1' + a_2 \cdot S_2' + a_3 \cdot S_3' + a_4 \cdot S_4'$$

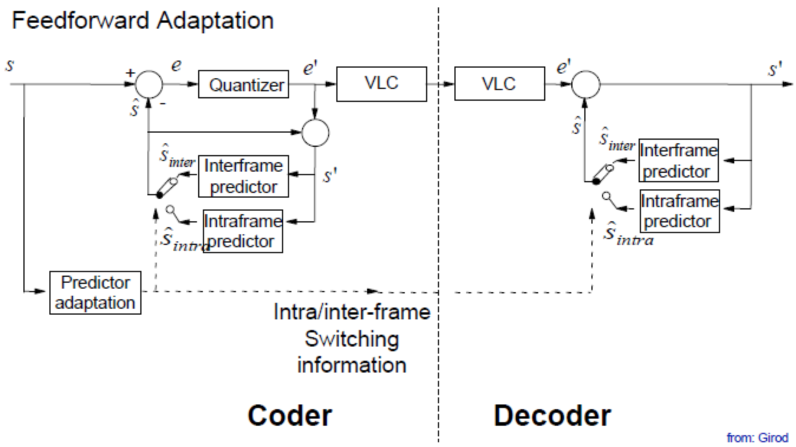
$$\hat{S}_{inter} = S'_{20}$$

from: G

25

Intra/Inter-Frame DPCM: Adaptation Strategies, I

Feedforward Adaptation



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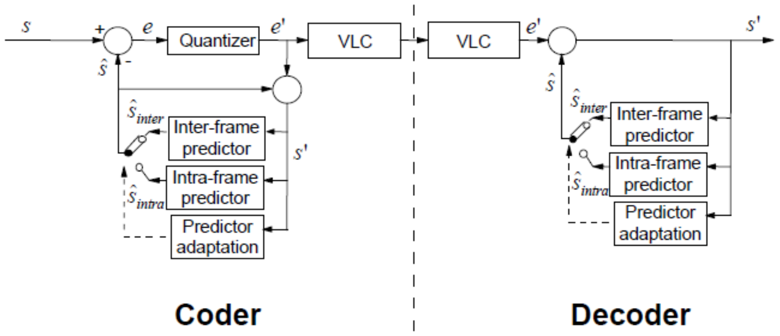
26

Intra/Inter-Frame DPCM: Adaptation Strategies, II

Feedback Adaptation



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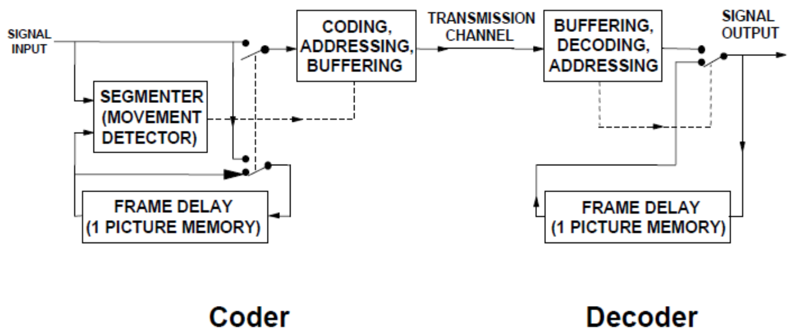
27

Principle of a Conditional Replenishment Coder



- Still areas: repeat from frame store
- Moving areas: transmit address and waveform

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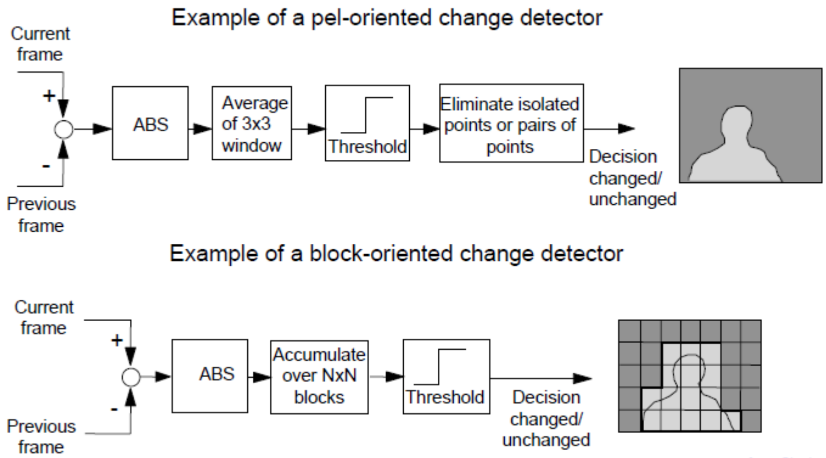
28

Change Detection



29

Summary

- Prediction: Estimation of random variable from past or present observable random variables
- Optimal prediction
- Optimal linear prediction
- Prediction in images: 1-D vs. 2-D prediction
- DPCM: Prediction from previously coded/transmitted samples (known at coder and decoder)
- DPCM and quantization
- DPCM and transmission errors
- Adaptive Intra/Inter-frame DPCM: forward adaptation vs. backward adaptation
- Conditional Replenishment: Only changed areas of image are transmitted

30