

# Lambda Calculus

## Alpha-Conversion

A not free (bound) variables  $x$  in a Lambda-Function can be replaced by a different variable  $y$  if  $y$  doesn't exist as a free variable in the expression (*nfin* meaning “not free in”):

$$\lambda x.M = \lambda y.[x := y]M \text{ if } (y \text{ nfin } \lambda x.M) \quad (1)$$

### Example

$$(\lambda x.x) = (\lambda y.y)$$

```
a = (\x -> x)
```

```
a = (\y -> y) -- α-Reduction
```

## Beta-Reduction

If a function is applied to an argument, that expression can be written as the body of the function with the input variable replaced by the argument:

$$(\lambda x.M)N = [x := N]M \quad (2)$$

### Example

$$(\lambda x.x)a = a$$

$$(\lambda x.xy)a = ay$$

```
a = (\x -> x) 5
```

```
a = 5 -- β-Reduction
```

## Eta-Reduction

$$(\lambda x.fx) = f \quad (3)$$

```
add5 xs = map (\x -> x + 5) xs
```

```
add5' = map (\x -> x + 5) -- η-Reduction
```