Problem Set for PDEs. Due Friday November 29

We'll be writing Poisson solvers to address both static and time-varying cases.

Let's start with a charged cylindrical conductor held at some potential V inside a box where the walls are held at zero potential.

1) With a method of relaxation solver, solve for the potential everywhere in space. What does the charge density look like in your solution? What is the charge per unit length? How does your solution compare to the analytic one you might have expected? As a reminder, the usual answer is $E = \frac{\lambda}{2\pi\epsilon_0 r}$ and $V = \frac{\lambda}{2\pi\epsilon_0}\log(r)$.

2) Update your solver to use conjugate gradient. How many steps does it take for the potential to converge to some threshold this way vs. the relaxation way from part 1)?

3) Now update your solver from part 2) to work on a range of resolutions. To do this, you'll need to evaluate the boundary conditions at a coarse resolution, solve that problem, then interpolate that solution into a higher-resolution box. Repeat until you have the resolution you want. How long does it take to get a global converged (again, to some threshold you set) vs. part 3?

4) Now add a bump, say 10% of the wire diameter. Repeat the solver from part 3) and look at the electric field (gradient of the potential). How big is the field near the bump vs. the surface of the wire away from the bump? Power companies make their transmission lines smooth to $\sim \mu m$ precision because of this.

5) Start with a box at T=0 everywhere. Then raise the temperature of one wall linearly with time. Solve for temperature as a function of time. Plot, say, the tepmerature along a line from the center of the heated side to the center of the opposite side. Are there any constants you need to set and/or different regimes you might expect your solution to be in?