CSCI 4100 Fall 2018
Assignment 2 Answers

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Exercise 1.8

$$P[\nu \leq 0.1] = 0.1^{10} + C_1^{10} \times 0.1^9 \times \mu = 0.1^{10} + C_{10}^{1} \times 0.1^9 \times 0.9 \leq 9.1 \times 10^{-9}$$

Exercise 1.9
Using Hoeffding Inequality:

$$P[|\mu - \nu| > \epsilon] \leq 2e^{-2\epsilon^2 N}$$

Then:

$$P[\nu \leq 0.1] = P[|0.9 - \nu| \geq 0.8]$$

Here $\mu = 0.9$ and $\epsilon = 0.8$, and because $N = 10$

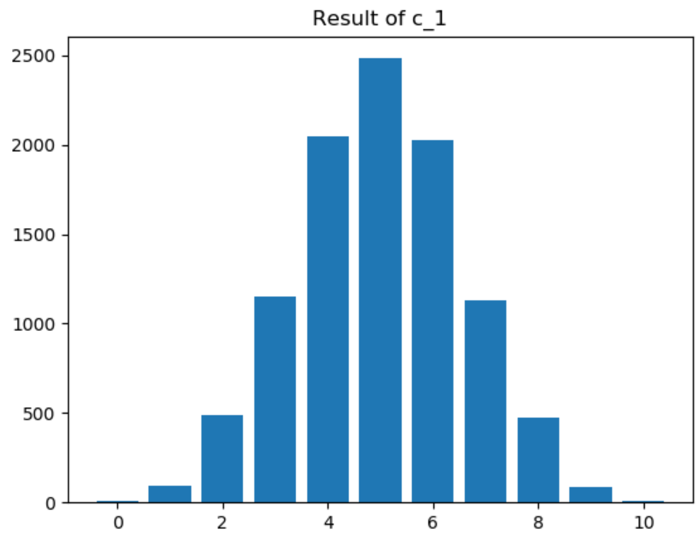
$$P[|0.9 - \nu| \geq 0.8] \leq 2e^{-2\times 0.8^2 \times 10} \leq 5.22 \times 10^{-6}$$

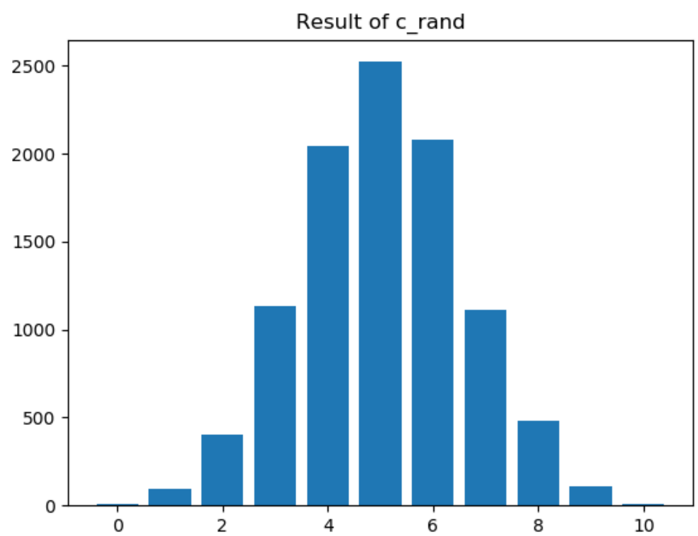
By using Hoeffding Inequality, the upper bound of probability is much larger than the upper bound calculated using binominal distribution.

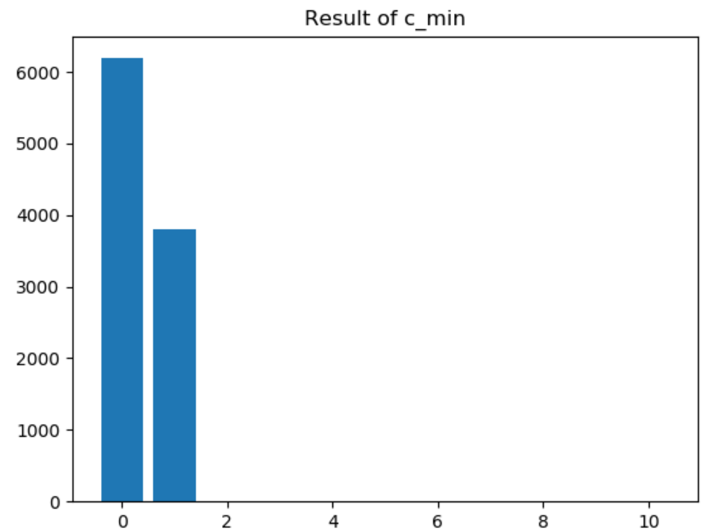
Exercise 1.10

(a) μ is 0.5 since the 3 coins are fair.

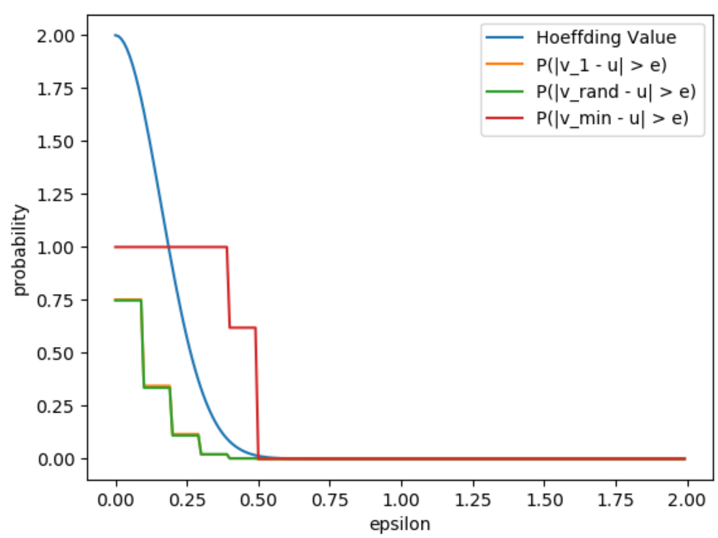
(b)







(c)



(d) According to the graph in part (c), coin c_1 and c_{rand} obey the Hoeffding bound because they decrease with Hoeffding Value decreases, but coin c_{min} does not, because there is a sudden drop on graph where the Hoeffding value decreases smoothly.

(e) The coin c_{min} is not a sample ramdonly chosen from the bottle. We cannot choose coin with specific property, and such coin does not obey the Hoeffding bound.

Exercise 1.11

(a) No, because using algorithm S only means it works well on the given training example set D, but the performance outside D is unsure.

(b) Yes, because performances of algorithm S and C is alway unsure outside the traing example set D whatever how they perform with D.

(c) $p = 0.9$, so $P(f_S = f) = p = 0.9$ and $P(f_S = f) = p = 0.1$.
Then:
$$P[P(f_S = f) > P(f_S = f)] = P[0.9 > 0.1] = 1$$

(d) The probability that C will produce better hypothesis than S is that $P[P(f_S = f) > P(f_S = f)] > 0.5$. To achieve this, p should be smaller than 0.5 because $P[P(f_S = f) > P(f_S = f)] = P(p > 1 - p)$, and when $p < 0.5$, $P(p > 1 - p) = 0$

Exercise 1.12
I would like to choose (c), because no one can promise the learning process can give a good hypothesis

Problem 1.3

(a) Because $y_n = sign(w^{*T} x_n)$, $y_n(w^{*T} x_n) = [sign(w^{*T} x_n)] w^{*T} x_n$.
Thus when $w^{*T} x_n > 0$, $y_n > 0$ and when $w^{*T} x_n < 0$, $y_n < 0$. So $y_n(w^{*T} x_n)$ is positive for $1 \leq n \leq N$ since N is limited.

(b) First show that $w^T(t)w^* \geq w^T(t-1)w^* + \rho$: LHS: according to rule 1.3 on page 7

$$\begin{aligned} w^T(t)w^* &= [w^T(t-1) + y(t-1)x_t] \times w^* \\ &= w^T(t-1)w^* + y(t-1)w^{*T} x_{t-1} \end{aligned}$$

RHS:
$$w^T(t-1)w^* + \rho = w^T(t-1)w^* + min_{1 \leq n \leq N} y(t-1)w^{*T} x_{t-1}$$

There is no doubt the LHS $\geq$ RHS.

Then, Prove $w^T(t)w^* \geq t\rho$:
Prove by induction:
Base case: when t = 0, $w^T(t)w^* = 0$ and $t\rho = 0$, so $w^T(0)w^* \geq 0\rho$ is true.
Assume for any P(t): $t \geq 0, w^T(t)w^* \geq t\rho$ is true
Show that P(t+1): $w^T(t+1)w^* \geq (t+1)\rho$:
$RHS = (t+1)\rho = t\rho + \rho$
And according to the first part, $LHS w^T(t+1)w^* = w^T(t)w^* + \rho$
The assumption states that $w^T(t)w^* \geq t\rho$, so P(t+1) is true
By induction, $w^T(t)w^* \geq t\rho$.

(c) According to the part (b):

$$\begin{aligned}
LHS &= ||w(t-1) + y(t-1)x(t-1)||^2 \\
&= ||w(t-1)||^2 + ||y(t-1)x(t-1)||^2 + 2y(t-1)w^T(t-1)x(t-1) \\
&\leq ||w(t-1)||^2 + ||y(t-1)x(t-1)||^2 \\
&\leq RHS, \ because\ of\ the\ hint.
\end{aligned}$$

(d) Prove by induction:
Base Case: P(1):

$$||w(1)||^2 \leq ||w(0)||^2 + ||x(0)||^2 \leq R^2$$

Assume P(t): $||w(t)||^2 \leq tR^2$ is true,
Show that P(t+1): $||w(t+1)||^2 \leq (t+1)R^2$ is also true:
LHS: according to part (c):

$$||w(t)||^2 \leq ||w(t+1)||^2 + ||x(t)||^2$$

Then due to assumption P(1), $||w(t+1)||^2 < tR^2$ and $||x(t)||^2$¡R:

$$LHS \leq tR^2 + R^2 = RHS$$

By induction, P(t+1) is true, and the assumption $||w(t)||^2 \leq tR^2$ is true.

(e) According part (b), $w^T(t)w^* \geq t\rho$.
According part (d), $||w(t)||^2 \leq tR^2$, so $||w(t)|| \leq \sqrt{t}R$ Therefore,

$$\frac{w^T(t)w^*}{||w(t)||} \geq \frac{t\rho}{\sqrt{t}R} = \frac{\sqrt{t}\rho}{R} = RHS$$

Show that $t \leq \frac{R^2||w^*||^2}{\rho^2}$:
According to the Cauthy-Schwarz Inequity,

$$w^T(t) \leq ||w(t)|| \cdot ||w^*||$$

So,

$$\sqrt{t}\frac{\rho}{R} \leq ||w^*||$$

$$t \leq \frac{||w^*||^2 R^2}{\rho^2}$$

Problem 1.7

(a) If $\mu = 0.05$:
$P[1\ coin] = (1 - 0.05)^{10} = 0.599$
$P[1000\ coin] = 1 - [1 - (1 - 0.05)^{10}]^{1000} = 1$
$P[1000000\ coin] = 1 - [1 - (1 - 0.05)^{10}]^{1000000} = 1$

If $\mu = 0.8$:
$P[1\ coin] = (1 - 0.8)^{10} = 1.024 \times 10^{-7}$
$P[1000\ coin] = 1 - [1 - (1 - 0.8)^{10}]^{1000} = 1.024 \times 10^{-4}$
$P[1000000\ coin] = 1 - [1 - (1 - 0.8)^{10}]^{1000000} = 0.0973$

(b)