Report of the homework

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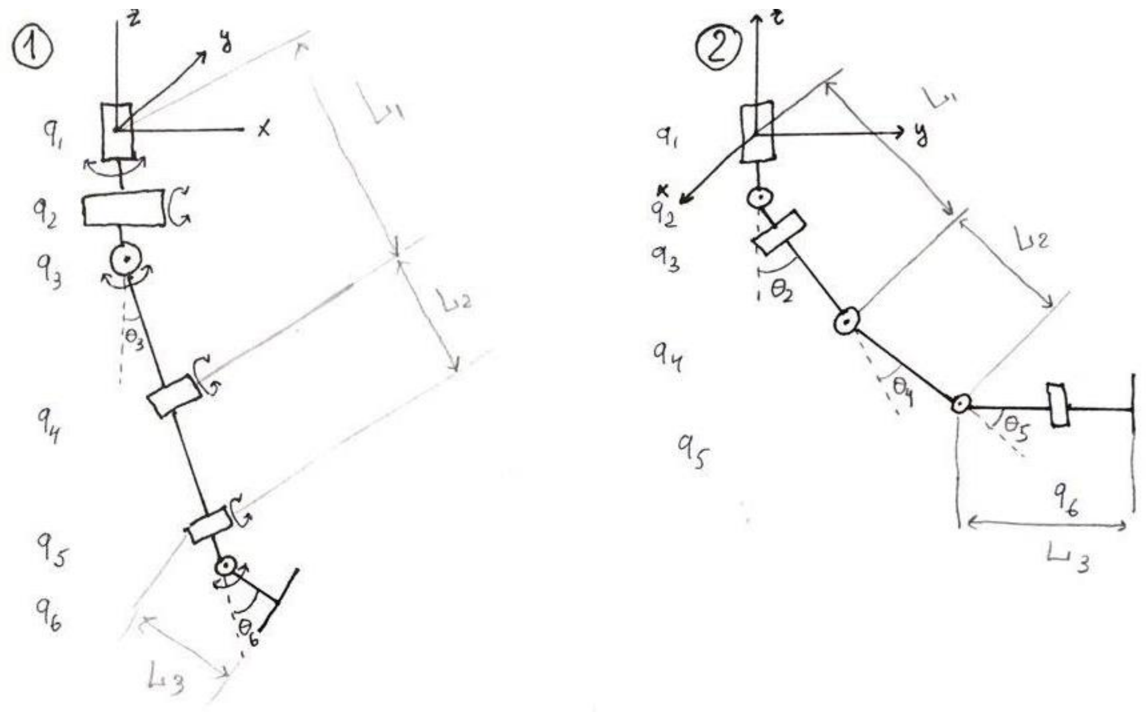
Language: Mathlab

Robot: AR-601 leg



Pic 1. Kinematic scheme of robot

Robot description

This is part of robot-humanoid AR-601 – leg. This leg has 3 links and 6 joints. 3 joints united in one spherical hip.

Forward kinematic

For solve this task was used Mathlab. I create matrices translation and rotation for every links and joints, where on the picture 1:

q1 – rotates around axis z

q2 – rotates around axis y

q3 – rotates around axis x

q4 – rotates around axis x

q5 – rotates around axis y

q6 – rotates around axis x

L1 – translation along the axis z

L2 – translation along the axis z

L3 – translation along the axis z

```
H = RzQ1*RyQ2*RxQ3*TzL1*RxQ4*TzL2*RyQ5*RxQ6*TzL3
```

```
RzQ1 =

[ cos(q1), -sin(q1), 0, 0]
[ sin(q1),  cos(q1), 0, 0]
[       0,        0, 1, 0]
[       0,        0, 0, 1]
```

```
RyQ2 =

[  cos(q2), 0, sin(q2), 0]
[        0, 1,       0, 0]
[ -sin(q2), 0, cos(q2), 0]
[        0, 0,       0, 1]
```

```
RxQ3 =

[ 1,       0,        0, 0]
[ 0, cos(q3), -sin(q3), 0]
[ 0, sin(q3),  cos(q3), 0]
[ 0,       0,        0, 1]
```

```
TzL1 =

[ 1, 0, 0,   0]
[ 0, 1, 0,   0]
[ 0, 0, 1, -L1]
[ 0, 0, 0,   1]
```

```
RxQ4 =

[ 1,       0,        0, 0]
[ 0, cos(q4), -sin(q4), 0]
[ 0, sin(q4),  cos(q4), 0]
[ 0,       0,        0, 1]
```

```
RyQ5 =

[  cos(q5), 0, sin(q5), 0]
[        0, 1,       0, 0]
[ -sin(q5), 0, cos(q5), 0]
[        0, 0,       0, 1]
```

```
RxQ6 =

[ 1,       0,        0, 0]
[ 0, cos(q6), -sin(q6), 0]
[ 0, sin(q6),  cos(q6), 0]
[ 0,       0,        0, 1]
```

```
TzL2 =

[ 1, 0, 0,   0]
[ 0, 1, 0,   0]
[ 0, 0, 1, -L2]
[ 0, 0, 0,   1]
```

```
TzL3 =

[ 1, 0, 0,   0]
[ 0, 1, 0,   0]
[ 0, 0, 1, -L3]
[ 0, 0, 0,   1]
```

Pic 2. Matrices of rotation and translation

These final formulas for coordinate in forward kinematic:

X = - L1*(sin(q1)*sin(q3) + cos(q1)*cos(q3)*sin(q2)) - L2*(cos(q4)*(sin(q1)*sin(q3) + cos(q1)*cos(q3)*sin(q2)) + sin(q4)*(cos(q3)*sin(q1) - cos(q1)*sin(q2)*sin(q3))) - L3*(sin(q6)*(cos(q4)*(cos(q3)*sin(q1) - cos(q1)*sin(q2)*sin(q3)) - sin(q4)*(sin(q1)*sin(q3) + cos(q1)*cos(q3)*sin(q2))) + cos(q6)*(cos(q5)*(cos(q4)*(sin(q1)*sin(q3) + cos(q1)*cos(q3)*sin(q2)) + sin(q4)*(cos(q3)*sin(q1) - cos(q1)*sin(q2)*sin(q3))) + cos(q1)*cos(q2)*sin(q5)))

Y = L1*(cos(q1)*sin(q3) - cos(q3)*sin(q1)*sin(q2)) + L2*(cos(q4)*(cos(q1)*sin(q3) - cos(q3)*sin(q1)*sin(q2)) + sin(q4)*(cos(q1)*cos(q3) + sin(q1)*sin(q2)*sin(q3))) + L3*(sin(q6)*(cos(q4)*(cos(q1)*cos(q3) + sin(q1)*sin(q2)*sin(q3)) - sin(q4)*(cos(q1)*sin(q3) - cos(q3)*sin(q1)*sin(q2))) + cos(q6)*(cos(q5)*(cos(q4)*(cos(q1)*sin(q3) - cos(q3)*sin(q1)*sin(q2)) + sin(q4)*(cos(q1)*cos(q3) + sin(q1)*sin(q2)*sin(q3))) - cos(q2)*sin(q1)*sin(q5)))

Z = L3*(cos(q6) *(sin(q2) *sin(q5) - cos(q5) *(cos(q2)*cos(q3)*cos(q4) - cos(q2)*sin(q3)*sin(q4))) + sin(q6)*(cos(q2)*cos(q3)*sin(q4) + cos(q2)*cos(q4)*sin(q3))) - L2*(cos(q2)*cos(q3)*cos(q4) - cos(q2)*sin(q3)*sin(q4)) - L1*cos(q2)*cos(q3)

I tried check me solve and enter all length of my 10, and enter all angels equal 0, that's mean that leg of my robot will be stretched along the z-axis and z = -10;

Editor - D:\Program Files\MATLAB\R2017b\bin\Projects\Forward kinematic\Forvard_Kinematic.m

Forvard_Kinematic.m | BLABLABLA.m | Inverse_Kinematic.m | C.m | untitled.m | cTestForware.m

```
A = fopen('Text_File.txt');
formatSpec = '%f';
mas = fscanf(A,formatSpec);
q1 = 0;q2 = 0; q3 = 0; q4 =0; q5 = 0; q6 = 0;
%syms q1 q2 q3 q4 q5 q6 L1 L2 L3;
test_file = fopen('Test_FK.txt');

% close the file
M = Matrix(q1,q2,q3,q4,q5,q6)
function M = Matrix(q1,q2,q3,q4,q5,q6)
L1 = 10; L2 =10; L3 = 10;
RzQ1 = [cos(q1) -sin(q1) 0 0;%rotation z on q1
        sin(q1) cos(q1) 0 0;
        0 0 1 0;
        0 0 0 1];
RyQ2 = [cos(q2) 0 sin(q2) 0;%rotation y on q2
        0 1 0 0;
        -sin(q2) 0 cos(q2) 0;
        0 0 0 1];
RxQ3 = [1 0 0 0;             %rotation x on q3
        0 cos(q3) -sin(q3) 0;
        0 sin(q3) cos(q3) 0;
        0 0 0 1];
TzL1 = [1 0 0 0;             %translation z on L1
        0 1 0 0;
        0 0 1 -L1;
        0 0 0 1];
RxQ4 = [1 0 0 0;             %rotation x on q4
        0 cos(q4) -sin(q4) 0;
        0 sin(q4) cos(q4) 0;
```

Command Window

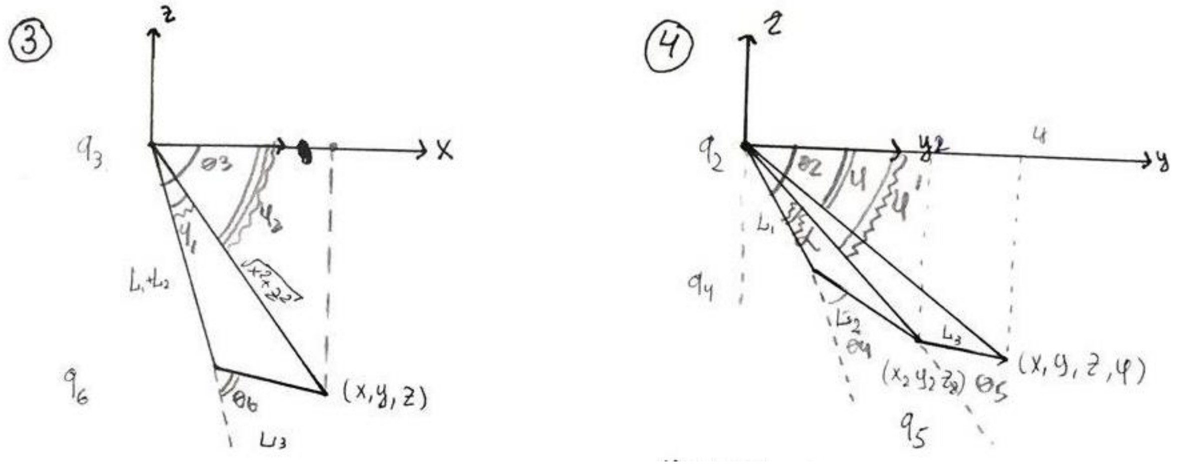
```
     1     0     0     0
     0     1     0     0
     0     0     1   -30
     0     0     0     1

fx >>
```

It was checked for y an z.

Inverse kinematic

For solve this problem I draw some pictures from different sides of my robot:



Pic 3. Active joints from different sides

We start derivation of equation for left picture on picture 3:

1) Find angle θ_6, using law of cosines formula 1, where x and z this coordinates end-effector, L1, L2, L3 – length of links. Result – formula 2.

$$x^2 + z^2 = (L_1 + L_2)^2 + L_3^2 + 2(L_1 + L_2) \cdot L_3 \cdot \cos\theta_6 \quad (1)$$

$$\theta_6 = \arccos\left(\frac{x^2 + z^2 - (L_1 + L_2)^2 - L_3^2}{2(L_1 + L_2) L_3} \right) \quad (2)$$

2) After that let's express the angle $\theta_3 = m*\varphi_1 + \varphi_2$. We find this using atan, because this allows you to consider extreme solutions. Find cos φ_1 and sin φ_1 form law of cosines and sinus:

$$L_3^2 = (L_1 + L_2)^2 + x^2 + z^2 - 2(L_1 + L_2) \cdot \sqrt{x^2 + z^2} \cdot \cos\varphi_1$$

$$\cos\varphi_1 = \left(\frac{L_3^2 - (L_1 + L_2)^2 - x^2 - z^2}{-2(L_1 + L_2)\sqrt{x^2 + z^2}} \right) \quad (3)$$

$$\sin \varphi_1 = \frac{L_3 \sin q_6}{\sqrt{x^2 + z^2}}$$

$$\varphi_1 = \operatorname{atan}\left(\frac{2 \sin \theta_6 \cdot L_3 \cdot (L_1 + L_2)}{x^2 + z^2 + (L_1 + L_2)^2 - L_3^2}\right) \quad (4)$$

For find φ_2 we use atan too and this is formula 5:

$$\varphi_2 = \operatorname{atan}\left(\frac{-z}{x}\right) \quad (5)$$

Result equation is formula 6:

$$q_3 = m \cdot \operatorname{atan}\left(\frac{2 \sin \theta_6 \cdot L_3 \cdot (L_1 + L_2)}{x^2 + z^2 + (L_1 + L_2)^2 - L_3^2}\right) + \operatorname{atan}\left(\frac{z}{x}\right) \quad (6)$$

Where m:

$$m = \begin{cases} -1 & \varphi_1 > 0 \\ +1 & \varphi_1 < 0 \end{cases} \quad (7)$$

For find the rest of assignment, I used second part of pic 3 and first of all express points y_2 and x_2. Formula 9:

$$
\begin{aligned}
&\varphi = \theta_2 + \theta_4 + \theta_5 \\
&z = z_2 + L_3 \cdot \sin\varphi \qquad y = y_2 + L_3 \cos\varphi \\
&z_2 = z - L_3 \sin\varphi \qquad y_2 = y - L_3 \cos\varphi
\end{aligned}
\tag{9}
$$

After that from law of cosines I find angle θ_4, formula 10:

$$
\begin{aligned}
&y_2^2 + z_2^2 = L_1^2 + L_3^2 + 2L_1L_2 \cdot \cos\theta_4 \\
&\theta_4 = \arccos\left(\frac{y_2^2 + z_2^2 - L_1^2 - L_3^2}{2L_1L_2}\right)
\end{aligned}
\tag{10}
$$

And after, like in previous example we find α angle with using atan. Find $\cos\varphi_1$ and $\sin\varphi_1$ from law of cosines and sinus:

$$
\begin{aligned}
&L_2 = y_2^2 + z_2^2 + L_1^2 - 2L_1 \cdot \sqrt{y_2^2 + z_2^2} \cdot \cos\alpha \\
&\cos\alpha = \frac{L_2^2 - y_2^2 - z_2^2 - L_1^2}{-2L_1\sqrt{y_2^2 + z_2^2}} \\
&\sin\alpha = \frac{L_2 \cdot \sin\theta_4}{\sqrt{y_2^2 + z_2^2}}
\end{aligned}
\tag{11}
$$

And using pic 4 part 2 we can take a formula for full angle θ_2:

$$\alpha = \operatorname{atan}\left(\frac{L_2 \sin\theta_4 \cdot 2L_1}{L_1^2 + z_2^2 + y_2^2 - L_2^2}\right)$$

$$\theta_2 = \alpha + \varphi'$$

$$\theta_2 = m \cdot \operatorname{atan}\left(\frac{L_2 \sin\theta_4 \cdot 2L_1}{L_1 + z_2^2 + y_2^2 - L_2^2}\right) + \operatorname{atan}\left(\frac{z_2}{y_2}\right) \quad (12)$$

That was cheeked, I wrote x = 0, y = 0, z = -30 and angles it give –pi/2 and pi/2, that’s true because when I derived equations for IK, angles were reported from the x-axes, and y-axes. 1.57 = pi/2/

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Forward_Kinematic.m | BLABLABLA.m | Inverse_Kinematic.m | C.m | untitled.m | cTestForware.m

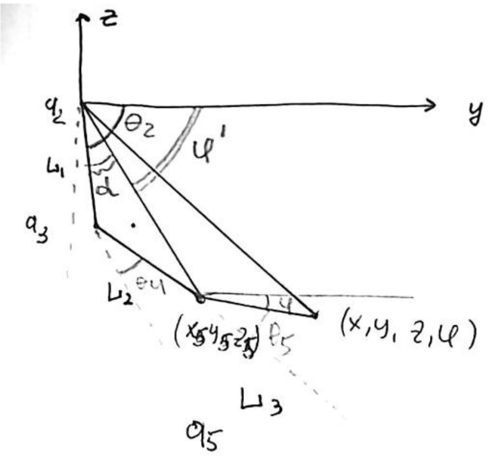
```
x = 0; y = 20; z = -10;
L1 = 10; L2 =10; L3 = 10;
teta6 = acos((x^2 + z^2 - (L1+L2)^2 - L3^2)/(2*(L1 + L2) * L3));
fi1 = atan2((2*sin(teta6)*L3*(L1+L2)),(x^2 + z^2 + (L1 + L2)^2 - L3^2));
m = -1;
if fi1 > 0
    m = -1;
elseif fi < 0
    m = 1;
end
teta3 = m * fi1 + atan2(z,x)

fi = -pi/2;%atan2(z,y);
z2 = z - L3*sin(fi);
y2 = y - L3*cos(fi);
fi_shtrih = atan2(z2,y2);
teta4 = acos((y2^2 + z2^2 - L1^2 - L2^2)/(2*L1*L2))
alfa = atan2((L2 * sin(teta4) * 2 * L1),(L1 + z2^2 + y2^2 - L2^2));
m_1 = 1;
if alfa > 0
    m_1 = 1;
elseif alfa < 0
    m_1 = -1;
end
teta2 = m_1*alfa + atan2(z2,y2)
teta5 = fi - teta2 - teta4
teta1 = atan2(y,x)
```

Workspace

Name	Value
alfa	0
fi	-1.5708
fi1	1.2246e-16
fi_shtrih	0
L1	10
L2	10
L3	10
m	-1
m_1	1
teta1	1.5708
teta2	0
teta3	-1.5708
teta4	0
teta5	-1.5708
teta6	3.1416
x	0
y	20
y2	20
z	-10
z2	0

And finally for check, I tried derived equation for IK again



$$y = L_1 \cos\theta_2 + L_2 \cdot \cos(\theta_2+\theta_4) + L_3 \cdot \cos(\theta_2+\theta_4+\theta_5)$$
$$z = -L_1 \sin\theta_2 - L_2 \sin(\theta_2+\theta_4) - L_3 \cdot \sin(\theta_2+\theta_4+\theta_5)$$
$$\varphi = \theta_2 + \theta_4 + \theta_5$$

$$y_3 = y - L_3 \cos(\varphi)$$
$$z_3 = z - L_3 \sin(\varphi)$$

$$y_3 = L_1 \cdot \cos\theta_2 + L_2 \cdot \cos(\theta_2+\theta_4)$$
$$z = L_1 \cdot \sin\theta_2 + L_2 \cdot \sin(\theta_2+\theta_4)$$

$$(y_3 - L_1 \cos\theta_2)^2 = (L_2 \cos(\theta_2+\theta_4))^2 \quad +$$
$$(z_3 - L_2 \sin\theta_2)^2 = (L_2 \sin(\theta_2-\theta_4))^2 \quad \ldots$$

$$(-2L_1 y_3)\cos\theta_2 + (-2L_1 z_3)\sin\theta_2 + (L_1^2 - L_1^2 + y_3^2 + z_3^2) = 0$$

$$P\cos\alpha + Q\sin\alpha + R = 0 \qquad \cos\gamma = \frac{P}{\sqrt{P^2+Q^2}} \qquad \sin\gamma = \frac{Q}{\sqrt{P^2+Q^2}}$$

$$\gamma = \operatorname{atan2}\left(\frac{Q}{\sqrt{P^2+Q^2}}, \frac{P}{\sqrt{P^2+Q^2}}\right) \qquad \cos\gamma \cdot \cos\alpha + \sin\gamma \cdot \sin\alpha + \frac{R}{\sqrt{P^2+Q^2}} = 0$$

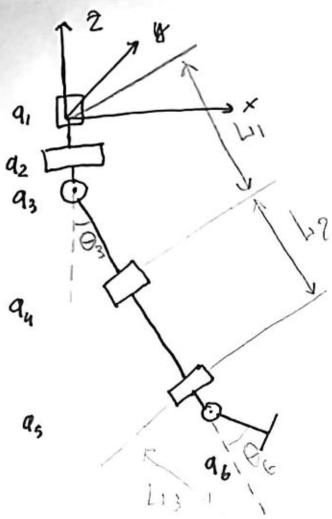
$$\cos(\alpha-\gamma) = \frac{-R}{\sqrt{P^2+Q^2}} \implies \alpha = \gamma + m \operatorname{acos}\left(\frac{-R}{\sqrt{P^2+Q^2}}\right) \quad m = \pm 1 \implies$$

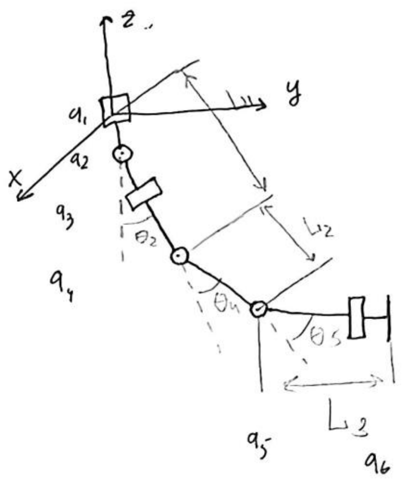
$$\theta_2 = \operatorname{atan2}\left(\frac{-z_3}{\sqrt{y_3^2+z_3^2}}, \frac{y_3}{\sqrt{y_3^2+z_3^2}}\right) + m \operatorname{acos}\left(\frac{-(y_3^2+z_3^2+L_1^2-L_2^2)}{2L_1\sqrt{y_3^2+z_3^2}}\right)$$

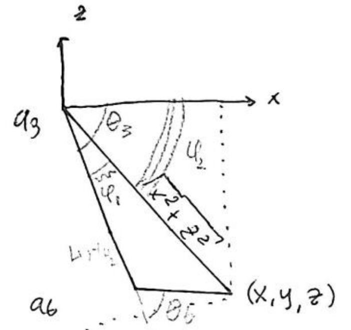
$$\cos(\theta_2-\theta_4) = \frac{y_3 - L_1\cos\theta_2}{L_2} \qquad \sin(\theta_2-\theta_4) = \frac{z_3 - L_1\sin\theta_2}{L_2}$$

$$\theta_4 = \operatorname{atan2}\left(\frac{z_3 - L_1\sin\theta_2}{L_2}, \frac{y_3 - L_1\cos\theta_2}{L_2}\right); \qquad \theta_5 = \varphi - \theta_4 - \theta_2;$$

And







$$x^2 + z^2 = (L_1 + L_2)^2 + L_3^2 + 2(L_1+L_2)\cdot L_3 \cdot \cos 6$$

$$\theta_6 = \arccos\left(\frac{x^2+z^2-(L_1+L_2)^2-L_3^2}{2(L_1+L_2)L_3} \right)$$

$$L_3^2 = (L_1+L_2)^2 + x^2 + z^2 - 2(L_1+L_2)\cdot\sqrt{x^2+z^2}\cdot\cos\varphi$$

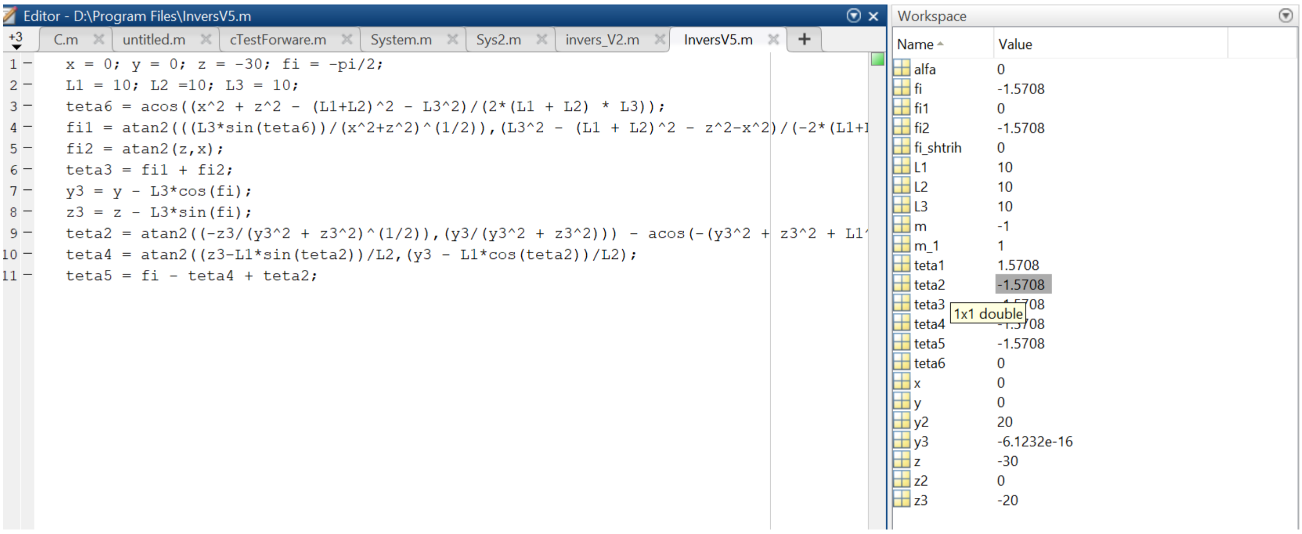
$$\cos\varphi_1 = \left(\frac{L_3^2 - (L_1+L_2)^2 - x^2 - z^2}{-2(L_1+L_2)\sqrt{x^2+z^2}} \right)$$

$$\sin\varphi_1 = \frac{L_3 \sin\theta_6}{\sqrt{x^2+z^2}} \qquad \varphi_2 = \operatorname{atan}\left(\frac{z}{x} \right)$$

$$\varphi_1 = \operatorname{atan}\left(\frac{2\sin\theta_6 \cdot L_3 \cdot (L_1+L_2)}{x^2+z^2+(L_1+L_2)^2-L_3^2} \right)$$

$$\theta_3 = m\varphi_1 + \varphi_2 \qquad m = \begin{cases} -1 & \varphi_1 > 0 \\ +1 & \varphi_1 < 0 \end{cases}$$

This calculation gave me following result:



I tried check all equation in extremely position and we take length all link = 10(for simplify check). This position for example when leg bent a knee: z = -10, x = 20.