Modern Complexity Theory (CS1.405)

Mid Semester Examination (Monsoon 2025)

International Institute of Information Technology, Hyderabad

Time: 1 hour and 30 minutes Total Marks: 40

Instructions: Answer *ANY FOUR* questions from the following FIVE questions.

This is a closed book and notes examination.

Regular calculator is allowed.

NO query in examination hall is allowed.

1. **(a)** Consider the following problem:

   BIGCLIQUE $:= \{\langle G \rangle |\ G$ has a clique of size $> 1 + |V|/2\}$,

   where $G = (V, E)$ is an undirected graph. Prove that BIGCLIQUE is NP-Complete.

   (Hint: Take CLIQUE $\leq_p$ BIGCLIQUE)

   (b) Write the differences between finite automata and Turing machines.

   (c) Define the configuration of a Turing machine (TM). Suppose that a, b, and $c \in \Gamma$ and u and $v \in \Gamma^*$, with states q_i and q_j, and you are given two configurations as $C_1 = uaq_ibv$ and $C_2 = uacq_jv$. Prove or disprove that C_1 "yields" C_2.

   [6 + 2 + 2 = 10]

2. **(a)** By "loop", we mean that a Turing machine simply does not halt. Consider the following statement: "Sometimes distinguishing a machine that is looping from one that is merely taking a long time is difficult".

   Give an example to support this statement.

   (b) Define the Traveling Salesperson Problem (TSP) formally. Prove that TSP is NP-Complete.

   [3 + (2 + 5) = 10]

3. **(a)** Let the language A be $A = \{0^k1^k | k \geq 0\}$. Design a Turing machine that can decide it in linear time.

   (b) Suppose you have a collection of integers $x_1, x_2, \ldots, x_k$ and a target number t. Consider the following problem:

   SUBSET-SUM $:= \{\langle S, t \rangle |\ S = \{x_1, x_2, \ldots, x_k\}$ and for some $\{y_1, y_2, \ldots, y_l\} \subseteq S, \sum y_i = t\}$.

   Show that SUBSET-SUM is in NP.

   [5 + 5 = 10]

4. **(a)** A linear bounded automaton (LBA) is exactly like a 1-tape Turing Machine, except that the input string $x \in \Sigma^*$ is enclosed in left and right endmarkers $\vdash$ and $\dashv$ which may not be overwritten (see Figure 1). The machine is constrained never to move left of $\vdash$ or right of $\dashv$. It is allowed to read/write between these markers. Give the formal definition of deterministic linearly bounded automaton, including the definitions of configurations and acceptance.

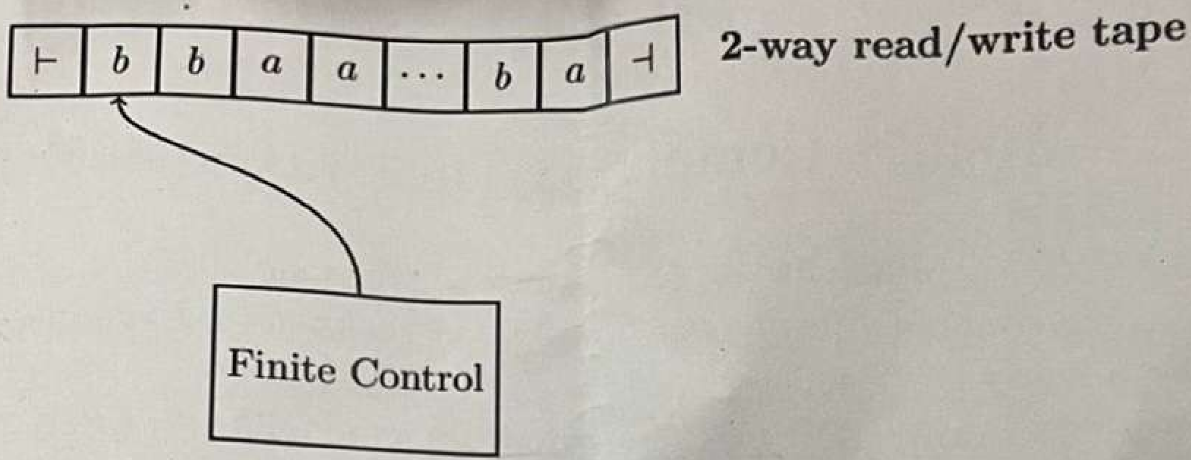

Figure 1: A linear bounded automaton (LBA)

(b) Let MODEXP $:= \{\langle a, b, c, p\rangle | a, b, c,$ and p are large binary (positive) integers such that $a^b = c \pmod{p}\}$. Show that MODEXP is in P. (Note that the most obvious algorithm does not run polynomial time. Hint: You may try it first where b is a power of 2.)

[5 + 5 = 10]

5. **(a)** Consider the problem UHAMPATH $:= \langle G, s, t\rangle |$ G is an undirected graph which has an (s, t)-Hamiltonian path$\}$. Prove that UHAMPATH is NP-Complete.

(b) Consider SET-SPLITTING $:= \{S, C\} | S$ is a finite set and $C = \{C_1, C_2, \cdots, C_k\}$ is a collection of subsets of S, for some $k > 0$, such that the elements of S can be colored *red* or *blue* so that no C_i has all its elements colored with the same color.$\}$

Prove or disprove that SET-SPLITTING is in P.

[5 + 5 = 10]

****************** End of Question Paper **********************