Homework 1

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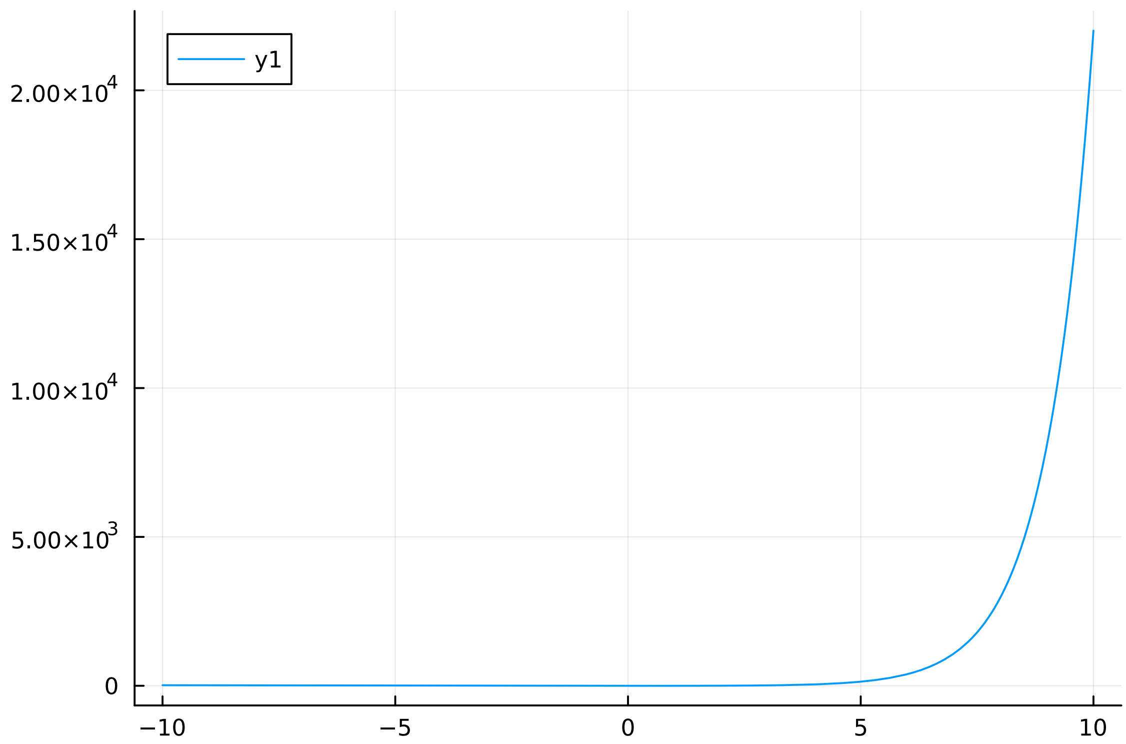
In [1]:
```
using Pkg
Pkg.add("Plots")
using Plots
using JuMP
using HiGHS
```

```
  Resolving package versions...
 No Changes to `C:\Users\damio\.julia\environments\v1.11\Project.toml`
 No Changes to `C:\Users\damio\.julia\environments\v1.11\Manifest.toml`
```

Question 1-1

In [2]:
```
f(x) = exp(x) - 1 - 2x
plot(f, -10, 10)
```

Out[2]:



This function is convex because for any line between 2 points there is a point under the line e^x is convex, and -2x is convex so therefore f(x) is convex

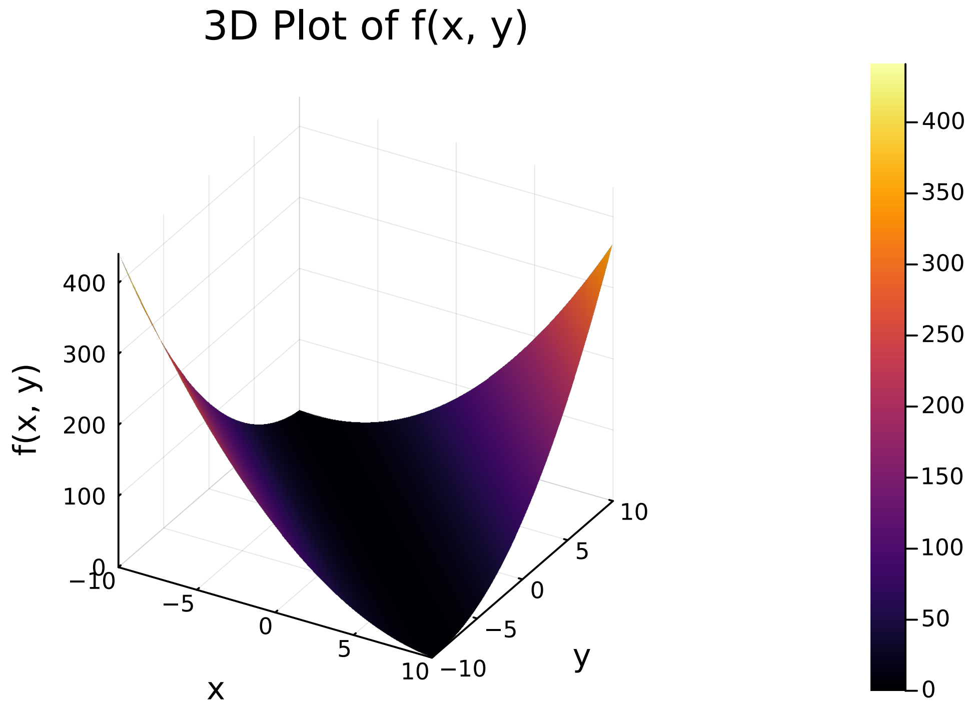
Question 1-2

In [3]:
```
f(x, y) = x^2 + y^2 + 2*x*y - 2*x - 2*y + 1
x_range = -10:0.5:10
y_range = -10:0.5:10
```

```
z_values = [f(x, y) for x in x_range, y in y_range]

surface(x_range, y_range, z_values, xlabel="x", ylabel="y", zlabel="f(x, y)", title
```

Out[3]:



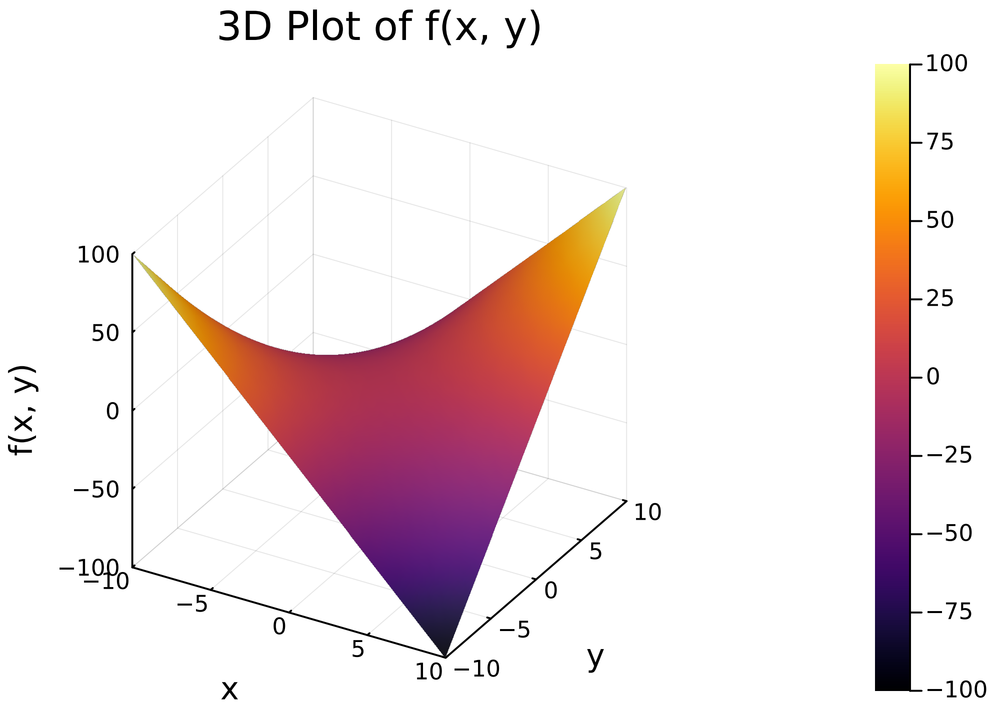
This function is convex because for any plane between 2 points lies above the surface x^2 is convex which is the simplification of $(x_1 + x_2 - 1)^2$

Question 1-3

In [4]:
```
f(x, y) = x*y
x_range = -10:0.5:10
y_range = -10:0.5:10
z_values = [f(x, y) for x in x_range, y in y_range]

surface(x_range, y_range, z_values, xlabel="x", ylabel="y", zlabel="f(x, y)", title
```

Out[4]:



This function is niether because there are sections that are convex and concave

Question 1-4

This function is both concave and convex because it is a line

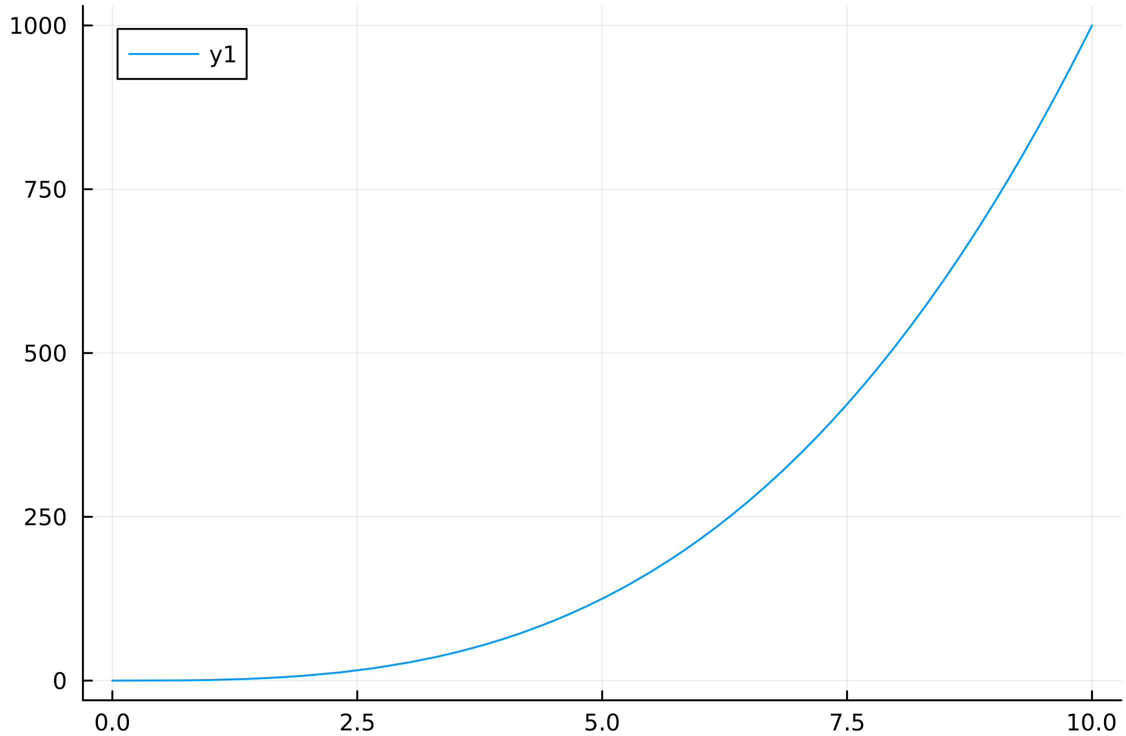
Question 1-5

This function will be concave because each function inside the min function is linear and the min of linear functions is concave

Question 1-6

In [5]:
```
f(x) = x^3
plot(f, 0, 10)
```

Out[5]:



This function is convex because we are only looking at the points $x \geq 0$ and $f''(x) = 6x$ which is non-negitive

Question 1-7

This function is netier because it is convex when $x \geq 0$ and concave otherwise

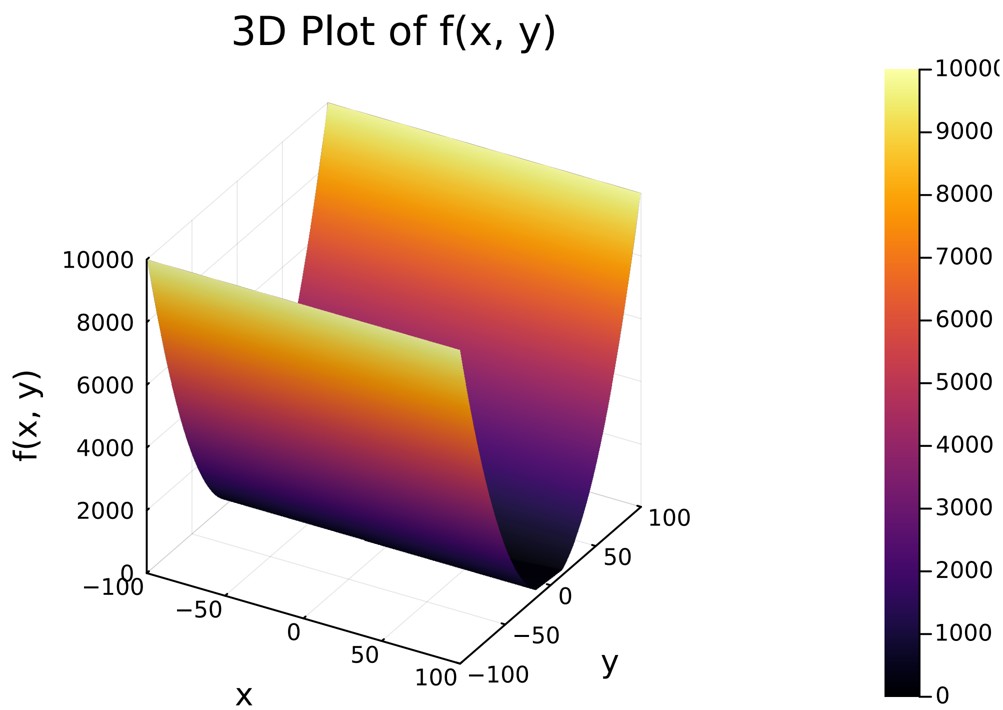
Question 1-8

In [6]:

```
f(x, y) = max(x^2, -3*x + 2*y + 9)
x_range = -100:0.5:100
y_range = -100:0.5:100
z_values = [f(x, y) for x in x_range, y in y_range]

surface(x_range, y_range, z_values, xlabel="x", ylabel="y", zlabel="f(x, y)", title
```

Out[6]:



This function is convex because it is the max of two convex functions

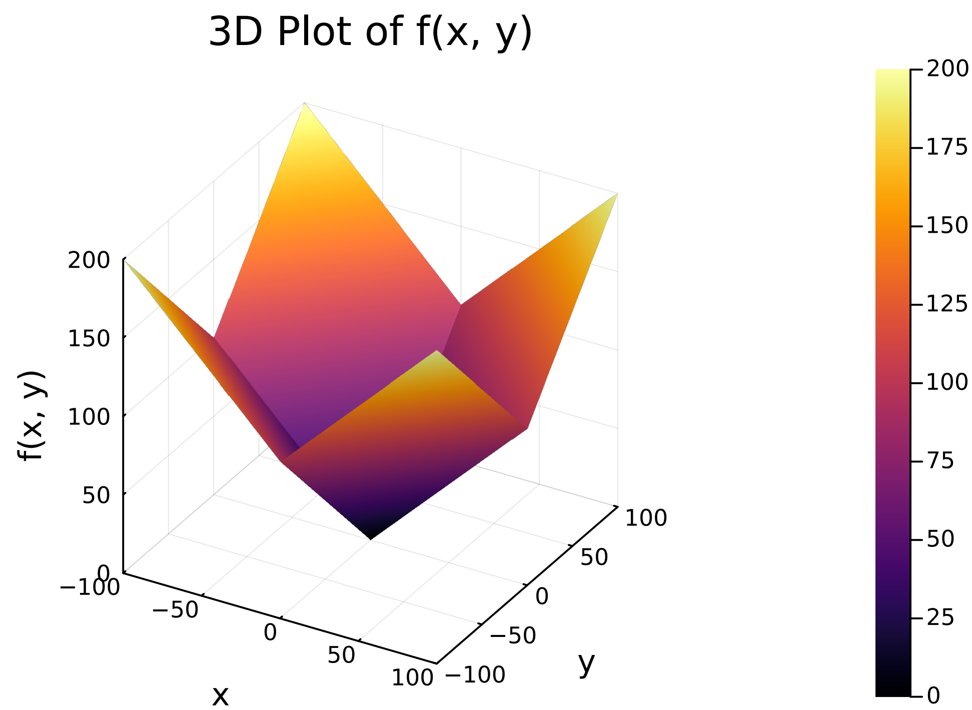
Question 1-9

In [7]:

```
f(x, y) = abs(x) + abs(y)
x_range = -100:0.5:100
y_range = -100:0.5:100
z_values = [f(x, y) for x in x_range, y in y_range]

surface(x_range, y_range, z_values, xlabel="x", ylabel="y", zlabel="f(x, y)", title
```

Out[7]:



This function is convex because $|x_1|$ and $|x_2|$ are convex so $|x_1| + |x_2|$ is convex

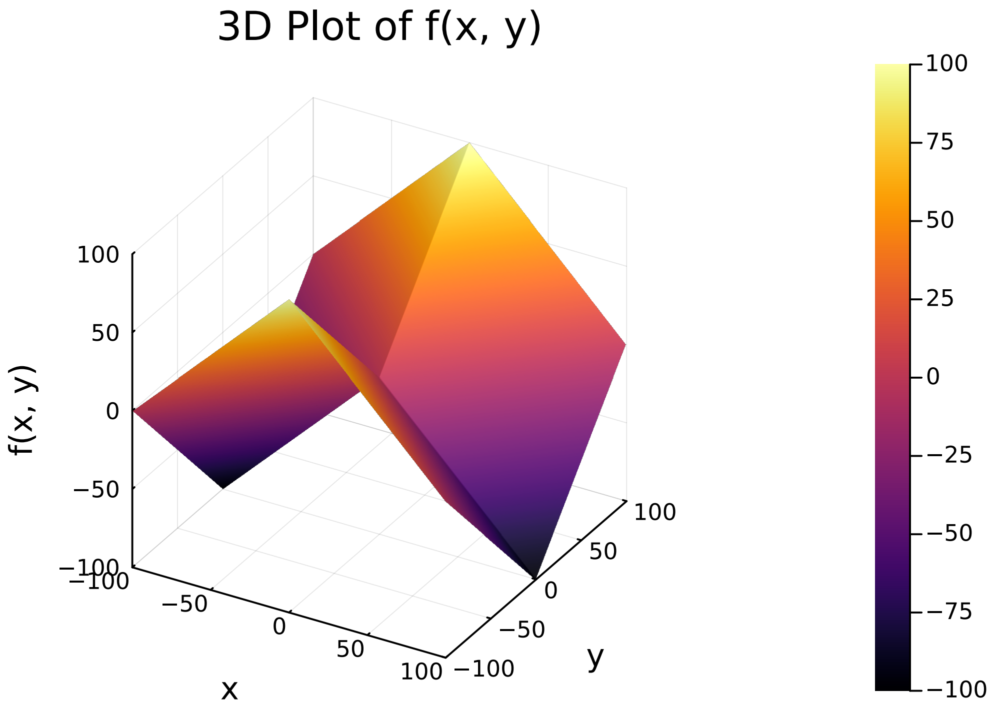
Question 1-10

In [8]:

```
f(x, y) = abs(x) - abs(y)
x_range = -100:0.5:100
y_range = -100:0.5:100
z_values = [f(x, y) for x in x_range, y in y_range]

surface(x_range, y_range, z_values, xlabel="x", ylabel="y", zlabel="f(x, y)", title
```

Out[8]:

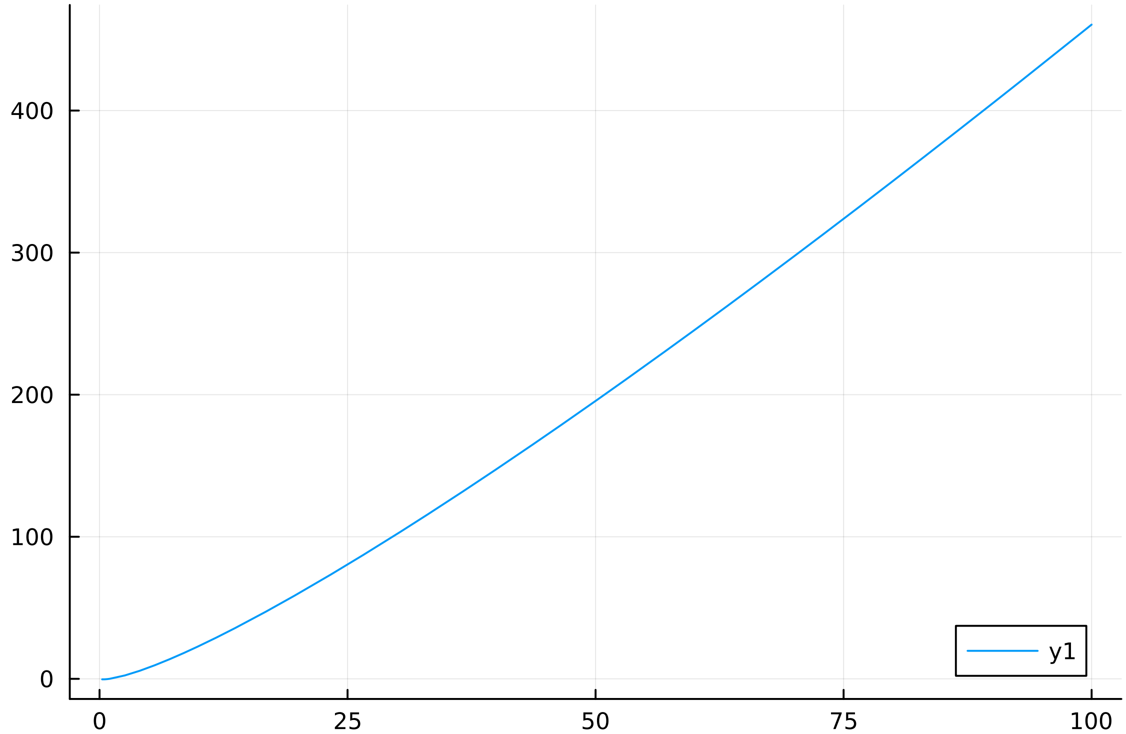


This function is neither because it is both convex and concave $|x_1|$ is convex and $-|x_2|$ is concave

Question 1-11

In [9]:
```
f(x) = x*log(x)
plot(f, 0, 100)
```

Out[9]:

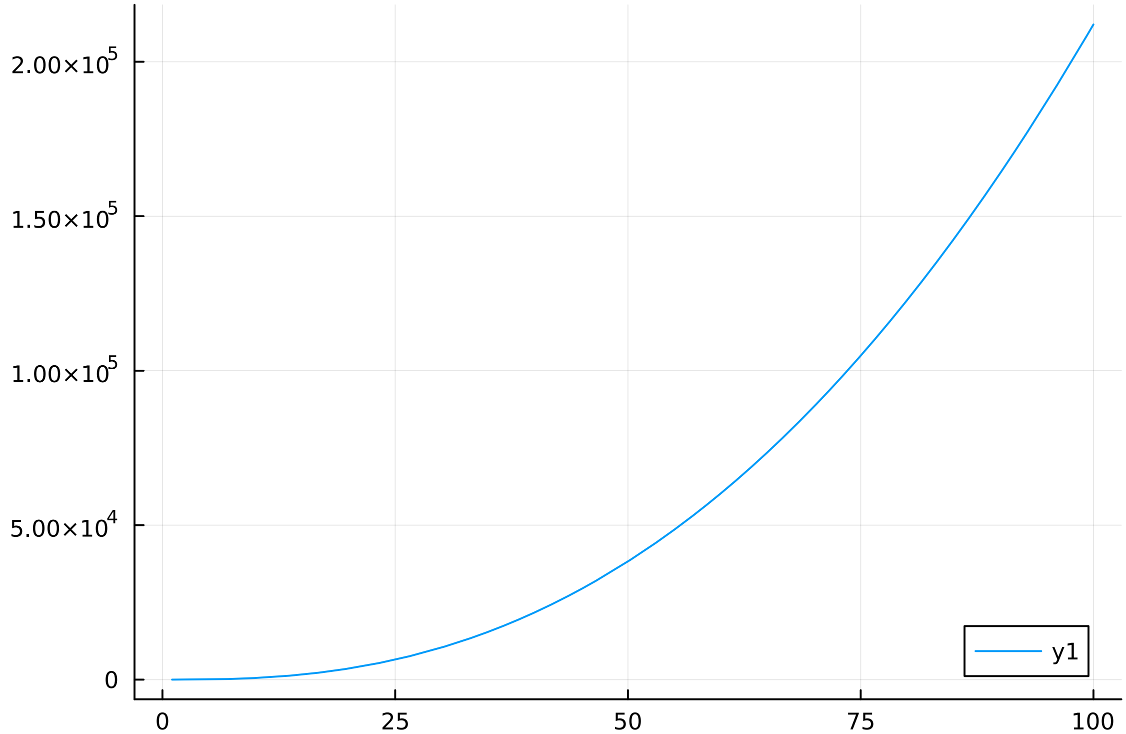


This function is convex

Question 1-12

In [10]:
```
f(x) = (x*log(x))^2
plot(f, 0, 100)
```

Out[10]:



This function is convex

Question 2-1

This set is convex because when graphed it is a circle, so any two points in the set can be connected with a line that doesnt leave the set
The conditonal $x_1^2 + x_2^2$ is convex so by definition f(x) $\geq$ 1 is convex

Question 2-2

This set is not convex because the set is the permimeter of a circle and not filled in, so a line between two points will not be contained in the set

Question 2-3

This set is convex because all points are included in the set, so any two points can be connected by a line contained in the set

Question 2-4

This is a convex set because any 2 x the satisfies the condition can be connected. Each point inbetween the two x will be in the set
If $Ax_1 \leq Ax_2 \leq b$ the all x between x_1 and x_2 are in the set

Question 2-5

This is a convex set because the empty set is convex

Question 2-6

g(x) is convex because it a a max of 3 convex functions, therefore the set is convex because we are constraining the set on g(x) $\leq$ 10 which by definition (slide 56) is convex

Question 2-7

g(x) is convex because it a a max of 3 convex functions, but this set is not convex because the set is in the form f(x) $\geq$ 10 and f is not concave (slide 57)

Question 3-1

$x_1^2 + 3x_2$ is a convex function
the constraint is a max of convex functions that are greater than 1 so that is not a convex contraint
therefore the problem is not a convex optimization problem

Question 3-2

$|x_1| + 3x_2$ is a convex function
the constaint is a max of convex functions that are less than 1 so that is a convex constraint
Therefore this problem is a convex optimization problem

Question 3-3

$|x_1| + 3x_2$ is a convex function
the constraint it convex (it is the same as the previous question)
however we are asked to maximize this function so this is NOT a convex optimization problem

Question 3-4

$-|3x_1 - 2x_2| + \sqrt(4x_1)$ is concave
the feasable region is convex because $x_1 \geq 0$ is convex and $-14x_1 + x_2 = 7$ is a line and is convex
therefore becauase we are maximizing a concave function over a convex feasiable region this is a convex optimization problem

Question 3-5

$c^T x$ is linear so it can be either convex or concave, because we are maximizing we will call it concave
the constraints are convex because they are linear

therefore because we are maximizing a concave function over a convex feasible region this is a convex optimization problem

Question 4-1

In [11]:
```
m = Model(HiGHS.Optimizer)

@variable(m, x1 >= 0)
@variable(m, x2 >= 0)

@constraint(m, 2*x1 + x2 >= 100)
@constraint(m, x1 + x2 >= 80)
@constraint(m, x1 >= 45)

@objective(m, Min, 3*x1 + 2*x2)

optimize!(m)

println("optimal value of x1: ", value(x1))
println("optimal value of x2: ", value(x2))
println("optimal objective value: ", objective_value(m))
```

```
Running HiGHS 1.9.0 (git hash: 66f735e60): Copyright (c) 2024 HiGHS under MIT licenc
e terms
Coefficient ranges:
  Matrix [1e+00, 2e+00]
  Cost   [2e+00, 3e+00]
  Bound  [0e+00, 0e+00]
  RHS    [4e+01, 1e+02]
Presolving model
2 rows, 2 cols, 4 nonzeros  0s
2 rows, 2 cols, 4 nonzeros  0s
Presolve : Reductions: rows 2(-1); columns 2(-0); elements 4(-1)
Solving the presolved LP
Using EKK dual simplex solver - serial
  Iteration        Objective     Infeasibilities num(sum)
          0     1.3500015164e+02 Pr: 2(45) 0s
          1     2.0500000000e+02 Pr: 0(0) 0s
Solving the original LP from the solution after postsolve
Model status        : Optimal
Simplex   iterations: 1
Objective value     :  2.0500000000e+02
Relative P-D gap    :  0.0000000000e+00
HiGHS run time      :          0.00
optimal value of x1: 45.0
optimal value of x2: 35.0
optimal objective value: 205.0
```

Question 5-2

In [12]:
```
m = Model(HiGHS.Optimizer)

@variable(m, light_beer >= 0)
@variable(m, pale_ale >= 0)
@variable(m, malt_liquor >= 0)
```

```
@constraint(m, 2*light_beer + 2*pale_ale + 3*malt_liquor <= 90)
@constraint(m, 3*light_beer + 2*pale_ale + 1.5*malt_liquor <= 65)
@constraint(m, 2*light_beer + 1.5*pale_ale + 2*malt_liquor <= 80)
;
```

Question 5-2

In [13]:

```
@objective(m, Max, 2.5*light_beer + 2*pale_ale + 2.5*malt_liquor)

optimize!(m)

println("optimal value of light_beer: ", value(light_beer))
println("optimal value of pale_ale: ", value(pale_ale))
println("optimal value of malt_liquor: ", value(malt_liquor))
println("optimal objective value (profit): ", objective_value(m))
```

```
Running HiGHS 1.9.0 (git hash: 66f735e60): Copyright (c) 2024 HiGHS under MIT licenc
e terms
Coefficient ranges:
  Matrix [2e+00, 3e+00]
  Cost   [2e+00, 2e+00]
  Bound  [0e+00, 0e+00]
  RHS    [6e+01, 9e+01]
Presolving model
3 rows, 3 cols, 9 nonzeros  0s
3 rows, 3 cols, 9 nonzeros  0s
Presolve : Reductions: rows 3(-0); columns 3(-0); elements 9(-0) - Not reduced
Problem not reduced by presolve: solving the LP
Using EKK dual simplex solver - serial
  Iteration        Objective     Infeasibilities num(sum)
          0    -6.9999926653e+00 Ph1: 3(19); Du: 3(6.99999) 0s
          2     8.3333333333e+01 Pr: 0(0) 0s
Model status        : Optimal
Simplex   iterations: 2
Objective value     :  8.3333333333e+01
Relative P-D gap    :  1.7053025658e-16
HiGHS run time      :          0.00
optimal value of light_beer: 10.0
optimal value of pale_ale: 0.0
optimal value of malt_liquor: 23.333333333333332
optimal objective value (profit): 83.33333333333333
```