Question 1

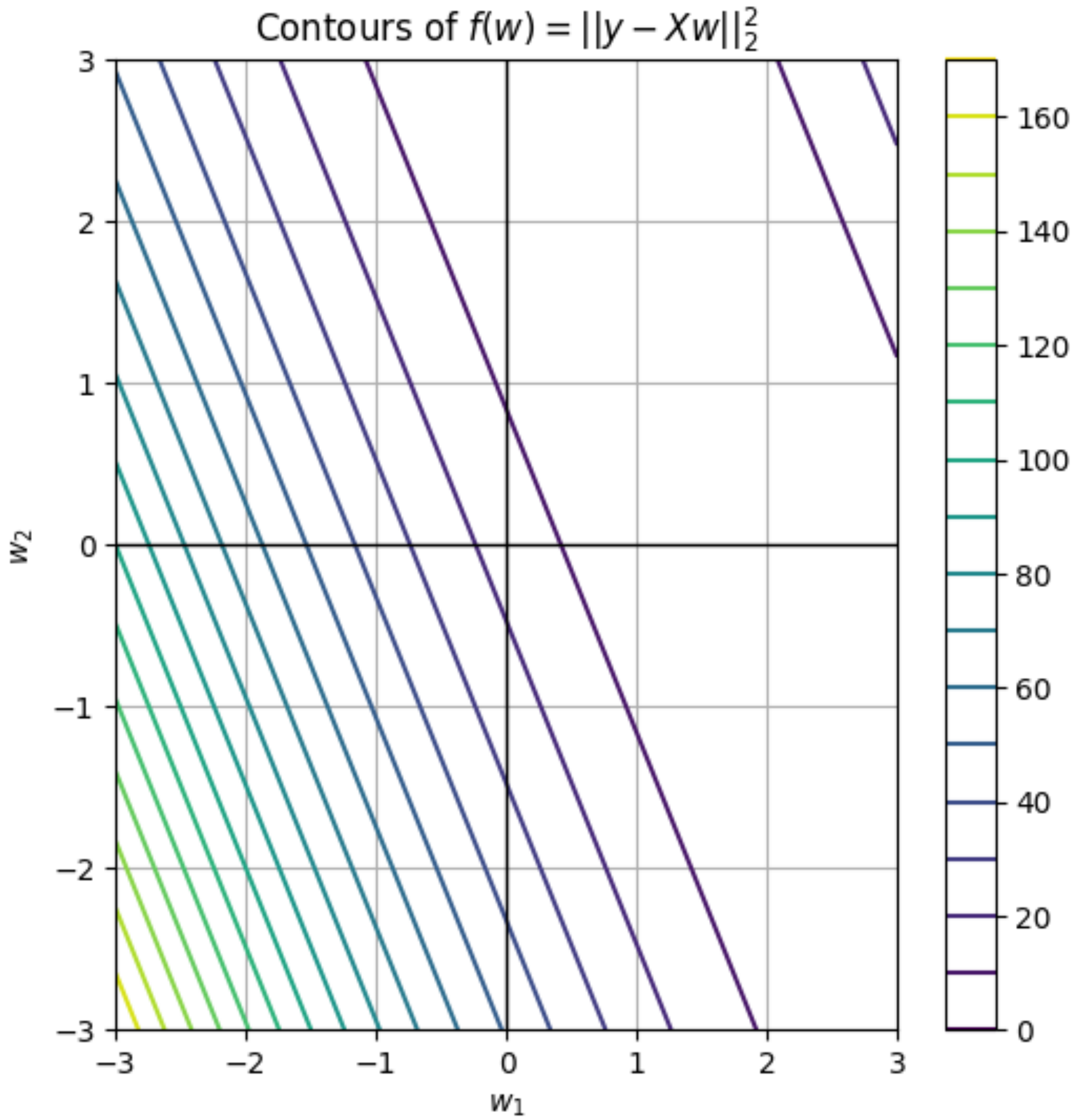
We could use LASSO to get the most insightful feautes and use them for prediction

Question 2-a

X is not full rank so there is not a unique solution

Questoin 2-b

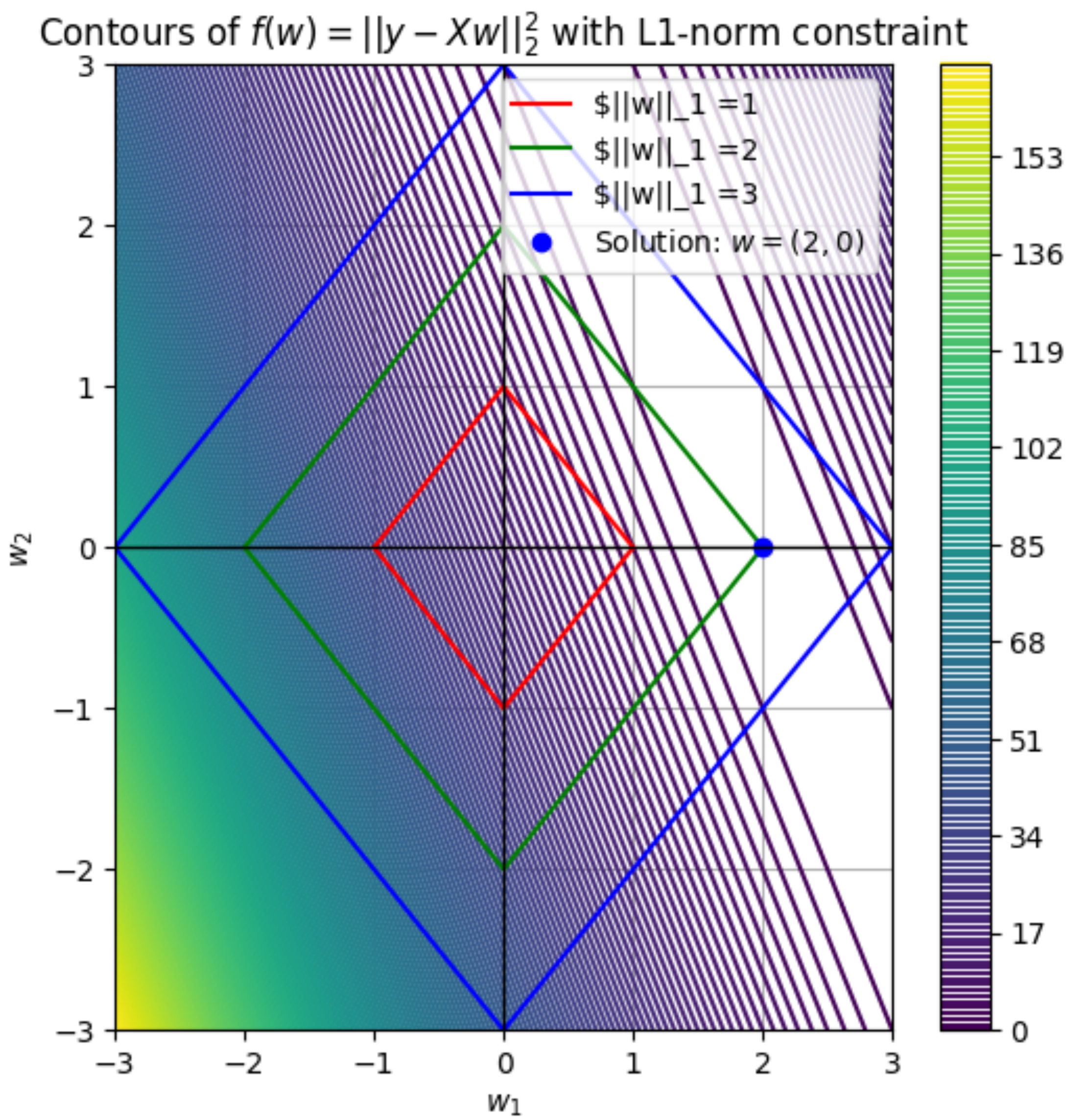
In [2]:
```
import numpy as np
import matplotlib.pyplot as plt

def cost_function(w1, w2):
    return (4 - 2*w1 - w2)**2

w1_vals = np.linspace(-3, 3, 400)
w2_vals = np.linspace(-3, 3, 400)
W1, W2 = np.meshgrid(w1_vals, w2_vals)

F = cost_function(W1, W2)

plt.figure(figsize=(6,6))
cp = plt.contour(W1, W2, F, levels=20, cmap='viridis')
plt.colorbar(cp)
plt.title(r'Contours of $f(w) = ||y - Xw||_2^2$')
plt.xlabel(r'$w_1$')
plt.ylabel(r'$w_2$')
plt.axhline(0, color='black',linewidth=1)
plt.axvline(0, color='black',linewidth=1)
plt.grid(True)
plt.show()
```



Question 2-c

In [11]:
```
import numpy as np
import matplotlib.pyplot as plt

def cost_function(w1, w2):
    return (4 - 2*w1 - w2)**2

def l1_norm_constraint(c):
    w1 = np.linspace(-c, c, 400)
    w2_upper = c - np.abs(w1)
    w2_lower = -(c - np.abs(w1))
    return w1, w2_upper, w2_lower

w1_vals = np.linspace(-3, 3, 400)
w2_vals = np.linspace(-3, 3, 400)
W1, W2 = np.meshgrid(w1_vals, w2_vals)

F = cost_function(W1, W2)
```

```python
plt.figure(figsize=(6,6))
cp = plt.contour(W1, W2, F, levels=200, cmap='viridis')
plt.colorbar(cp)

for c, color in zip(range(1, 4), ['r', 'g', 'b']):
    w1, w2_upper, w2_lower = l1_norm_constraint(c)
    plt.plot(w1, w2_upper, color=color, label=r'$||w||_1 ='+str(c))
    plt.plot(w1, w2_lower, color=color)

plt.plot(2, 0, 'bo', label=r'Solution: $w = (2,0)$')
plt.title(r'Contours of $f(w) = ||y - Xw||_2^2$ with L1-norm constraint')
plt.xlabel(r'$w_1$')
plt.ylabel(r'$w_2$')
plt.axhline(0, color='black',linewidth=1)
plt.axvline(0, color='black',linewidth=1)
plt.grid(True)
plt.legend(loc='best')
plt.show()
```

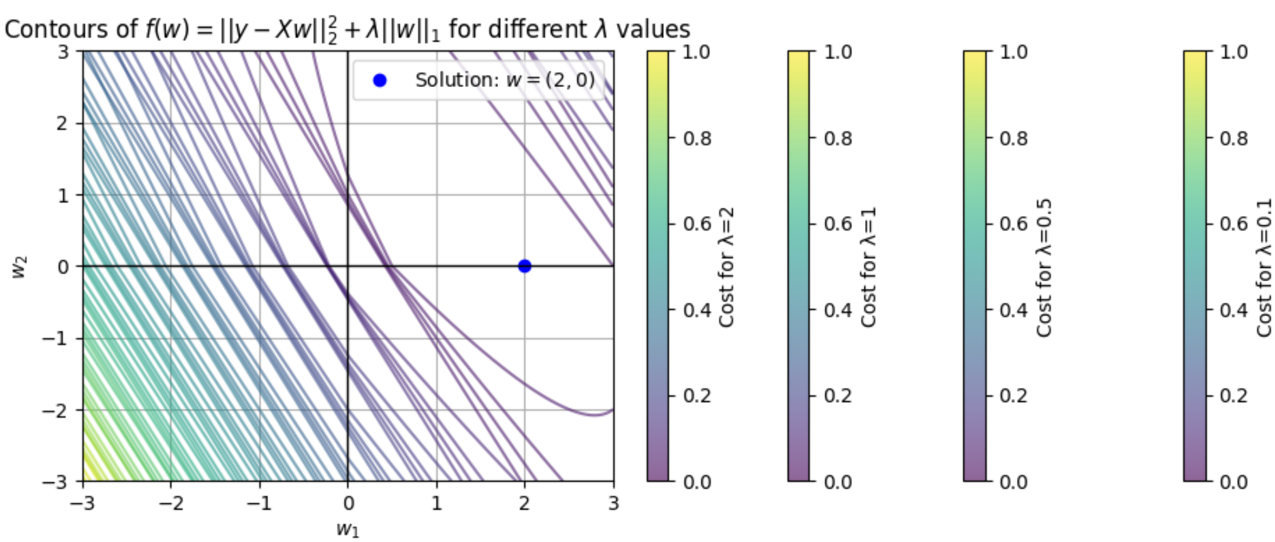


Question 2-d

In [16]:

```
import numpy as np
import matplotlib.pyplot as plt

def cost_function(w1, w2, lam=0.1):
    """Combined cost function for LASSO: ||y - Xw||_2^2 + lambda ||w||_1"""
    residual = 4 - (2 * w1 + w2)
    l1_norm = np.abs(w1) + np.abs(w2)
    return residual**2 + lam * l1_norm

w1_vals = np.linspace(-3, 3, 400)
w2_vals = np.linspace(-3, 3, 400)
W1, W2 = np.meshgrid(w1_vals, w2_vals)

# Plot the cost function for different values of lambda
lambda_vals = [0.1, 0.5, 1, 2]
plt.figure(figsize=(12,4))

for lam in lambda_vals:
    F = cost_function(W1, W2, lam)
    cp = plt.contour(W1, W2, F, levels=20, cmap='viridis', alpha=0.6)
    plt.colorbar(cp.collections[0], label=f'Cost for λ={lam}')

# Plot the solution point for visualization
plt.plot(2, 0, 'bo', label=r'Solution: $w = (2,0)$')

plt.title(r'Contours of $f(w) = ||y - Xw||_2^2 + \lambda ||w||_1$ for differ
plt.xlabel(r'$w_1$')
plt.ylabel(r'$w_2$')
plt.axhline(0, color='black',linewidth=1)
plt.axvline(0, color='black',linewidth=1)
plt.grid(True)
plt.legend(loc='best')
plt.show()
```

```
/var/folders/tn/v9tpvrrs4qgdbw0xd1q0l8qh0000gn/T/ipykernel_81974/3853232607.
py:21: MatplotlibDeprecationWarning: The collections attribute was deprecate
d in Matplotlib 3.8 and will be removed in 3.10.
  plt.colorbar(cp.collections[0], label=f'Cost for λ={lam}')
```



```
In [18]: def prxgraddescent_l1(X,y,tau,lam,w_init,it):

         ## compute it iterations of L2 proximal gradient descent starting at w1
         ## w_{k+1}= (w_k - tau*X'*(X*w_k - y)/(1+lam*tau)
         ## step size tau
             W = np.zeros((w_init.shape[0], it+1))
             Z = np.zeros((w_init.shape[0], it+1))
             W[:,[0]] = w_init
             for k in range(it):
                 Z[:,[k+1]] = W[:,[k]] - tau * X.T @ (X @ W[:,[k]] - y);
                 W[:,[k+1]] = np.sign(Z[:,[k+1]])* np.clip(np.abs(Z[:,[k+1]])-lam*tau

             return W,Z

         ## Proximal gradient descent trajectories
         ## Least Squares Problem
         X = np.array([[2, 1]])
         y = np.array([[4]])

         ### Find values of f(w), the contour plot surface for
         w1 = np.arange(-1,3,.1)
         w2 = np.arange(-1,3,.1)
         fw = np.zeros((len(w1), len(w2)))
         for i in range(len(w2)):
             for j in range(len(w1)):
                 w = np.array([ [w1[j]], [w2[i]] ])
                 fw[i,j] = (X @ w - y)**2
```

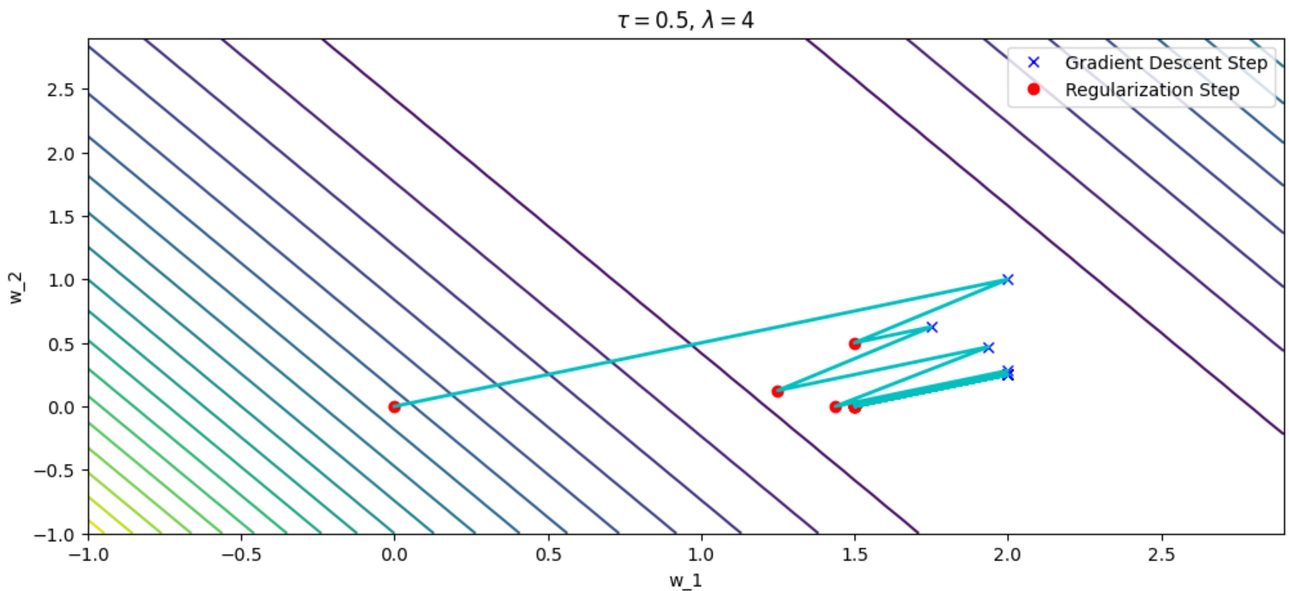
```
/var/folders/tn/v9tpvrrs4qgdbw0xd1q0l8qh0000gn/T/ipykernel_81974/3034412313.
py:27: DeprecationWarning: Conversion of an array with ndim > 0 to a scalar
is deprecated, and will error in future. Ensure you extract a single element
from your array before performing this operation. (Deprecated NumPy 1.25.)
  fw[i,j] = (X @ w - y)**2
```

Questoin 3-a

```
In [20]: ## Find and display weights generated by gradient descent

         w_init = np.array([[0],[0]])
         lam = 4;
         it = 10
         tau = 0.25
         W,Z = prxgraddescent_l1(X,y,tau,lam,w_init,it)
         # Concatenate gradient and regularization steps to display trajectory
         G = np.zeros((2,0))
         for i in range(it):
             G = np.hstack((G,np.hstack((W[:,[i]],Z[:,[i+1]]))))

         plt.figure(figsize=(12,5))
         plt.contour(w1,w2,fw,20)
         plt.plot(Z[0,1::],Z[1,1:],'bx',linewidth=2, label="Gradient Descent Step")
         plt.plot(W[0,:],W[1,:],'ro',linewidth=2, label="Regularization Step")
```

```
plt.plot(G[0,:],G[1,:],'-c',linewidth=2)
plt.legend()
plt.xlabel('w_1')
plt.ylabel('w_2')
plt.title('$\\tau = $'+str(.5)+', $\lambda = $'+str(lam));
```

```
<>:21: SyntaxWarning: invalid escape sequence '\l'
<>:21: SyntaxWarning: invalid escape sequence '\l'
/var/folders/tn/v9tpvrrs4qgdbw0xd1q0l8qh0000gn/T/ipykernel_81974/1140934211.
py:21: SyntaxWarning: invalid escape sequence '\l'
  plt.title('$\\tau = $'+str(.5)+', $\lambda = $'+str(lam));
```



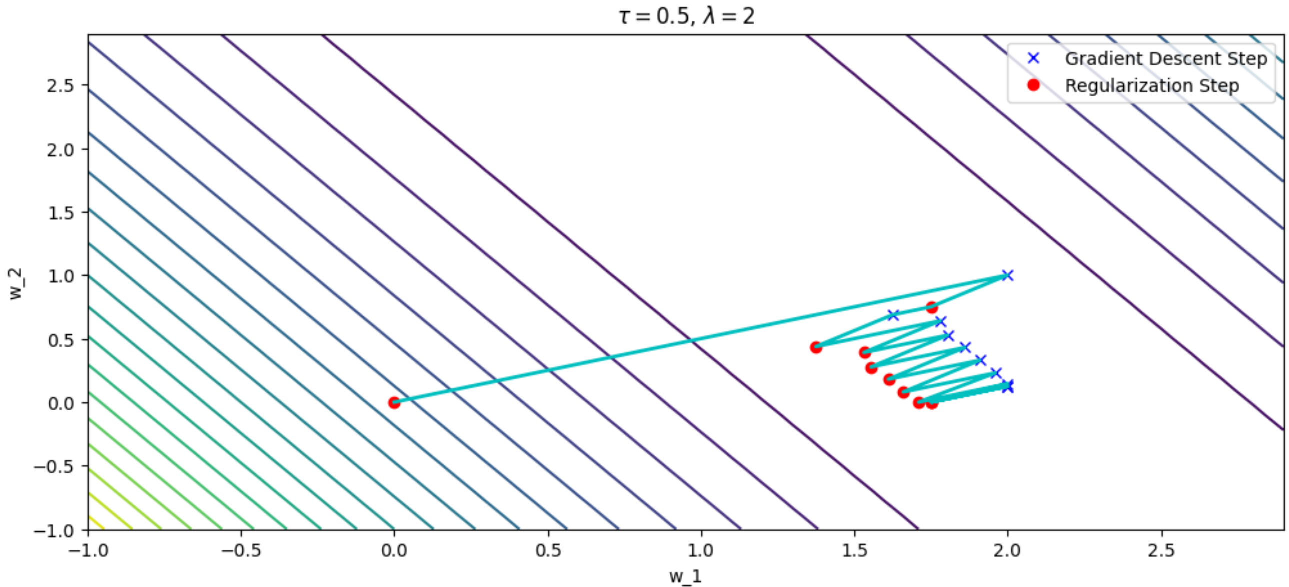
About 5 iterations for convergance

Question 3-b

In [21]:
```
## Find and display weights generated by gradient descent

w_init = np.array([[0],[0]])
lam = 2;
it = 10
tau = 0.25
W,Z = prxgraddescent_l1(X,y,tau,lam,w_init,it)
# Concatenate gradient and regularization steps to display trajectory
G = np.zeros((2,0))
for i in range(it):
    G = np.hstack((G,np.hstack((W[:,[i]],Z[:,[i+1]]))))

plt.figure(figsize=(12,5))
plt.contour(w1,w2,fw,20)
plt.plot(Z[0,1::],Z[1,1:],'bx',linewidth=2, label="Gradient Descent Step")
plt.plot(W[0,:],W[1,:],'ro',linewidth=2, label="Regularization Step")
plt.plot(G[0,:],G[1,:],'-c',linewidth=2)
plt.legend()
plt.xlabel('w_1')
plt.ylabel('w_2')
```

```
plt.title('$\\tau = $'+str(.5)+', $\lambda = $'+str(lam));
```

```
<>:21: SyntaxWarning: invalid escape sequence '\l'
<>:21: SyntaxWarning: invalid escape sequence '\l'
/var/folders/tn/v9tpvrrs4qgdbw0xd1q0l8qh0000gn/T/ipykernel_81974/1212030886.
py:21: SyntaxWarning: invalid escape sequence '\l'
  plt.title('$\\tau = $'+str(.5)+', $\lambda = $'+str(lam));
```



About 9 iterations for convergance

Question 3-c

The weights that are less than a specified threshold are set to zero and the rest are scalled down

Question 4-a

$$\tau < \frac{1}{||X||_{op}^2}$$

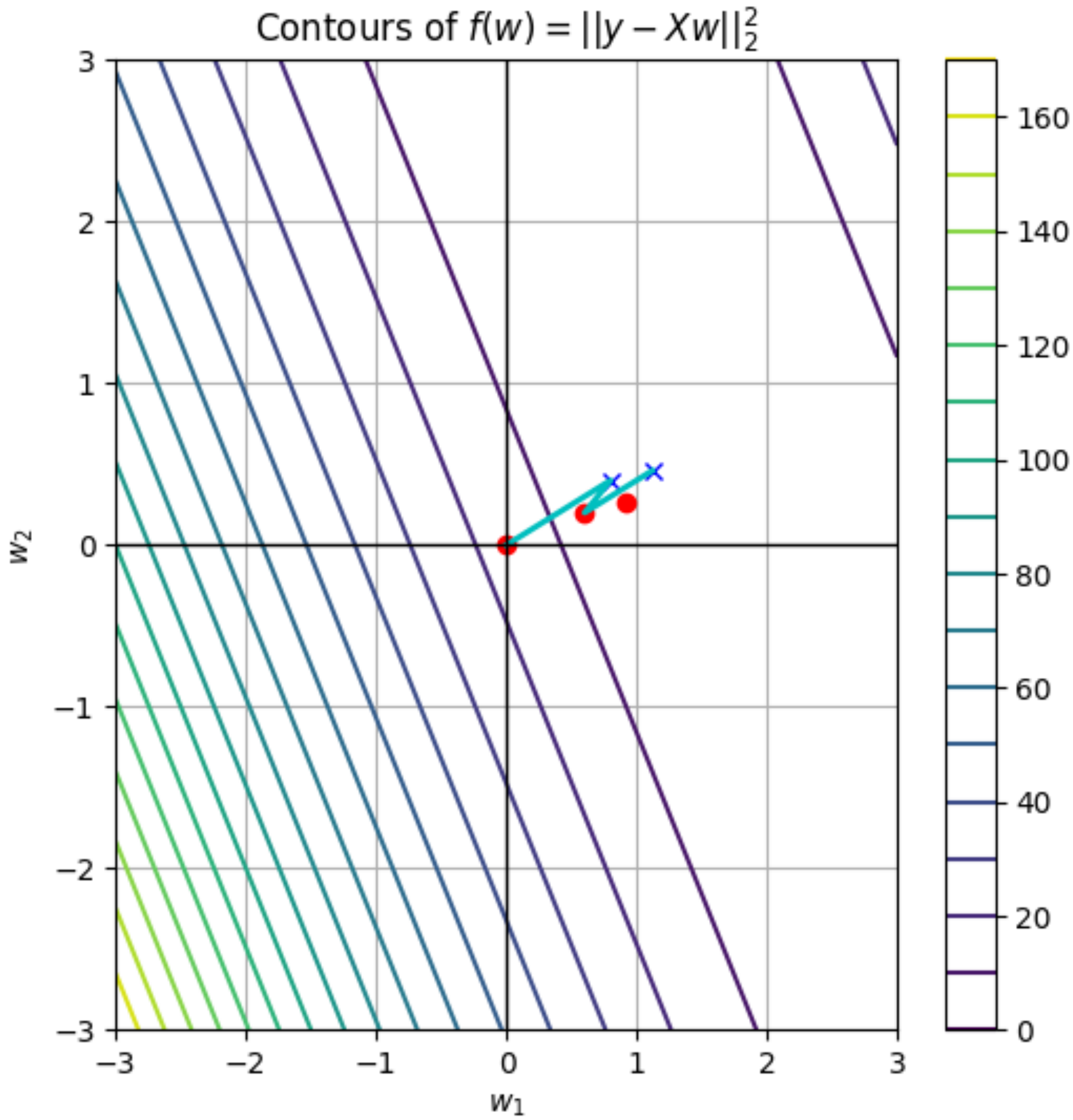
In [26]:
```
operator_norm = np.linalg.norm(X, ord=2)
print(1 / (operator_norm*operator_norm))
```

```
0.19999999999999996
```

Question 4-b

In [27]:
```
## Find and display weights generated by gradient descent
X = np.array([[2, 1]])
y = np.array([[4]])


w_init = np.array([[0],[0]])
```

```python
lam = 4;
it = 2
tau = 0.1 # TODO
W,Z = prxgraddescent_l1(X,y,tau,lam,w_init,it)
# Concatenate gradient and regularization steps to display trajectory
G = np.zeros((2,0))
for i in range(it):
    G = np.hstack((G,np.hstack((W[:,[i]],Z[:,[i+1]]))))

def cost_function(w1, w2):
    return (4 - 2*w1 - w2)**2

w1_vals = np.linspace(-3, 3, 400)
w2_vals = np.linspace(-3, 3, 400)
W1, W2 = np.meshgrid(w1_vals, w2_vals)

F = cost_function(W1, W2)

plt.figure(figsize=(6,6))
cp = plt.contour(W1, W2, F, levels=20, cmap='viridis')
plt.colorbar(cp)
plt.plot(Z[0,1::],Z[1,1:],'bx',linewidth=2, label="Gradient Descent Step")
plt.plot(W[0,:],W[1,:],'ro',linewidth=2, label="Regularization Step")
plt.plot(G[0,:],G[1,:],'-c',linewidth=2)
plt.title(r'Contours of $f(w) = ||y - Xw||_2^2$')
plt.xlabel(r'$w_1$')
plt.ylabel(r'$w_2$')
plt.axhline(0, color='black',linewidth=1)
plt.axvline(0, color='black',linewidth=1)
plt.grid(True)
plt.show()
```




```
In [ ]:
```