Activity 12

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Question 1-a

$$Z = X^T = V\Sigma U^T$$

Question 1-b

rank 1 approx.

$$\sigma^1 V_1 U_1^T$$

Question 2-a

The LS is not unique when rank(X) $< p$

Question 2-b

We can create new variables for X and y:

$$\hat{X} = \begin{bmatrix} X \\ \sqrt{\lambda} I \end{bmatrix} \hat{y} = \begin{bmatrix} y \\ 0 \end{bmatrix}$$

$$\min_w ||\hat{y} - \hat{X}w||_2^2$$

Question 2-c

For $\min_w ||\hat{y} - \hat{X}w||_2^2$ to have a unique solution $\hat{X}$ must have $rank = p$, λ must also be > 0

Question 3-a

$$X^T X = V\Sigma U^T U \Sigma V^T = V \Sigma^2 V^T$$

$$(X^T X + \lambda I)^{-1} X^T = V(\Sigma^2 + \lambda I)^{-1} \Sigma U^T$$

$$(\Sigma^2 + \lambda I)^{-1} = \frac{1}{\sigma_i^2 + \lambda}$$

$$(X^TX + \lambda I)^{-1}X^T = \sum_{i=1}^{p} \frac{1}{\sigma_i^2 + \lambda} v_i u_i^T$$

Question 3-b

$$X^\dagger = \lim_{\lambda \to 0^+} (X^TX + \lambda I)^{-1}X^T$$

As $\lambda \to 0$

$$(X^TX + \lambda I)^{-1} = (X^TX)^{-1}$$

Then

$$X^\dagger = (X^TX)^{-1}X^T$$

Question 3-c

When X is sqaure

$$(X^TX)^{-1} = X^{-1}X^{-T}$$

$$X^{-T}X^T = I$$

$$X^\dagger = X^{-1}$$

Question 3-d

When r < p and only the first r singular values are nonzero so any terms calculated in the sum past r are 0 and basically ignored.
So we can simplify the expression from 3-a to only take the sum from 1 to r

Question 3-e

$$X^\dagger = (X^TX)^{-1}X^T$$

As $\lambda \to 0$

$$X^\dagger = \sum_{i=1}^{r} \frac{1}{\sigma_i} v_i u_i^T$$

Therefore

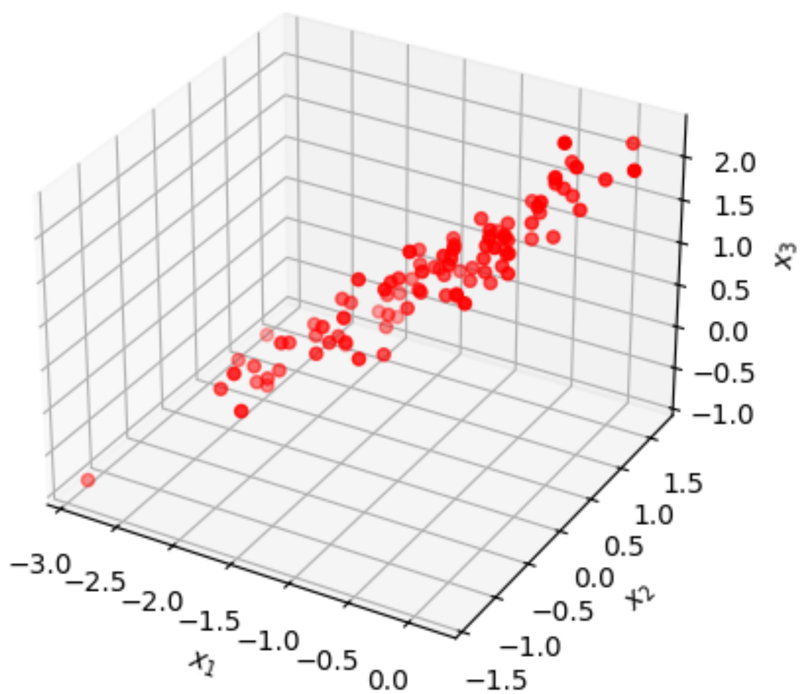
$$X^\dagger = V\Sigma_r^{-1}U^T$$

Question 4-a

In [17]:
```python
# Enable interactive rotation of graphs
%matplotlib widget

import numpy as np
from scipy.io import loadmat
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Load data for activity
X = loadmat('PCA_Activity.mat')['X']
rows, cols = np.array(X.shape)
x, y, z = X

print('Rows of X = ',rows)
print('Cols of X = ',cols)
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

ax.scatter(x, y, z, c='r', marker='o')

ax.set_xlabel('$x_1$')
ax.set_ylabel('$x_2$')
ax.set_zlabel('$x_3$')

plt.show()
```

```
Rows of X =  3
Cols of X =  100
```

Figure

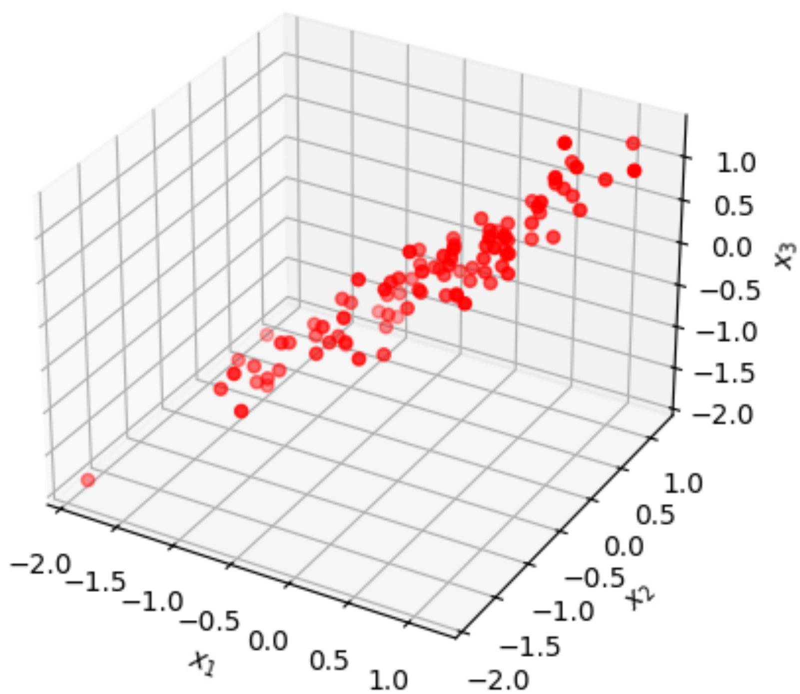


The graph seems pretty one dimensional, it all lies on a line, it doesnt not appear to be perfectly 0 mean though

Question 4-b

In [18]:
```
# Subtract mean
X_m = X - np.mean(X, 1).reshape((3,1))
x_m, y_m, z_m = X_m
# display zero mean scatter plot

fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

ax.scatter(x_m, y_m, z_m, c='r', marker='o')

ax.set_xlabel('$x_1$')
ax.set_ylabel('$x_2$')
ax.set_zlabel('$x_3$')

plt.show()
```

Figure



In [21]:
```
# Use SVD to find first principal component

U,s,VT = np.linalg.svd(X_m,full_matrices=False)

# complete the next line of code to assign the first principal component to
a = U[:, 0]

print(a)
```

```
[-0.58277194 -0.57701087 -0.57221964]
```

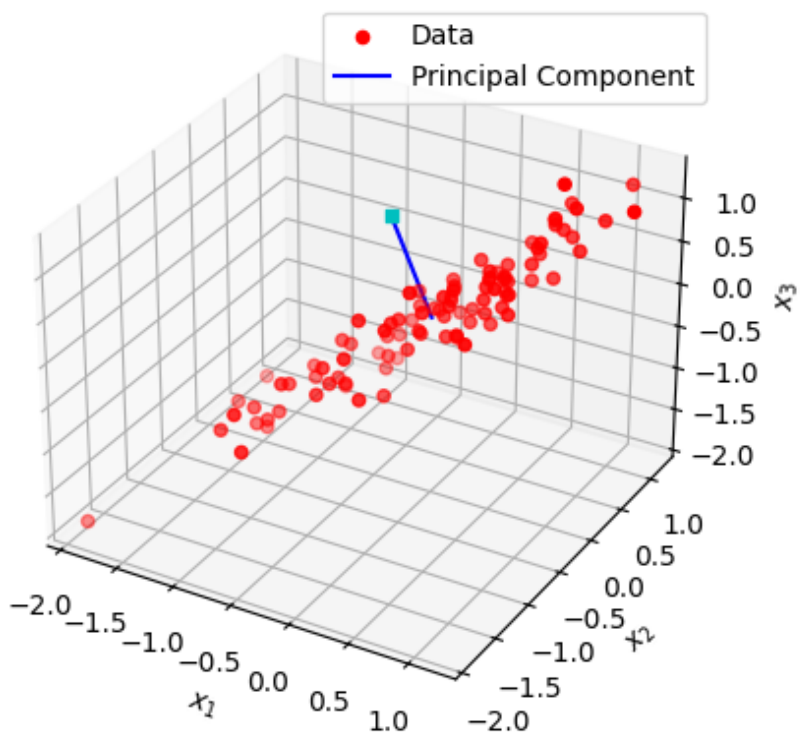
A one dimensional subspace is resonable to account for most of the error/variation

Question 2-c

In [24]:
```
# display zero mean scatter plot and first principal component

U,s,VT = np.linalg.svd(X,full_matrices=False)

# complete the next line of code to assign the first principal component to
a = U[:, 0]
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
```

```
ax.scatter(x_m, y_m, z_m, c='r', marker='o', label='Data')

ax.scatter(a[0],a[1],a[2], c='c', marker='s')

ax.set_xlabel('$x_1$')
ax.set_ylabel('$x_2$')
ax.set_zlabel('$x_3$')

ax.plot([0,a[0]],[0,a[1]],[0,a[2]], c='b',label='Principal Component')

ax.legend()
plt.show()
```

Figure



When we dont remove the mean the PC direction is not good at all for aligning wit the dominant feature direction