In [1]:
```python
import numpy as np
import math
import scipy.stats
from sklearn.metrics import roc_curve, auc
# makes printing more human-friendly
np.set_printoptions(precision=3,suppress=True)
```

In [2]:
```python
# Prior probabilities
P_pro = 0.1
P_amateur = 0.9

# Parameters for normal distribution
mean_pro = 2.5
std_dev_pro = 0.2
mean_amateur = 4
std_dev_amateur = 0.5
```

Question 1a

In [3]:
```python
def normal_distribution(x, mean, std_dev):
    """
    Compute the probability density function for the normal distribution.
    """
    p = (1 / (std_dev * np.sqrt(2 * np.pi))) * np.exp(-0.5 * ((x - mean) / s
    return p

t = np.array([2.5, 3., 3.5])
print('P(t|pro) \t', normal_distribution(t, mean_pro, std_dev_pro))
print('P(t|amateur) \t', normal_distribution(t, mean_amateur, std_dev_amateu
```

```
P(t|pro)         [1.995 0.088 0.   ]
P(t|amateur)     [0.009 0.108 0.484]
```

Question 1b

In [4]:
```python
def posterior_probability(time):
    """
    Compute the posterior probability of a runner being a 'pro' or 'amateur'
    """
    # Calculate likelihoods
    L_pro = normal_distribution(time, mean_pro, std_dev_pro)
    L_amateur = normal_distribution(time, mean_amateur, std_dev_amateur)

    P_time = L_pro * P_pro + L_amateur * P_amateur

    # Calculate posteriors
    P_pro_given_time = (L_pro * P_pro) / P_time                    # P(pro|t
    P_amateur_given_time = (L_amateur * P_amateur) / P_time        # P(amate

    return P_pro_given_time, P_amateur_given_time
```

```python
P_pro_given_time, P_amateur_given_time = posterior_probability(t)
print('P(pro|t) \t', P_pro_given_time)
print('P(amateur|t) \t', P_amateur_given_time)
```

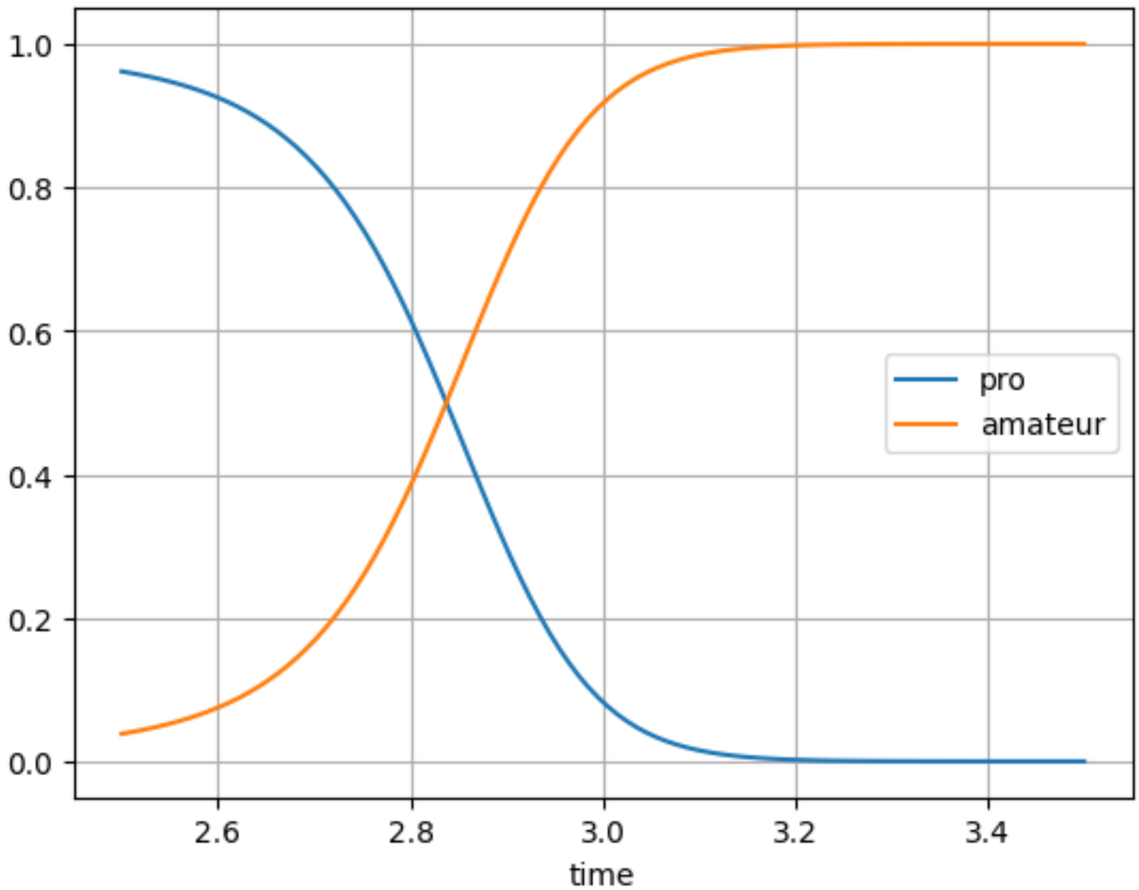
```
P(pro|t)         [0.962 0.083 0.   ]
P(amateur|t)     [0.038 0.917 1.   ]
```

Question 1c

In [5]:
```python
def classify_runner(time):
    """
    Classify a runner as 'pro' or 'amateur' based on their marathon time usi
    """
    # Calculate posteriors
    P_pro_given_time, P_amateur_given_time = posterior_probability(time)

    # Classify based on the highest posterior
    preds = []
    for i, t in enumerate(time):
        decision = "pro" if P_pro_given_time[i] > P_amateur_given_time[i] el
        preds.append(decision)
    return preds

# Test the function
print('P(pro|t) \t', classify_runner(t))
```

```
P(pro|t)         ['pro', 'amateur', 'amateur']
```

In [6]:
```python
from matplotlib import pyplot as plt
t = np.linspace(2.5, 3.5, 100)  # 100 times between 2.5 and 3.5
P_pro_given_time, P_amateur_given_time = posterior_probability(t)

plt.plot(t, P_pro_given_time, label='pro')
plt.plot(t, P_amateur_given_time, label='amateur')
plt.xlabel('time')
plt.grid('on')
plt.legend()
plt.show()
```



Question 1d

In [7]:
```
def sample_amateur_times(n):
    return np.random.randn(n)*std_dev_amateur+mean_amateur

def sample_pro_times(n):
    return np.random.randn(n)*std_dev_pro+mean_pro

n = 1000
amateur_times = sample_amateur_times(n)
decisions = classify_runner(amateur_times)
P_miss_cls = decisions.count('pro') / len(decisions)
print('Prob Misclassifying Amateurs', P_miss_cls)

pro_times = sample_pro_times(n)
decisions = classify_runner(pro_times)
P_miss_cls = decisions.count('amateur') / len(decisions)
print('Prob Misclassifying Pros', P_miss_cls)
```

```
Prob Misclassifying Amateurs 0.01
Prob Misclassifying Pros 0.043
```

Question 2a

In [8]:
```
num_spam_results = 19
```

```
num_spam_results
```

Out[8]: 19

Question 2b

In [9]:
```
P_TPR = 15 / (15 + 5)
P_TPR
```

Out[9]: 0.75

Question 2c

In [10]:
```
P_FPR = 4 / (4 + 16)
P_FPR
```

Out[10]: 0.2

Question 3a

b = 0.3
| Index (k) | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | |-------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------| | $P(y(k) = 1|x(k))$ | **0.05** | **0.15** | **0.40** | **0.55** | **0.25** | **0.45** | **0.48** | **0.62** | **0.67** | **0.75** | | True Label | **0** | **0** | **0** | **0** | **1** | **1** | **1** | **1** | **1** | **1** | | **Predicted Label (b=0.3)** | **0** | **0** | **1** | **1** | **0** | **1** | **1** | **1** | **1** | **1** |

Question 3b

$$C = \begin{bmatrix} TP & FN \\ FP & TN \end{bmatrix}$$

From the table:

- **TP:** 5 (indices: 6, 7, 8, 9, 10)
- **FP:** 2 (indices: 3, 4)
- **TN:** 2 (indices: 1, 2)
- **FN:** 1 (index: 5)

Thus, the confusion matrix:

$$C = \begin{bmatrix} 5 & 1 \\ 2 & 2 \end{bmatrix}$$

Question 3c

1. **Sensitivity:**
   $\text{TPR} = \frac{TP}{TP+FN} = \frac{5}{5+1} = \frac{5}{6} \approx 0.8333$

2. **Specificity:**
   $\text{TNR} = \frac{TN}{TN+FP} = \frac{2}{2+2} = \frac{2}{4} = 0.5$

3. **False Alarm:**
   $\text{FPR} = \frac{FP}{FP+TN} = \frac{2}{2+2} = \frac{2}{4} = 0.5$

4. **Miss:**
   $\text{FNR} = \frac{FN}{TP+FN} = \frac{1}{1+5} = \frac{1}{6} =\approx 0.167$

5. **Precision:**
   $\text{Precision} = \frac{TP}{TP+FP} = \frac{5}{5+2} = \frac{5}{7} \approx 0.714$

6. **Recall:**
   $\text{Recall} = \frac{TP}{TP+FN} = \frac{5}{5+1} = \frac{5}{6} \approx 0.8333$

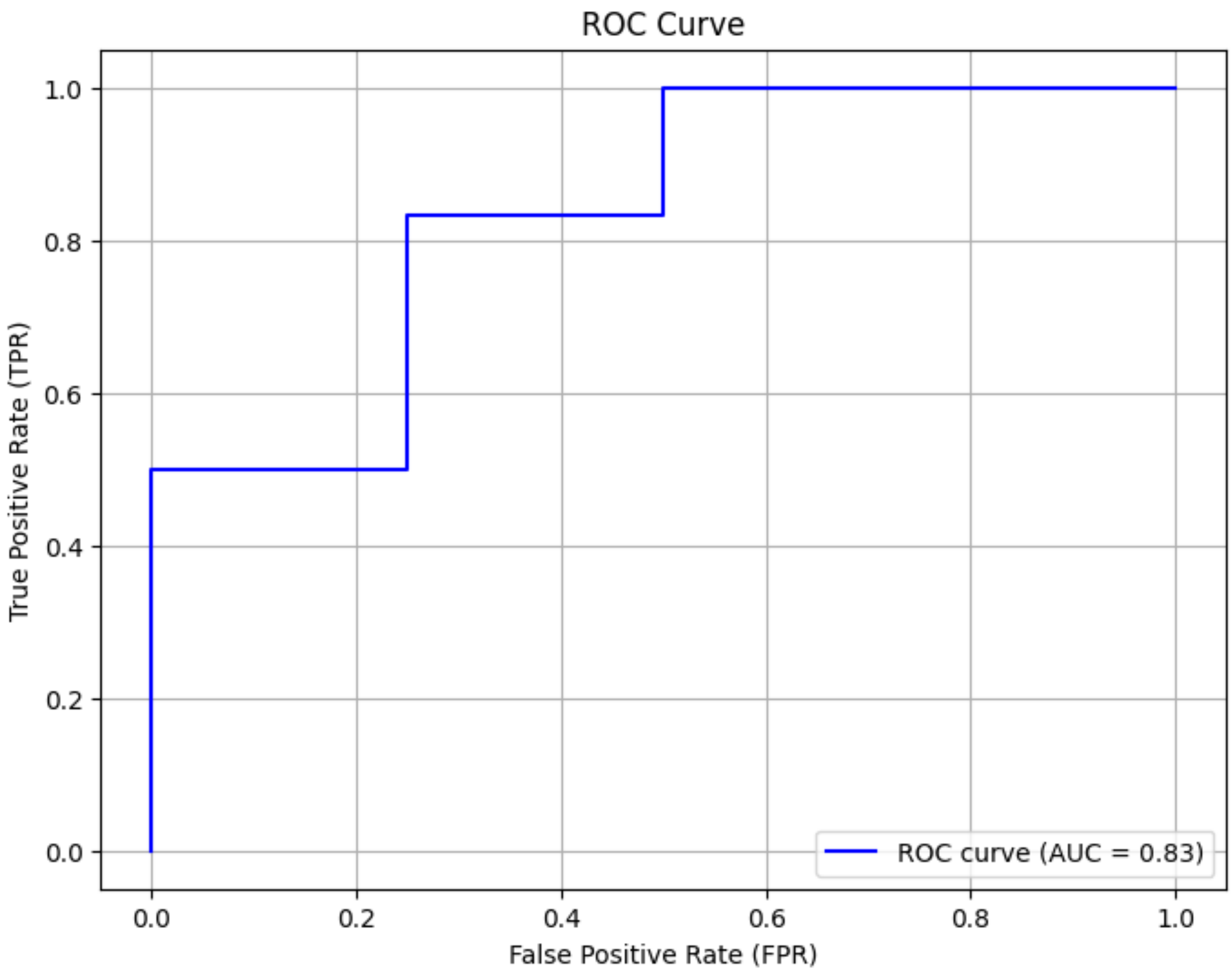
7. **Accuracy:**
   $\text{Accuracy} = \frac{TP+TN}{TP+TN+FP+FN} = \frac{5+2}{10} = \frac{7}{10} = 0.7$

Question 3d

In [11]:

```
P_y_given_x = [0.05, 0.15, 0.40, 0.55, 0.25, 0.45, 0.48, 0.62, 0.67, 0.75]
true_labels = [0, 0, 0, 0, 1, 1, 1, 1, 1, 1]

def calculate_tpr_fpr(P_y_given_x, true_labels, threshold):
    # y(k) = 0 if P (y(k) = 1|x(k)) ≤ b;
    predicted_labels = [1 if p > threshold else 0 for p in P_y_given_x]

    TP = sum((pred == 1) and (true == 1) for pred, true in zip(predicted_lab
    FP = sum((pred == 1) and (true == 0) for pred, true in zip(predicted_lab
    TN = sum((pred == 0) and (true == 0) for pred, true in zip(predicted_lab
    FN = sum((pred == 0) and (true == 1) for pred, true in zip(predicted_lab

    TPR = TP / (TP + FN) if (TP + FN) != 0 else 0
    FPR = FP / (FP + TN) if (FP + TN) != 0 else 0

    return TPR, FPR

thresholds = np.linspace(0, 1, 1000)
tpr_values = []
fpr_values = []

for threshold in thresholds:
    tpr, fpr = calculate_tpr_fpr(P_y_given_x, true_labels, threshold)
    tpr_values.append(tpr)
```

```
    fpr_values.append(fpr)

plt.figure(figsize=(8, 6))
plt.plot(fpr_values, tpr_values, color='b', label='ROC curve (AUC = {:.2f})'
plt.title('ROC Curve')
plt.xlabel('False Positive Rate (FPR)')
plt.ylabel('True Positive Rate (TPR)')
plt.legend(loc='lower right')
plt.grid(True)
plt.show()

roc_auc = auc(fpr_values, tpr_values)
print(f"Area Under the Curve (AUC): {roc_auc.round(4)}")
```




```
Area Under the Curve (AUC): 0.8333
```