Questions 1 and 3

In [1]:
```
import random
import numpy as np

# for easier reading np
np.set_printoptions(precision=3,suppress=True)
```

Question 1-a

$$W = [\, -M + .5 \quad 1 \quad 1 \quad \dots \quad 1 \,]$$

Question 1-b

$$W = [\, -.5 \quad 1 \quad 1 \quad \dots \quad 1 \,]$$

Question 1-c

$$W = [\, .5 \quad -1 \,]$$

Question 1-d

$$W = \left[\, \frac{M}{2} \quad 1 \quad 1 \quad \dots \quad 1 \,\right]$$

Question 2-a

In [45]:
```
# Data is just a grid of points in 2D
X1, X2 = np.meshgrid(
    np.linspace(-1, 1, 20),
    np.linspace(-1, 1, 20)
)
X = np.stack((X1.flatten(), X2.flatten()), axis=1)
print('X', X.shape)
```

```
X (400, 2)
```

In [46]:
```
# Part a) Specify weights and biases.
w1 = np.array([[5,2, -5 ],[-5, 5, -1]], dtype=np.float32)
b1 = np.array([[1, 5, 4]], dtype=np.float32)
w2 = np.array([[1],[1],[1]], dtype=np.float32)
b2 = np.array([-2.5], dtype=np.float32)
print('w1', w1.shape) # w1 should be a 2x3 matrix
print('b1', b1.shape) # b1 should be a 1x3 matrix or a 1d vector with 3 elem
print('w2', w2.shape) # w2 should be a 3x1 matrix
```

```
print('b2', b2.shape) # b2 should be a 1x1 matrix or a 1d vector with 1 elem
```

```
w1 (2, 3)
b1 (1, 3)
w2 (3, 1)
b2 (1,)
```

In [47]:

```
# Convert to your favorite library
import torch
X = torch.from_numpy(X).to(dtype=torch.float32)
w1 = torch.from_numpy(w1).to(dtype=torch.float32)
b1 = torch.from_numpy(b1).to(dtype=torch.float32)
w2 = torch.from_numpy(w2).to(dtype=torch.float32)
b2 = torch.from_numpy(b2).to(dtype=torch.float32)
```

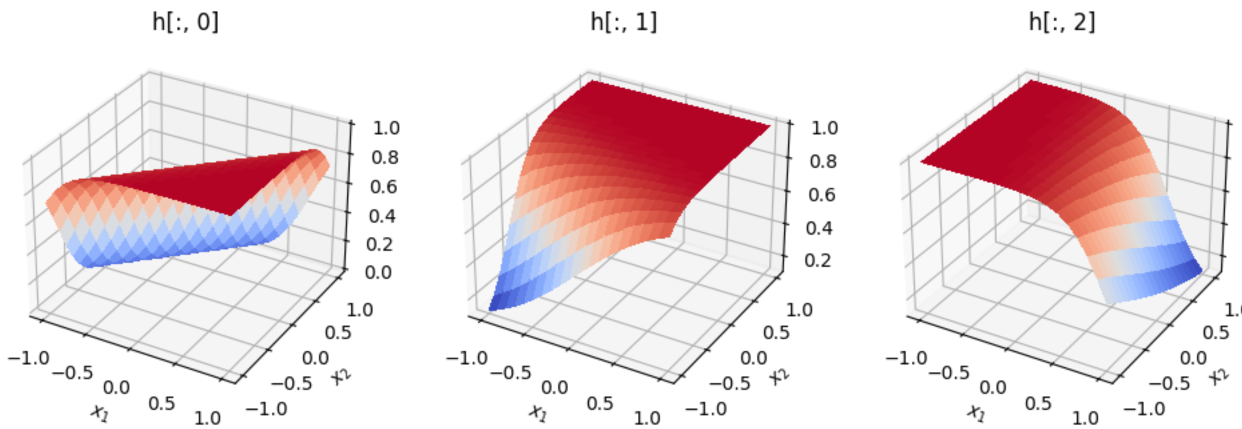
In [48]:

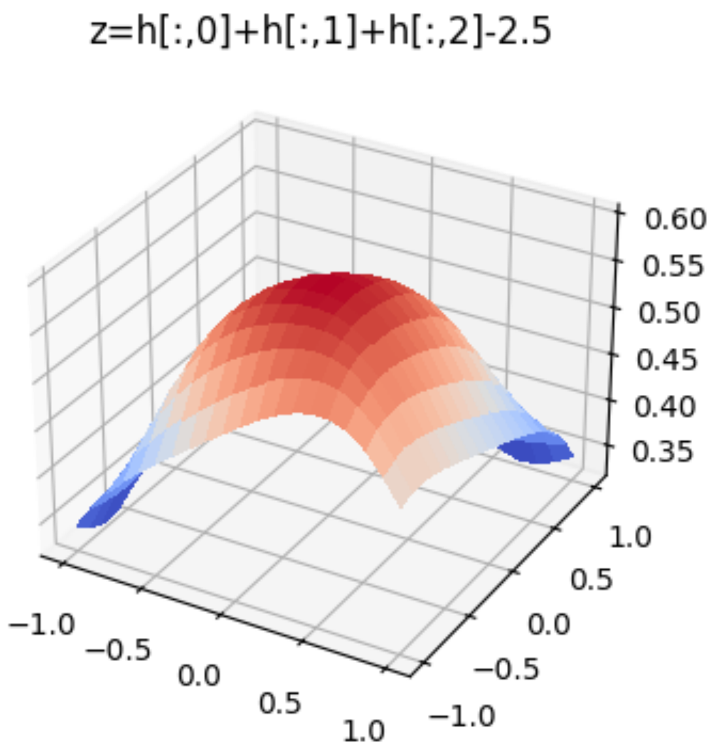
```
# Part b) Compute the output of the network for inputs given by X.

# Hints: pytorch. Refer to:
# torch.sigmoid
# '@' implements matrix multiplication

# Compute outputs for hidden layer
h = torch.sigmoid(X @ w1 + b1)
print('h', h.shape)  # h should be a 400x3 matrix

# Compute outputs for output layer
z = torch.sigmoid(h @ w2 + b2)
print('z', z.shape)  # z should be a 400x1 matrix
```

```
h torch.Size([400, 3])
z torch.Size([400, 1])
```

In [49]:

```
# Part c) Nothing to code.
from matplotlib import cm, pyplot as plt
fig, ax = plt.subplots(1, 3, figsize=(12, 4), subplot_kw={"projection": "3d"
ax[0].plot_surface(X1, X2, h[:, 0].view(20, 20),
                   cmap=cm.coolwarm, linewidth=0, antialiased=False)
ax[1].plot_surface(X1, X2, h[:, 1].view(20, 20),
                   cmap=cm.coolwarm, linewidth=0, antialiased=False)
ax[2].plot_surface(X1, X2, h[:, 2].view(20, 20),
                   cmap=cm.coolwarm, linewidth=0, antialiased=False)
for i in range(3):
    ax[i].set_title(f'h[:, {i}]')
    ax[i].set_xlabel('$x_1$')
    ax[i].set_ylabel('$x_2$')

fig, ax = plt.subplots(figsize=(4, 4), subplot_kw={"projection": "3d"})
ax.plot_surface(X1, X2, z[:, 0].view(20, 20),
                   cmap=cm.coolwarm, linewidth=0, antialiased=False)
ax.set_title('z=h[:,0]+h[:,1]+h[:,2]-2.5');
```



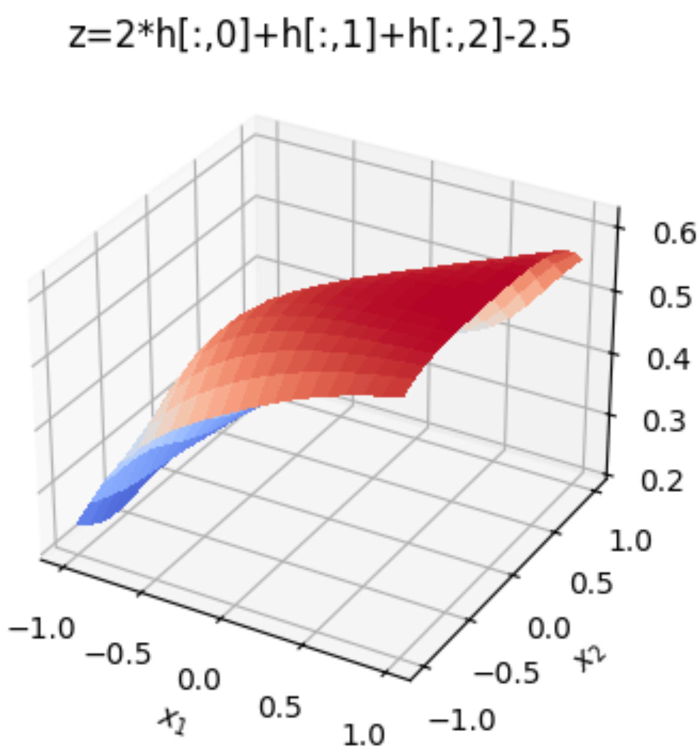


Question: How do the directions of increase of the hidden neurons relate to the weight matrix?

Answer:
For H1 when x_1 is postive and x_2 is negative the function is at a max
For H2 when x_1 is postive and x_2 is postive the function is at a max
For H3 when x_1 is negative and x_2 is negative the function is at a max

In [52]:
```
# Part d) How to increase output of the model at x=(1,-1) by changing the we
w1 = np.array([[5,2, 5 ],[-5, 5, -1]], dtype=np.float32)
b1 = np.array([[1, 5, 4]], dtype=np.float32)
w2 = np.array([[1],[1],[1]], dtype=np.float32)
b2 = np.array([-2.5], dtype=np.float32)

# Recompute outputs (reuse code from part b)
u1 = X@w1 + b1
h = torch.sigmoid(u1)
```

```python
u2 = h @ w2 + b2
z = torch.sigmoid(u2)

# Plot again to vizualize the results. Did the output at x=(1,-1) increased?
fig, ax = plt.subplots(figsize=(4, 4), subplot_kw={"projection": "3d"})
ax.plot_surface(X1, X2, z[:, 0].view(20, 20),
                   cmap=cm.coolwarm, linewidth=0, antialiased=False)
ax.set_title('z=2*h[:,0]+h[:,1]+h[:,2]-2.5');
ax.set_xlabel('$x_1$');
ax.set_ylabel('$x_2$');
```



We can make the weights going into the h[2] from x_1 neuron positive so that more positive weight is added from that neuron

In [ ]: