Problem 1: Polynomial model order determination

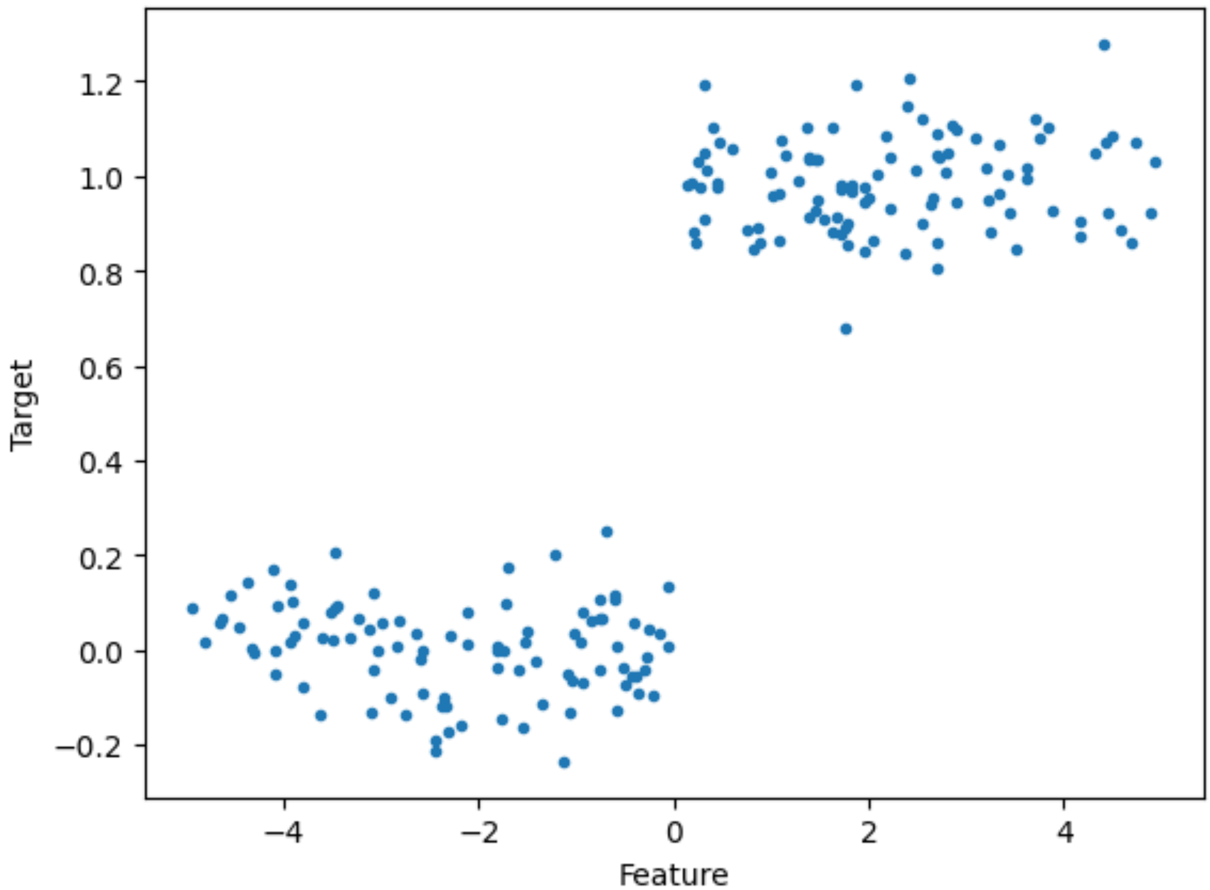
In [28]:
```
import matplotlib.pyplot as plt
import numpy as np
from sklearn.neighbors import KNeighborsClassifier
import sklearn.metrics as metrics
from sklearn.model_selection import cross_val_score, KFold
from sklearn.preprocessing import PolynomialFeatures
from sklearn.pipeline import make_pipeline
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import train_test_split
import random
import torch
from sklearn import datasets

# for easier reading np
np.set_printoptions(precision=3,suppress=True)
```

In [29]:
```
with open('./re_dat.csv', 'r') as f:
  X = np.genfromtxt(f,delimiter=',',skip_header=1)
  X, y = X[:,:-1], X[:,-1]

plt.plot(X[:, 0], y, '.')
plt.xlabel('Feature')
plt.ylabel('Target')
```

Out[29]:
```
Text(0, 0.5, 'Target')
```



Question 1-a

In [30]:
```
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.1)
```

Questoin 1-b

In [31]:
```
kf = KFold(n_splits=5, shuffle=True)
max_order = 15
best_score = -np.inf
best_p = 1

for p in range(1, max_order + 1):
    model = make_pipeline(PolynomialFeatures(p), LinearRegression())
    scores = cross_val_score(model, X_train.reshape(-1, 1), y_train, cv=kf,
    mean_score = np.mean(scores)

    if mean_score > best_score:
        best_score = mean_score
        best_p = p

print(f"Best polynomial order: P* = {best_p}, Validation R^2 = {best_score:.
```

```
Best polynomial order: P* = 15, Validation R^2 = 0.9095
```

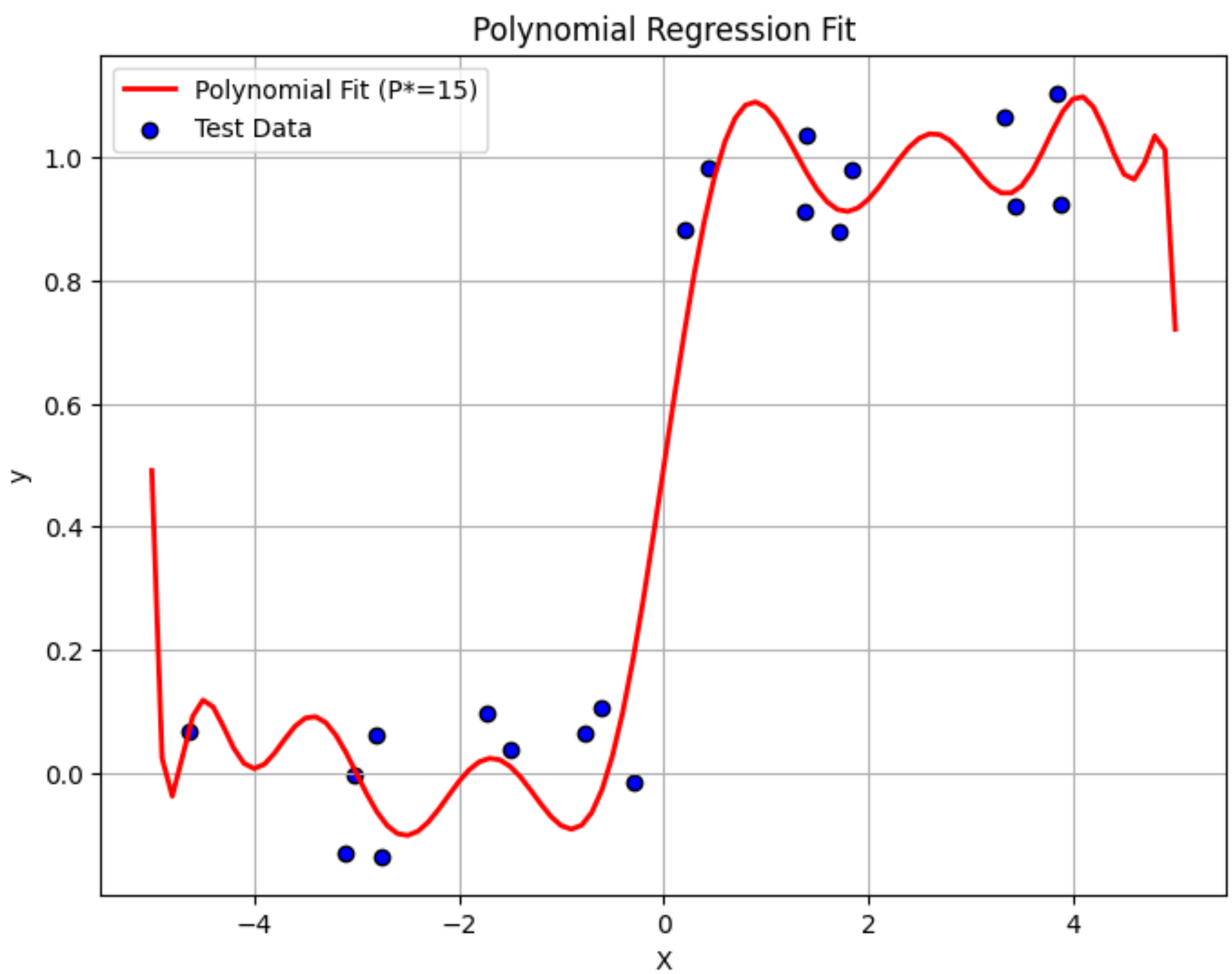
Questoin 1-c

In [32]:
```python
best_model = make_pipeline(PolynomialFeatures(best_p), LinearRegression())
best_model.fit(X_train.reshape(-1, 1), y_train)
r2_score_test = best_model.score(X_test.reshape(-1, 1), y_test)
print(f"Test R^2 score: {r2_score_test:.4f}")
```

```
Test R^2 score: 0.9529
```

In [33]:
```python
# Generate a smooth curve for the polynomial function
X_range = np.arange(-5, 5.1, 0.1).reshape(-1, 1)  # Grid from -5 to 5 with s
y_pred = best_model.predict(X_range)  # Predict values

# Plot the trained model
plt.figure(figsize=(8, 6))
plt.plot(X_range, y_pred, label=f"Polynomial Fit (P*={best_p})", color='red'
plt.scatter(X_test, y_test, label="Test Data", color='blue', edgecolor='blac
plt.xlabel("X")
plt.ylabel("y")
plt.title("Polynomial Regression Fit")
plt.legend()
plt.grid(True)
plt.show()
```



Question 2-a

In [34]:
```
y
```

Out[34]:
```
array([ 0.088,  0.018,  0.055,  0.068,  0.117,  0.048,  0.141,  0.002,
       -0.005,  0.17 , -0.051,  0.   ,  0.092,  0.015,  0.14 ,  0.103,
        0.029, -0.078,  0.057, -0.138,  0.024,  0.081,  0.021,  0.088,
        0.204,  0.092,  0.027,  0.064,  0.043, -0.131, -0.042,  0.122,
       -0.004,  0.058, -0.101,  0.006,  0.06 , -0.136,  0.035, -0.018,
       -0.094, -0.004, -0.19 , -0.213, -0.118, -0.099, -0.117, -0.173,
        0.029, -0.159,  0.011,  0.079,  0.   ,  0.009, -0.038, -0.148,
       -0.004,  0.096,  0.174, -0.043, -0.163,  0.017,  0.038, -0.023,
       -0.115,  0.202, -0.236, -0.051, -0.132, -0.064,  0.032,  0.014,
       -0.071,  0.078,  0.062,  0.065, -0.043,  0.105,  0.066,  0.251,
        0.106,  0.116,  0.005, -0.129, -0.037, -0.076, -0.056,  0.056,
       -0.056, -0.09 , -0.041, -0.016,  0.041, -0.095,  0.032,  0.008,
        0.132,  0.979,  0.987,  0.883,  0.861,  1.031,  0.975,  1.05 ,
        0.911,  1.191,  1.012,  1.105,  0.977,  0.984,  1.069,  1.056,
        0.888,  0.847,  0.89 ,  0.858,  1.006,  0.959,  0.963,  0.864,
        1.078,  1.044,  0.991,  1.102,  0.913,  1.041,  1.035,  1.035,
        0.927,  1.033,  0.949,  0.91 ,  0.88 ,  1.104,  0.915,  0.983,
        0.879,  0.97 ,  0.677,  0.891,  0.857,  0.899,  0.979,  0.967,
        1.194,  0.943,  0.975,  0.843,  0.952,  0.866,  1.003,  1.085,
        1.04 ,  0.93 ,  0.837,  1.146,  1.205,  1.012,  0.901,  1.12 ,
        0.941,  0.953,  1.089,  0.861,  0.804,  1.042,  1.04 ,  1.01 ,
        1.05 ,  1.108,  1.097,  0.943,  1.081,  1.017,  0.949,  0.881,
        1.065,  0.965,  1.005,  0.921,  0.845,  1.017,  0.994,  1.12 ,
        1.08 ,  1.105,  0.925,  0.906,  0.873,  1.05 ,  1.279,  1.073,
        0.923,  1.084,  0.887,  0.858,  1.072,  0.922,  1.032])
```

In [35]:
```
# Load Iris dataset
iris = datasets.load_iris()
X = torch.tensor(iris.data[:, :4], dtype=torch.float32)  # Use all 4 feature
y = torch.tensor((iris.target == 2).astype(float), dtype=torch.float32)  # B

# Split dataset
X_train, X_test, y_train, y_test = train_test_split(X, y, train_size=0.8, sh

# Data iterator
def data_iter(batch_size, features, labels):
    num_examples = len(features)
    indices = list(range(num_examples))
    random.shuffle(indices)
    for i in range(0, num_examples, batch_size):
        j = indices[i:i + batch_size]
        yield features[j], labels[j]

# Initialize model parameters
w = torch.nn.Parameter(torch.zeros((4, 1)), requires_grad=True)
b = torch.nn.Parameter(torch.zeros((1, 1)), requires_grad=True)
torch.nn.init.normal_(w, mean=0, std=0.01)

# Define the logistic regression model
def logistic_regression(X, w, b):
    return torch.sigmoid(X @ w + b)  # Sigmoid function for probability outp
```

```python
# Binary cross-entropy loss
def binary_cross_entropy(y_hat, y):
    return -torch.mean(y * torch.log(y_hat + 1e-8) + (1 - y) * torch.log(1 -

# Stochastic gradient descent optimizer
def sgd(params, grads, lr):
    for p, g in zip(params, grads):
        p.data -= lr * g

# Training setup
lr = 0.1
batch_size = 10
num_epochs = 50
net = logistic_regression
loss = binary_cross_entropy

# Train model
for epoch in range(num_epochs):
    for X_batch, y_batch in data_iter(batch_size, X_train, y_train):
        y_hat = net(X_batch, w, b).squeeze()  # Get predictions
        l = loss(y_hat, y_batch)  # Compute loss
        l.backward()  # Backpropagation
        sgd([w, b], [w.grad, b.grad], lr)  # Update weights
        w.grad = None
        b.grad = None

    # Evaluate model
    with torch.no_grad():
        y_train_pred = (net(X_train, w, b) > 0.5).squeeze().float()
        y_test_pred = (net(X_test, w, b) > 0.5).squeeze().float()
        train_acc = (y_train_pred == y_train).float().mean()
        test_acc = (y_test_pred == y_test).float().mean()

    print(f'Epoch {epoch+1:03d}, Train Accuracy: {train_acc:.3f}, Test Accur

# Final parameters
print('Intercept =', b.detach().numpy())
print('Coefficients =', w.detach().numpy())
```

```
Epoch 001, Train Accuracy: 0.667, Test Accuracy: 0.700
Epoch 002, Train Accuracy: 0.658, Test Accuracy: 0.700
Epoch 003, Train Accuracy: 0.667, Test Accuracy: 0.700
Epoch 004, Train Accuracy: 0.658, Test Accuracy: 0.700
Epoch 005, Train Accuracy: 0.942, Test Accuracy: 0.967
Epoch 006, Train Accuracy: 0.992, Test Accuracy: 0.967
Epoch 007, Train Accuracy: 0.775, Test Accuracy: 0.800
Epoch 008, Train Accuracy: 0.950, Test Accuracy: 0.967
Epoch 009, Train Accuracy: 0.933, Test Accuracy: 0.967
Epoch 010, Train Accuracy: 0.933, Test Accuracy: 0.933
Epoch 011, Train Accuracy: 0.783, Test Accuracy: 0.833
Epoch 012, Train Accuracy: 0.942, Test Accuracy: 0.967
Epoch 013, Train Accuracy: 0.933, Test Accuracy: 0.833
Epoch 014, Train Accuracy: 0.850, Test Accuracy: 0.833
```

```
Epoch 015, Train Accuracy: 0.925, Test Accuracy: 0.967
Epoch 016, Train Accuracy: 0.917, Test Accuracy: 0.833
Epoch 017, Train Accuracy: 0.950, Test Accuracy: 0.933
Epoch 018, Train Accuracy: 0.992, Test Accuracy: 0.933
Epoch 019, Train Accuracy: 0.967, Test Accuracy: 0.933
Epoch 020, Train Accuracy: 0.900, Test Accuracy: 0.933
Epoch 021, Train Accuracy: 0.950, Test Accuracy: 0.933
Epoch 022, Train Accuracy: 0.950, Test Accuracy: 1.000
Epoch 023, Train Accuracy: 0.950, Test Accuracy: 1.000
Epoch 024, Train Accuracy: 0.925, Test Accuracy: 0.967
Epoch 025, Train Accuracy: 0.975, Test Accuracy: 0.933
Epoch 026, Train Accuracy: 0.983, Test Accuracy: 0.933
Epoch 027, Train Accuracy: 0.950, Test Accuracy: 0.933
Epoch 028, Train Accuracy: 0.958, Test Accuracy: 0.933
Epoch 026, Train Accuracy: 0.983, Test Accuracy: 0.933
Epoch 027, Train Accuracy: 0.950, Test Accuracy: 0.933
Epoch 028, Train Accuracy: 0.958, Test Accuracy: 0.933
Epoch 029, Train Accuracy: 0.950, Test Accuracy: 1.000
Epoch 030, Train Accuracy: 0.992, Test Accuracy: 0.933
Epoch 031, Train Accuracy: 0.967, Test Accuracy: 0.933
Epoch 032, Train Accuracy: 0.992, Test Accuracy: 0.933
Epoch 033, Train Accuracy: 0.992, Test Accuracy: 0.933
Epoch 034, Train Accuracy: 0.967, Test Accuracy: 0.933
Epoch 035, Train Accuracy: 0.933, Test Accuracy: 0.867
Epoch 036, Train Accuracy: 0.933, Test Accuracy: 0.867
Epoch 037, Train Accuracy: 0.983, Test Accuracy: 0.933
Epoch 038, Train Accuracy: 0.975, Test Accuracy: 1.000
Epoch 039, Train Accuracy: 0.933, Test Accuracy: 0.933
Epoch 040, Train Accuracy: 0.950, Test Accuracy: 0.933
Epoch 041, Train Accuracy: 0.975, Test Accuracy: 0.933
Epoch 042, Train Accuracy: 0.975, Test Accuracy: 0.933
Epoch 043, Train Accuracy: 0.967, Test Accuracy: 0.933
Epoch 044, Train Accuracy: 0.975, Test Accuracy: 0.933
Epoch 045, Train Accuracy: 0.933, Test Accuracy: 0.967
Epoch 046, Train Accuracy: 0.992, Test Accuracy: 0.933
Epoch 047, Train Accuracy: 0.975, Test Accuracy: 1.000
Epoch 048, Train Accuracy: 0.992, Test Accuracy: 0.933
Epoch 049, Train Accuracy: 0.950, Test Accuracy: 0.933
Epoch 050, Train Accuracy: 0.975, Test Accuracy: 0.933
Intercept = [[-0.86]]
Coefficients = [[-1.777]
 [-1.607]
 [ 2.696]
 [ 2.288]]
```

Question 3-a

$$X = \begin{bmatrix} 1 & 0.5 & 0.8 \\ 1 & -1 & 0.6 \end{bmatrix}$$

Question 3-b

$$z = Xw = \begin{bmatrix} 1 & 0.5 & 0.8 \\ 1 & -1 & 0.6 \end{bmatrix} \begin{bmatrix} 0.4 \\ -1.1 \\ 0.6 \end{bmatrix} = \begin{bmatrix} 0.33 \\ 1.86 \end{bmatrix}$$

Question 3-c

$$I(\mathbf{w}) = \frac{1}{2K} \sum_{k=1}^{K} (u(k) - y(k))$$

$$I(\mathbf{w}) = \frac{1}{4}\left[(0.33-1)^2 + (1.86-0)^2\right] = \frac{1}{4}[0.4489 + 3.4596] = \frac{3.9085}{4} = 0.977125$$

Question 3-d

$$\mathbf{e} = \begin{bmatrix} f(u(1)) - y(1) \\ f(u(2)) - y(2) \end{bmatrix} = \begin{bmatrix} 0.33 - 1 \\ 1.86 - 0 \end{bmatrix} = \begin{bmatrix} -0.67 \\ 1.86 \end{bmatrix}$$

Question 3-e

$$p = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Question 3-f

$$\nabla_{\mathbf{w}} I(\mathbf{w}) = \frac{1}{K} \mathbf{X}^T (\mathbf{e} \odot \mathbf{p})$$

$$\mathbf{e} \odot \mathbf{p} = \begin{bmatrix} -0.67 \\ 1.86 \end{bmatrix} \odot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.67 \\ 1.86 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 1 & 1 \\ 0.5 & -1 \\ 0.8 & 0.6 \end{bmatrix} \begin{bmatrix} -0.67 \\ 1.86 \end{bmatrix} = \begin{bmatrix} 0.595 \\ -1.0975 \\ 0.29 \end{bmatrix}$$