In [2]:
```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn.linear_model import LinearRegression
from sklearn.metrics import r2_score

# makes printing more human-friendly
np.set_printoptions(precision=3, suppress=True)
```

In [3]:
```python
data1 = {
    'x': np.array([10, 8, 13, 9, 11, 14, 6, 4, 12, 7, 5])[:, None],
    'y': np.array([8.04, 6.95, 7.58, 8.81, 8.33, 9.96, 7.24, 4.26, 10.84, 4.
}

data2 = {
    'x': np.array([10, 8, 13, 9, 11, 14, 6, 4, 12, 7, 5])[:, None],
    'y': np.array([9.14, 8.14, 8.74, 8.77, 9.26, 8.10, 6.13, 3.10, 9.13, 7.2
}

data3 = {
    'x': np.array([10, 8, 13, 9, 11, 14, 6, 4, 12, 7, 5])[:, None],
    'y': np.array([7.46, 6.77, 12.74, 7.11, 7.81, 8.84, 6.08, 5.39, 8.15, 6.
}

data4 = {
    'x': np.array([8, 8, 8, 8, 8, 8, 8, 19, 8, 8, 8])[:, None],
    'y': np.array([6.58, 5.76, 7.71, 8.84, 8.47, 7.04, 5.25, 12.50, 5.56, 7.
}
```

In [4]:
```python
# 1a) Fit the data using the LinearRegression model
model1 = LinearRegression()
model1.fit(data1['x'], data1['y'])
print(model1.coef_, model1.intercept_)
```

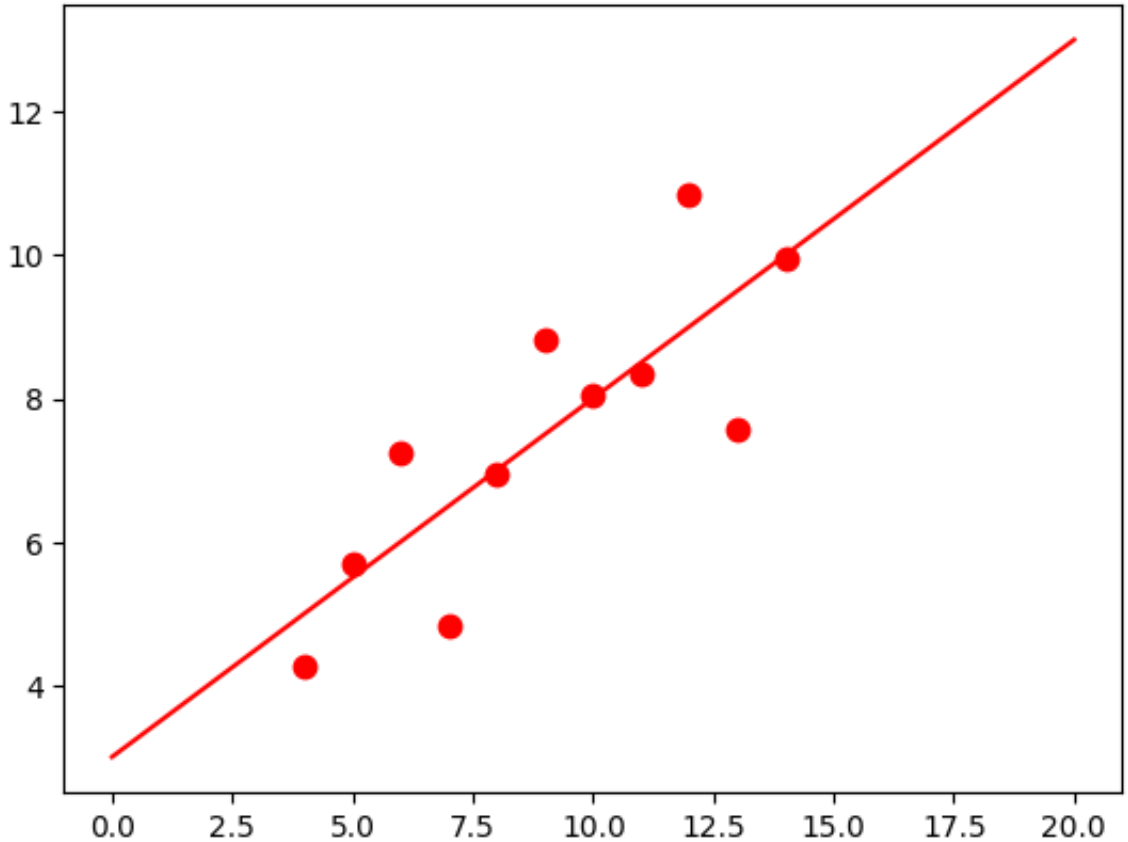
```
[0.5] 3.0000909090909103
```

In [5]:
```python
# 1b) Predict and measure R2 scores
yhat1 = model1.predict(data1['x'])   # Refer to LinearRegression.predict()
rsqr1 = r2_score(data1['y'], yhat1)   # Compute R-square between ground-trut
w1, b1 = model1.coef_, model1.intercept_
print(f'Model 1: y = {w1[0]:.2f}x + {b1:.2f} \t R2 = {rsqr1:.2f}')
```

```
Model 1: y = 0.50x + 3.00        R2 = 0.67
```

In [6]:
```python
# 1c) Plot the data and predictions
x_range = np.linspace(0, 20, 100)[:, None]
plt.plot(data1['x'], data1['y'], 'r.', ms=15);
plt.plot(x_range, model1.predict(x_range), 'r-')
```

Out[6]:
```
[<matplotlib.lines.Line2D at 0x16115e390>]
```

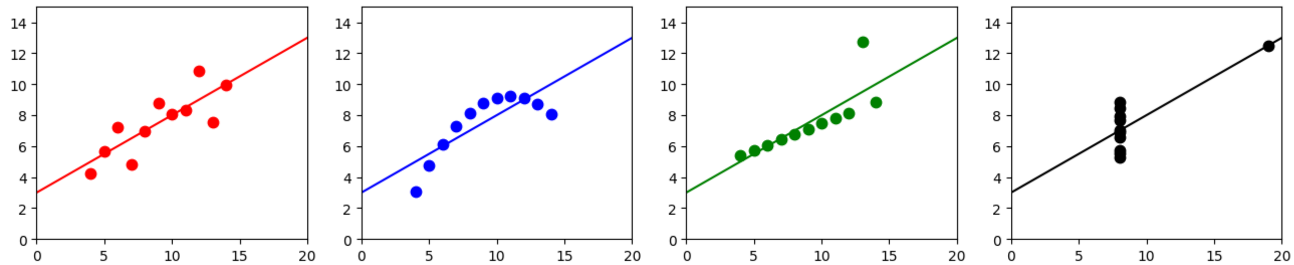



In [7]:
```
# 1d) Compute the linear regression for other datasets
model2 = LinearRegression().fit(data2['x'], data2['y'])
yhat2 = model2.predict(data2['x'])
rsqr2 = r2_score(data2['y'], yhat2)
w2, b2 = model2.coef_, model2.intercept_
print(f'Model 2: y = {w2[0]:.2f}x + {b2:.2f} \t R2 = {rsqr2:.2f}')

model3 = LinearRegression().fit(data3['x'], data3['y'])
yhat3 = model3.predict(data3['x'])
rsqr3 = r2_score(data3['y'], yhat3)
w3, b3 = model3.coef_, model3.intercept_
print(f'Model 3: y = {w3[0]:.2f}x + {b3:.2f} \t R2 = {rsqr3:.2f}')

model4 = LinearRegression().fit(data4['x'], data4['y'])
yhat4 = model4.predict(data4['x'])
rsqr4 = r2_score(data4['y'], yhat4)
w4, b4 = model4.coef_, model4.intercept_
print(f'Model 4: y = {w4[0]:.2f}x + {b4:.2f} \t R2 = {rsqr4:.2f}')
```

```
Model 2: y = 0.50x + 3.00        R2 = 0.67
Model 3: y = 0.50x + 3.00        R2 = 0.67
Model 4: y = 0.50x + 3.00        R2 = 0.67
```

In [8]:
```
# e) Plot all datasets and linear regression line
x_range = np.linspace(0, 20, 100)[:, None]

f, ax = plt.subplots(1, 4, figsize=(16, 3))
ax[0].plot(data1['x'], data1['y'], 'r.', ms=15)
```

```
ax[0].plot(x_range, model1.predict(x_range), 'r-')
ax[1].plot(data2['x'], data2['y'], 'b.', ms=15)
ax[1].plot(x_range, model2.predict(x_range), 'b-')
ax[2].plot(data3['x'], data3['y'], 'g.', ms=15)
ax[2].plot(x_range, model3.predict(x_range), 'g-')
ax[3].plot(data4['x'], data4['y'], 'k.', ms=15)
ax[3].plot(x_range, model4.predict(x_range), 'k-')
for axes in ax:
    axes.set_xlim(0, 20);
    axes.set_ylim(0, 15);
```



# Discuss observations

The $R^2$ values are the same for all models
Not all models seem like good fits, for example data2 looks like an exponential curve that could be fit better
I would not be able to tell if linear regression is a good fit without plotting
A exponential model would better fit data2

# Question 4a-d

## Question 4a:

A logistic regression model estimates the probability that a given input $x$ belongs to the positive class $Y = 1$ using the **sigmoid function**:

$$P(Y = 1|x) = \sigma(w^T x + b) = \frac{1}{1 + e^{-(w^T x + b)}}$$

where:

- $w$ is the vector of coefficients,
- $b$ is the bias term,
- $x$ is the input feature vector,
- $\sigma(z) = \frac{1}{1+e^{-z}}$ is the sigmoid function.

## Question 4b:

In cases where the cost of a **false negative** is much higher than that of a **false positive**, we should **lower** the decision threshold $p_{\text{thr}}$.

- The default threshold is $p_{\text{thr}} = 0.5$.
- Lowering $p_{\text{thr}}$ (e.g., to 0.3) means the model will classify more samples as **positive**, reducing false negatives.
- This ensures that more potentially positive cases (such as patients with a serious medical condition) are flagged.

## Question 4c:

The decision boundary is the set of points where the model predicts the probability to be exactly ( p_{\text{thr}} ):

$$P(Y = 1|x) = p_{\text{thr}}$$

Substituting the sigmoid function:

$$\frac{1}{1 + e^{-(w^T x + b)}} = p_{\text{thr}}$$

Rearrange:

$$1 + e^{-(w^T x + b)} = \frac{1}{p_{\text{thr}}}$$

$$e^{-(w^T x + b)} = \frac{1}{p_{\text{thr}}} - 1$$

Taking the natural logarithm on both sides:

$$-(w^T x + b) = \ln\left(\frac{1}{p_{\text{thr}}} - 1\right)$$

$$w^T x + b = -\ln\left(\frac{1}{p_{\text{thr}}} - 1\right)$$

Since this equation is of the form $w^T x + b = \text{constant}$, it represents a **hyperplane**, confirming that the decision boundary is always linear.

## Question 4d:

For a 2D feature space where $x = (x_1, x_2)$, the decision boundary equation is:

$$w_1 x_1 + w_2 x_2 + b = -\ln\left(\frac{1}{p_{\text{thr}}} - 1\right)$$

Solving for $x_2$ in terms of $x_1$:

$$x_2 = -\frac{w_1}{w_2} x_1 - \frac{b + \ln\left(\frac{1}{p_{\text{thr}}} - 1\right)}{w_2}$$

Comparing with the standard line equation $x_2 = m x_1 + c$, we get:

- **Slope**: $m = -\frac{w_1}{w_2}$
- **Intercept**: $c = -\frac{b + \ln\left(\frac{1}{p_{\text{thr}}} - 1\right)}{w_2}$

This equation defines a straight line, confirming the linear decision boundary.