ECE/CS/ME 539: Intro to Artificial Neural Networks
Homework 2

Problem 1: Maximum A Posteriori Classification

(4pts) Suppose you want to classify marathon runners participating on the next Madison marathon into two categories – professionals (pro), and amateurs (amateur) – based on their marathon times. After some research, you found that marathon times among pros follow a Gaussian distribution with mean $\mu_{\text{pro}} = 2.5$ and standard deviation $\sigma_{\text{pro}} = 0.2$ and amateurs times follow a Gaussian distribution with mean $\mu_{\text{amateur}} = 4$ and standard deviation $\sigma_{\text{amateur}} = 0.5$. Furthermore, you know that in the Madison marathon, only about 10% of the runners are professional. You may find `scipy.stats.norm` useful to solve the problem computationally.

(a) **(1pt)** Compute the likelihood of three participant's times provided that they were either amateurs or pros.

Participant's time (t)	2.5	3	3.5
$P(t\|\text{pro})$			
$P(t\|\text{amateur})$			

(b) **(1pt)** For each of the three participants in the table above, compute the posterior probabilities $P(\text{pro}|t)$ and $P(\text{amateur}|t)$.

Participant's time (t)	2.5	3	3.5
$P(\text{pro}\|t)$			
$P(\text{amateur}\|t)$			

(c) **(1pt)** Apply the MAP decision rule to decide which participants are pros and which are amateurs.

Participant's time (t)	2.5	3	3.5
MAP Decision			

(d) **(1pt)** Estimate the probability of misclassification among amateurs and among pro runners through simulation. To accomplish this, begin by sampling $N = 1000$ marathon times for each class from the underlying Gaussian distributions (sampling function is already provided in file "hw2_starter_code.ipynb"). After classification, count the number of mistakes made by the MAP classifier.

Problem 2: Confusion Matrix

(3pts) A new test is developed to detect spam emails. After an experiment, the confusion matrix of this test is reported below:

Actual / Predicted	**P**	**N**
P	15	5
N	4	16

(a) **(1pt)** How many spam (P) results does this new test report? _____

(b) **(1pt)** What percentage of actual spam emails are correctly identified as spam in this test? _____

(c) **(1pt)** What is the false positive rate, defined as the fraction of products that are reported as defective but are actually non-defective, among all negative tests? _____

Problem 3: Performance Metrics

(3pts) Consider 10 feature-label pairs $(x(k), y(k); 1 \leq k \leq 10)$. Assume that $y(k) \in \{0, 1\}$. The posterior probability $P(y(k) = 1|x(k))$ is given in the 2nd row of the table below, and the corresponding ground truth label is given in the third row. Here, a class label = 0 means Negative, and a class label = 1 means Positive.

Index (k)	1	2	3	4	5	6	7	8	9	10
$P(y(k) = 1\|x(k))$	0.05	0.15	0.40	0.55	0.25	0.45	0.48	0.62	0.67	0.75
True Label	0	0	0	0	1	1	1	1	1	1
Predicted label $y(k)$										

Given a threshold b, we set the predicted label $y(k) = 0$ if $P(y(k) = 1|x(k)) \leq b$; and $= 1$ otherwise.

(a) **(0.5pt)** If $b = 0.3$, fill in the predicted label in the 4th row of the above table.

(b) **(0.5pt)** Compute the confusion matrix C with $b = 0.3$.

(c) **(1pt)** With $b = 0.3$, compute the following quantities: sensitivity (sen), specificity (spe), Pr. False Alarm (pfa), Pr. Miss (pmiss), precision (pre), recall, and accuracy.

(d) **(1pt)** For the value of threshold b varying from 0 to 1, compute the list of distinct pairs of (TPR, FPR) and then plot the ROC curve and calculate the area under the ROC curve (AUC).