

# Gapless Superconductivity Overview

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## Abstract

This document contains the brief overview of the gapless superconductivity results shown in Maki's review.

## 1 Introduction

Superconducting energy gap - it's energy range of suppressed density of the electrons states around Fermi energy, this feature usually considered as key attribute of the superconductivity. Abrikosov and Gor'kov had discovered the "gapless behavior" during the study of the effect of magnetic impurities on superconductivity.

Common aspects:

1. The external perturbation breaks the time-reversal symmetry of the electron system.
2. Dissipation mechanism, mixing the time-reversed states is involved.

## 2 Time-reversal symmetry

Superconductivity is understood as the correlation of electrons in pairs formed with mutually time-reversed states(i.e., opposite momenta and spins). Term "time-reversal symmetry", used here, referring to the symmetry of the electron system.

### 2.1 4D-Representation

Let's use a spinor representation of the single-particle (or hole) state as follows:

$$\Psi(x) = \begin{pmatrix} \psi_{\uparrow}(x) \\ \psi_{\downarrow}(x) \\ \psi_{\uparrow}^{\dagger}(x) \\ \psi_{\downarrow}^{\dagger}(x) \end{pmatrix}, \text{ and } \Psi^{\dagger}(x) = \begin{pmatrix} \psi_{\uparrow}^{\dagger}(x) & \psi_{\downarrow}^{\dagger}(x) & \psi_{\uparrow}(x) & \psi_{\downarrow}(x) \end{pmatrix} \quad (1)$$

In terms of these field operators the BCS Hamiltonian is written.

$$H_0 = \sum_{\sigma} \int \psi_{\sigma}^{\dagger} \left( -\frac{1}{2m} \nabla^2 - \mu \right) \psi_{\sigma} d^3x + V \int (\psi^{\dagger}(x)\psi(x))(\psi^{\dagger}(x)\psi(x)) d^3x \quad (2)$$