Gapless Superconductivity Overview

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Abstract

This document contains the brief overview of the gapless superconductivity results shown in Maki's review.

1 Introduction

Superconducting energy gap - it's energy range of supressed density of the electrons states around Fermi energy, this feature usually considered as key attribute of the superconductivity. Abrikosov and Gor'kov had discovered the "gapless behavior" during the study of the effect of magnetic impurities on superconductivity.

Common aspects:

1. The external perturbation breaks the time-reversal symmetry of the electron system.
2. Dissipation mechanism, mixing the time-reversed states is involved.

2 Time-reversal symmetry

Superconductivity is understood as the corellation of electrons in pairs formed with mutually time-reversed states(i.e., opposite momenta and spins). Term "time-reversal symmetry", used here, referring to the symmetry of the electron system.

2.1 4D-Repersentation

Reduced Hamiltonian:

$$H = \sum_{k,\sigma} \epsilon_k n_{k\sigma} + \sum_{k,l} V_{k,l} c^{\dagger}_{k,\uparrow} c^{\dagger}_{-k,\downarrow} c_{-l,\uparrow} c_{l,\downarrow} \tag{1}$$

Assuming that states for the fermions are given:

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \text{ and } |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2}$$

Than operator of quantity of the particles in state s should be given:

$$n_s = n_{k\sigma} = c^\dagger_{k,\sigma} c_{k,\sigma} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{3}$$

Creation, annihilation operators are:

$$c^\dagger_{k,\sigma} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \text{ and } c_{k,\sigma} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{4}$$

Field operators of these states should be given as the Fourier-transform.

$$\psi^\dagger(x) = \int \frac{d^3k}{(2\pi)^{\frac{1}{2}}} e^{-ikx} c^\dagger_k, \text{ and } \psi(x) = \int \frac{d^3k}{(2\pi)^{\frac{1}{2}}} e^{ikx} c_k \tag{5}$$

Let's use a spinor representation of the single-particle (or hole) state as follows:

$$\Psi(x) = \begin{pmatrix} \psi_\uparrow(x) \\ \psi_\downarrow(x) \\ \psi^\dagger_\uparrow(x) \\ \psi^\dagger_\downarrow(x) \end{pmatrix}, \text{ and } \Psi^\dagger(x) = \begin{pmatrix} \psi^\dagger_\uparrow(x) & \psi^\dagger_\downarrow(x) & \psi_\uparrow(x) & \psi_\downarrow(x) \end{pmatrix} \tag{6}$$

Hamiltonian in terms of the field operators.

$$\mathcal{H}_0 = \sum_{\sigma=\uparrow,\downarrow} \int \psi^\dagger_\sigma \left(-\frac{1}{2m} \nabla^2 - \mu \right) \psi_\sigma d^3x + \mathrm{V} \int (\psi^\dagger(x)\psi(x))(\psi^\dagger(x)\psi(x)) d^3x \tag{7}$$