Theoretical part

3.1

1) $n^2+10n\cdot\log n+50n+100$

$n^2+10n\log n+50n+100 \leq n^2+10n^2+50n^2+100n^2 \leq$

$\leq 161n^2$

$n^2+10n\log n+50n+100 \leq 161n^2 \quad , \quad c=161 \ \forall n \geq 1$

$n^2+10n\log n+50n+100 = O(n^2)$

2) $n^{\frac{7}{2}}+7n^3\log n+n^2$

$n^3\sqrt{n}+7n^3\log n+n^2 \leq n^3\sqrt{n}+7n^3\sqrt{n}+n^3\sqrt{n} \leq 9n^3\sqrt{n}$

$n^3\sqrt{n} \geq n^3\log n \geq n^2$

$n^3\sqrt{n}+7n^3\log n+n^2 \leq 9n^3\sqrt{n} \quad , \quad c=9 \ , \ \forall n \geq 1$

$n^3\sqrt{n}+7n^3\log n+n^2 = O(n^{\frac{7}{2}})$

3) $6^{n+1}+6(n+1)!+24n^{42}$

$6\cdot6^n+6(n+1)!+24n^{42}$

For $n \geq 5$: $(n+1)! \geq 6^{n-4}\cdot5!$

For $n \geq 1295$: $(n+1)! \geq 6^{n-4}\cdot5!\cdot(6)\cdot(36)\cdot(216)$

$(n+1)! \geq 6^{n+2}\cdot5!$

$6\cdot6^n+6(n+1)!+24n^{42} \leq 36(n+1)! \quad , \quad c=36 \ \forall n \geq 1295$

$6\cdot6^n+6(n+1)!+24n^{42} = O((n+1)!)$

Answer : 1) $O(n^2)$ 2) $O(n^{\frac{7}{2}})$ 3) $O((n+1)!)$

3.2

1) First of all, I checked complexity of checkpoint

a) for (int i = 1 ; i < number ; i = i * 3) - goes $\lceil \log_3 n \rceil$ times and once we check that i ≥ number

Therefore : times = $\lceil \log_3 n \rceil + 1$

b) System . out . print - goes $\lceil \log_3 n \rceil$ times

checkpoint:

cost	times
C_4	$\log_3 n + 1$
C_5	$\log_3 n$

2) For calculate Complexity:

a) for (int i=n; i>0 ; i--)

goes n and +1 to check that i = 0 => n+1

b) for (int j=0; j<i; j++)

goes 1, 2 ..., n and + 1 to check

Therefore : $\sum_{i=2}^{n+1} t_i$

c) checkpoint (n)

goes $\sum_{i=1}^{n}$

Therefore : Times = $\left(\sum_{i=1}^{n} t_i\right)(c_4 \cdot \log_3 n + c_5 \log_3 n + c_4)$

calculate C:

cost	times
C_1	$n+1$
C_2	$\sum_{i=2}^{n+1} t_i$
C_3	$\left(\sum_{i=1}^{n} t_i\right)(C_4 + C_5) \cdot (\log_3 n) + \left(\sum_{i=1}^{n} t_i\right) \cdot (+C_4)$

Complexity : $C_1 \cdot (n+1) + C_2\left(\sum_{i=2}^{n+1} t_i\right) + C_3\left(\sum_{i=1}^{n} t_i\right) \cdot (C_4 \log_3 n + C_5 \log_3 n + C_4) = n+1 + \frac{(n+1)(n+2)}{2} - 1 + \frac{n(n+1)}{2}(2 \log_3 n + 1) = O(n^2 \log n)$

Answer: $O(n^2 \log n)$

3.3

$$T(n) = \sqrt{k}\, T\left(\frac{n}{k^2}\right) + c\cdot\sqrt[4]{n} \qquad T(1)=0$$

$$a=\sqrt{k},\quad b=k^2;\quad f(n)=c\cdot\sqrt[4]{n}$$

$$\log_b a = \log_{k^2}(\sqrt{k}) = \frac{1}{4}\log_k k = \frac{1}{4}$$

$$f(n) = c\cdot\sqrt[4]{n} = c\cdot n^{\frac{1}{4}}$$

$f(n) = \Theta(n^{\frac{1}{4}})$ Reflexive Property

Second case of Master Theorem

Therefore: $T(n) = \Theta(n^{\log_b a}\log n) = \Theta(n^{\frac{1}{4}}\log n)$

Answer: $\Theta(n^{\frac{1}{4}}\log n)$