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Question ①

f = flux variable. ϕ = potential variable.
s = Source / Sink

:Could be wrong:

$\nabla \cdot f - s = 0 \cdots$ equ 1

$f = -\alpha \nabla \phi \cdots$ equ 2 Δf = high − low.

α = material conductivity.

$\nabla \cdot (\alpha \nabla \phi) + s = 0$

@ 'a' being homogenous and $s = 0$.

$\nabla^2 \phi = 0$

a.

ⓘ steady state groundwater flow;

flux variable (f): Darcy flux (q) [m/s]

Potential variable (ϕ): Hydraulic head (m)

Material parameter (α): Hydraulic Conductivity (K) (m/s)

ⓘⓘ steady state Electrical conduction:

flux variable (f): Current density (J) A/m²

Potential variable (ϕ): Electric potential (V) (volt)

Material parameter (α): Electric conductivity (σ) (S/m)

b.

Isotropy: properties are the same in all directions.

Homogeneity: Properties are the same throughout the material.

In anisotropic "α" becomes a tensor and in the heterogenous case "α" varies with position.

c

ⓘ ① Equipotentials.

These are lines along a surface where the total hydraulic head is constant. There is no difference in potential energy in the flow direction.

② Flow lines;

These are paths along which water flows. They indicate the direction of water movement. They generally tell about direction of seepage and ground water movement.

(ii) Equipotentials and Flow lines are always assumed to be perpendicular to one another ensuring that water flows from one equipotential to the next along the flow lines. They are also contain uniform Spacing ensuring we have a uniform potential difference across the flow net and around the region we are observing.

(iii) (a) Impermeable boundary (No Flow):

This is a point where there is no flow and flow lines run perpendicular to the boundary.

(b) Free surface:

The boundary representing open water surface where our equipotential lines run parallel to the surface.

(c) Constant head

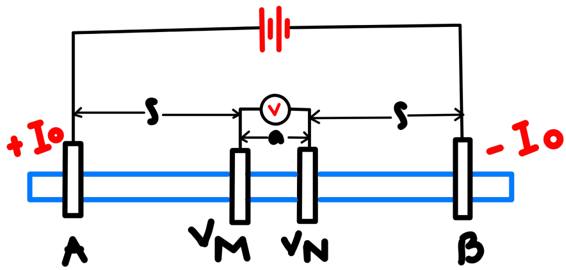
(d) Fixed potential

(e) Source/Sink

(iv) * Homogenous and isotropic material or condition.

Q2 Schlumberger Array:

a



Deriving geometric factor:

$$V_M = V_{AM} + V_{BM} = \frac{I_0 \rho_a}{2\pi r_{AM}} - \frac{I_0 \rho_a}{2\pi r_{BM}}$$

$$V_N = V_{AN} + V_{BN} = \frac{I_0 \rho_a}{2\pi r_{AN}} - \frac{I_0 \rho_a}{2\pi r_{BN}}$$

$$\Delta V = V_M - V_N = \left(\frac{I_0 \rho_a}{2\pi r_{AM}} - \frac{I_0 \rho_a}{2\pi r_{BM}}\right) - \left(\frac{I_0 \rho_a}{2\pi r_{AN}} - \frac{I_0 \rho_a}{2\pi r_{BN}}\right)$$

$$= \frac{I_0 \rho_a}{2\pi}\left(\frac{1}{r_{AM}} - \frac{1}{r_{BM}}\right) - \frac{I_0 \rho_a}{2\pi}\left(\frac{1}{r_{AN}} - \frac{1}{r_{BN}}\right)$$

$$= \frac{I_0 \rho_a}{2\pi}\left(\frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}}\right)$$

Secluding $I_0 \rho_a$

$$\frac{\Delta V}{I_0 \rho_a} = \frac{1}{2\pi}\left(\frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}}\right)$$

$$\frac{\Delta V}{I_0 \rho_a} = G \quad \therefore \quad G_{Schlumb} = \frac{1}{2\pi}\left(\frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}}\right)$$

$$G = \frac{1}{2\pi}\left(\frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}}\right)$$

$$G = \frac{1}{2\pi}\left(\frac{1}{s} - \frac{1}{s+a} - \frac{1}{s+a} + \frac{1}{s}\right)$$

$$G = \frac{1}{\pi}\left(\frac{a}{s(s+a)}\right)$$

$z = 0,\ z = S = 2a$

@ $z = 0$, $r = 2.5a$

recall that $J_r = \dfrac{I_0}{2\pi (r)^2}$

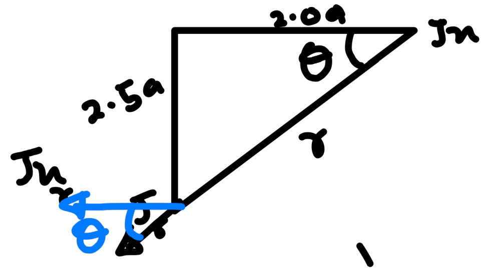
$$\therefore J_r = \frac{I_0}{2\pi (2.5)^2}$$

@ $z = S = 2a$, $r = ?$

$$r^2 = (2.5a)^2 + (2a)^2$$

$$r^2 = (6.25a^2) + (4a^2)$$

$$r^2 = 10.25a^2$$



$$J_x = \frac{I_0 \cos\theta}{2\pi (r)^2}$$

$J_x (z \cdots) \cos\theta$

Calculating θ

$$\tan\theta = \frac{2a}{2.5a}$$

$$\theta = \tan^{-1}\left\{ \tfrac{2}{2.5} \right\}$$

$$\theta = 38.6598083$$

$$\boxed{J_x = J_r \cos\theta \qquad \frac{J_x}{J_r} = \cos\theta}$$

$$J_x = \frac{I_0 \cos(38.65)}{2\pi (10.25a^2)}$$

$$\frac{J_x (z=2a)}{J_x (z=0)} = \frac{\dfrac{I_0 \cos(38.65)}{2\pi (10.25a^2)}}{\dfrac{I_0}{2\pi (2.5a^2)}}$$

$$\frac{J_x (z=S) \cos\theta}{J_x (z=0)} = \frac{\cancel{I_0} \cos(38.65)}{\cancel{2\pi}(10.25a^2)} \times \frac{\cancel{2\pi}(2.5a^2)}{\cancel{I_0}}$$

$$= \frac{2.5\cancel{a^2} \cos(38.65)}{10.25\cancel{a^2}}$$

$$= \frac{2.5 \cos(38.65)}{10.25}$$

$$= 0.19048$$

$z = 0,\ z = s = 15a.$

@ $t = 0$, $r = 15.5a$ recall that $J_r = \dfrac{I_o}{2\pi (r)^2}$

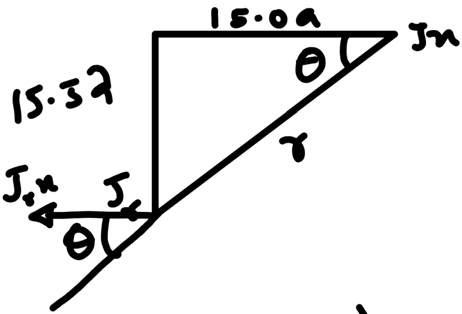
$\therefore J_n = \dfrac{I_o}{(15.5a)^2} =$

@ $z = s = 15a$, $r = 15.5a$

$r^2 = (15a)^2 + (15.5a)^2$

$r^2 = 225a^2 + 240.25a^2$

$r^2 = 465.25a^2$



$J_n = \dfrac{I_o \cos\theta}{2\pi (r)^2}$

$J_n(z \ldots) \cos\theta$

$\dfrac{r}{n} = \cos\theta = \dfrac{2I\cos\theta}{2\pi r^2}$

Calculating θ

$\tan\theta = \dfrac{15\cancel{a}}{15.5\cancel{a}}$

$\theta = \tan^{-1}\left\{\dfrac{15}{15.5}\right\}$

$\theta = 44.06°$

$$\boxed{J_n = J_r \cos\theta \qquad \dfrac{J_n}{J_r} = \cos\theta}$$

$J_n = \dfrac{I_o \cos(44.06)}{2\pi\ (465.25a^2)}$

$\dfrac{J_n(z=s)}{J_n(z=0)} = \dfrac{\dfrac{I_o \cos(44.06)}{2\pi\ (465.25a^2)}}{\dfrac{I_o}{2\pi\ (15.5a^2)}}$

$\dfrac{J_n(z=s)\cos\theta}{J_n(z=0)} = \dfrac{\cancel{I_o}\cos(44.06)}{\cancel{2\pi}\ (465.25a^2)} \times \dfrac{\cancel{2\pi}(15.5a^2)}{\cancel{I_o}}$

$= \dfrac{15.5\cancel{a^2}\cos(44.06)}{465.25\cancel{a^2}}$

$= \dfrac{15.5\cos(44.06)}{465.25}$

$= 0.0239$

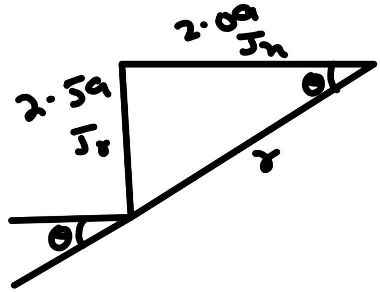
$z = z = s = 2a$

$z = 0,\ r = 2.5a$ $\qquad J_r = \dfrac{I_0}{2\pi r^2}$

$$J_n = \frac{2I_0}{2\pi r^2} \quad \therefore \quad J_n = \frac{\cancel{2} I_0}{\cancel{2}\pi (2.5a)^2} = \frac{I_0}{\pi\, 6.25a^2}$$

$z = 2.0a,\ r = ?$

Using pythagoras theorem:



$$r^2 = (2.5a)^2 + (2.0a)^2$$

$$r^2 = (6.25a^2) + (4a^2)$$

$$r^2 = 10.25a^2$$

$$\frac{n}{r} = \cos\theta = \frac{2I_0 \cos\theta}{2\pi r^2}$$

$$= \frac{\cancel{2} I_0 \cos\theta}{\cancel{2}\pi (10.23a^2)}$$