ICPC Workshop 1

Graph Theory

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Internet: Statement

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5 4
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4 6
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Sample Output

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Constraints

$N, M \leq 200000$

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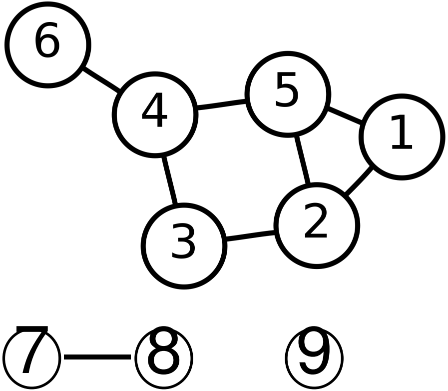
Diagram



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Representing graphs

- A graph is an abstraction of the town, as simply a set of objects in which some pairs of the objects are in some sense “related”
- Houses correspond to **nodes** and cables correspond to **edges**
- How can we represent a graph mathematically? Computationally?
- Adjacency list
  - For each node, store a list (vector in C++) of adjacent nodes
  - Implementation: see code

Depth-first search

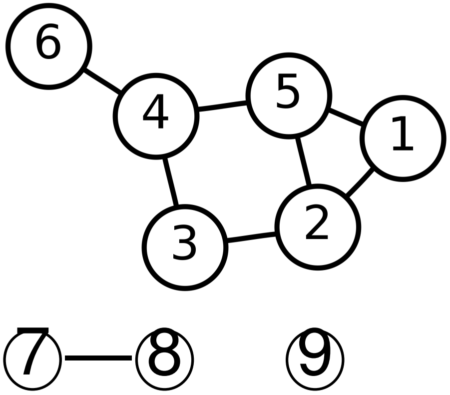
- To "process" a node, just "process" each of its neighbours
- But never "process" a node more than once
- Implementation: see code

Theorem (Fundamental Theorem of DFS)

A DFS initiated at a node u will process a node v exactly when there exists a path between u and v.

- So our problem can be solved by running a DFS from node 1, then checking which nodes have been processed. We'll come back to how exactly to code a solution up and submit it.

DFS Walkthrough



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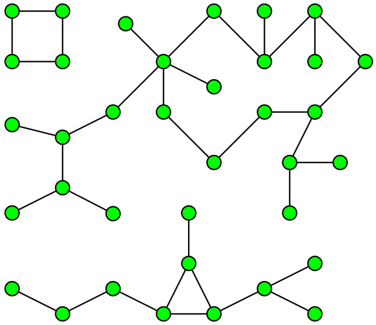
Example



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Thinking time

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- Implementation

Lab

- Join the vjudge group: https://vjudge.net/group/unswicpc
- Go to the contest for this workshop
- If you need help, or don't know what to do, message me or Angus
- **A: A+B** — solve this first if you haven't used vjudge before
- **B: Internet** — implement the first problem from today
- **C: Cards** — implement the third problem from today
- **D: Paradox** — second problem from today (two colouring), but a bit harder
- **E: Maze** and **F: Graph** — harder problems
- Angus will go over Graph at 1:40