Exercise: Pass-through MBS pricing and duration

Consider a MBS pass through with principal $600 million, the original mortgage pool has a WAM = 360 months, and WAC = 6.5%, and the pass-through security pays a coupon equal to r^{PT}_{12} = 6%, lower than the average coupon rate of the mortgage pool, both to ensure there is enough cash available for coupon payments, and also to provide a compensation for the MBS issuer (e.g., Fannie Mae or Freddie Mac)

Task 1) Compute the value and the duration (ignoring changes in PSA) of the pass-through security.

Notes:

- We can use the PSA level to determine the speed of prepayment, and therefore the timing and size of future cash flows
- In particular, given a PSA level, for instance 200% PSA, we obtain the CPR_t for each month t and thus the corresponding monthly prepayment rate: $p_t = 1 - (1 - CPR_t)^{1/12}$
- Given that the PSA level determines exactly the amount of principal that is paid back, we can compute the value of the pass-through security by first computing the sequence of cash flows, and then, treating these as certain cash flows from a highly rated company, we discount them to today using the appropriate discount rate
- Agency MBS are essentially default risk free, implying that the coupons will be paid to the investors

- To compute the sequence of cash flows, consider a given time t during the life of the mortgage pool in which L_t is the outstanding principal at the beginning of the period. From this value, we can compute the following quantities for time t:

Mortgage interest payment:	I_t	=	$L_t \times r^m_{12} / 12$
Scheduled principal:	Pay^{sch}_t	=	$C_t - I_t$
Principal payment:	Pay^{prp}_t	=	$p_t \times L_t$

- Given the scheduled principal payments and prepayments, we can finally update both the outstanding principal and the total coupon flow at the beginning of the following month t +1:
  - Outstanding principal: $L_{t+1} = L_t - Pay^{sch}_t - Pay^{prp}_t$
  - Update of scheduled coupon: $C_{t+1} = (1 - p_t) \times C_t$
- In particular, the new total flow from the pool coupons equals the previous month coupon flow adjusted for the fraction of prepaid mortgages. For instance, if 100% of homeowners prepay their mortgages at time t, then $p_t = 1$, and the coupon at time t+1 is zero, as we would expect. Conversely, if nobody repays the mortgages, then $p_t = 0$, and $C_{t+1} = C_t$, that is, the total coupon flow is constant.
- If we think of these securities having zero or small default risk, then we can use the Treasury discount curve to discount these cash flows. Assume for instance a flat term structure with constant (c.c.) 5% yield.

Task 2: Accounting for the fact that changes in interest rates affect prepayment levels, compute the effective duration of the pass-through security.

We so far assumed that the current PSA level of PSA = 200% is unaffected by the change in interest rate. In this case, because the pass-through MBS has a constant coupon, we can compute its duration as the weighted average time to receive cash flows.

Consider now the case in which if the interest rate moves down from 5% to 4.50%, the PSA increases from 200% to 250%, while if the interest rate moves up from 5% to 5.50%, the PSA decreases from 200% to 150%

We can compute the duration of the security by using its definition

$$D = -\frac{1}{P}\frac{dP}{dr}$$

Which allows us to approximate duration while taking into consideration changes in PSA levels as

$$D = -\frac{1}{P}\frac{P(+50bps) - P(-50bps)}{2 \times 50bps}$$

where *P* is the current value of the pass-through security and *P*(+50*bps*) and *P*(−50*bps*) are the values of the same pass-through security when we increase and decrease the interest rate by 50 basis points, respectively, and the PSA levels accordingly.

Task 3: To visualize the convexity of the security, plot its value as a function assuming a) a constant PSA of 200% b) a PSA that decreases in interest rates in line with the numbers below.

PSA=200		Variable PSA		
Int. Rate	Value	Int. Rate	PSA	Value
2.0%		2.0%	500	
3.0%		3.0%	400	
4.0%		4.0%	300	
5.0%		5.0%	200	
6.0%		6.0%	100	
7.0%		7.0%	80	
8.0%		8.0%	60	