Practica_19_Casas_Mercade

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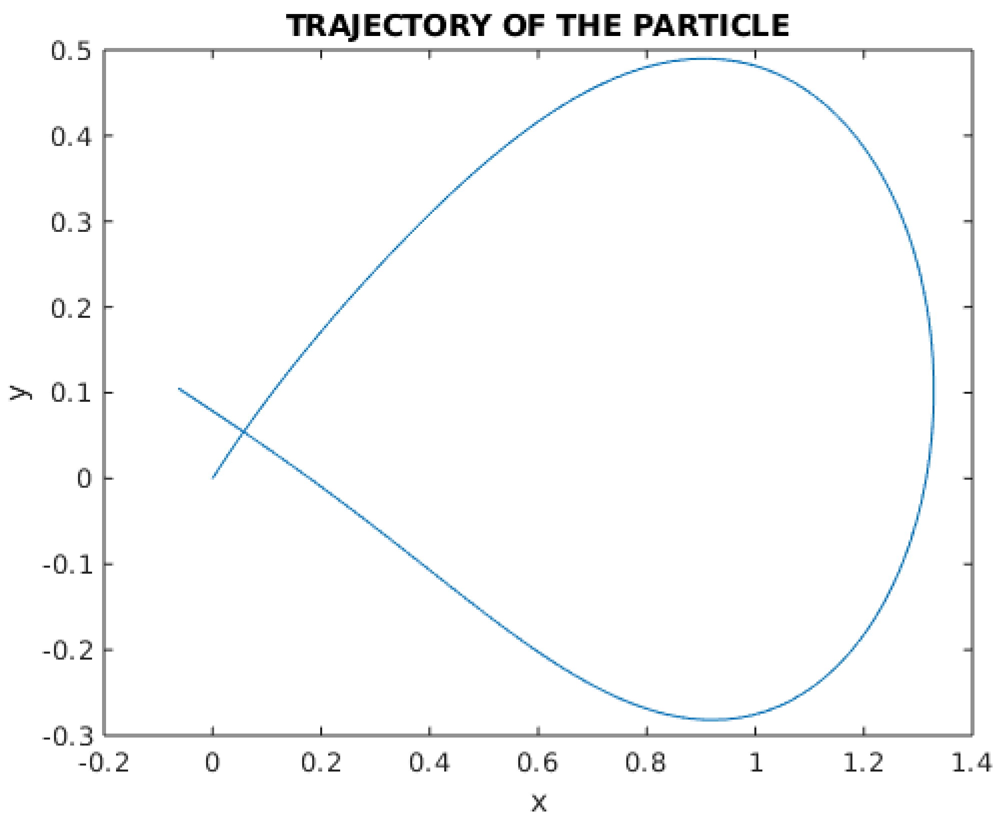
Section A

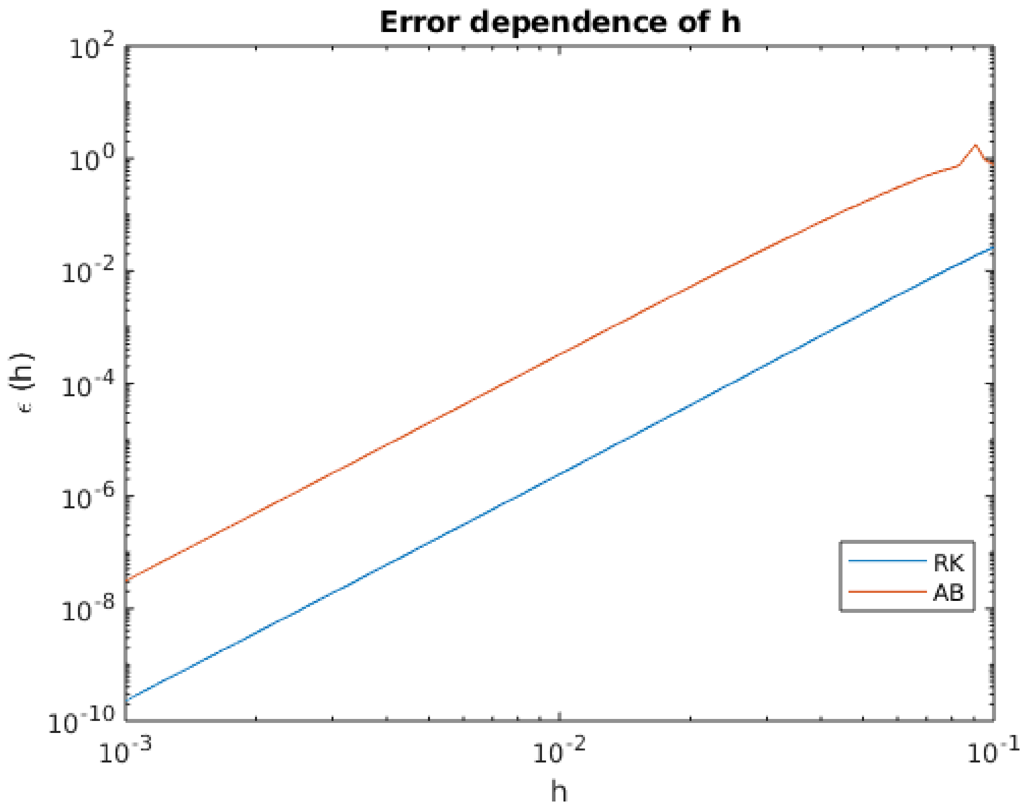
```
% x = (v,r)
% fun: (v,r) --> (dr/dt, dv/dt)
format long g;
close all;
clc;
clear all;

h = 1e-2;
time = 2;
points = time / h + 1;
initial = [0, 0, 1, 1]';

%With RK4 we get the addition steps for the AB4
solutionRK = RK4(initial, h, @gravFunctionV, points);
aditionalSteps = solutionRK(:, 1:4);

%With AB4 we get the the positions and velocities of m during the
 first two
%seconds since its launch
solutionAB = AB4(aditionalSteps, h, @gravFunctionV, points);
figure;
plot(solutionAB(1, :), solutionAB(2, :));
title('TRAJECTORY OF THE PARTICLE')
xlabel('x')
ylabel('y')

% To calucate the error we first calculate the exact solution, which
 we
% will consider that is the one obtained for h=1e-4
hext = 1e-4;
points = time / hext +1;
exactSolutionRK = RK4(initial, hext, @gravFunctionV, points);
rext = exactSolutionRK(1:2, end);

%Now we find the position at t=2 for values of h bigger than
 hext=1e-4, and
%calculate the diffrence with the exact solution. We do this with RK4
 and
```

```
%AB4 to see the order of the error obtained with each method.

errorsRK = [];
errorsAB = [];
hs = [];
t=2;
% The maximum and the minimum h:
minSteps = t/1e-1;
maxSteps = t/1e-3;

% We will test for every h possible:
for steps = minSteps:1:maxSteps
    points = steps + 1;
    h = t/steps;
    hs = [hs h];
    solutionRK = RK4(initial, h, @gravFunctionV, points);
    r = solutionRK(1:2, end);
    errorsRK = [errorsRK norm(r - rext)];

    %As done before, we use the first three steps provided by RK plus
    %the intial condition
    solutionAB = AB4(solutionRK(:, 1:4), h, @gravFunctionV, points);
    r2 = solutionAB(1:2, end);
    errorsAB = [errorsAB norm(r2 - rext)];
end


figure;
loglog(hs, errorsRK)
hold on
loglog(hs, errorsAB)
hold off
title('Error dependence of h')
xlabel('h')
ylabel('\epsilon (h)')
legend('RK', 'AB', 'Location', 'best');

% Codes:
%{
  function v = RK4(vn0, h, fun, desiredPoints)
    % Algoritme per resoldre ODEs de PVI.
    %
    % Inputs:
    %   vn0:introduim vn0 columna
    %   h: increment de temps. Estara equiespaiat
    %   fun: Funcio f que dona la derivada: dv/dt = f(t, v(t))
    %   desiredPoints: nombre de punts que treura (comptant el que ja
 li
    %   donem). Es en realitat desiredPoints = steps-1 on steps son
 els pasos que fara.
    % Outputs:
    %   v: matriu amb els punts com a columnes
```

```
    % Prellocating memory to gain speed
    v = zeros(length(vn0), floor(desiredPoints));
    v(:,1) = vn0;
    %v = [vn0];
    vn = vn0;

    for i = 1:desiredPoints-1 % Si li demanem un punt fara 0
 iteracions
        a = h * fun(vn);
        b = h * fun(vn +a / 2);
        c = h * fun(vn + b / 2);
        d = h * fun(vn + c);

        vn1 = vn + (1/6).* (a + 2*b + 2*c + d);
        v(:, i+1) = vn1; %cada columna es un "punt"
        vn = vn1;
    end
end


function V = AB4(vn, h, fun, desiredPoints)
    % Algoritme multistep per solucionar ODEs de IVP. Més rapid que
 RK4.
    % Input:
    %   vn: [vn, vn+1, vn+2, vn+3] matriu de vectors columna (els 4
 punts anteriors al desitjat). Calculats amb RK4 per exemple.
    %       ELS 4 RESULTATS HAN DETAR SEPARATS H.
    %   h: incrament de temps equiespaiats
    %   fun: Funcio f que dona la derivada: dv/dt = f(t, v(t)). "Part
 dreta de una edo de mm1"
    %   desiredPoints: nombre de punts que treura (comptant el que ja
 li
    %       donem).
    % Outputs:
    %   v: matriu amb els punts com a columnes


    % Utilitzem els coeficients ja calculats a clase:
    % b0 = -3/8; b1 = 37/24 ; b2 = -59/24; b3 = 55/24;
    betas = [-3/8; 37/24; -59/24; 55/24];
    V = vn;

    % Per evitar evaluar la fun multiples vegades farem un cua en un
 vector: (FIFO: first in first out)
    queue = [fun(V(:, end - 3)), fun(V(:, end - 2)), fun(V(:, end -
 1)), fun(V(:, end))];

    for i = 1:desiredPoints-4
        vNext = V(:, end) + h .* (queue*betas); % Que era en realitat:
 (betas(1) .* fun(V(:, end - 3)) + betas(2) .* fun(V(:, end - 2)) +
 betas(3) .* fun(V(:, end - 1)) + betas(4) .* fun(V(:, end)))
        %Actualizem cua: el rendiment sera pobre pero es problema del
 matlab:
        queue(:, 1) = [];
```

```
        a = fun(vNext);
        queue(1:4, end+1) = fun(vNext);
        V = [V, vNext]; %cada columna es un punt (x, y,vx,vy) i per
 tant V sera una matriu de 4 files
    end
end


%}


% Questions:
% Looking at the logaritmic plot of the error we can see that
% AB4 requires h = 0.008 to reach the precision of 10^-4 and
% RK4 needs h 0.25.

% RK4 should be slower because it requires 4 evaluations of the
 function while
% AB4 only needs one evaluation of the function with the
 optimitzations done.
```





Section B)

Since for theta=pi/4 it was seen that the time taken was less than t=2 we keep the angle but reduce the time for the inital guess

```
z0 = [pi / 4, 1.8]';
%funForNewton takes the inital angle and the time and returns de
 distance
%to the origin at the final point.
% We use the tolerance specified on newtonn te get the tolerance
 required
% by the problem
[XK, resd, it] = newtonn(z0, 1e-8, 100, @funForNewton);
disp('The inital angle is')
disp(XK(1, end))
disp('The time of arrival is')
disp(XK(2, end))

%{
function r = funForNewton(z)
    % Function to launch newtonn
    % Input: z is a vector of two components, the first one
 corresponds to
    % the launch angle of the particle and the second one to the time
 that it
    % takes to get to the origin
```

```
    % Output: the distance to the origin. Newton will make it 0.
    initial = [0; 0; sqrt(2) .* cos(z(1)); sqrt(2) .* sin(z(1))]; %
 initial point to launch RK4
    steps = 20000; %h will be smaller than 0.0001 since we the time to
 be < 2

    if 0 < z(2) < 2
        h = z(2) / steps;
        sol = RK4(initial, h, @gravFunctionV, steps + 1);
        r = sol(1:2, end);
    else % If time is greater than 2, we will introduce an "artificial
 slope" to help newton to converge to r = (0, 0)
        % If we launch newton to the correct point we will not reach
 this code:
        disp('Surpasing t = 2');
        r = [1; 1] * z(2);
    end

end

%}

%{
function [XK, resd, it] = newtonn(x0, tol, itmax, fun)
    % This code is the newton method for nonlionear systems, is an
 iterative
    % method that allows you to approximate the solution of the system
 with a
    % precision tol

    % INPUTS:
    % x0 = initial guess  --> column vector
    % tol = tolerance so that ||x_{k+1} - x_{k} || < tol
    % itmax = max number of iterations allowed
    % fun = @ ffunction_handler
    % OUTPUT:
    % XK = matrix where the xk form 0 to the last one are saved (the
 last
    % one is the solution) --> saved as columns
    % Resd = resulting residuals of iteration: ||F_k||, we want it to
 be 0,
    % as we are looking for f(x)=0
    % it = number of required iterations to satisfy tolerance

    xk = [x0];
    XK = [x0];
    resd = [norm(feval(fun, xk))];
    it = 1;
    tolk = 1;

    while it < itmax && tolk > tol
        J = jaco(fun, xk); % Jacobia en la posicio anterior
        fk = feval(fun, xk);
        [P, L, U] = PLU(J);
```

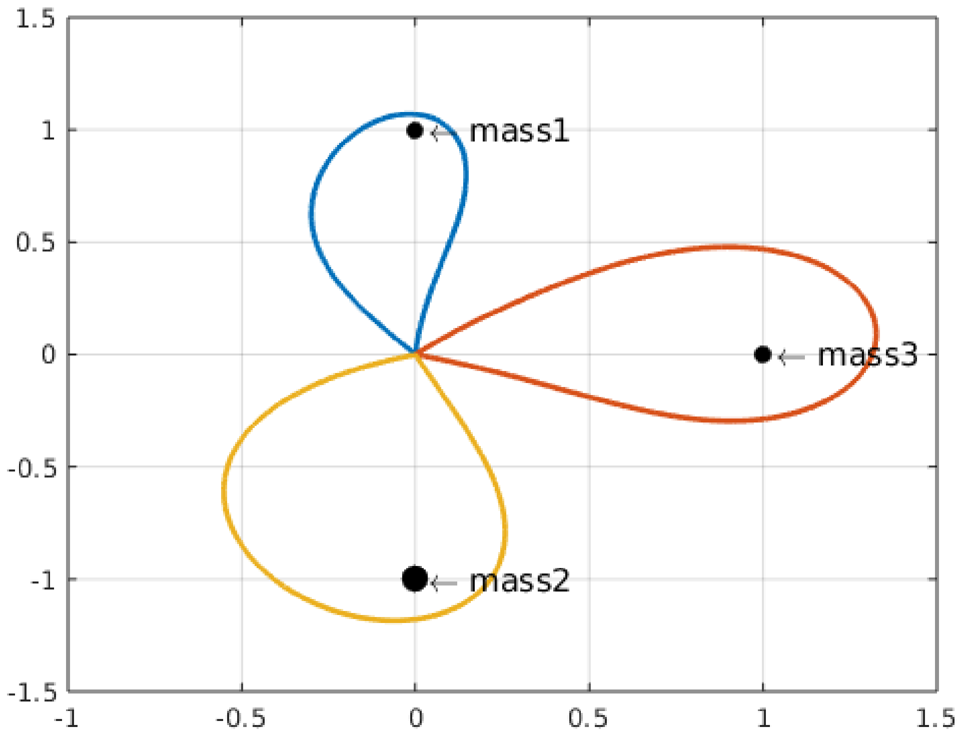
```
        Dx = pluSolve(L, U, P, (-fk)'); %Solucio de la ecuacio J*Dx =
 -fk
        %Matlab linear sistem solving
        %Dx = J\(-fk)';
        xk = xk + Dx;
        XK = [XK, xk];
        resd = [resd, norm(fk)];
        tolk = norm(XK(:, end) - XK(:, end - 1));
        it = it + 1;
    end

    %}
```

```
The inital angle is
          0.77745497923704

The time of arrival is
          1.91158621395717
```

Section C

```
    h = 1e-3;
    v = sqrt(2);
    iTime = 1.8;

    thetas = [pi / 2, 0, -pi / 2];
    %The intial theta guess is based on the problem's symmetry

    figure;

    for ii = thetas
        z0 = [pi / 4 + ii, 1.8]';
        %We use newton to find which is the launch angle and the
 required
        %to get to the origin again
        [XK, resd, it] = newtonn(z0, 1e-8, 100, @funForNewton);
        %Once it has been obatined we use those results to know how
 many steps
        %are required and the components of the inital velocity, and
 with this
        %information we can proced as in section A in order to do the
 plot of
        %the tragectory followed by the particle
        points = XK(2, end) / h + 1; %+1 sempre!
        initial = [0, 0, v * cos(XK(1, end)), v * sin(XK(1, end))]';
        solutionRK = RK4(initial, h, @gravFunctionV, points);
        plot(solutionRK(1, :), solutionRK(2, :), 'LineWidth', 2)
        grid on
        hold on
    end

    massPositions = [0, 1; 1, 0; 0, -1];
```

```
   plot(massPositions(1:2, 1), massPositions(1:2,
2), 'o', 'MarkerEdgeColor', 'k', 'MarkerFaceColor', 'k')
   plot(massPositions(3, 1), massPositions(3, 2), '.r', 'MarkerSize',
35, 'MarkerEdgeColor', 'k', 'MarkerFaceColor', 'k')
   text(massPositions(1, 1), massPositions(1, 2), ' \leftarrow
mass1', 'FontSize', 12)
   text(massPositions(3, 1), massPositions(3, 2), ' \leftarrow
mass2', 'FontSize', 12)
   text(massPositions(2, 1), massPositions(2, 2), ' \leftarrow
mass3', 'FontSize', 12)

   % The three diferent trajectories found make a turn around one of
the masses.
   % All of the trajectores have a teardrop shape.
   % Finally, another property in common is that they do the loop
anti-clockwise
   % around the masses-
```



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