University of Idaho
Department of Computer Science
Coeur d'Alene

Lecture #13

Contours, Image Moments, and Hough Transforms

Garrett Wells
revised September 17, 2024

2024-09-17

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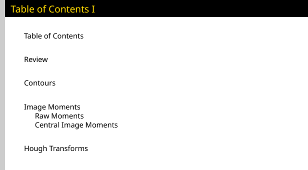
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Review

Edge Detection

1. Sobel

Review

Edge Detection

1. Sobel

Review

Edge Detection

1. Sobel
   - gives us too many lines

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└Review

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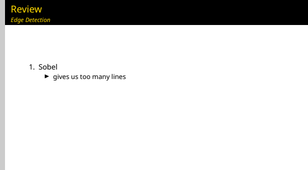
Review
Edge Detection

1. Sobel
   - gives us too many lines

Review

Edge Detection

1. Sobel
2. Canny

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└Review

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Review
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1. Sobel
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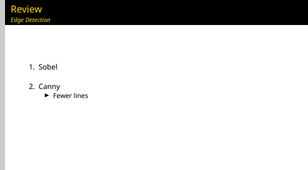
Review

Edge Detection

1. Sobel

2. Canny
   - Fewer lines



Review

Edge Detection

1. Sobel

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Review

Edge Detection

1. Sobel

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   - Fewer lines
   - Hysteresis thresholding allows tuning

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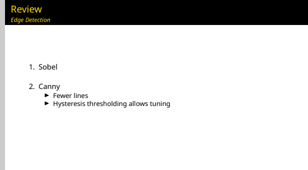

Review
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1. Sobel

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Review

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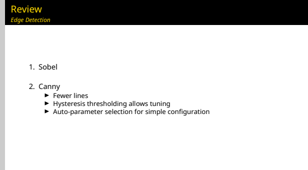
   - Fewer lines
   - Hysteresis thresholding allows tuning
   - Auto-parameter selection for simple configuration



Review
Edge Detection

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   - Fewer lines
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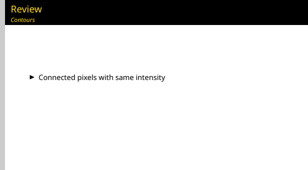
   - Auto-parameter selection for simple configuration

Review

Contours

- Connected pixels with same intensity

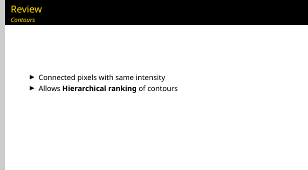


Review
Contours
- Connected pixels with same intensity

Review

Contours

- Connected pixels with same intensity
- Allows **Hierarchical ranking** of contours



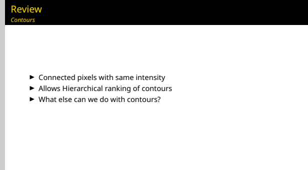
Review
Contours

- Connected pixels with same intensity
- Allows **Hierarchical ranking** of contours

Review

Contours

- Connected pixels with same intensity
- Allows Hierarchical ranking of contours
- What else can we do with contours?



Review
Contours
- Connected pixels with same intensity
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- What else can we do with contours?

Contours

We can. . .

Contours

We can. . .

Contours

We can. . .

- get helpful shape information. . .

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└Contours

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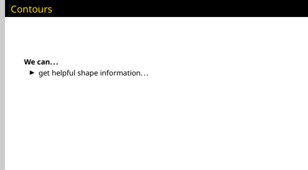

Contours

We can. . .

- get helpful shape information. . .

Contours

We can…

- get helpful shape information…
- area

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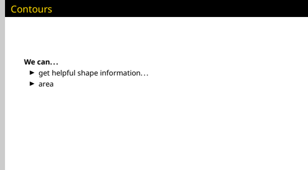

Contours

We can…

- get helpful shape information…
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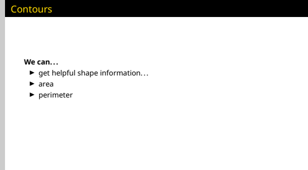
Contours

We can…

- get helpful shape information…
- area
- perimeter



Contours

We can…

- get helpful shape information…
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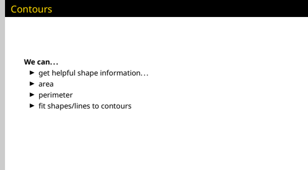
- perimeter

Contours

We can...

- get helpful shape information...
- area
- perimeter
- fit shapes/lines to contours



Contours

We can...

- get helpful shape information...
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Contours: Code Example

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cv.contourArea(...)
cv.arcLength(...)
cv.approxPolyDP(...)
cv.convexHull(...)
cv.isContourConvex(...)
cv.boundingRect(...)
cv.minAreaRect(...)
cv.minEnclosingCircle(...)
cv.fitEllipse(...)
cv.fitLine(...)
```

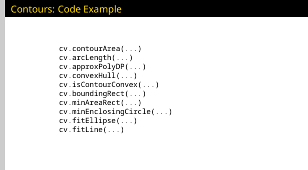

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Image Moments

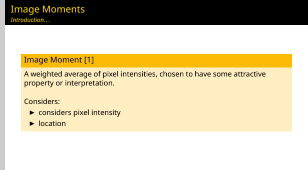
Introduction. . .

Image Moment [1]

A weighted average of pixel intensities, chosen to have some attractive property or interpretation.

Considers:

- considers pixel intensity
- location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1. Considering location & intensity means that we are deriving information that can be used to identify or compare objects.

Image Moments

Introduction...

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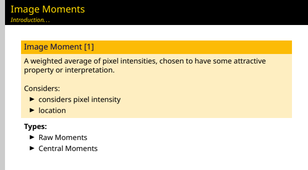
Types:

- Raw Moments
- Central Moments



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Moments

Moment [2]

The moments of a function are certain quantitative measures related to the shape of the function's graph.

Moment Order	Mass	Probability Density	Pixel Intensity
0	total mass	expected value	area/total intensity
1	center of mass	variance	center of intensity
2	moment of inertia	skewness	c

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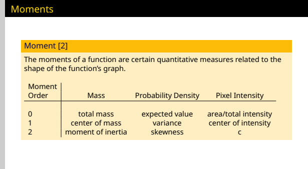
└Image Moments
└Moments



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Raw Moments

Raw Moment [1]

For continuous functions (*order* $= p + q$):

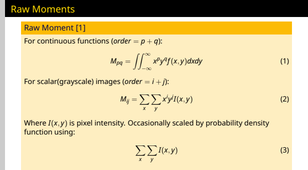
$$M_{pq} = \iint_{-\infty}^{\infty} x^p y^q f(x,y) dxdy \tag{1}$$

For scalar(grayscale) images (*order* $= i + j$):

$$M_{ij} = \sum_x \sum_y x^i y^j I(x,y) \tag{2}$$

Where $I(x,y)$ is pixel intensity. Occasionally scaled by probability density function using:

$$\sum_x \sum_y I(x,y) \tag{3}$$



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1. LOTS of information, just know that we are (a) summing values, (b) averaging/scaling values.

Raw Moments

What can we get from a raw moment…

- area
- centroid of shape

Centroid [3]

The geometric center of a figure. It is the arithmetic mean position of all the points in the surface of the figure.

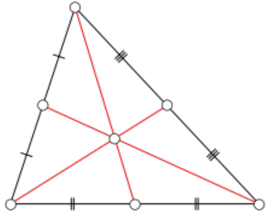
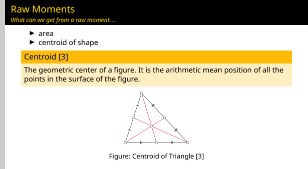


Figure: Centroid of Triangle [3]



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Raw Moments

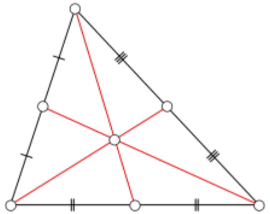
Centroids…

Area

$$M_{00} \tag{4}$$

Centroid

$$\{\bar{x}, \bar{y}\} = \left\{ \frac{M_{10}}{M_{00}}, \frac{M_{01}}{M_{00}} \right\} \tag{5}$$



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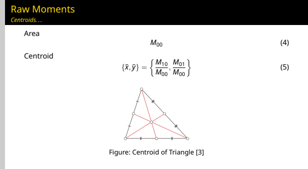

Raw Moments
Centroids…
Area
M_{00} (4)
Centroid
$\{\bar{x}, \bar{y}\} = \left\{ \frac{M_{10}}{M_{00}}, \frac{M_{01}}{M_{00}} \right\}$ (5)
Figure: Centroid of Triangle [3]

1. Centroid components, $\bar{x}$ and $\bar{y}$, are used in calculating central image moments.

Central Image Moments

Central Moment [1]

For continuous functions:

$$\mu_{pq} = \iint_{-\infty}^{\infty} (x - \bar{x})^p (y - \bar{y})^q f(x, y) dxdy \tag{6}$$

For digital images:

$$\mu_{pq} = \sum_x \sum_y (x - \bar{x})^p (y - \bar{y})^q f(x, y) \tag{7}$$

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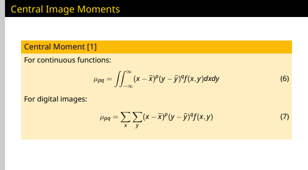
└Image Moments
└Central Image Moments
└Central Image Moments



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Central Image Moments

Properties

- Translation invariant

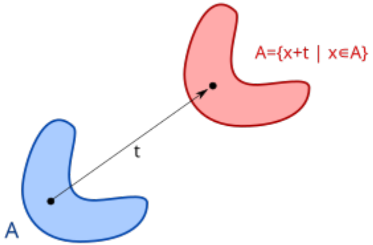
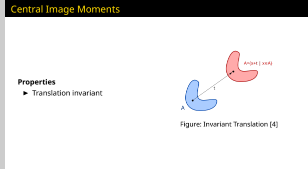


Figure: Invariant Translation [4]


Central Image Moments

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1. *"Translation Invariance"* or *"continuous translational symmetry"* meas that translation of the figure does not change the object [4].

Central Image Moments

Properties

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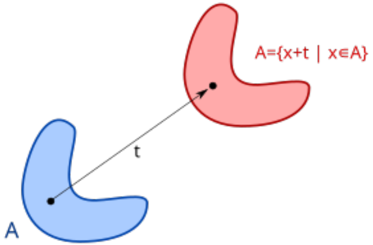

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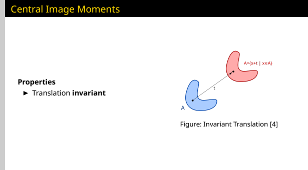
Central Image Moments

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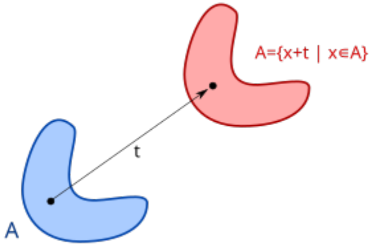
Central Image Moments

Properties

- ► Translation invariant

Invariance

1. The condition or quality of unchanging; consistency.
2. The property of being mathematically invariant.



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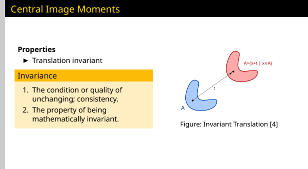
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Invariant Moments

Moments can be used to derive **invariants**.

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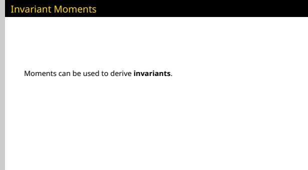
└─Image Moments
  └─Central Image Moments
    └─Invariant Moments



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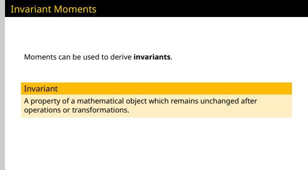
Invariant

A property of a mathematical object which remains unchanged after operations or transformations.



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1. Invariants are appealing because they represent information about an object that doesn’t change under transformation and therefore can be used for comparison.

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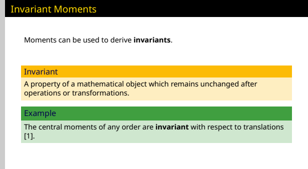
A property of a mathematical object which remains unchanged after operations or transformations.

Example

The central moments of any order are **invariant** with respect to translations [1].



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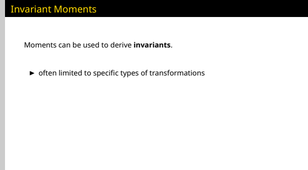
Invariant Moments

Moments can be used to derive **invariants**.

- often limited to specific types of transformations



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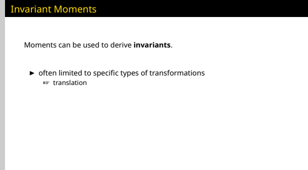

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Invariant Moments

Moments can be used to derive **invariants**.

- ► often limited to specific types of transformations
  - ☞ translation
  - ☞ scale

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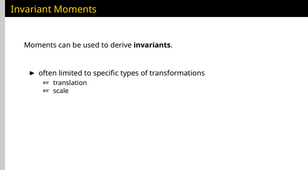
  └─Central Image Moments
    └─Invariant Moments



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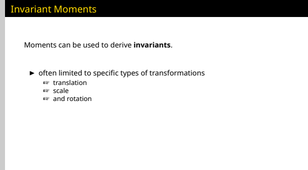

Invariant Moments

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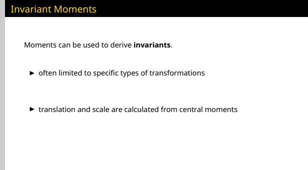
- often limited to specific types of transformations
  - ☞ translation
  - ☞ scale
  - ☞ and rotation

Invariant Moments

Moments can be used to derive **invariants**.

- often limited to specific types of transformations

- translation and scale are calculated from central moments



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Moments can be used to derive **invariants**.

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- translation and scale are calculated from central moments
- discrete image transformation are generally an *approximation*

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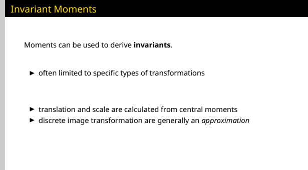
  └─Central Image Moments
    └─Invariant Moments



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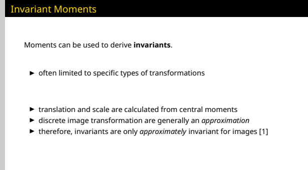
- therefore, invariants are only *approximately* invariant for images [1]



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- often limited to specific types of transformations

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Hu Moments

Justification for Hu Moments. . .

- Goal is to perform shape matching (comparison)

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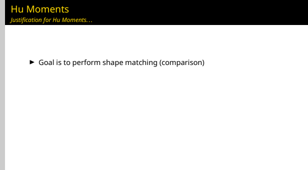
Lecture #13
└Image Moments
 └Central Image Moments
  └Hu Moments



Hu Moments

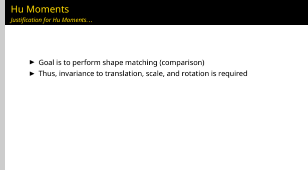
Justification for Hu Moments. . .

- Goal is to perform shape matching (comparison)

Hu Moments

Justification for Hu Moments...

- Goal is to perform shape matching (comparison)
- Thus, invariance to translation, scale, and rotation is required



Hu Moments

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Hu Moments

Justification for Hu Moments…

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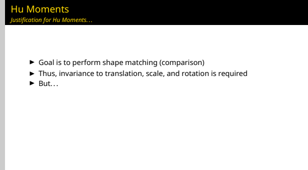
- But…



Hu Moments

Justification for Hu Moments…

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Hu Moments

Justification for Hu Moments…

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- But…
  - raw moments…

2024-09-17

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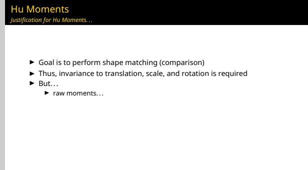
  └─Central Image Moments
    └─Hu Moments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Hu Moments

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- But. . .
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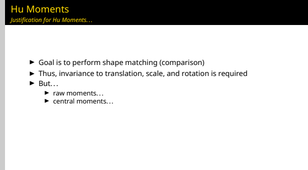
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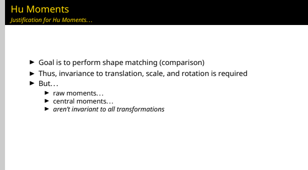
- But...
  - raw moments...
  - central moments...
  - *aren't invariant to all transformations*



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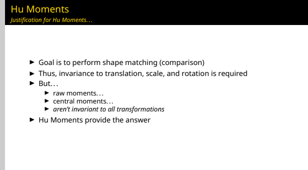

Hu Moments
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- Goal is to perform shape matching (comparison)
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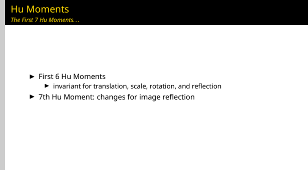
Hu Moments

The First 7 Hu Moments. . .

- First 6 Hu Moments
  - invariant for translation, scale, rotation, and reflection
- 7th Hu Moment: changes for image reflection



Hu Moments
The First 7 Hu Moments. . .

- First 6 Hu Moments
  - invariant for translation, scale, rotation, and reflection
- 7th Hu Moment: changes for image reflection

Hu Moments

Calculating invariants from central moments(μ) with:

$$\eta_{ij} = \frac{\mu_{ij}}{\mu_{00}^{\left(1+\frac{i+j}{2}\right)}} \tag{8}$$

$$\begin{aligned}
I_1 &= \eta_{20} + \eta_{02} \\
I_2 &= (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2 \\
I_3 &= (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 \\
I_4 &= (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \\
I_5 &= (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \\
I_6 &= (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}) \\
I_7 &= (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] - (\eta_{30} - 3\eta_{12})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2].
\end{aligned}$$

Figure: Hu Moments [1]

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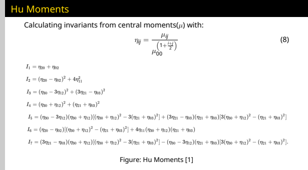
Lecture #13
└Image Moments
  └Central Image Moments
    └Hu Moments


Hu Moments

Calculating invariants from central moments(μ) with:

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Figure: Hu Moments [1]

Hu Moments in OpenCV

```
# image or array of 2D points, calculates moments
moments = cv.moments(
        array,      # input image/array of 2D points
        binaryImage # flag, if true, all non-zero pixels set to 1
        )
# moments from image (cv.moments), calculates Hu Moments (list)
hu_moments = cv.HuMoments(
            moments, # moments from image
            )
```

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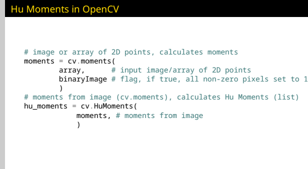
Lecture #13
└Image Moments
  └Central Image Moments
    └Hu Moments in OpenCV



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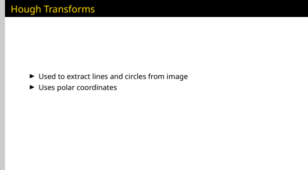
Hough Transforms

- Used to extract lines and circles from image
- Uses polar coordinates



Hough Transforms

- Used to extract lines and circles from image
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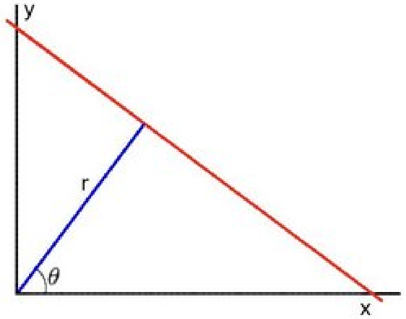
Hough Transforms



Cartesian Representation:

$$y = mx + b \tag{9}$$

Polar Representation:

$$r = x\cos\theta + y\sin\theta \tag{10}$$

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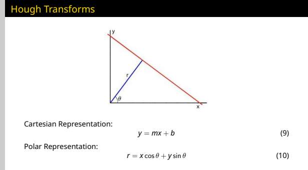
Lecture #13
└Hough Transforms

└Hough Transforms



Hough Transforms

Cartesian Representation:

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Hough Transforms

Hough Algorithm…

1. take point (x, y)

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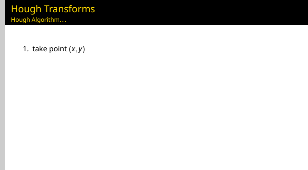
Lecture #13
└Hough Transforms
└Hough Transforms



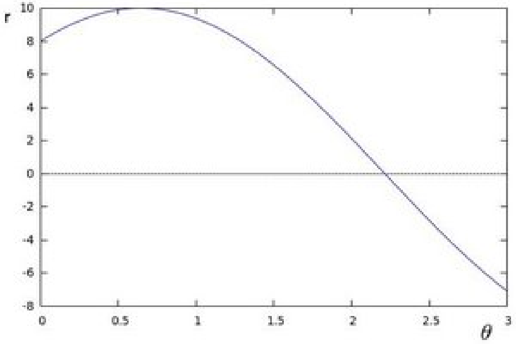
Hough Transforms
Hough Algorithm…

1. take point (x, y)

Hough Transforms

Hough Algorithm…

1. take point (x, y)
2. graph all lines that could go through that point
    - taking $r_\theta = x_0 \cdot \cos\theta + y_0 \cdot \sin\theta$
    - graph r_θ for $0 < \theta < 2\pi$



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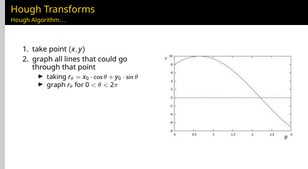
Lecture #13
└Hough Transforms

└Hough Transforms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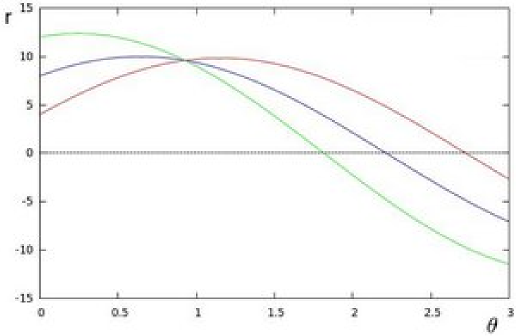
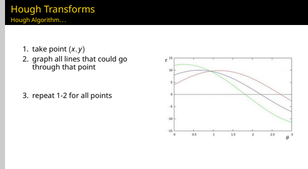
Hough Transforms

Hough Algorithm…

1. take point (x, y)
2. graph all lines that could go through that point
3. repeat 1-2 for all points




Hough Transforms
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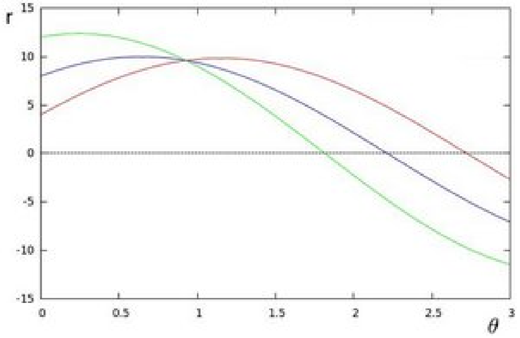
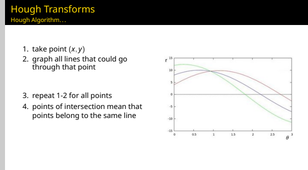
Hough Transforms

Hough Algorithm. . .

1. take point (x, y)
2. graph all lines that could go through that point
3. repeat 1-2 for all points
4. points of intersection mean that points belong to the same line

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Hough Transforms

Hough Algorithm…

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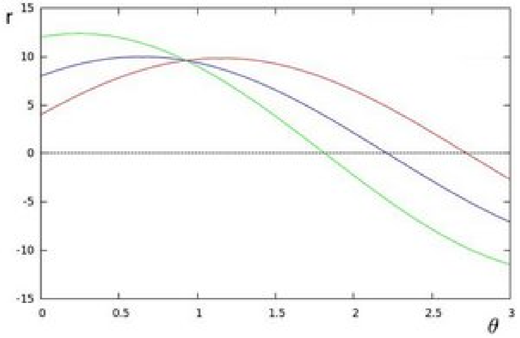
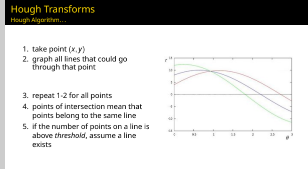
5. if the number of points on a line is above *threshold*, assume a line exists

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Hough Transforms in OpenCV

```
lines = cv.HoughLines(
    image,  # input image
    rho,    # distance resolution (pixels)
    theta,  # angle resolution (radians)
    threshold, # votes required for line status
    ...
)

# probabilistic Hough Transform
lines = cv.HoughLinesP(
    image,  # input image (binary)
    rho,    # distance resolution
    theta,  # angle resolution
    threshold, # return lines with this many votes
)
```

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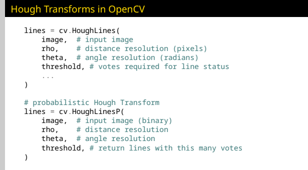
Lecture #13
└Hough Transforms
  └Hough Transforms in OpenCV



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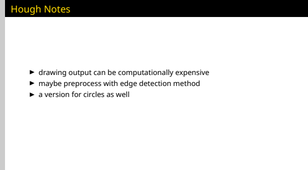
Hough Notes

- drawing output can be computationally expensive
- maybe preprocess with edge detection method
- a version for circles as well



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[2] *Moment (mathematics)*, in *Wikipedia*, Jun. 17, 2024. [Online]. Available: `https://en.wikipedia.org/w/index.php?title=Moment_(mathematics)&oldid=1229572079` (visited on 09/17/2024).

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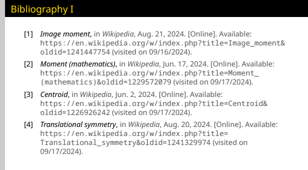
[4] *Translational symmetry*, in *Wikipedia*, Aug. 20, 2024. [Online]. Available: `https://en.wikipedia.org/w/index.php?title=Translational_symmetry&oldid=1241329974` (visited on 09/17/2024).



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