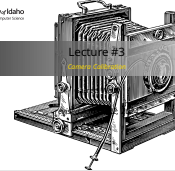


## Lecture #3

### *Camera Calibration*

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## Lecture #3



Lecture #3

Camera Calibration

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└

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- ▶ Camera Obscura
- ▶ Modern Cameras
  - 1. Aperture
  - 2. Focal Length

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└ Review

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- ▶ Camera Obscura
- ▶ Modern Cameras
  - 1. Aperture
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  - 3. **Shutter**

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Lecture #3

- └ Review
- └ Cameras

- ▶ Camera Obscura
- ▶ Modern Cameras
  - 1. Aperture
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  - 3. **Shutter**
  - 4. **Sensor**

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Lecture #3

- └ Review
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- Review
- ▶ Camera Obscura
  - ▶ Modern Cameras
    - 1. Aperture
    - 2. Focal Length
    - 3. **Shutter**
    - 4. **Sensor**

# Cameras and Reality

*Cameras see the world...*

## Camera

Translates between the 3D world and a 2D image [1].

$$x = PX$$

Our goal is to use features in the 2D image(lines) to describe 3D objects.

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## Lecture #3

### └ Cameras and Reality

#### └ Cameras and Reality

Cameras and Reality  
*Cameras see the world...*

## Camera

Translates between the 3D world and a 2D image [1].

$$x = PX$$

Our goal is to use features in the 2D image(lines) to describe 3D objects.

# Cameras and Reality

*Cameras tend to lie about the world...*

- ▶ Cameras depend on lenses to bend and focus light
- ▶ Lenses are designed and manufactured by humans
- ▶ Lenses are prone to distortion

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## Lecture #3

### └─ Cameras and Reality

#### └─ Cameras and Reality

- ▶ Cameras depend on lenses to bend and focus light
- ▶ Lenses are designed and manufactured by humans
- ▶ Lenses are prone to distortion

# Distortion

## Warping Reality

- Distortion reduces fidelity
- Reference lines become curved

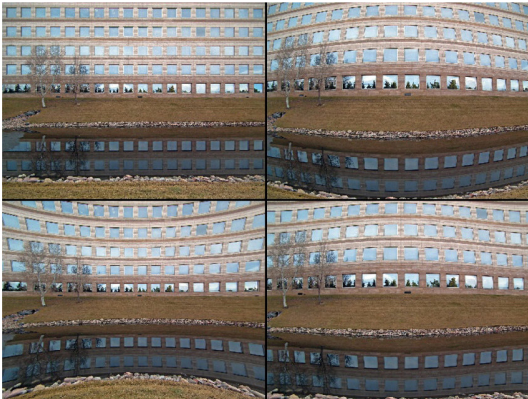


Figure: Examples of Distortion

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## Lecture #3

### └─ Distortion

### └─ Distortion

**Distortion**  
*Warping Reality*

- Distortion reduces fidelity
- Reference lines become curved

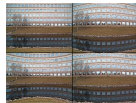


Figure: Examples of Distortion



# Distortion

*Types of Distortion*

## Radial Distortion

- ▶ Light bends more at edges than center
- ▶ Pincushion, Barrel, Mustache(Complex)

## Tangential

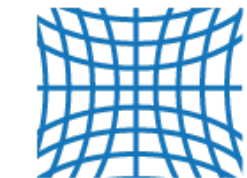
- ▶ Image sensor isn't parallel to image plane

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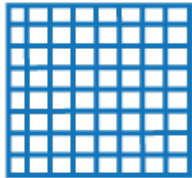
Lecture #3  
└─ Distortion  
    └─ Distortion

Distortion
Types of Distortion
Radial Distortion
▶ Light bends more at edges than center
▶ Pincushion, Barrel, Mustache(Complex)
Tangential
▶ Image sensor isn't parallel to image plane

# Radial Distortion



**Pincushion distortion**  
Positive radial displacement



**No distortion**



**Barrel distortion**  
Negative radial displacement

Figure: Radial Distortion

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## Lecture #3

└ Distortion

└ Radial

└ Radial Distortion

Radial Distortion



**Pincushion distortion**  
Positive radial displacement



**No distortion**



**Barrel distortion**  
Negative radial displacement

Figure: Radial Distortion

1. Radial distortion occurs when light rays bend more near the edges of a lens than they do at its center

# Tangential Distortion



Figure: Tangential Distortion Example

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Lecture #3  
└─ Distortion  
    └─ Tangential  
        └─ Tangential Distortion

Tangential Distortion

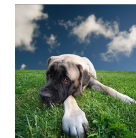


Figure: Tangential Distortion Example

# Tangential Distortion

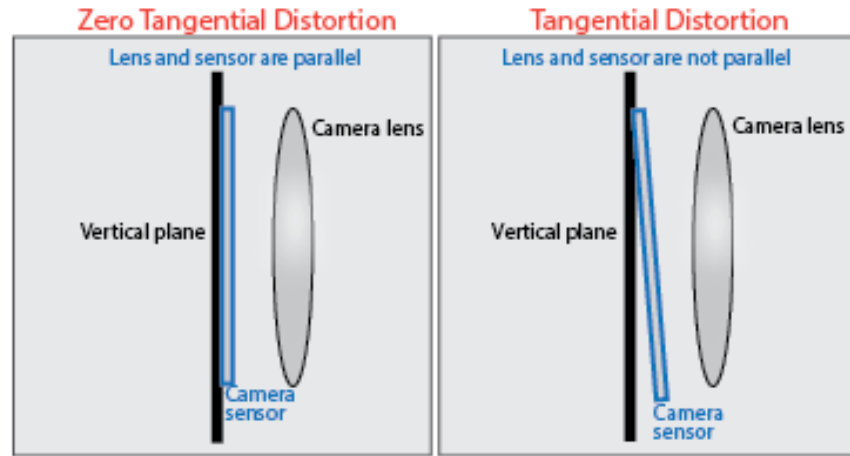


Figure: Tangential Distortion: Lens Sensor Misalignment

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## Lecture #3

- └ Distortion
  - └ Tangential
    - └ Tangential Distortion

Tangential Distortion

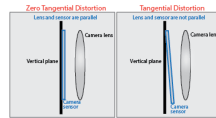


Figure: Tangential Distortion: Lens Sensor Misalignment

1. Occurs when the lens and sensor are not parallel.

# Intentional Radial Distortion

*Fish-eye Lenses*

Fish-eye lenses produce intentional distortion



Figure: Fish-Eye Lens



Figure: Artistic Fish-eye Perspective

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- Lecture #3
  - Distortion
    - Intentional Distortion
      - Intentional Radial Distortion

Intentional Radial Distortion

Fish-eye Lenses

Fish-eye lenses produce intentional distortion



Figure: Fish-Eye Lens



Figure: Artistic Fish-eye Perspective

# Intentional Tangential Distortion

*Tilt-Shift Lenses*

Tilt-shift lenses allow you to change lens alignment



Figure: Tilt Shift Lens



Figure: Tilted View of Hong Kong

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- Lecture #3
  - Distortion
    - Intentional Distortion
      - Intentional Tangential Distortion

Intentional Tangential Distortion

*Tilt-Shift Lenses*

Tilt-shift lenses allow you to change lens alignment



Figure: Tilt Shift Lens

Figure: Tilted View of Hong Kong

# Intentional Tangential Distortion

*Tilt-Shift Lenses*

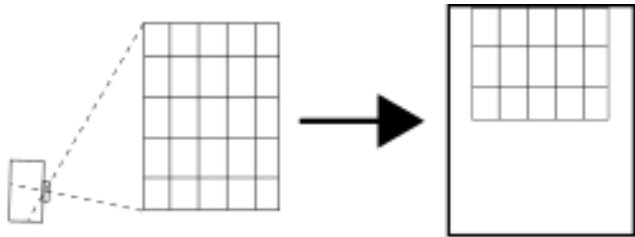


Figure: Level Camera

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- Lecture #3
  - Distortion
    - Intentional Distortion
      - Intentional Tangential Distortion

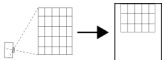


Figure: Level Camera

# Intentional Tangential Distortion

*Tilt-Shift Lenses*

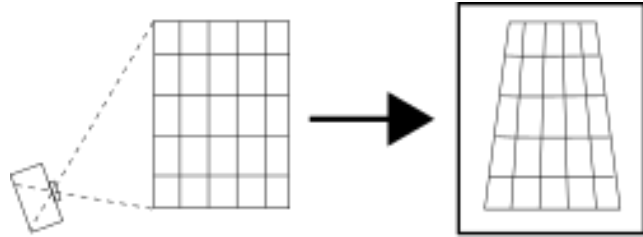


Figure: Tilted Camera

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## Lecture #3

- └ Distortion

- └ Intentional Distortion

- └ Intentional Tangential Distortion

Intentional Tangential Distortion  
Tilt-Shift Lenses

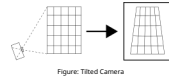


Figure: Tilted Camera



# Intentional Tangential Distortion

*Tilt-Shift Lenses*

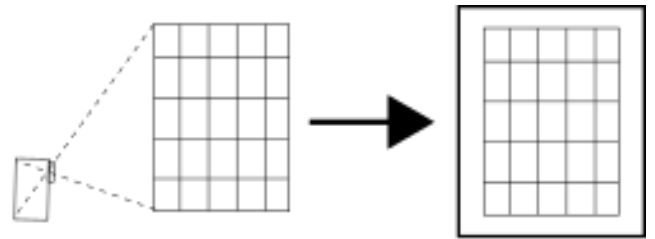


Figure: Shifted Camera

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- Lecture #3
  - Distortion
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      - Intentional Tangential Distortion

Intentional Tangential Distortion

Tilt-Shift Lenses

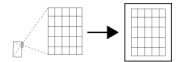


Figure: Shifted Camera



1. Camera Coordinate System
2. Image Sensor's Coordinate System
3. World Space

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1. **Intrinsic Matrix:** camera  $\rightarrow$  image sensor
2. **Extrinsic Matrix:** world  $\rightarrow$  camera

*How do we map between our 3D world and the camera's 2D perspective?*

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Lecture #3

└ Camera Calibration

└ Calibration Matrices

1. **Intrinsic Matrix:** camera  $\rightarrow$  image sensor

2. **Extrinsic Matrix:** world  $\rightarrow$  camera

*How do we map between our 3D world and the camera's 2D perspective?*

# Ideal Case

## Pinhole Camera

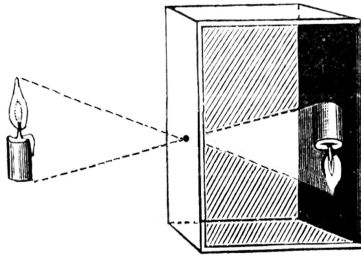


Figure: Pinhole Camera

- Straight line from image plane to world point
- Easy, how can we describe this “projection”?

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## Lecture #3

- └ Camera Calibration
  - └ Ideal Case

Ideal Case  
Pinhole Camera



Figure: Pinhole Camera

- Straight line from image plane to world point
- Easy, how can we describe this “projection”?

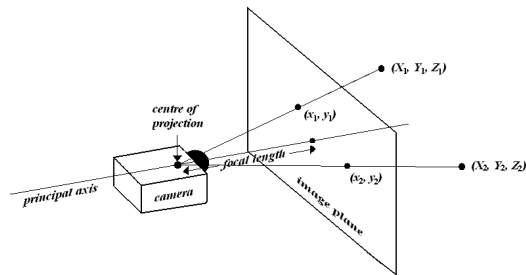


Figure: Camera, Image Plane, and World Points

**principal axis**

*axis extending perpendicular to the aperture*

**image plane**

*where image is rendered upside down on the sensor*

**principal point**

*point where principal axis intersects the image plane/sensor*

**camera/virtual plane**

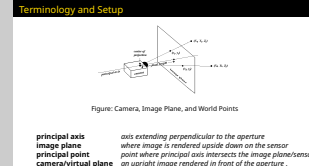
*an upright image rendered in front of the aperture .*

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## Lecture #3

### └ Background Theory

### └ Terminology and Setup



1. For a pinhole camera(ideal case) the image plane is constructed at the aperture/point of convergence. Which should match the focal length.
2. **Correction:** Contrary to this diagram, we are referring to the “image plane” as the virtual image plane/camera.
3. **Virtual Plane:** must be  $x f_{focallength}$  distance in front of the aperture, with scale  $x : 1$ (or  $1 : 1$  at focal length), following rules of projective geometry [2].

# Projective Geometry

## Projective Geometry

1. A system specialized for applications such as computer graphics.
2. System for scaling points in images(2D) or 3D models.



Figure: Projector Close

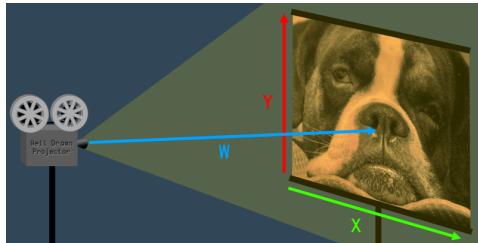


Figure: Projector Far: Where are our points?

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## Lecture #3

### └ Background Theory

### └ Projective Geometry

#### Projective Geometry

1. A system specialized for applications such as computer graphics.
2. System for scaling points in images(2D) or 3D models.



Figure: Projector Close

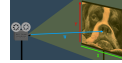


Figure: Projector Far: Where are our points?

1. **Summary:** *Homogeneous coordinates allow any cartesian point to be projected when multiplied by a scalar. This is important because image coordinate  $(x,y)$  corresponds to a world coordinate  $(x,y,z)$*
2. If you think it is weird to include technology such as a projector in a mathematical system, remember, this geometry is made for applications such as computer graphics where scaling/projecting is common/necessary.
3. For our purposes,  $W$  is the principal axis extending from, and perpendicular to the aperture

# Homogeneous Coordinates

## Projective Geometry

A four dimensional space consisting of  $X$ ,  $Y$ ,  $Z$ , and  $W$  [3], [4].

## Homogeneous Coordinate

A coordinate (3D or 4D) with a dimension  $W$ , which represents projective space.

$$(X, Y) \rightarrow (X \times W, Y \times W, W)$$

$$(X, Y, Z) \rightarrow (X \times W, Y \times W, Z \times W, W)$$

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- Lecture #3
  - Background Theory
    - Homogeneous Coordinates

Homogeneous Coordinates
Projective Geometry
A four dimensional space consisting of $X$ , $Y$ , $Z$ , and $W$ [3], [4].
Homogeneous Coordinate
A coordinate (3D or 4D) with a dimension $W$ , which represents projective space. $(X, Y) \rightarrow (X \times W, Y \times W, W)$ $(X, Y, Z) \rightarrow (X \times W, Y \times W, Z \times W, W)$



# Homogeneous Coordinates

## Example

2D  $\rightarrow$  3D Homogeneous  
 $(X, Y) \rightarrow (X \times W, Y \times W, W)$   
 $(1, 3) \ \& \ W = 1 \rightarrow (1, 3, 1)$

## Example

3D  $\rightarrow$  4D Homogeneous  
 $(X, Y, Z) \rightarrow (X \times W, Y \times W, Z \times W, W)$   
 $(5, 2, 1) \ \& \ W = 3 \rightarrow (15, 6, 3, 3)$

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Lecture #3

└ Background Theory

└ Homogeneous Coordinates

Homogeneous Coordinates

Example

2D  $\rightarrow$  3D Homogeneous  
 $(X, Y) \rightarrow (X \times W, Y \times W, W)$   
 $(1, 3) \ \& \ W = 1 \rightarrow (1, 3, 1)$

Example

3D  $\rightarrow$  4D Homogeneous  
 $(X, Y, Z) \rightarrow (X \times W, Y \times W, Z \times W, W)$   
 $(5, 2, 1) \ \& \ W = 3 \rightarrow (15, 6, 3, 3)$

# Homogeneous Coordinates

*Intuitive/Weird Homogeneous Coordinate Properties*

- ☞ Original scale(1:1), is when  $W = 1$
- ☞  $W < 0$  flips upside-down and back-to-front
- ☞  $W$  cannot equal 0

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## Lecture #3

└ Background Theory

└ Homogeneous Coordinates

Homogeneous Coordinates

*Intuitive/Weird Homogeneous Coordinate Properties*

- ☞ Original scale(1:1), is when  $W = 1$
- ☞  $W < 0$  flips upside-down and back-to-front
- ☞  $W$  cannot equal 0

$$x = PX$$

Figure: World To Image Coordinates

- ☞  $x$  denotes image coordinate (homogeneous)
- ☞  $X$  is the same coordinate, but in world space (homogeneous)
- ☞ *What is  $P$ ?*

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## Lecture #3

### └ Camera Matrix

### └ Camera Matrix

$$x = PX$$

Figure: World To Image Coordinates

- ☞  $x$  denotes image coordinate (homogeneous)
- ☞  $X$  is the same coordinate, but in world space (homogeneous)
- ☞ *What is  $P$ ?*

$$x = PX$$

Figure: World To Image Coordinates

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

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## Lecture #3

└ Camera Matrix

└ Camera Matrix

$$x = PX$$

Figure: World To Image Coordinates

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

1. **Math Check:**  $[3 \times 4] \times [4 \times 1] = [3 \times 1]$ , which is 3D vector.

$$\begin{bmatrix} f_x & 0 & 0 & 0 \\ 0 & f_y & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Figure: Camera Matrix: Image & Camera Same Origin

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## Lecture #3

└ Camera Matrix

└ Camera Matrix

$$\begin{bmatrix} f_x & 0 & 0 & 0 \\ 0 & f_y & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Figure: Camera Matrix: Image & Camera Same Origin

1. **Note:**  $f_x, f_y$ , are the same for the ideal case (pinhole camera/camera obscura) but as soon as we introduce a sensor/film, there is the possibility of misalignment.
2. According to [2],  $f_x, f_y$  differ because of sensor flaws, non-uniform scaling, etc.
3. Some texts use single focal length and an “aspect ratio” to describe the amount of deviation from a square pixel. This approach separates camera geometry from distortion.

# Camera Matrix

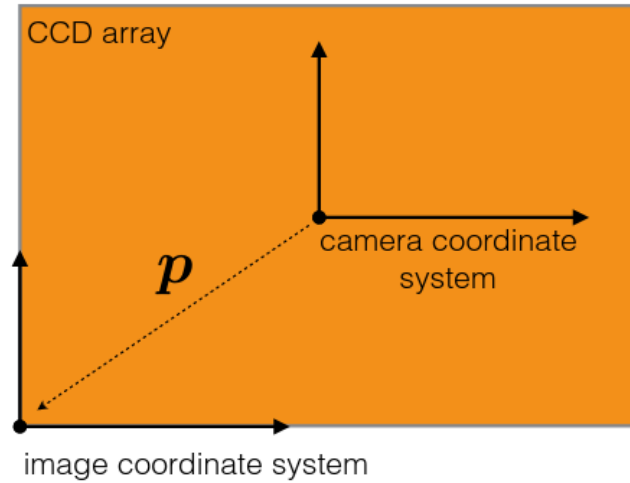


Figure: Difference in Image & Camera Coordinate System

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## Lecture #3

└ Camera Matrix

└ Camera Matrix

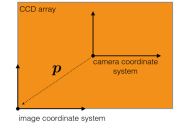


Figure: Difference in Image & Camera Coordinate System

$$\begin{bmatrix} f_x & 0 & p_x & 0 \\ 0 & f_y & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Figure: Camera Matrix: Image Offset from Camera

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## Lecture #3

└ Camera Matrix

└ Camera Matrix

$$\begin{bmatrix} f_x & 0 & p_x & 0 \\ 0 & f_y & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Figure: Camera Matrix: Image Offset from Camera

1. Here, we add  $p_x, p_y$ , which are the optical center/principal point offsets
2. These are used to shift the location of the center of the film/sensor relative to the pinhole/aperture.

# Remember The Goal

## Definition

**Camera** Translates between the 3D world and a 2D image [1].

$$x = PX$$

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## Lecture #3

- └ Camera Matrix

- └ Camera Matrix Decomposition

- └ Remember The Goal

Remember The Goal

Definition

**Camera** Translates between the 3D world and a 2D image [1].

$$x = PX$$

1. So far we have constructed a camera matrix that accounts for the internal geometry of the camera.
2. It doesn't model any of our lens distortion, or relate world space to the virtual image.



# Camera Matrix Decomposition

*Adding distortion correction to our model...*

## Camera Matrix Decomposed

$$x = PX$$



$$P = KR[I \mid -C]$$

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## Lecture #3

└ Camera Matrix

└ Camera Matrix Decomposition

└ Camera Matrix Decomposition

Camera Matrix Decomposition

*Adding distortion correction to our model...*

Camera Matrix Decomposed

$$x = PX$$



$$P = KR[I \mid -C]$$

1. This is how we break down the camera matrix to model *intrinsic*(interior) and *extrinsic*(exterior/world space) relationships.

# Camera Matrix Decomposition

*In more detail*

$$P = KR[I \mid -C]$$

- $K$  **intrinsic matrix** of the camera
- $R[I \mid -C]$  **extrinsic matrix**
- $R$  **rotation** from world to camera coordinate system
- $I$  identity matrix
- $-C$  **translation** from camera to world coordinate

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Lecture #3

└ Camera Matrix

└ Camera Matrix Decomposition

└ Camera Matrix Decomposition

Camera Matrix Decomposition

*In more detail*

$$P = KR[I \mid -C]$$

$K$   
 $R[I \mid -C]$   
 $R$   
 $I$   
 $-C$

**intrinsic matrix** of the camera  
**extrinsic matrix**  
**rotation** from world to camera coordinate system  
identity matrix  
**translation** from camera to world coordinate

# Intrinsic Matrix

$$\begin{bmatrix} f_x & s & p_x \\ 0 & f_y & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

Figure: Intrinsic Matrix,  $K$

- Represent optical center and focal length of camera [5].
- $s$  represents *skew*
- $f_x, f_y$ , represent *focal length in pixels*
- $p_x, p_y$ , represent optical center (in pixels)

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## Lecture #3

└ Camera Matrix

└ Intrinsic Matrix

└ Intrinsic Matrix

$$\begin{bmatrix} f_x & s & p_x \\ 0 & f_y & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

Figure: Intrinsic Matrix,  $K$

- Represent optical center and focal length of camera [5].
- $s$  represents *skew*
- $f_x, f_y$ , represent *focal length in pixels*
- $p_x, p_y$ , represent optical center (in pixels)

# Extrinsic Matrix

$$R \left[ \begin{array}{ccc|c} 1 & 0 & 0 & -C_x \\ 0 & 1 & 0 & -C_y \\ 0 & 0 & 1 & -C_z \end{array} \right]$$

Figure: Extrinsic Matrix,  $R[I] - C$

- Represents the location of the camera in world space [5].
- $R$  is the rotation matrix(3x3), values defined in [6].
- Translate → rotate

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## Lecture #3

└ Camera Matrix

└ Intrinsic Matrix

└ Extrinsic Matrix

Extrinsic Matrix

$$R \left[ \begin{array}{ccc|c} 1 & 0 & 0 & -C_x \\ 0 & 1 & 0 & -C_y \\ 0 & 0 & 1 & -C_z \end{array} \right]$$

Figure: Extrinsic Matrix,  $R[I] - C$

- Represents the location of the camera in world space [5].
- $R$  is the rotation matrix(3x3), values defined in [6].
- Translate → rotate

# Extrinsic Matrix: Camera to World

## Coordinate Systems

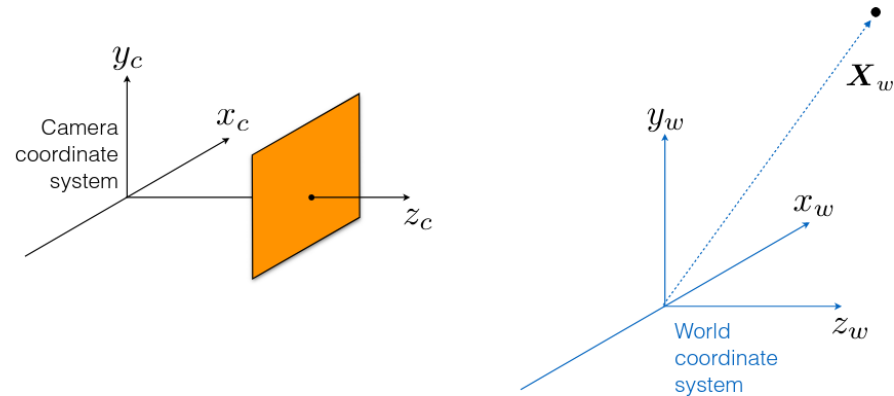


Figure: Camera, Virtual Plane, and World Coordinate Systems (K. Kitani)

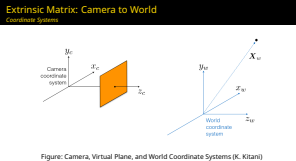
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## Lecture #3

### Camera Matrix

### Intrinsic Matrix

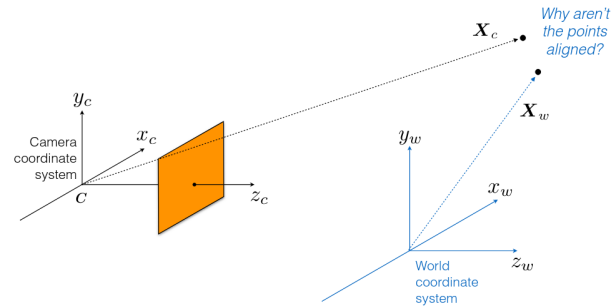
### Extrinsic Matrix: Camera to World



1. Figures from [1].

# Extrinsic Matrix: Camera to World

## Step 1: Translation



$$(\mathbf{X}_w - \mathbf{C})$$

Translate

Figure: Translation

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- Lecture #3
  - Camera Matrix
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    - Extrinsic Matrix: Camera to World

Extrinsic Matrix: Camera to World

Step 1: Translation

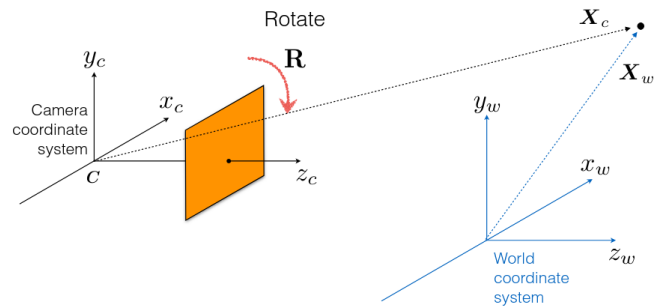
$(\mathbf{X}_w - \mathbf{C})$   
Translate

Figure: Translation

1. Figures from [1].

# Extrinsic Matrix: Camera to World

## Step 2: Rotation



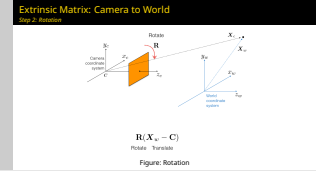
$$\mathbf{R}(X_w - C)$$

Rotate Translate

Figure: Rotation

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- Lecture #3
  - Camera Matrix
    - Intrinsic Matrix
    - Extrinsic Matrix: Camera to World



1. Figures from [1].

# Extrinsic Matrix: Camera to World

## Step 2: Rotation

$$R = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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### Lecture #3

└ Camera Matrix

└ Intrinsic Matrix

└ Extrinsic Matrix: Camera to World

Extrinsic Matrix: Camera to World  
Step 2: Rotation

$$R = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1. This is a rotation about the Z-axis (axis of projection) only.
2. Figures from [1].



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└ Camera Matrix

└ Intrinsic Matrix

└ Camera Matrix As Sequence of Operations

$$P = \underbrace{\begin{pmatrix} f_x & 0 & x_0 \\ 0 & f_y & y_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Intrinsic Matrix}} \times \underbrace{\begin{pmatrix} R & \mathbf{t} \end{pmatrix}}_{\text{Extrinsic Matrix}}$$

$$= \underbrace{\begin{pmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{2D Translation}} \times \underbrace{\begin{pmatrix} f_x & 0 & 0 \\ 0 & f_y & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{2D Scaling}} \times \underbrace{\begin{pmatrix} 1 & s/f_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{2D Shear}} \times \underbrace{\begin{pmatrix} I & \mathbf{t} \end{pmatrix}}_{\text{3D Translation}} \times \underbrace{\begin{pmatrix} R & 0 \\ 0 & 1 \end{pmatrix}}_{\text{3D Rotation}}$$

Figure: Sequence of Operations (Simek, K.)

$$P = \underbrace{\begin{pmatrix} f_x & 0 & x_0 \\ 0 & f_y & y_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Intrinsic Matrix } K} \times \underbrace{\begin{pmatrix} R & \mathbf{t} \end{pmatrix}}_{\text{Extrinsic Matrix}}$$

$$= \underbrace{\begin{pmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{2D Translation}} \times \underbrace{\begin{pmatrix} f_x & 0 & 0 \\ 0 & f_y & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{2D Scaling}} \times \underbrace{\begin{pmatrix} 1 & s/f_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{2D Shear}} \times \underbrace{\begin{pmatrix} I & \mathbf{t} \end{pmatrix}}_{\text{3D Translation}} \times \underbrace{\begin{pmatrix} R & 0 \\ 0 & 1 \end{pmatrix}}_{\text{3D Rotation}}$$

Figure: Sequence of Operations (Simek, K.)

1. Another perspective on what is happening, what types of operations are being performed under the hood here.

# Extrinsic Matrix: Alternative Notation

**Key Difference:** Rotates → translates

$$P = K[R|t]$$

where

$$t = -RC$$

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- └ Camera Matrix

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- └ Extrinsic Matrix: Alternative Notation

Extrinsic Matrix: Alternative Notation

Key Difference: Rotates → translates

$$P = K[R|t]$$

where

$$t = -RC$$

# Distortion Coefficients

Real lenses usually have some distortion, mostly radial distortion and slight tangential distortion. So, the above model is extended as:

$$\begin{aligned} \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= R \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + t \\ x' &= x/z \\ y' &= y/z \\ x'' &= x' \frac{1+k_1r^2+k_2r^4+k_3r^6}{1+k_4r^2+k_5r^4+k_6r^6} + 2p_1x'y' + p_2(r^2 + 2x'^2) + s_1r^2 + s_2r^4 \\ y'' &= y' \frac{1+k_1r^2+k_2r^4+k_3r^6}{1+k_4r^2+k_5r^4+k_6r^6} + p_1(r^2 + 2y'^2) + 2p_2x'y' + s_3r^2 + s_4r^4 \\ \text{where } r^2 &= x'^2 + y'^2 \\ u &= f_x * x'' + c_x \\ v &= f_y * y'' + c_y \end{aligned}$$

$k_1, k_2, k_3, k_4, k_5$ , and  $k_6$  are radial distortion coefficients.  $p_1$  and  $p_2$  are tangential distortion coefficients.  $s_1, s_2, s_3$ , and  $s_4$ , are the thin prism distortion coefficients. Higher-order coefficients are not considered in OpenCV.

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└ Distortion Coefficients

└ Distortion Coefficients

1. These images were included by the previous instructor, but I couldn't find a good link to the cited OpenCV documentation so they will appear uncited here. A close second might be the description in [7].
2. You won't need to do anything with these for this class. OpenCV calculates them during calibration.

Distortion Coefficients

Real lenses usually have some distortion, mostly radial distortion and slight tangential distortion. So, the above model is extended as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + t$$
$$x' = x/z$$
$$y' = y/z$$
$$x'' = x' \frac{1+k_1r^2+k_2r^4+k_3r^6}{1+k_4r^2+k_5r^4+k_6r^6} + 2p_1x'y' + p_2(r^2 + 2x'^2) + s_1r^2 + s_2r^4$$
$$y'' = y' \frac{1+k_1r^2+k_2r^4+k_3r^6}{1+k_4r^2+k_5r^4+k_6r^6} + p_1(r^2 + 2y'^2) + 2p_2x'y' + s_3r^2 + s_4r^4$$

where  $r^2 = x'^2 + y'^2$

$$u = f_x x'' + c_x$$
$$v = f_y y'' + c_y$$

$k_1, k_2, k_3, k_4, k_5$ , and  $k_6$  are radial distortion coefficients.  $p_1$  and  $p_2$  are tangential distortion coefficients.  $s_1, s_2, s_3$ , and  $s_4$  are the thin prism distortion coefficients. Higher-order coefficients are not considered in OpenCV.

# Distortion Coefficients Continued

$$s \begin{bmatrix} x''' \\ y''' \\ 1 \end{bmatrix} = \begin{bmatrix} R_{33}(\tau_x, \tau_y) & 0 & -R_{13}(\tau_x, \tau_y) \\ 0 & R_{33}(\tau_x, \tau_y) & -R_{23}(\tau_x, \tau_y) \\ 0 & 0 & 1 \end{bmatrix} R(\tau_x, \tau_y) \begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix}$$
$$u = f_x * x''' + c_x$$
$$v = f_y * y''' + c_y$$

where the matrix  $R(\tau_x, \tau_y)$  is defined by two rotations with angular parameter  $\tau_x$  and  $\tau_y$ , respectively,

$$R(\tau_x, \tau_y) = \begin{bmatrix} \cos(\tau_y) & 0 & -\sin(\tau_y) \\ 0 & 1 & 0 \\ \sin(\tau_y) & 0 & \cos(\tau_y) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\tau_x) & \sin(\tau_x) \\ 0 & -\sin(\tau_x) & \cos(\tau_x) \end{bmatrix} = \begin{bmatrix} \cos(\tau_y) & \sin(\tau_y) \sin(\tau_x) & -\sin(\tau_y) \cos(\tau_x) \\ 0 & \cos(\tau_x) & \sin(\tau_x) \\ \sin(\tau_y) & -\cos(\tau_y) \sin(\tau_x) & \cos(\tau_y) \cos(\tau_x) \end{bmatrix}.$$

In the functions below the coefficients are passed or returned as

$$(k_1, k_2, p_1, p_2[, k_3[, k_4, k_5, k_6[, s_1, s_2, s_3, s_4[, \tau_x, \tau_y]]]])$$

vector. That is, if the vector contains four elements, it means that  $k_3 = 0$  . The distortion coefficients do not depend on the scene viewed. Thus, they also belong to the intrinsic camera parameters. And they remain the same regardless of the captured image resolution. If, for example, a camera has been calibrated on images of 320 x 240 resolution, absolutely the same distortion coefficients can be used for 640 x 480 images from the same camera while  $f_x$ ,  $f_y$ ,  $c_x$ , and  $c_y$  need to be scaled appropriately.

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- Distortion Coefficients
- Distortion Coefficients Continued

Distortion Coefficients Continued

$$s \begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \begin{bmatrix} R_{11}(\tau_x, \tau_y) & 0 & -R_{12}(\tau_x, \tau_y) \\ 0 & R_{22}(\tau_x, \tau_y) & -R_{21}(\tau_x, \tau_y) \\ 0 & 0 & 1 \end{bmatrix} R(\tau_x, \tau_y) \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

where the matrix  $R(\tau_x, \tau_y)$  is defined by two rotations with angular parameter  $\tau_x$  and  $\tau_y$ , respectively

$$R(\tau_x, \tau_y) = \begin{bmatrix} \cos(\tau_y) & 0 & -\sin(\tau_y) \\ 0 & 1 & 0 \\ \sin(\tau_y) & 0 & \cos(\tau_y) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\tau_x) & \sin(\tau_x) \\ 0 & -\sin(\tau_x) & \cos(\tau_x) \end{bmatrix} = \begin{bmatrix} \cos(\tau_y) & \sin(\tau_y) \sin(\tau_x) & -\sin(\tau_y) \cos(\tau_x) \\ 0 & \cos(\tau_x) & \sin(\tau_x) \\ \sin(\tau_y) & -\cos(\tau_y) \sin(\tau_x) & \cos(\tau_y) \cos(\tau_x) \end{bmatrix}.$$

in the functions below the coefficients are passed or returned as

$[k_1, k_2, p_1, p_2[, k_3[, k_4, k_5, k_6[, s_1, s_2, s_3, s_4[, \tau_x, \tau_y]]]]$

vector. That is, if the vector contains four elements, it means that  $k_3 = 0$  . The distortion coefficients do not depend on the scene viewed. Thus, they also belong to the intrinsic camera parameters. And they remain the same regardless of the captured image resolution. If, for example, a camera has been calibrated on images of 320 x 240 resolution, absolutely the same distortion coefficients can be used for 640 x 480 images from the same camera while  $f_x$ ,  $f_y$ ,  $c_x$ , and  $c_y$  need to be scaled appropriately.

[1] K. Kitani, "Camera Matrix," Sep. 15, 1975. PMID: 11.

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- └ Camera Matrix
  - └ Distortion Coefficients
    - └ Bibliography

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